FROM WHAT WORKS TO SCALING UP: IMPROVING MENTAL STRATEGIES IN SOUTH AFRICAN GRADE 3 CLASSES

Mike Askew, Mellony Graven, Hamsa Venkat

University of Witwatersrand, Rhodes University, University of Witwatersrand

This paper shares results from a national 'familiarisation trial' of a mental mathematics intervention focused on assessing and encouraging strategic calculation methods with Grade 3 students in South Africa. Successful smaller pilots refined the intervention into 6 foci and this paper draws on assessment results from the four provinces that trialled one focus: adding and subtracting using jump strategies. Findings from pre- and post- test results of 1379 students show statistically significant gains in both the fluencies underlying calculating strategically and in items assessing strategic competency. The results indicate that scaling up this model into national implementation is feasible, and that the intervention package can support improvements in mental mathematics learning outcomes.

INTRODUCTION AND CONTEXT

There is a body of evidence on the importance of the interaction between procedural fluency and strategic competence (Mulligan and Mitchelmore (2009), with some researchers demonstrating that attainment in reasoning about number relations in primary school is a better predictor than arithmetical (procedural) fluency of later mathematical attainment (Nunes, Bryant, Sylva, & Barros 2009). Despite such arguments, in some educational systems, including South Africa where we work, teaching continues to prioritise developing students' fluency in mathematical procedures (with a main emphasis, in primary schools, on algorithms for multi-digit calculations). One argument for the continued importance of teaching procedures is that fluency in these leads to structural understanding (a core aspect of strategic competence) since algorithms are rooted in the base-ten system.

Even if it were the case that a procedural-fluency-first approach does lead to understanding structure (a claim that we, the authors, think is debatable), the situation in South Africa is compounded by the emphasis on working procedurally not balanced by an equal emphasis on developing algorithmic fluency. A wealth of research has shown that when working on multi-digit calculations South African students (particularly those in historically disadvantaged schools) reliance on unit counting approaches continues well beyond when counting is appropriate (Schollar, 2008). The students thus do not engage with the structural aspects of the number system. This lack of structural understanding is regarded a critical reason for the continued low standards of attainment in South Africa for many students (Spaull, & Kotze, 2015).

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For over ten years two South African Numeracy Research and Development Chair projects have been exploring ways to change this situation. In the first five years of each initiative, the main emphasis was on ‘what works’; developing didactic approaches and professional development programmes that address teaching for structural understanding whilst fitting with curriculum and inspection constraints and adapting to the dominant, largely teacher centred, pedagogies. The challenge for the second five years of these initiatives was to explore how approaches developed that had been shown to work on a small scale could be scaled up nationally. One such project developed as a collaboration between the two Chairs – the Mental Starters Assessment Project (MSAP). In this paper we report on how this project is being scaled up nationally through collaboration with South Africa’s Department of Basic Education (DBE) and the results of a national ‘familiarisation trial’ that built on early pilot testing and provides a bridge into national adoption.

THEORETICAL BACKGROUND

A focus of the Numeracy Chair initiatives has been on mental calculation skills, chosen not only because this is a required emphasis in the SA curriculum but also because we deemed it a way to wean students off relying on unit counting. The curriculum notes the role of mental processes to “enhance logical and critical thinking, accuracy and problem solving” (DBE 2011, 8–9), with examples of such mental processes including strategies like *bridging through ten* (36 + 9 = 36 + 4 + 5) or *compensation* (36 – 9 = 36 – 10 + 1). To be effective and efficient, such strategies for adding and subtracting mentally require a structural understanding of part-part-whole relations (for example, that 9 can comprise parts of 4 and 5, not simply a collection of 9 single units).

As Askew (2009) notes, a strategy like *bridging through ten*, while drawing on part-part-whole understanding, is only strategic when supported by fluency in number bonds: to efficiently calculate the answer to 36 + 9, rapid recall that 4 is the missing part in 40 = 36 + [ ] and coordinating that with knowing, again rapidly, that 4 and 5 comprise 9, underpin carrying out the strategy. Thus, as well as attending to strategic calculating, our work with teachers needed to focus on students’ rapid recall of number bonds for single digit and multiples of ten addition and subtraction. In addition to rapid recall and strategic calculating we were interested in *strategic reasoning* - reasoning about structural relations between numbers that does not rely on finding specific answers to calculations. Strategic competence is thus a blend of fluency, strategic calculating and strategic reasoning. We chose to work with Grade 3 students as is a year when the move from counting to strategies is needed to ground going forward.

To design a teaching intervention supporting moves into strategic competence we drew on the stream of research demonstrating the importance of using representations that mirror the desired underlying mathematical structure, such as part-whole bar models and empty number lines (see, for example, Van den Heuvel-Panhuizen, 2008). For the instructional part of the lesson starter, the final model comprised teacher led working on fluency in underpinning number bonds and then working strategically through two calculations and then student individual working on a set of three examples.

The intervention overall covered six different strategic 'foci': bridging through ten, jump strategy, doubling and halving, re-ordering, compensation and understanding the relationship between addition and subtraction. These six titles were taken from the Curriculum and Assessment Policy Statement (CAPS) (DBE 2011), the main curriculum document from which teachers plan. We thus expected teachers would recognise these strategies as part of what they were expected to be teaching. Pragmatically, six foci allowed teachers to work on two foci in each of the three terms in the teaching year.

Here we focus on the jump strategy, that is, for a calculation like 36 + 28 only partitioning the 28 into 20 + 8 and adding 20 to 36 and then 8 to 56 (using bridging through 10), initially with the support of an empty number line. This strategy was not widely used in our schools where the dominant approach was to partition both 36 and 28 and add the tens, add the ones and then add the two answers. Not only is the jump strategy a little more efficient, it also transfers more easily to subtraction.

MODEL OF INTERVENTION AT SCALE

An intervention that could effectively be scaled required teachers to perceive the materials as both easy to manage and fitting with curriculum requirements. Such fit with existing circumstances and constraints would be central to any intervention's success, given the evidence of lack of take-up of many previous initiatives as a result of expectations being too far from the 'ground' of South African schools. Here we outline the final intervention model, with brief reasons for the design decisions; for a fuller account of the origins of the model see Graven & Venkat (2021).

CAPS sets out an expectation that each lesson should begin with 10 minutes of oral and mental work, so the materials were designed to fit within those 'lesson starters'. There is a national mathematics workbook that the vast majority of teachers work through with their classes in the part of the lesson following the 10-minute starter: we knew our work in schools that the intervention would fail if we expected teachers either to replace workbook time with other tasks, or to add in extra materials.

Each of the six units was designed to be a three-week cycle comprising a pre-intervention assessment, guidance and materials for eight lesson 'starters' and a post-intervention assessment. While in theory, the two assessments and eight starters could be completed in two weeks, the provision of three weeks meant teachers could extend or revisit any of the starter ideas if they thought their students needed more work on these.

Pre- and post-unit assessments were designed to be easily administered to classes in a time-limited form. Given the evidence that the standard measure of progress in mathematics in many South African classrooms is recording a correct answer to a calculation, irrespective of the means of arriving at that answer (inefficient or copied), we used the low-stakes time-limited format to develop the teachers' awareness of the importance of fluency of basic mathematical facts, and use of time efficient strategies.

The final assessments developed comprised two pages. A first page had 20 rapid recall fluency items, to be completed in two minutes: simple, core number bonds that we expected should be well within the capabilities of being quickly answered by most Grade 3 learners. For the jump-strategy items, typical items included:

57 – 10 = [ ]   79 – 40 [ ]

The second page (to be completed in three minutes) had 10 questions that were a combination of strategic calculating and strategic reasoning items, each of which could be reasonably easily and quickly answered if students had some awareness of mathematical structure. For example, strategic calculating items drawing on jump strategies included:

57 + 26 = [ ]   83 – 24 = [ ]

We expected the strategic reasoning items to be a 'stretch' for both teachers and students as questions not leading to a closed numerical answer are rare in our context, but included these to raise participants' awareness of the importance of structural thinking. Typical jump strategy items included:

61 – 32 = 61 – [ ] – 2   74 – [ ] = 74 – 20 – 5.

Booklets provided to each gave guidance on running for each of eight ten-minute lesson starters, and also included all support materials. The materials were translated and made available in all 11 of South Africa's official languages. As well as print materials, each starter outline had a link, via a QR code, to a short, two to three-minute video demonstrating how to model the strategy, using the empty number line in the case of jump strategies. As noted, each starter had the common format of brief practice of rapid recall items, for example adding a small multiple of ten to a two-digit number (23 + 30; 23 + 50..), then the teacher using the focal strategy to model, with representations, solving two calculations, such as 47 + 21 43 + 24 in case of jump strategies, with students then. individually working on at three similar examples.

In summary, the run of each cycle, over three weeks comprised

- The pre-assessment
- Ten-minute lesson starter teaching aimed at developing fluencies and strategies across the 2–3 weeks following the pre-assessment
- Re-assessment providing feedback on learning.

RESEARCH METHODS

The basic model of the intervention was refined over three phases. The first, design phase comprised a small scale pilot involving three classes across two provinces. In this phase, members of the research and development team worked closely with the teachers to support the enactment of the intervention and to refine the model.

The second phase was a collaborative scaling-up with the national Department of Basic Education (DBE) in a trial that worked across three provinces – nationally, a high socio-economic status (SES) province, a mid-level SES province and a low SES province. In this second trial the teachers' support was 'once-removed' from that provided in the first trial: rather than support from the research and development team, local subject advisors were trained in the use of the materials and they then supported the teachers. Thus, the research question driving this first level of scaling-up was to understand the extent to which the intervention, now mediated by district subject advisers (supported in turn by the two research teams), could produce pre- to post-test gains on two units. The data from the pre- and post-assessments of in this trial showed that such a model could produce good learning gains (see Graven & Venkat, 2021). These with the outcomes pointed to a proof of possibility – that the materials could be used by teachers in the system at large to produce learning gains.

The third phase of development, a *national familiarisation trial,* provided the data reported on here. In this phase, the intervention involved all the early grades' subject advisers in all nine South African provinces, each advisor working with one or two Grade 3 teachers in a school in their own district. As in the second phase, the advisers supported teachers with the rollout of the intervention, including pre- and post-test administration and collating test responses. Now we turn to look at the results from the four of the provinces providing data on jump strategies.

FINDINGS

Figure 1 is a box and whisker plot for the percentage scores on all items pre- and post-test across all four provinces doing jump strategies (matched learners, n= 1379). As Figure 1 shows, the median post-test score was 43%, which was only in the top quartile for the pre-test. The first quartile cut off on the post-test was 20%, which was the same as the median score of 20% on the pre-test. In the pre-test, a quarter of learners scored less than 10% - that cut off point improved to 20% in the post-test. So, on the post-test over half of the learners performed at a level that fewer than a quarter of the learners attained on the pre-test. Also, three-quarters of the learners on the post-test performed at a level that only a half of learners performed at on the pre-test. Post-test, the mean score was 44% compared with 28% in the pre-test.

Table 1 sets out the two-tailed *t*-test results of changes in student scores from the pre-test to the post-test. The provinces are ordered in terms of SES from the one with the highest SES (P1) to that with the lowest (P4). In each case the calculated *t*-score (*t*-cal) is statistically significantly above the t-critical (*t*-crit) score. As can be seen, across all four provinces the gains made were statistically significant.

Table 2 disaggregates the pre- to post-test gains across parts one (fluency) and part 2 (strategic calculating/reasoning) of the assessment. There is no clear pattern of whether gains were more due to improvement on either part: in two provinces (P1 and P3) gains were lower on part 2 of the test, with this pattern reversed in the other two provinces

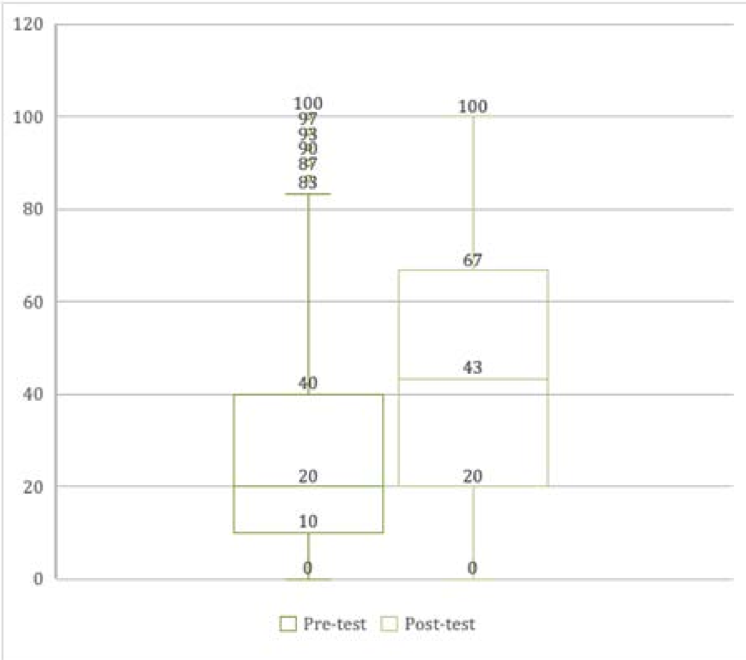

Figure 1. Box and whisker comparing pre- and post-test scores (%) n=1379

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Table 3 presents the effect sizes for each province, and 'levelled' on the commonly used interpretation of effect sizes as small (d = 0.2), medium (d = 0.5), and large (d = 0.8). With an effect size above 0.4 deemed worthy of consideration, the effect sizes in each of the provinces easily meet that criterion.

Province	N	Mean gain	SD	*t*-cal	*t*-crit	df	p
P1	443	15.12	20.52	15.51	1.97	442	<0.001.
P2	46	25.80	21.52	8.13	2.01	45	<0.001
P3	366	12.65	18.84	13.09	1.97	365	<0.001
P4	524	19.45	20.63	21.58	1.96	523	<0.001

Table 1. *t*-test Pre- and Post-Test Gains (%) Jump Strategies

Province	N		Mean gain	SD	*t*-cal	*t*-crit	df	p
P1	443	Part 1	16.95	20.53	17.38	1.97	442	<0.001
		Part 2	11.47	26.31	9.17	1.97	442	<0.001
P2	46	Part 1	22.61	25.60	5.99	2.01	45	<0.001
		Part 2	32.17	26.83	8.13	2.01	45	<0.001
P3	366	Part 1	16.02	22.23	13.78	1.97	365	<0.001
		Part 2	5.90	19.28	5.85	1.97	365	<0.001
P4	524	Part 1	19.40	22.30	19.91	1.96	523	<0.001
		Part 2	19.54	27.81	16.09	1.96	523	<0.001

Table 2. *t*-test Part 1 & 2 Pre- and Post-Test Gains (%) Jump Strategies

Province	N	Pre-test Mean %	Post-test Mean %	Cohen's D	Level
P1	443	18	33	1.2	Large
P2	46	34	59	1.07	Large
P3	366	26	39	0.47	Medium
P4	524	37	57	0.78	Medium

Table 3. Effect sizes for each province

DISCUSSION

The statistical significance of the pre- and post-test gains made on the assessments, together with the effect sizes, indicates that the jump strategy intervention raised attainment above what might be expected to have come about simply from the usual teaching that may have taken place over that time. We note that mean gain of 12 percentage points from the pre- to post- amounts to students answering about 4 more questions (out of 30) correctly. But it is also worth noting that the total time teaching underpinning this gain was only around 80 minutes (on the assumption that the teachers carried out eight 10-minute mental starters). Differences in patterns of gains across the four provinces are worthy of further investigations. For example, we might have expected that, given its high SES, gains in P1 would have been substantially higher than in the other provinces. The reasons why two of the provinces show strong gains in the Part 2 strategic competence items are also worth of further inquiry.

In short, across these four provinces, notwithstanding the differences in gains across provinces, there is evidence that the intervention was successful in supporting improved learner performance on adding and subtracting two-digit numbers. Given the many Covid challenges and disruptions that teachers and learner faced we have been especially pleased to note these improvements.

CONCLUSION

It is beyond the scope of this paper to report on the data from other strategies and other provinces though similar gains were noted. National rollout by the DBE, with the support of the Chair teams, is planned for 2022. While continued disruptions to schooling continue and there is a focus on 'catch-up' in relation to weak curriculum coverage over the past two years we have been pleased to hear general buy-in from teachers and provincial co-ordinators and advisors in relation to both the value of the intervention and the quality and ease of use of the support materials. We expect that some of the success of the intervention, despite many challenges relating to curriculum coverage and 'catch-up' concerns is that the intervention has not 'interfered' with the main body of the teaching time of lessons and has focused on supporting more strategic use of the ten-minute warm up session at the start of lessons to promote number sense and structural reasoning around number.

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