TIMSS 2019 South African Item Diagnostic Report: Grade 9 Mathematics

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Acronyms

CAPS	Curriculum and Assessment Policy Statement
CRQ	Constructed Response Question
DBE	Department of Basic Education
FET	Further Education and Training
HSRC	Human Sciences Research Council
IEA	International Association for the Evaluation of Educational Achievement
LoLT	Language of Learning and Teaching
MCQ	Multiple-choice Question
SE	Standard Error
TIMSS	Trends in International Mathematics and Science Study
TIMSS-SA	TIMSS in South Africa

Preface

The Human Sciences Research Council (HSRC) released the South African results of the 2019 Trends in International Mathematics and Science Study (TIMSS) in December 2020. TIMSS is a cross-national assessment of mathematics and science of Grade 4 or 5 and Grade 8 or 9 learners from the participating countries. TIMSS was developed by the International Association for the Evaluation of Educational Achievement (IEA) to allow participating nations to compare their learners' educational achievement across borders.

Two reports containing the highlights of the Grade 5[1] and Grade 9[2] TIMSS 2019 results were published in December 2020. Two reports with the full analyses, The South African TIMSS 2019 Grade 9 Results[3] and The South African TIMSS 2019 Grade 5 Results[4], were published in 2022.

This report is one of four educator resource documents. The four reports contain diagnostic analyses of restricted use items for TIMSS Grade 5 Mathematics, TIMSS Grade 5 Science, TIMSS Grade 9 Mathematics and TIMSS Grade 9 Science.

These reports, together with additional resources, are available on the TIMSS SA website[5].

This report was compiled by Dr Lynn Bowie and Prof Hamsa Venkat with Sylvia Hannan and Dr Palesa Sekhejane. The report is best described as a resource for educators that will contribute to their understanding of what mathematics our Senior Phase learners know and can do, and how to support the successful teaching and learning of mathematics constructs through the recommendations made.

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Human Sciences Research Council

1 https://www.timss-sa.org/publication/timss-2019-highlights-of-south-african-grade-5-results-in-mathematics-and-science.
2 https://www.timss-sa.org/publication/timss-2019-highlights-of-south-african-grade-9-results-in-mathematics-and-science.
3 https://www.timss-sa.org/publication/the-south-african-timss-2019-grade-9-results.
4 https://www.timss-sa.org/publication/the-south-african-timss-2019-grade-5-results.
5 https://www.timss-sa.org/.

How do I use this report?

This report can be used by all mathematics educators, although it specifically focuses on Grade 9 learners. It does not replace or contradict any official Department of Basic Education (DBE) policies or documents, particularly those related to assessment and the delivery of the intended curriculum.

This report is presented in two sections:

1. Part A presents the introduction and background and highlights some broad performance trends from the analysis of the TIMSS 2019 Grade 9 mathematics restricted use items.
2. Part B presents the analysis of individual mathematics restricted use items.
3. In Part C, items are grouped according to concept or content domains, drawing useful insights to inform classroom practice.

When an educator or DBE official receives this report, an easy way to navigate it is through the steps below. This sequence of steps has been outlined to assist educators in helping their learners and/or to assist DBE officials in their mentoring, coaching, training and support of educators.

STEP 1 | Scan the table of contents and the three parts to familiarise yourself with this report.

STEP 2 | Read through the introduction and background to TIMSS in Part A. This will provide you with an understanding of the context of the assessment.

STEP 3 | Work through the item-by-item analysis in Part B. Both constructed response and multiple-choice items are reported. In this part there is an analysis of learner responses for each item.

STEP 4 | In Part C, items are grouped according to concept or content domains and the analysis provides useful insights to inform classroom practice.

STEP 5 | Based on the item-by-item analysis, identify and pursue remedial actions specific to your learners and your schools.

Part A: Introduction and Background

A.1. Introduction

The purpose of this item diagnostic report is to help educators improve their mathematics teaching. The report analyses South African Grade 9 learners' performance on the restricted use mathematics items from the 2019 Trends in International Mathematics and Science Study (TIMSS), identifies where learners are going wrong, and provides guidance on how to help them.

We begin by giving you some background information about TIMSS. Further details are available on the TIMSS-SA website[6]. We then go on to comment on the ways in which TIMSS reports on mathematics items and how these items differ from the items that we have commonly used in South African assessments. This is followed by a summary of South African Grade 9 learners' overall patterns of performance on the TIMSS 2019 restricted use items.

In Part B of the report, we provide the item level analysis for each restricted use item. We provide a summary of learner performance on each of the items, including the Curriculum and Assessment Policy Statement (CAPS) Grade level of the item, how CAPS and TIMSS describe the difficulty of the item, and performance levels on the item.

In Part C of the report, we have clustered the restricted use TIMSS items into four key strands – geometry and measurement, algebra, data handling and probability, and multiplicative reasoning – that we can comment on. We introduce each of the strands and how they feature in the Senior Phase. We then comment on ways to help learners build towards answering this set of questions correctly through tasks, representations and educator talk that educators – across the Foundation and Intermediate Phases – can use to support their teaching.

Please note, in this report, for ease of reading, the learner frequency responses were rounded to whole numbers.

[6] https://www.timss-sa.org/.

A.2. What is TIMSS?

TIMSS is an assessment of the mathematics and science knowledge and skills of Grade 4 or 5 (Intermediate Phase) and Grade 8 or 9 (Senior Phase) learners around the world. TIMSS was developed by the International Association for the Evaluation of Educational Achievement (IEA) to allow participating nations to compare their learners' educational achievement within and across borders. The goal of TIMSS is to help countries make informed decisions about how to improve teaching and learning in mathematics and science.

In South Africa, the Human Sciences Research Council (HSRC), with the support of the Department of Basic Education (DBE), has conducted the TIMSS assessment since 1995, administering the test at the Grade 8 or 9 levels in 1995, 1999, 2003, 2011, 2015 and 2019. In 2015, South Africa participated in TIMSS-Numeracy at the Grade 5 level and in 2019 South Africa continued TIMSS participation at this level, testing both mathematics and science.

A.3. Who participates in TIMSS?

TIMSS is meant to be written by children in the eighth year of formal schooling, which is Grade 8 in South Africa. However, South African children wrote the test when they were in Grade 9, a year later than other countries. In September 2019, 519 schools, representing a cross-section of schools across South Africa, took part in TIMSS 2019. The selected schools included rural and urban schools, quintiles 1 to 5 and independent schools from all nine provinces. Altogether 20 829 Grade 9 South African learners wrote the test in 2019.

Worldwide, 39 countries participated in TIMSS 2019 at Grade 9 level. South African learners did not perform very well. In fact, South Africa had the second lowest score out of the 39 countries. You can read more about South African learners' performance in the TIMSS 2019 Highlights of South African Grade 9 Results in Mathematics and Science[7].

The TIMSS 2019 Grade 9 results indicated that about 59 percent of our learners had not acquired basic mathematical knowledge and only 3 percent showed they can apply their mathematical knowledge and understanding in complex or problem-solving situations.

Some countries wrote an electronic version of the test whilst others, including South Africa, wrote a paper-based version of the test. In the analysis of the restricted use items, we provide a comparison of South African Grade 9 learners' performance with the average of learners from the other countries that wrote the paper-based version of the test. It is important to note that, although we term this the international average, it is limited to the set of 17 countries that wrote the paper-based version of the test. These include some countries that performed above the TIMSS centrepoint of 500 (Australia, Cyprus, Ireland and Japan), eight of the ten lowest performing countries (Morocco, South Africa, Saudi Arabia, Kuwait, Oman, Egypt, Jordan, Lebanon) and others that performed below the TIMSS centrepoint (Bahrain, Islamic Republic of Iran, Kazakhstan, New Zealand and Romania).

A.4. Who sets the TIMSS items and what are their key features?

The TIMSS achievement booklets contain both trend and non-trend items. The trend items are included in each cycle and form an anchor that allows for estimating achievement over time. The non-trend items are new items generated for each cycle and are subjected to extensive validation processes. For more details on the assessment frameworks and matrix design, refer to the TIMSS 2019 Assessment Frameworks[8].

The TIMSS items are set in English by an international panel of experts. The items are then translated by expert translators into the language of instruction in the participating countries. There are two languages of instruction in South Africa at the Grade 9 level – English and Afrikaans. Most South African learners wrote the test in English, with a few writing it in Afrikaans.

There are two types of items – multiple-choice questions (MCQ) and items that are called constructed response questions (CRQs). The CRQs mostly required learners to give numeric or algebraic answers, a few items required learners to give a reason and others were matching type questions. The items are based on a curriculum decided by what is taught in the majority of countries participating in TIMSS.

An item may consist of only one question, or it may involve sub-questions with the score sheet indicating the criteria for the partial or full allocation of marks.

[7] https://www.timss-sa.org/publication/timss-2019-highlights-of-south-african-grade-9-results-in-mathematics-and-science.
[8] http://timssandpirls.bc.edu/timss2019/frameworks/.

A.5. What is the TIMSS curriculum for Grade 9 mathematics?

A.5.1. Content Domains

TIMSS Grade 9 items are drawn from the Content Domains of Number, Algebra, Measurement and Geometry, and Data and Probability. The levels of overlap between the TIMSS Content Domains and the South African Curriculum and Assessment Policy Statement (CAPS) are generally high. Reddy et al. (2022) report the levels of overlap for each of the Content Domains alongside the average mathematics scale scores in Table 1.

Table 1: Percentage match between TIMSS and CAPS, and average mathematics score

Content Domains	Percentage of items	Percentage match between TIMSS and CAPS	Mathematics scale score (SE) (difference from overall score)
Number	30%	97%	390 (2.3) (+1 points)
Algebra	30%	78%	401(2.5)* (+12 points)
Measurement and Geometry	20%	86%	376 (2.7)* (-13 points)
Data and Probability	20%	54%	370 (2.4)* (-19 points)
All mathematics items	100%	76%	389 (2.3)

*Statistically significant difference from overall mean.
Source: Reddy et al. (2022).

The levels of overlap between the TIMSS Content Domains and the South African CAPS were generally high. This means that learners should, in the case of most items, have encountered those topics in their schooling before taking the TIMSS tests. The overall match between TIMSS and CAPS was high in the domains of Number and Geometry, and reasonable in Algebra, but low in the Data and Probability domains. In Part C we provide more detail on this and on the extent of overlap between the restricted use items and CAPS.

Performance in the Algebra Content Domain was significantly higher than the overall score, while achievement in Geometry, as well as Data and Probability content, were significantly lower than the overall score. Achievement in the Number Content Domain was the same as the overall score. There did not seem to be any consistent relationship between the extent of curriculum coverage and achievement.

A.5.2. Cognitive Domains

Items are also spread across a range of what TIMSS refers to as Cognitive Domains. These are a range of thinking skills that include attention to learners' ability to apply what they have learned, solve problems, and use analysis and logical thinking to reason through situations. The TIMSS Cognitive Domains are Knowing, Applying and Reasoning.

The balance of items in the overall test across the Cognitive Domains are shown in Table 2 below.

Table 2: Percentage of questions at each Cognitive Domain

Cognitive Domains	Percentages
Knowing	35%
Applying	40%
Reasoning	25%

A.6. Restricted use items

After each TIMSS cycle the IEA releases a number of TIMSS assessment items – called restricted use items. Twenty-seven Grade 9 mathematics items were released after the TIMSS 2019 cycle. One item had two questions which were marked individually. This means that we look at the results of 28 questions in this analysis. Ten of these items were MCQ, one was a series of statements that needed to be judged as true or false, and the remaining 17 questions required learners to write their answer (i.e. CRQs).

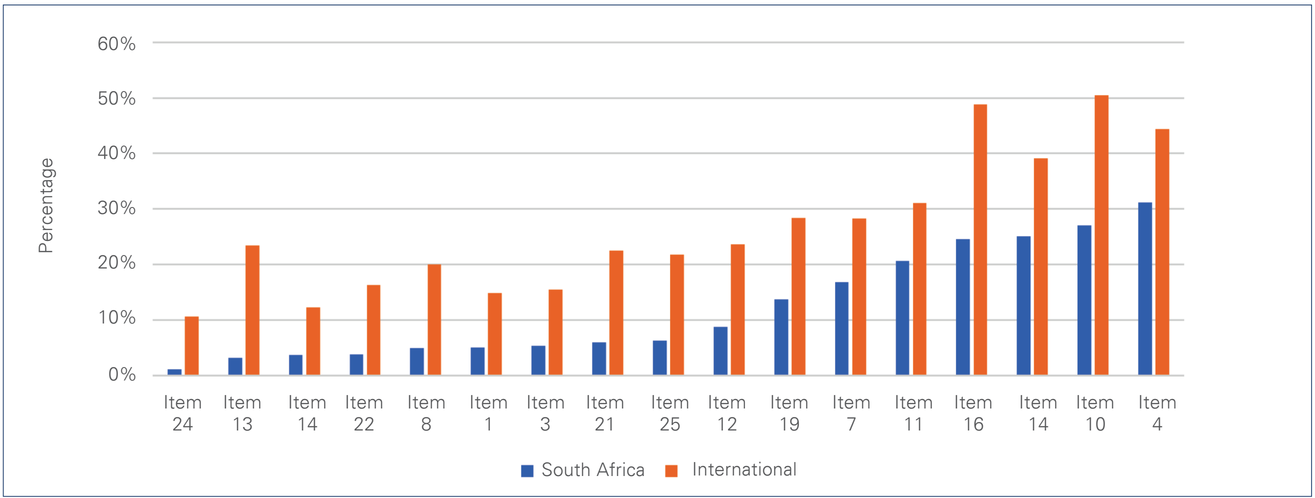
Although restricted use items will not be used again in the TIMSS assessment, they can help us understand what types of difficulties learners have and where they have gaps in their knowledge.

A.7. Some broad performance trends based on restricted use items

A.7.1. Overall patterns of performance

Figure 1 paints the picture of South African Grade 9 learner and average international performance (17 countries who wrote paper TIMSS) on the constructed response questions (CRQs).

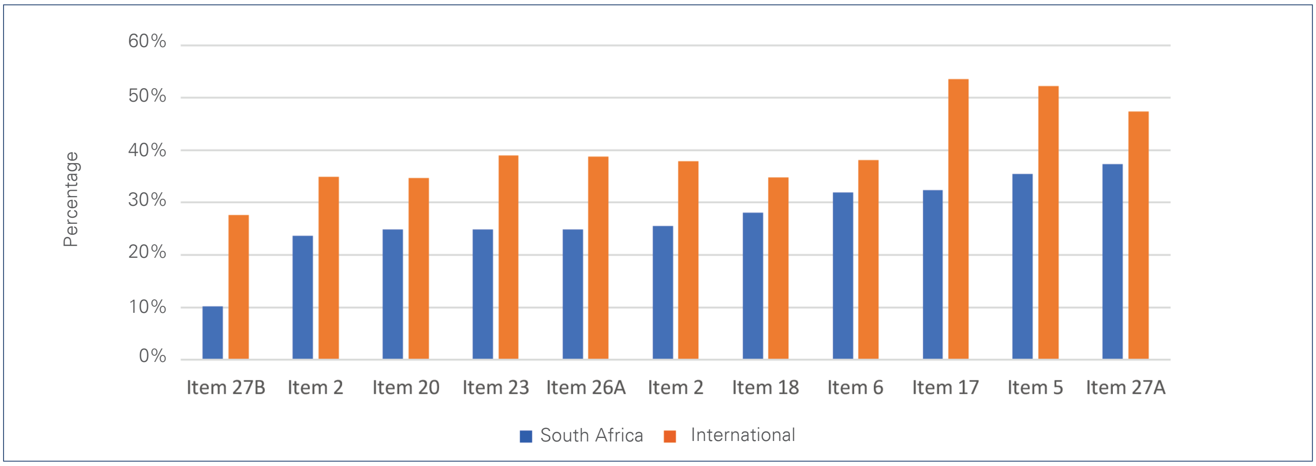
Figure 1: South African and international performance on restricted use CRQs



Although the performance of Grade 9 South African learners was lower than that of the average international for all items, it followed a similar pattern, i.e. questions that South African learners found more difficult, learners internationally also found more difficult. There were however a few items (e.g. **Item 13**) where there was a particularly large difference between the South African performance and the international performance on an item. This is noted and discussed in the individual item analysis in Part B.

Figure 2 illustrates the South African and average international performance on the MCQs. Again, as expected the performance of South African learners was lower than that of their international counterparts for all items. Both South African learners and their international counterparts generally performed better on the MCQs than on the CRQs.

Figure 2: South African and international performance on restricted use MCQs



Performance on restricted use MCQs and CRQs was different (Table 3). In both South Africa and other countries, performance on MCQs was higher than on CRQs. This can possibly be attributed to learners guessing an answer or being cued towards the correct answer in MCQs. In most cases the proportion of South African learners getting the answer to MCQs correct was close to 25 percent, which would be the proportion we would expect if learners were randomly guessing the answer.

Table 3: South African and average international percentage correct on MCQs and CRQs

Question type	South Africa	International average
CRQs	12%	27%
MCQs	27%	40%

A.7.2. Omitted questions and cued responses

Table 4 shows that South African and international learners were unlikely to omit MCQs, which suggests they might well have guessed the answers. However, there were interesting differences in the rates of omission for the CRQ items, with South African learners far more likely to write something, rather than leaving the answer space blank. This mirrors earlier research findings in South Africa noting that random guessing of answers is a widespread way of working in mathematics classrooms (Hoadley, 2007).

Table 4: Percentage of learners who omitted questions

Question type	South Africa	International
CRQs	8%	14%
MCQs	3%	4%

In many instances we saw evidence of learners picking up terms or quantities stated in the question presentation, and using or selecting these terms/quantities for the answer instead of making sense of the underlying mathematics. **Item 17** exemplifies this kind of cued response.

Which ratio is equivalent to 3:2?

- Ⓐ 18:6
- Ⓑ 12:8
- Ⓒ 5:1
- Ⓓ 2:3

Here, 46 percent of the learners chose option D (incorrect) as the answer because it had the same numbers that were in the stem of the question.

Part B: Analysis of restricted use items

In this section we look at each restricted use item. We provide the Cognitive and Content Domain as specified by Trends in International Mathematics and Science Study (TIMSS), and give the Curriculum and Assessment Policy Statement (CAPS) Content Domain it links to along with the CAPS Cognitive Domain.

We then provide the data on learner performance on the question, followed by a brief commentary on the question and on common errors where those were available.

To give a quick sense of learner performance on the questions we have used the following colour coding of percent correct:

0% - 15%
16% - 30%
31% - 45%

Item 1

If a is an integer, are these statements true for **all** values of a?

Shade one circle for each statement.

	True	False
$a^2 = 2a$	Ⓐ	Ⓑ
$a + 2 = 2 - (-a)$	Ⓐ	Ⓑ
$a - 2 = -2 + a$	Ⓐ	Ⓑ
$\frac{a+3}{2} = a + \frac{3}{2}$	Ⓐ	Ⓑ
$\frac{a \times 3}{2} = a \times \frac{3}{2}$	Ⓐ	Ⓑ

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Number: Integers	Knowing	Grade 8: Patterns, functions and algebra	Routine procedure

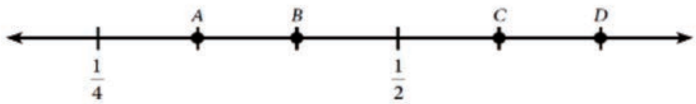
Percentages of learner responses

	Correct	Partially Correct	Incorrect	Omitted
South Africa	5%	12%	82%	1%
International average (n=17)	15%	16%	65%	4%

These are basic algebra facts that learners should know. Learners' poor performance on these items is cause for concern and supports the comments in the Department of Basic Education's (DBE) Matriculation examination diagnostic report on the 2021 results that suggest learners' basic algebra is weak (DBE, 2022).

Item 2

Which point shows $\frac{5}{12}$ on the number line?



Ⓐ *A*
Ⓑ *B*
Ⓒ *C*
Ⓓ *D*

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Number: Fractions and decimals	Applying	Grade 6: Number operations and relationships	Complex procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	12%	26%	20%	40%	3%
International average (n=17)	13%	38%	19%	25%	5%

Only a quarter of the learners were able to recognise that as 5 is just under $\frac{1}{2}$ of 12, that $\frac{5}{12}$ should be just a little smaller than $\frac{1}{2}$. This suggests learners do not have a good sense of fractions and have not mastered the multiplicative reasoning underlying the fraction concepts. The fact that 40 percent of South African learners chose the point furthest to the right (Option D) indicates that learners might be thinking that since 5 and 12 are big numbers (relative to 1 and 2), that $\frac{5}{12}$ is a large number.

Item 3

In the square below:

- The numbers in each row add up to 1,
- The numbers in each column add up to 1, and
- The numbers in both diagonals add up to 1.

$\frac{8}{15}$		$\frac{2}{5}$
$\frac{1}{5}$	X	

What is the value of X ?

X = ________________

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Number: Fractions and decimals	Reasoning	Grade 6: Number operations and relationships	Complex procedure

Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	5%	85%	10%
International average (n=17)	16%	65%	19%

The poor performance on this item was not surprising given learners' weak performance on the basic fraction question (**Item 2**). This question required learners to work strategically and be proficient in adding and subtracting fractions.

Item 4

Alice has a bracelet with blue, red, and white beads in a ratio of 2:3:1. There are 12 beads on the bracelet.

How many beads are blue?

Answer: ________________

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Number: Ratio, proportion, and percent	Knowing	Grade 7: Number operations and relationships	Routine procedure

Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	31%	65%	4%
International average (n=17)	44%	46%	10%

About one third of the learners were able to correctly answer this basic ratio question. This aligns with learners' performance on the other basic ratio question (**Item 16**). Learners' performance on **Item 16** suggests that learners did not have an understanding of what a basic ratio means and thus the performance we see on this question is likely to be as a result of that lack of understanding, rather than as a result of a calculation error.

Item 5

Which is the value of $2(6x - 3y)$ when $x = 3$ and $y = 2$?

Ⓐ 6

Ⓑ 12

Ⓒ 24

Ⓓ 30

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Algebra: Expressions, operations, and equations	Knowing	Grade 7: Patterns, functions and algebra	Routine procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	31%	22%	36%	9%	2%
International average (n=17)	16%	23%	52%	7%	2%

This is a routine question with small numbers and uncomplicated calculations, so the fact that only 36 percent of learners were correct is worrying. The most commonly chosen incorrect answer was 6 (Option A), which could be obtained by reversing the values for x and y, which is careless. The second most common incorrect answer was 12, which could be obtained if learners forgot to multiply by 2.

Item 6

Which expression is equivalent to $2y + 6xy^2$?

Ⓐ $2y(1 + 3xy)$

Ⓑ $2y(1 + 6xy)$

Ⓒ $2y(1 + 3x)$

Ⓓ $1 + 3xy$

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Algebra: Expressions, operations, and equations	Knowing	Grade 7: Patterns, functions and algebra	Routine procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	32%	52%	9%	6%	1%
International average (n=17)	38%	41%	12%	6%	3%

Over 50 percent of learners chose option B. This suggests that learners were factorizing the $2y$ out of the first term but not the second.

Item 7

The stopping distance (d) metres depends on the speed (v) metres per second of the car when the brakes are applied. A formula for calculating this distance is:

$$d = \frac{2v + v^2}{20}$$

What is the stopping distance when $v = 20$?

$d =$ _______________ m

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Algebra: Expressions, operations, and equations	Applying	Grade 7: Patterns, functions and algebra	Routine procedure

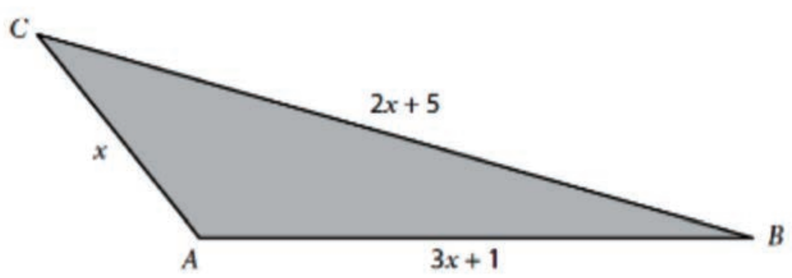
Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	17%	77%	6%
International average (n=17)	28%	56%	15%

Although this question, like **Item 5**, simply involves substitution and calculation, the performance on this question was roughly half that of the performance on **Item 5**. The particularly poor performance on this question was thus probably due to the fact that this question was set in a specific context, rather than being a standalone equation, and learners might have found the language demands of this question high.

Item 8

The perimeter of triangle ABC is 21 cm.



What is the value of x?

$x =$ _______________ cm

TIMSS Domain and link to CAPS

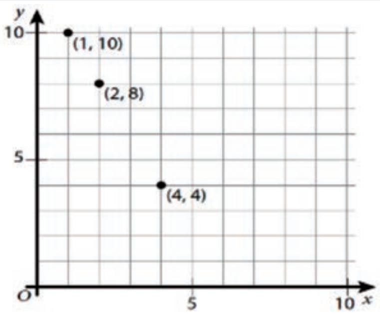
Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Algebra: Expressions, operations, and equations	Applying	Grade 8: Measurement, patterns, functions and algebra	Complex procedure

Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	5%	90%	5%
International average (n=17)	20%	67%	13%

Very few learners answered this question correctly. South African learners' performance was much lower than the international average. This question combines algebra and measurement, content areas we know learners find more difficult. We also know that our learners have a surprisingly weak understanding of perimeter (DBE, 2015). These factors might account for the poor performance on this item.

Item 9



A straight line can be drawn through the points on the graph.
Which point is on the same straight line?

Ⓐ (2, 5)
Ⓑ (3, 5)
Ⓒ (5, 0)
Ⓓ (5, 2)

TIMSS Domain and link to CAPS

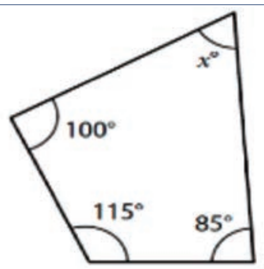
Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Algebra: Expressions, operations, and equations	Applying	Grade 8: Patterns, functions and algebra	Routine procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	20%	24%	29%	24%	3%
International average (n=17)	16%	25%	20%	35%	5%

This question could be answered by drawing a line through the three points and then seeing which of the other points lie on the straight line. The fact that the most popular answer was Option C (5, 0) might be as a result of the fact that the 5 on the x-axis is a labelled point on the graph (very) roughly in line with the three given points. This again indicates that learners seem to have given cued responses rather than trying to make sense of the question.

Item 10



What is the value of x ?

$x =$ ____________________

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Geometry: Geometric shapes and measurement	Applying	Grade 8: Space and shape (Geometry)	Routine procedure

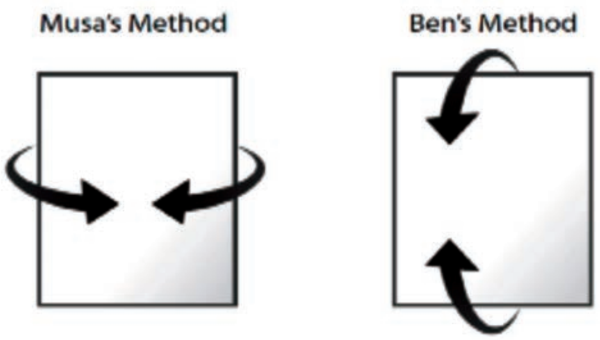
Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	27%	70%	3%
International average (n=17)	51%	42%	7%

It is not clear why learners got this question wrong. Learners should know that the sum of angles in a quadrilateral add up to 360° and so this should have been a straight-forward application of a known fact.

Item 11

Musa and Ben have identical rectangular pieces of paper. They use different ways to roll their papers into cylinders so that the opposite sides of the paper touch as shown below.



Compare the properties of the two cylinders.

Use >, <, or = for each.

Height
Musa's cylinder ____________ Ben's cylinder

Diameter
Musa's cylinder ____________ Ben's cylinder

Surface Area (open ends)
Musa's cylinder ____________ Ben's cylinder

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Geometry: Geometric shapes and measurement	Reasoning	Grade 9: Measurement (Cylinders may not have been covered before the TIMSS assessment was written)	Complex procedure

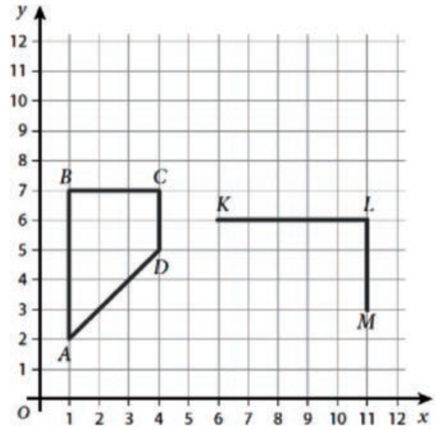
Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	21%	76%	4%
International average (n=17)	31%	62%	7%

South African learners may not have worked with cylinders before writing the TIMSS assessment which could explain some of the poor performance on this question. The question is, however, a nice question and one that would be useful for learners to do whilst working on cylinders. It also highlights the importance of making links between the work on 3D geometry, the attributes of solids, and work on surface area and volume.

Item 12

Siya drew trapezoid *ABCD*. He then started drawing a **congruent** trapezoid *KLMN*.



What will be the coordinates of point *N* when Siya completes the figure?

Answer: (______________ , ______________)

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Geometry: Geometric shapes and measurement	Reasoning	Grade 8: Space and shape (Geometry)	Complex procedure

Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	9%	80%	11%
International average (n=17)	24%	60%	16%

Learners performed poorly on this question. However, as we do not have access to common errors, it is unclear if they were unable to draw the congruent shape, did not know what congruent means or whether they struggled to read off the point. The question does use the word trapezoid which would not be familiar to South African learners, but this should not have deterred them from answering the question as they were told ABCD is a trapezoid.

Item 13

Sophie recorded the temperature (°C) at the same time each day for 5 days:

–2, 1, 3, 2, 3

What is the mean of these 5 temperatures?

Answer: ______________ °C

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Data and probability: Data	Knowing	Grade 7: Data handling	Routine procedure

Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	3%	89%	8%
International average (n=17)	23%	64%	13%

This question was extremely poorly answered and the performance of South African learners was much lower than that of their international peers. It is not clear whether it was the idea of mean or the inclusion of a negative number that caused the difficulty; but given learners poor performance on the other item involving negative numbers, and indications of a large proportion of learners struggling with the notion of mean in **Item 27**, it might well be that both contributed to the poor performance. One thing this does suggest is that we should look for places to combine topics so learners get used to seeing negative numbers, fractions or decimals appearing in the data section.

Item 14

Lee wants to make three graphs to show information about his town.

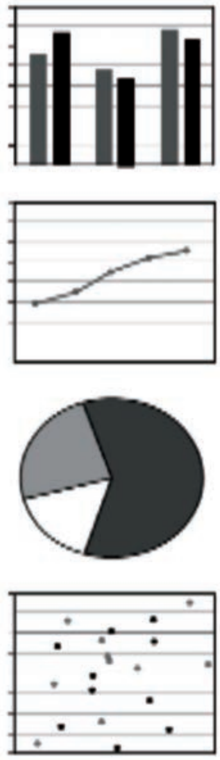
Which type of graph is best for each title?

Draw a line to match each title to the best type of graph.

Job Types of Workers in Town

The Number of Girls and Boys Born Each Year

Town Population Over Time



TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Data and probability: Data	Applying	Grade 9: Data handling	Complex procedure

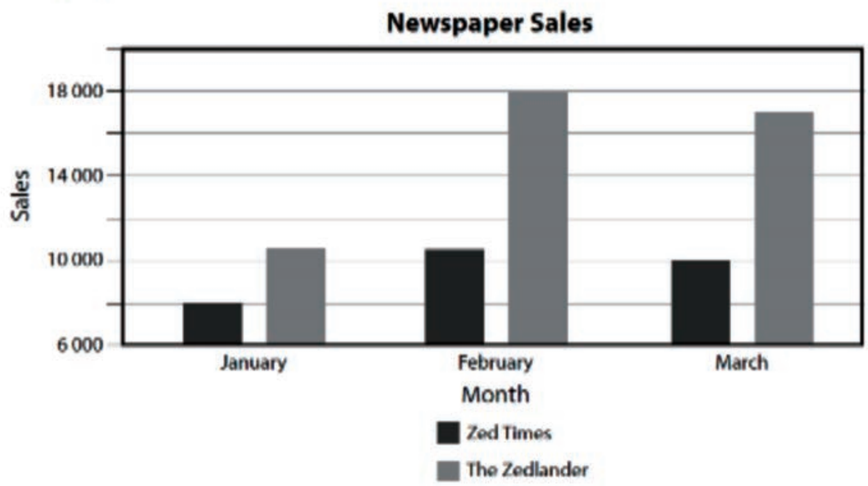
Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	25%	62%	13%
International average (n=17)	39%	51%	10%

Learners may not have studied scatter plots before writing the TIMSS assessment so they may have been unfamiliar with this representation. CAPS emphasises the "critical" reading and interpretation of data and asks learners to engage with different kinds of representations, so this kind of question should be one our learners are exposed to. We need to ensure that we pay attention to this alongside the more routine exercises of calculating statistics or drawing graphs.

Item 15

The graph shows the sales of two Zedland newspapers over three months.



Nancy claimed that sales for the Zedlander were more than double the sales for Zed Times each month.

Explain why Nancy's claim is **incorrect**.

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Data and probability: Data	Reasoning	Grade 7: Data handling	Complex procedure

Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	4%	84%	15%
International average (n=17)	12%	60%	27%

As with the previous question, this kind of critical analysis of data representations is emphasised in CAPS yet learners performed very poorly on this question. This may have been exacerbated by the fact that getting this answer correct relies on learners having a good sense of multiplicative reasoning.

Item 16

On Thursday, the lowest temperature in City X was 6 °C and the lowest temperature in City Y was −3 °C. What was the difference between the lowest temperatures in the cities?

Answer: ________________°C

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Number: Integers	Knowing	Grade 7: Numbers, operations and relations	Routine procedure

Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	25%	71%	4%
International average (n=17)	49%	43%	8%

The fact that only 25 percent of learners got this question correct is worrying. This is a very basic question involving negative numbers which should have been mastered in Grade 7. This indicates that we need to pay attention to learners' understanding of negative numbers and cannot assume they have mastered it by the time they reach Grade 9.

Item 17

Which ratio is equivalent to 3:2?

- (A) 18:6
- (B) 12:8
- (C) 5:1
- (D) 2:3

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Number: Ratio, proportion, and percent	Knowing	Grade 7: Numbers, operations and relationships	Routine procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	8%	32%	12%	46%	2%
International average (n=17)	10%	54%	8%	26%	3%

As commented previously, about one third of the learners could answer this question, suggesting they were able to work with basic ratios. However, the fact that 46 percent of learners chose 2:3 as the ratio equivalent to 3:2 indicates a large proportion of learners do not understand the concept of ratio and were simply choosing the option with the same numbers they saw in the question.

Item 18

Nozazi wants to enlarge this photo keeping the same proportion between height and width.

height = 20 cm

width = 10 cm

She wants her new photo to have a width of 25 cm. What will be the height of her new photo?

- (A) 50 cm
- (B) 45 cm
- (C) 40 cm
- (D) 35 cm

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Number: Ratio, proportion and percent	Applying	Grade 8: Space and shape (Geometry) and whole numbers	Routine procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	28%	22%	7%	42%	1%
International average (n=17)	35%	16%	6%	42%	1%

The most common error learners made here was to use additive reasoning rather than multiplicative reasoning, i.e. they reasoned that because the width increased by 15 cm, the height should also increase by 15 cm to be 35 cm, instead of noting that the width was enlarged by 2.5 and so the height should also be enlarged by a factor of 2.5. Learners' difficulties in moving to multiplicative reasoning is well-documented. Working with enlargements provides an important opportunity to demonstrate the difference between additive and multiplicative reasoning.

Item 19

Write each of the digits 1, 2, 3, and 4 in a box below to make the smallest product. Each digit may only be used once.

☐ ☐ X ☐ ☐

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Number: Integers	Reasoning	Grade 6: Number, operations and relationships	Problem solving

Percentages of learner responses

	Correct	Incorrect 14×23 23×14	Incorrect 32×41; 41×32 31×42; 42×31	Incorrect	Omitted
South Africa	14%	11%	20%	50%	6%
International average (n=17)	28%	19%	11%	36%	5%

This question only required knowledge of place value and multiplication, and so should have been well within the reach of Grade 9 learners. Fourteen percent of learners got the answer correct and a further 11 percent correctly identified that having the digits 1 and 2 in the tens place was important. However, the majority of learners did not solve the problem even partially. It is important that we build learners' problem-solving skills. Familiar contexts like basic number work are useful starting points for this.

Item 20

Refilwe worked 4 hours each day on Monday through Friday and earned 7 zeds per hour. She worked 6 hours on Saturday and earned 10 zeds per hour.

Which expression shows how to calculate Refilwe's earnings?

Ⓐ $(5 \times 4) + 6$

Ⓑ $(4 \times 5) + (6 \times 7)$

Ⓒ $(4 \times 7) + (6 \times 10)$

Ⓓ $(4 \times 7 \times 5) + (6 \times 10)$

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Algebra: Expressions, operations and equations	Applying	Grade 7: Patterns, functions and algebra	Routine procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	7%	10%	57%	25%	1%
International average (n=17)	7%	11%	45%	35%	2%

The majority of learners incorrectly chose option C, ignoring the fact that it was 4 hours **each day**. This suggests that the learners did not read the question carefully and were cued by the numbers that were visible in the question.

Item 21

Marius buys cell phones for x zeds each and sells them to make a profit. He determines his selling price for each phone, y zeds, by doubling the price he paid and subtracting 3 zeds.

Write an equation that shows y in terms of x.

Equation: ___________________________

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Algebra: Expressions, operations and equations	Applying	Grade 8: Patterns, functions and algebra	Routine procedure

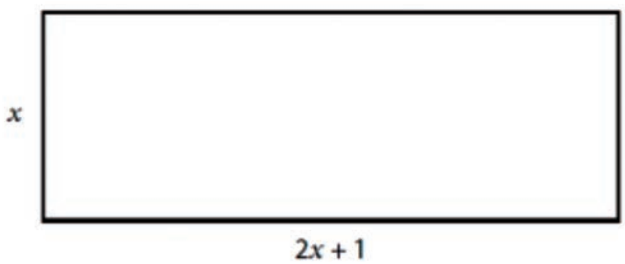
Percentages of learner responses

	Correct	Incorrect	Omitted
South Africa	6%	84%	10%
International average (n=17)	23%	56%	22%

Learners performed very poorly on this question. This is likely due to the difficulty learners may have with the Language of Learning and Teaching (LoLT), but also because the question required learners to translate from written language into the language of algebra.

Item 22

The perimeter of the rectangle below is 20 cm. What is the area of the rectangle?



Answer: ____________________ cm^2

TIMSS Domain and link to CAPS

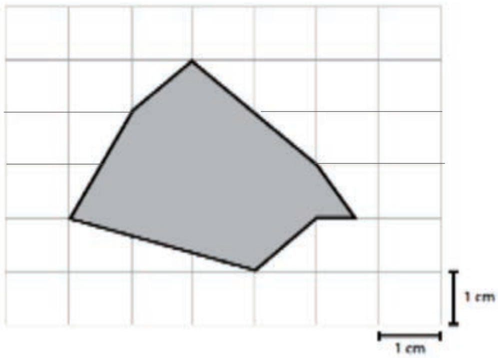
Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Algebra: Expressions, operations and equations	Reasoning	Grade 8: Measurement, patterns, functions and algebra	Complex procedure

Percentages of learner responses

	Correct	Partially correct	Incorrect	Omitted
South Africa	4%	4%	86%	6%
International average (n=17)	16%	4%	66%	14%

This question is similar to **Item 8** and very few learners got it correct. This question combines algebra and measurement, which we know learners find more difficult than working with either topic area on its own. We also know that our learners have a surprisingly weak understanding of perimeter. This question also required learners to work out the area. These factors may account for learners' poor performance on this question.

Item 23



Which is the best estimate of the shaded area?

Ⓐ 6 cm^2

Ⓑ 8 cm^2

Ⓒ 10 cm^2

Ⓓ 12 cm^2

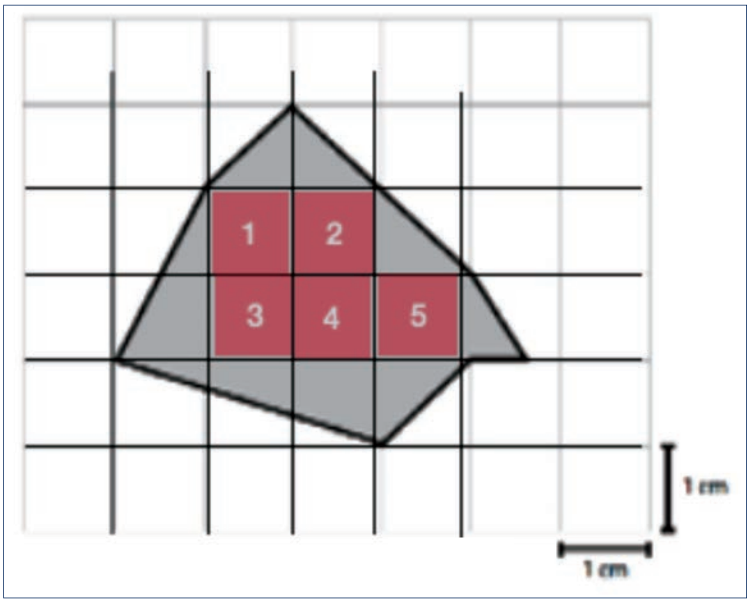
TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Geometry: Geometric shapes and measurements	Applying	Grade 7: Measurement	Routine procedure

Percentages of learner responses

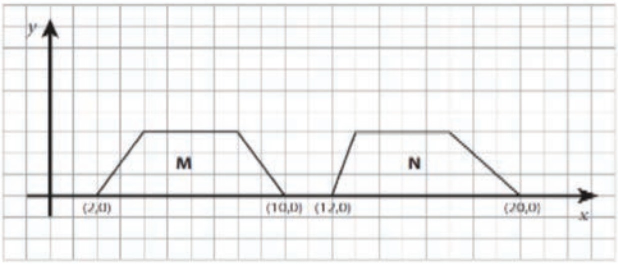
	A	B	C	D	Omitted
South Africa	22%	21%	25%	29%	3%
International average (n=17)	13%	22%	39%	22%	4%

Learners' poor performance on this straightforward question highlights that learners have not developed a conceptual understanding of area. The fact that almost a quarter of the learners chose an answer of 6 cm^2 suggests a poor understanding of area. Counting the whole squares contained in the shape (see below) makes it very clear that the answer could not be 6 cm^2.



Item 24

Two trapezoids, M and N, are shown on the grid below.



Extend the non-parallel sides of M to form a triangle. Also extend the non-parallel sides of N to form a triangle.

Give the coordinates of each triangle's third vertex.

A. Vertex of triangle formed from Figure M: ______________

B. Vertex of triangle formed from Figure N: ______________

TIMSS Domain and link to CAPS

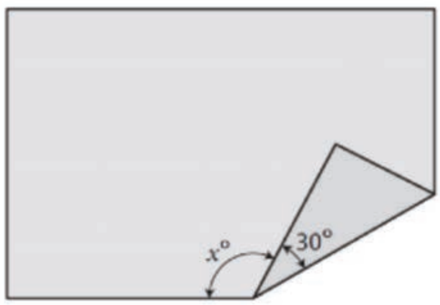
Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Geometry: Geometric shapes and measurements	Applying	Grade 8: Space and shape	Complex procedure

Percentages of learner responses

	Correct (20)	Partially correct (10)	Partially correct (11)	Incorrect	Omitted
South Africa	1%	1%	0,2%	89%	9%
International average (n=17)	11%	4%	1%	62%	22%

Almost no learners were able to correctly answer this question. We do not have information on common errors to help us discern if the problem lay in learners' geometric understanding or the ability to figure out the coordinates. However, the one issue this question does highlight is some of the complexity relating to language in geometry. This is something we need to pay explicit attention to with our learners.

Item 25



A rectangular piece of paper is folded at one corner, as shown above. What is the value of x ?

Answer: ______________

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Geometry: Geometric shapes and measurements	Reasoning	Grade 8: Space and shape (Geometry)	Problem-solving

Percentages of learner responses

	Correct	Incorrect (Gave 150°)	Incorrect other	Omitted
South Africa	6%	12%	75%	7%
International average (n=17)	22%	20%	45%	12%

Very few learners managed to get this question correct. The question would have been an unfamiliar question. Twelve percent of learners calculated that $x°=150°$ by incorrectly applying the notion that the sum of angles on a straight line add up to 180° in a situation where the angles did not lie on a straight line. However, more than three quarters of the learners gave some other incorrect answer.

Although the question is an unusual one, if learners had physically folded a piece of paper, it would not have been difficult for them to see that on unfolding the paper there would be two angles of 30° alongside the x° on the straight edge of the piece of paper. This underscores the importance of learners creating, manipulating and playing with geometric shapes and observing their properties.

Item 26

A bag contains 24 marbles, some white and some black.

A marble is chosen at random, its colour is noted, and the marble is placed back into the bag. This is done 120 times, and a white marble appears 70 times.

How many white marbles are likely to be in the bag?

Ⓐ 7
Ⓑ 10
Ⓒ 12
Ⓓ 14

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Data and probability: Probability	Applying	Grade 8: Data handling	Routine procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	23%	16%	32%	25%	5%
International average (n=17)	15%	16%	26%	39%	4%

This is a probability question and some learners might have struggled as it asked for the number of marbles rather than giving the number of marbles, asking them to find the relative frequency. However, given the poor performance of learners on straightforward ratio questions (**Item 4** and **Item 17**) it is likely that this played a part in learners' difficulty with this question. The most popular choice of distractor was 12 which is half of 24, which suggests that learners made a guess that was not based on the scenario presented.

Item 27A

A relay team for a 400 m race has 4 runners. They took 12 seconds, 13 seconds, 11 seconds, and 13 seconds, respectively, to complete their legs of the race.

A. What is the mean time it takes the runners to complete their legs?

Ⓐ 13,0 sec.
Ⓑ 12,5 sec.
Ⓒ 12,25 sec.
Ⓓ 11,5 sec.

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Data and probability: Data	Knowing	Grade 7: Data handling	Routine procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	33%	16%	37%	8%	6%
International average (n=17)	19%	21%	47%	8%	5%

This is a straight-forward question requiring learners to calculate the mean. Although 37 percent of learners selected the correct answer, almost as many (33%) selected the option of 13,0 seconds. A learner with an understanding of the concept would immediately recognise this was not a feasible mean when two of the times were 13 seconds and two were less than 13 seconds.

Item 27B

B. In the next race, 2 of the runners each improved their times by 2 seconds, and the other 2 had the same times as before. By how many seconds did the team's mean running time improve?

Ⓐ 0 sec.

Ⓑ 1 sec.

Ⓒ 2 sec.

Ⓓ 4 sec.

TIMSS Domain and link to CAPS

Content Domain TIMSS	Cognitive Domain TIMSS	CAPS	CAPS Cognitive Demand
Data and probability: Data	Applying	Grade 7: Data handling	Complex procedure

Percentages of learner responses

	A	B	C	D	Omitted
South Africa	6%	10%	30%	48%	6%
International average (n=17)	6%	28%	32%	29%	5%

Close to half of the learners chose 4 seconds as the correct answer. They probably came to this answer by simply adding on the 2 seconds of each of the 2 runners, showing a lack of understanding of the concept of mean.

In **Part C**, we group items according to concept or content domain in order to gain insights into improving mathematics teaching and learning in the classroom.

Part C: Learnings from the restricted use items

In Part B we provided an analysis of each individual item. However, we found that by grouping items according to concept or content domain we could extract useful insights from the analyses for classroom use.

C.1. Geometry and measurement

One of the TIMSS content areas is Geometric Shapes and Measurement. In the Senior Phase CAPS this is separated into two content areas: Space and Shape (Geometry) and Measurement. In this report we look at Geometry and Measurement together because we see similar key ideas underlying the work of both areas in CAPS in the Senior Phase.

The TIMSS content area Geometry consists of one topic area, Geometric Shapes and Measurements, which encompasses 2D shapes (triangles, quadrilaterals and some other polygons); 3D shapes (prisms, pyramids, cones, cylinders and spheres); angles, lines and the relationships between angles on lines and in geometric figures; transformation geometry and work on the Cartesian planes; problems involving perimeter, areas and the Pythagorean Theorem; and problems involving surface area and volume. This aligns well with the content stipulated in CAPS.

The Human Sciences Research Council's (HSRC) analysis (Reddy et al., 2022) of the 43 Geometry and Measurement items in TIMSS 2019 suggested an overall match of 86 percent with CAPS in this content area. Eight out of nine of the restricted use items should have been covered in CAPS prior to the administration of the TIMSS Grade 9 assessment. Thus, learners should have been able to tackle most of the questions.

However, this content area was the area in which South African Grade 9 learners performed the worst out of the four TIMSS areas, with an average of 15 percent correct for questions in this strand, compared to 28 percent for the international group (Reddy et al., 2022). The mathematics scale score for this section was significantly lower than the overall scale score. Table 5 shows the match between the TIMSS items and CAPS, and learner performance on all Geometry and Measurement items, and on the restricted use items in this area.

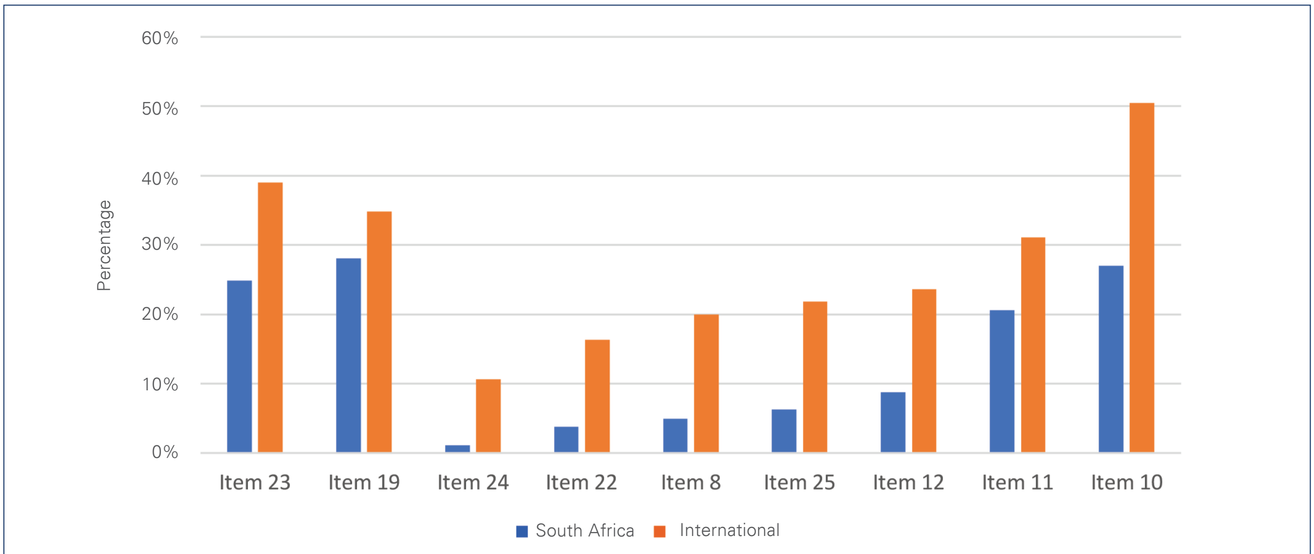
Table 5: Geometric shapes and measurement in TIMSS Grade 9 overall and restricted use items

Geometric shapes and measurement items in TIMSS 2019				Restricted use geometry and measurement items		
No. of items	Percentage match between TIMSS and CAPS	Mathematics scale score (SE) (difference from overall score)	Average percent correct	No. of items	Percentage match between restricted use items and CAPS	Average percent correct
43	86%	376 (2.7)* (-13 points)	15%	9	89%	14%

*Statistically significant difference from overall mean.

Nine of the restricted use items were from this content area. Figure 3 shows the weak performance of South African learners on these items, where performance in over half of the restricted use items was below ten percent correct. For all items, the performance of South African learners was substantially below that of their international counterparts.

Figure 3: Grade 9 performance on restricted use geometric shapes and measurement items



In the following sections, we discuss three key ideas in geometry and measurement that emerged from our analysis of the TIMSS restricted use items: (i) developing the geometric eye by pulling apart and putting together shapes, (ii) noting variance and invariance in transformations and construction, and (iii) decoding geometric language.

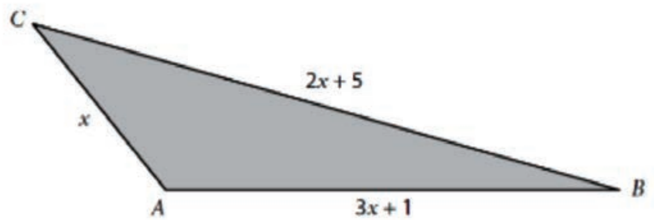
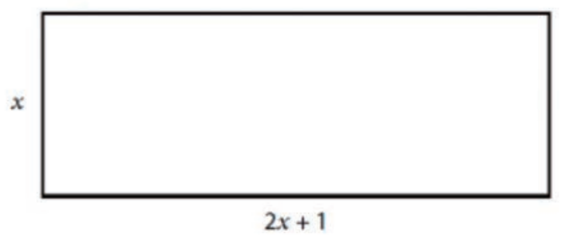
We discuss each of these ideas and provide possible activities for the mathematics classroom.

C.1.1. Developing the geometric eye through pulling apart and putting together shapes

In order for learners to be able to tackle the more formal geometry of the Further Education and Training (FET) Phase (Grade 7 to 9) or to make use of their geometric understanding in work or further studies, learners need to develop their geometric eye. The notion of the geometric eye was introduced by Godfrey who was an author of a geometry textbook published in 1903 (Jones and Fujita, 2002). He described it as "the power of seeing geometrical properties detach themselves from a figure". Jones and Fujita (2002: 16) explain that this means "we would not solve geometrical problems unless we could create proper geometrical images in the mind". The geometric eye is not just about being able to picture shapes in one's mind, but also being able to see the geometric properties of those shapes. One of the ways we can help learners develop their geometric eye is by giving them experience in pulling apart and putting together shapes, in seeing shapes within shapes and using these to explore the properties of the shapes.

Where do we see this in the TIMSS restricted use items and what might we do about it in our classrooms?

There were two TIMSS items that required learners to combine their knowledge of measurement with algebra. These are shown below:

Question	Item number and South African percent correct
The perimeter of triangle *ABC* is 21 cm.  What is the value of x ? x = ______________ cm	**Item 8** **5%**
The perimeter of the rectangle below is 20 cm. What is the area of the rectangle?  Answer: ________________ cm^2	**Item 22** **4%**

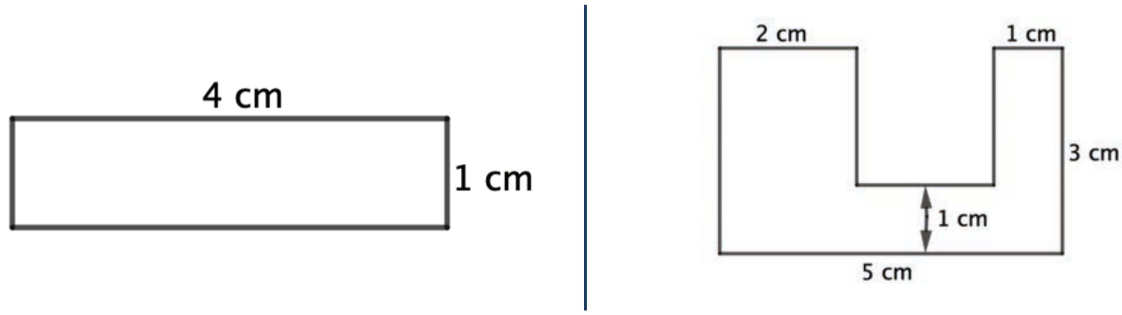
No common errors were reported for these questions so we cannot tell whether the poor performance was primarily due to issues with the perimeter/area concept or with the algebra. However, what is clear, is that very few learners could correctly work with a question that combined algebra and measurement. In addition, the analysis of the restricted use items for TIMSS 2015 (Mosimege et al., 2016) showed that a large proportion of South African learners did not understand perimeter and simply added together the visible numbers in the question to calculate perimeter.

There are two implications for classroom teaching:

1. The importance of regularly providing learners with opportunities to combine algebra and geometry/measurement.
2. The need to help learners build a proper understanding of the concept of perimeter. Part of doing this is providing learners with the opportunity to put together and pull apart shapes.

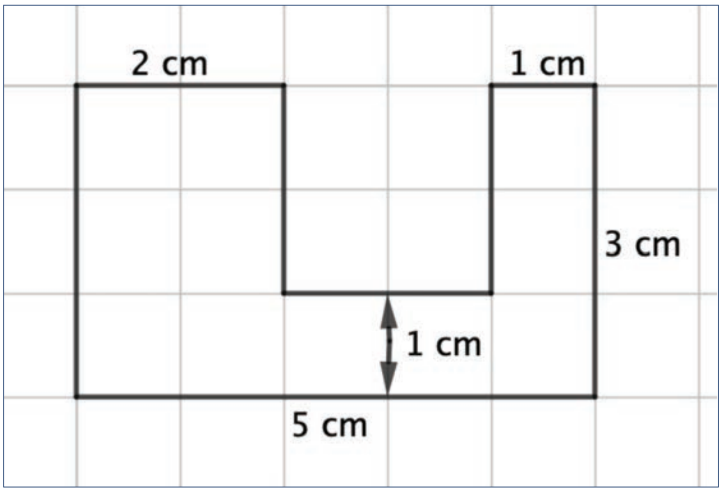
For perimeter in particular it is worthwhile to provide learners with explicit practice of simply filling in all the missing sides in a diagram before working with perimeter or area. For example:

Fill in all the missing lengths on the diagrams below:



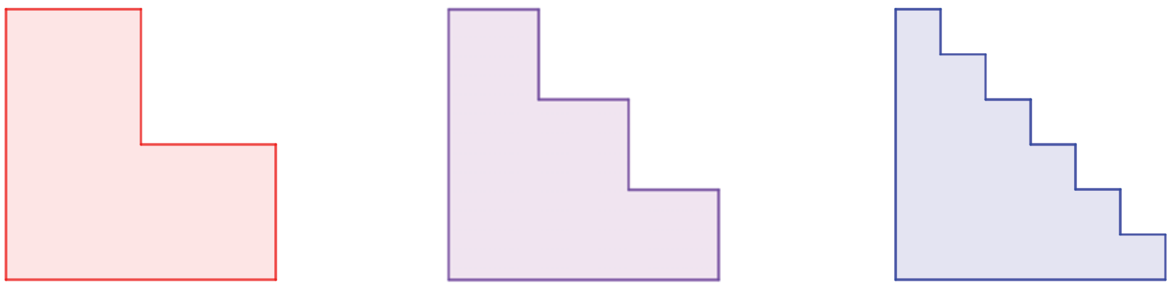
(Note: in these diagrams all angles that look like right angles are right angles)

It might be necessary for some learners to begin this kind of exploration on grid paper, as learners who have insufficient experience in playing with shapes in primary school might require the help of the grid initially to find the missing sides. This scaffold should, of course, be removed over time, as learners at this level need to ultimately be able to work without the grid background.



A nice geometric reasoning activity related to this is the following:

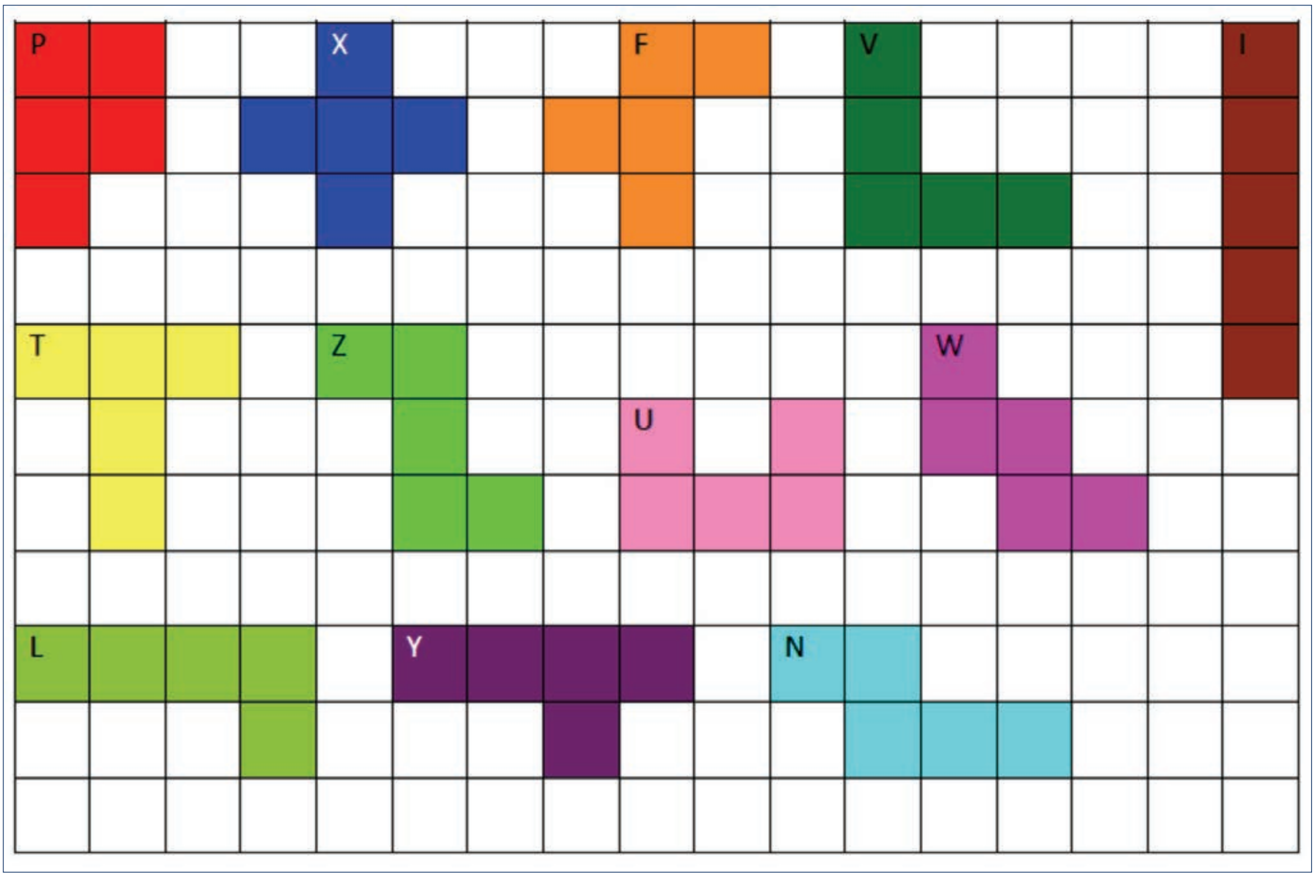
Compare the red, purple and blue shapes. The long vertical sides and horizontal sides are all the same. Which has the biggest area? Which has the biggest perimeter? Show why you say so.



Note that in this case, no actual dimensions are given as we want the learners to reason geometrically.

Learners need plenty of experience playing with area and perimeter prior to working with the formulae.

For learners who have access to technology, play with the simulation from the following website https://phet.colorado.edu/en/simulations/area-builder. Learners without access to technology can get a similar experience playing with pentominoes (five equal sized squares connected edge-to-edge) that have been cut out of a piece of cardboard.



Ask learners to find different ways of creating a rectangle using 3 pentominoes. Each of these rectangles will have an area of 15 square units.

How do their perimeters compare?

Then ask them to make 2 rectangles of 20 square units and then 2 rectangles of 25 square units using the pentominoes and to determine the perimeter in each case.

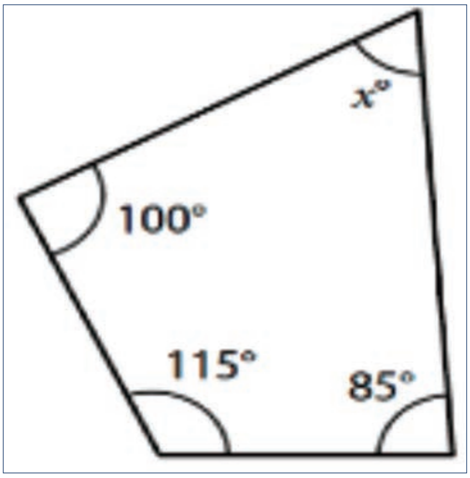
Although this kind of playing can seem time-consuming, learners need this kind of hands-on experience to build their geometric eye and develop a good understanding of the concepts of perimeter and area.

The importance of learners getting regular practice putting together and pulling apart shapes whilst attending to the properties of these shapes was underscored by learners' performance on **Item 22**, where only one quarter of the learners could correctly select the best estimate for the area and almost a quarter of the learners selected option A, which was clearly an underestimate.

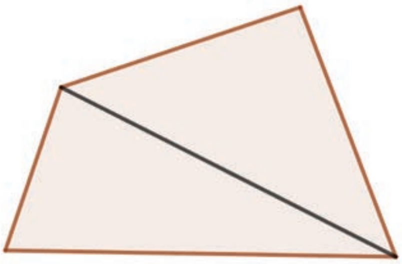
The following video clip provides an overview of some of the conceptual issues related to perimeter and area highlighted through our analysis of the TIMSS restricted use items and provides some suggested classroom activities.

Video clip 1: Perimeter and area
https://youtu.be/hl2THNekb_E

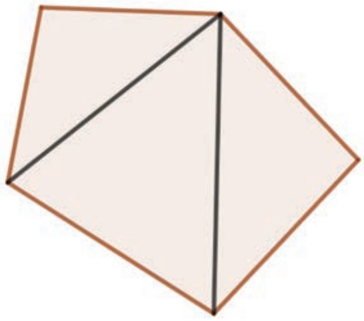
However, the importance of seeing shapes within shapes is not limited to measurement alone. Only 27 percent of South African learners could correctly find the size of the fourth angle in a quadrilateral in **Item 9**.



Although learners could simply be taught that the sum of the angles in a quadrilateral is 360°, this type of fact is more likely to be remembered if it is learnt in a connected and coherent way. Learners should see that polygons can be broken down into triangles and then the sum of angles in the polygon can be deduced from the fact that the sum of angles in a triangle is 180°:



A quadrilateral can be split into 2 triangles.
Sum of angles in quadrilateral = $2 \times 180° = 360°$



A pentagon can be split into 3 triangles.
Sum of angles in quadrilateral = $3 \times 180° = 540°$

Exploring the sum of the interior angles of a polygon with different numbers of sides can be done as a nice investigation that combines both geometry and pattern work. Learners can derive the general formula for the sum of interior angles of a polygon with n sides.

This video clip illustrates how a polygon can be broken down into rectangles to derive the formula for the sum of angles in a polygon.

Video clip 2: Angles in polygons
https://www.youtube.com/watch?v=dyLEVLVpnHE

C.1.2. Noting variance and invariance in transformations and constructions as ways to develop the geometric eye

In the Senior Phase, constructions and transformations are a key part of the geometry curriculum. These should not just be done as routine exercises. We should not think that what we need to do is teach learners how to construct perpendicular lines, for example, by giving them a series of steps to do this or that all we need to do to teach reflection is to show learners how to reflect a shape.

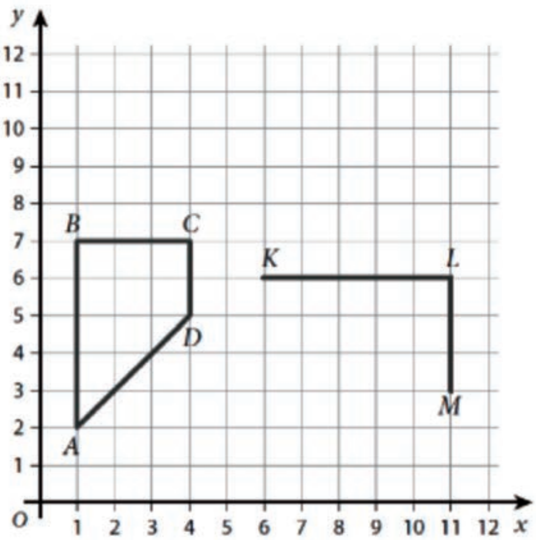
Instead, constructions and transformations should provide powerful experiences for exploring properties of shapes and developing a geometric eye. We need to move away from simply seeing transformations and constructions as topics to be covered, but rather as important activities for the development of strong mental imagery and the ability to see geometric properties. It is tempting to leave these kinds of activities out as they are not easy to include in the end-of-year exams, but we need to be aware that if we do not provide learners with these opportunities they will struggle to work with diagrams and geometry.

In this section we exemplify the kind of difficulties we saw Grade 9 learners experience in the TIMSS 2019 restricted use items and suggest some activities for working with transformations and constructions in a powerful way in the Senior Phase.

Where do we see this in TIMSS 2019 restricted use items and what might we do about it in our classrooms?

Only nine percent of South African learners were able to answer **Item 12** correctly:

Siya drew trapezoid *ABCD*. He then started drawing a **congruent** trapezoid *KLMN*.

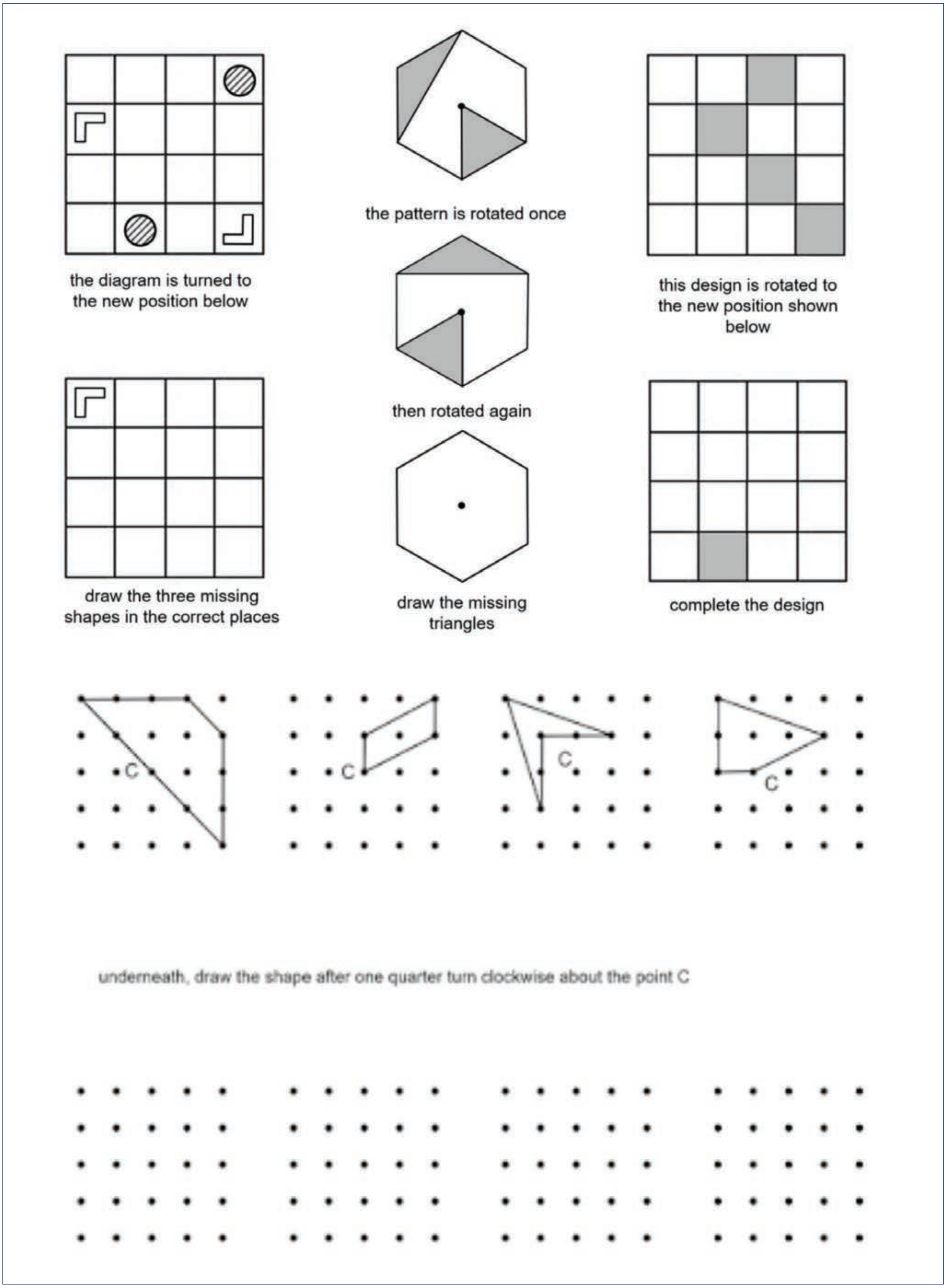


What will be the coordinates of point *N* when Siya completes the figure?

Answer: (______________, ______________)

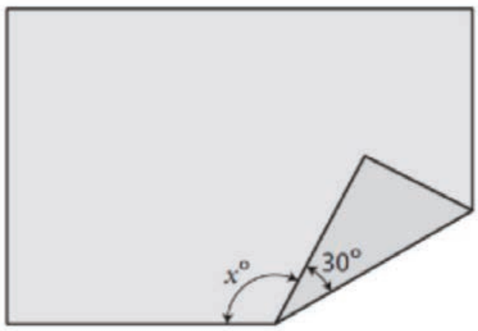
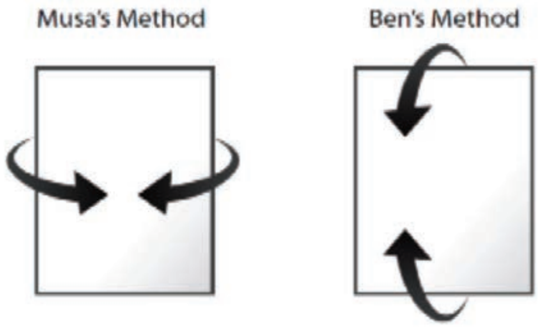
Although learners might not have been familiar with the word "trapezoid", they were told that it is shape ABCD, so they simply needed to create a congruent copy of that shape. The learners were also not told how ABCD had been transformed so if they have been taught tricks for the standard transformations (e.g. if you reflect in the x-axis then all you need to do is change the sign of the y-coordinate of the point) it would not help them here. What learners would need here is practice transforming shapes, developing the ability to see congruent shapes in different orientations, and developing the ability to mentally manipulate visual images.

Activities like the ones shown below are potentially very powerful in this regard.



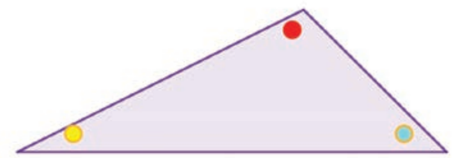
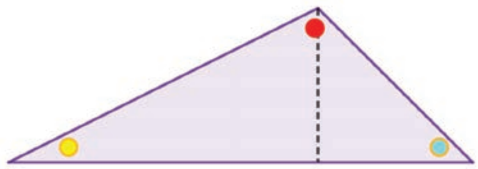
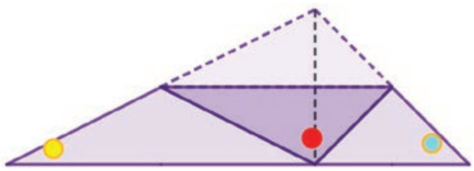
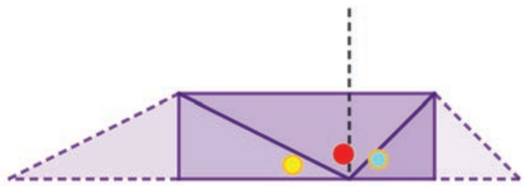
Source: donsteward.blogspot.com.

In the restricted use items we saw two questions that involved paper folding:

Question	Item number and South African percent correct
 A rectangular piece of paper is folded at one corner, as shown above. What is the value of x ? Answer: ________________	**Item 25** **6%**
Musa and Ben have identical rectangular pieces of paper. They use different ways to roll their papers into cylinders so that the opposite sides of the paper touch as shown below.  Compare the properties of the two cylinders. Use $>$, $<$, or $=$ for each. **Height** Musa's cylinder __________ Ben's cylinder **Diameter** Musa's cylinder __________ Ben's cylinder **Surface Area (open ends)** Musa's cylinder __________ Ben's cylinder	**Item 11** **21%**

If learners had physically done the paper folding, they should have found both of these questions fairly straight-forward. The fact that learners' performance was poor on these items suggests that they did not do this. Although paper-folding is not an official part of the curriculum it can provide an accessible route to seeing properties or developing an understanding of shapes, and thus play a similar role to constructions and transformations.

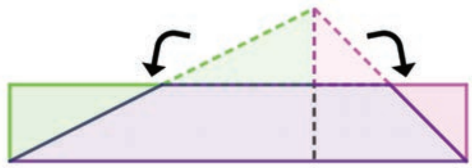
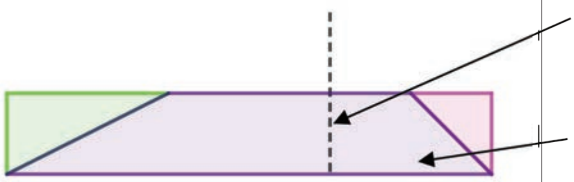
For example, paper-folding can be used to show why the angles in a triangle add up to 180°:

1. Cut out a triangle and put different coloured dots in the corners.	2. Fold the paper to find a perpendicular line from a vertex to the base.
	
3. Fold the triangles so the vertex ends up on the base.	4. Now fold in the other vertices. We see that the yellow, red and turquoise angles lie on a straight line to add up to 180°.
	

This video clip illustrates how we can find the sum of angles in a triangle by paper-folding.

Video clip 3: Sum of angles in a triangle
https://youtu.be/-LZ5V0Takq0

The same paper-folding exercise can be used to show how we get the formula for the area of a triangle. However, it is probably easier to see if we do some cutting and moving:

1. Start in the same way (steps 1 to 3 above), but this time cut off the triangle you folded down.	2. Cut that cut-off triangle along the perpendicular line and rearrange the pieces as shown to make a rectangle.
	
3. Now we can see the whole triangle has the same area as this rectangle below.	4. So area of triangle = base of triangle × ½ height of triangle = ½ h × b which we can rewrite as ½ bh.
	½ of the height of the triangle = width of the rectangle The base of the triangle = length of the rectangle

In both cases it is worth encouraging learners, after they initially physically manipulate a triangle, to then try and do it more abstractly, for example, by taking another triangle and picturing the same process rather than physically doing it. In this way we help them develop their geometric eye.

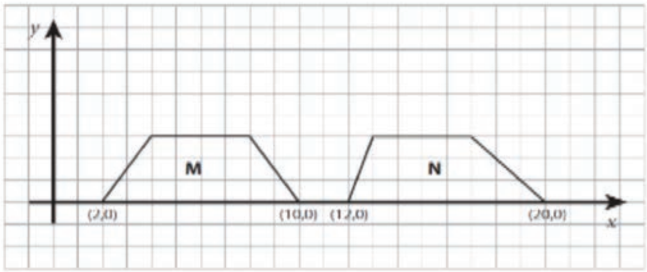
This video clip illustrates how we can derive the formula for the area of a triangle in a slightly different way to the example discussed above by cutting up and rearranging pieces of the triangle.

Video clip 4: Area of triangles
https://youtu.be/ljDJBAdbvik

C.1.3. Decoding geometric language

The final issue raised by the analysis of the TIMSS restricted use geometry items was that of language. Learners performed extremely poorly on **Item 24**, with only two percent of learners getting at least some part of the answer correct (the comparative statistic for the international group was 16 percent). The question required learners to extend straight lines and read off where they cross. A factor that could have contributed to learners' poor performance was a difficulty in decoding the language.

Two trapezoids, M and N, are shown on the grid below.



Extend the non-parallel sides of M to form a triangle. Also extend the non-parallel sides of N to form a triangle.

Give the coordinates of each triangle's third vertex.

A. Vertex of triangle formed from Figure M: ______________

B. Vertex of triangle formed from Figure N: ______________

The language or notational demands in geometry are numerous. They include:

- specific words like vertex, perpendicular, and isosceles to name a few that learners may not be familiar with and need to learn;
- words that learners may be familiar with in an everyday context, but that have a specific meaning in mathematics, for example produce, adjacent, corresponding;
- the way we name parts of diagrams, for example <ABC;
- often fairly condensed ways of expressing facts, for example "both pairs of opposite angles are equal" or "extend the non-parallel sides of M to form a triangle".

All of these language demands occur in the context where the majority of learners are learning mathematics in a language that is not their home language. Thus, we need to pay particular attention to the language demands in geometry in our teaching.

C.2. Algebra

The TIMSS content area of Algebra is divided into two areas:

1. **Expressions, operations and equations:** This includes evaluating expressions, simplifying expressions, and determining equivalence of expressions, writing expressions and equations to represent situations and solving linear equations and inequalities.
2. **Relationships and functions:** This includes interpreting, relating and generating representations of linear functions in tables, graphs or words. It includes doing the same for simple non-linear functions (e.g. quadratics) and generalising pattern relationships.

This content area aligns reasonably well with the content stipulated in CAPS, although some items with functions are beyond the scope of Grade 9 CAPS. However, all of the eight restricted use items relating to algebra should have been covered by South African Grade 9 learners at the time of writing the TIMSS assessment.

This content area was an area in which South African learners performed better than other TIMSS content areas. However, with a South African average of 21 percent correct for questions in this key topic area (compared to the international average of 33 percent correct), there is still room for improvement (Reddy et al., 2022). For all the restricted use items South African learners performed below their international counterparts.

Table 6 shows the match between the TIMSS items and CAPS, and learner performance on all algebra items, and on the restricted use items in this area.

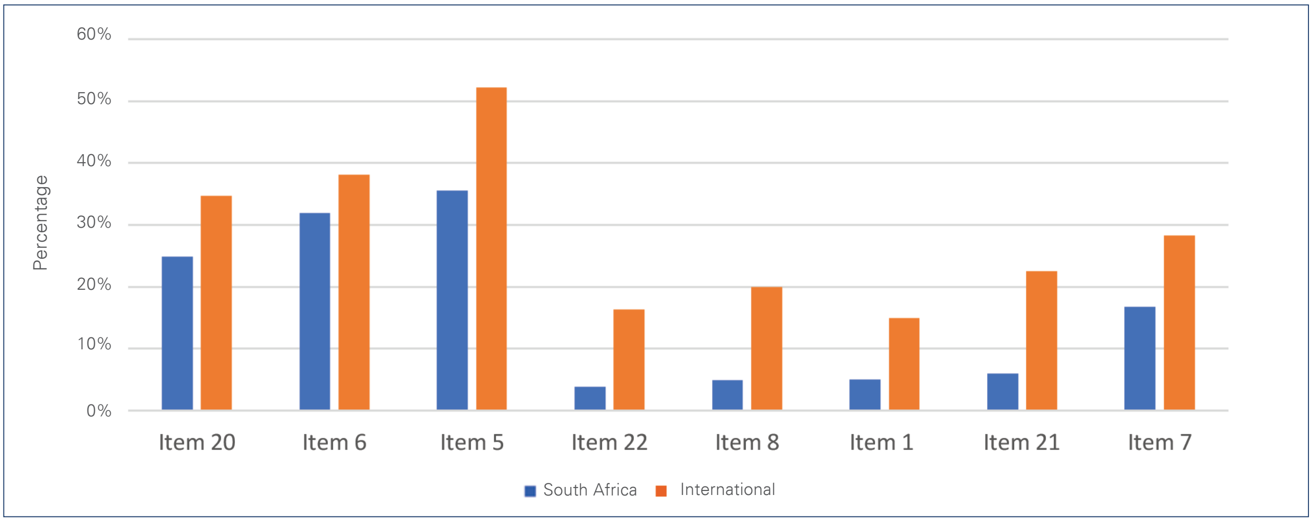
Table 6: Algebra in TIMSS Grade 9 overall and restricted use items

Algebra questions in TIMSS 2019				Restricted use Algebra questions		
No. of items	Percentage match between TIMSS and CAPS	Mathematics scale score (SE) (difference from overall score)	Average	No. of items	Percentage match between restricted use items and CAPS	Average
61	78%	401(2.5)* (+12 points)	21%	8	100%	16%

*Statistically significant difference from overall mean.

Figure 4 shows the performance of South African learners on these items, where performance in half of the restricted use items was below ten percent correct. For all items, the performance of South African learners was substantially below that of their international counterparts.

Figure 4: Grade 9 performance on restricted use algebra items



The small proportion of restricted use items in this section (8 out of 61 algebra items) means that the themes we are able to extract are limited and unlikely to encompass all the issues relating to algebra that might need attention. However, a look across the restricted use algebra items did suggest three key areas of concerns: (i) language and reading questions, (ii) the language of algebra, and (iii) expressing generality.

C.2.1. Language and reading questions

Our analysis of the restricted use items suggested that learners had difficulty working with items that are language-heavy.

Both questions shown in the table below required learners to substitute into a formula, but learner performance on the second question was substantially lower than on the first.

Question	Item number and South African percent correct
Which is the value of $2(6x - 3y)$ when $x = 3$ and $y = 2$? Ⓐ 6 Ⓑ 12 Ⓒ 24 Ⓓ 30	**Item 5** **36%**
The stopping distance (d) metres depends on the speed (v) metres per second of the car when the brakes are applied. A formula for calculating this distance is: $d = \frac{2v + v^2}{20}$ What is the stopping distance when $v = 20$? $d =$ ________________ m	**Item 7** **17%**

Learners were able to use calculators when answering questions so the calculations with a bigger number in the second question should not have been a major factor. We speculate that the fact that the second question was embedded in a context and thus required more reading and interpretation from learners may have contributed to the lower performance.

Linked to this was learners' common mistake on **Item 20** (shown below) where 57 percent of learners incorrectly chose option C, ignoring the fact that the question stated that it was 4 hours per day for the five weekdays. This indicates learners might not be taking sufficient care when reading questions and also, perhaps, dealing only with information that is immediately visible in the item presentation.

Refilwe worked 4 hours each day on Monday through Friday and earned 7 zeds per hour. She worked 6 hours on Saturday and earned 10 zeds per hour.

Which expression shows how to calculate Refilwe's earnings?

Ⓐ $(5 \times 4) + 6$

Ⓑ $(4 \times 5) + (6 \times 7)$

Ⓒ $(4 \times 7) + (6 \times 10)$

Ⓓ $(4 \times 7 \times 5) + (6 \times 10)$

Learners' performance on these restricted use items suggests we need to pay attention to helping our learners read and comprehend mathematics questions. To do this, you can:

a) Include more contextual questions
Because our learners often struggle with word problems it is tempting to skip over them. However, the only way learners will be able to solve contextual or word problems is if they do more of them. Learners therefore need regular exposure to contextual problems. These can be done in all topics, not just algebra. In particular when learners are working on number topics, contextual problems can be a useful way to give meaning to numbers and operations.

For example:
- 4 friends want to share 8 cakes equally. How much cake does each person get?
- 16 friends want to share 8 cakes equally. How much cake does each person get?
- 80 friends want to share 8 cakes equally. How much cake does each person get?

This set of examples is useful in providing an understanding of division as sharing, but also shows the link between fractions and division, i.e. we divide the number of cakes by the number of people to see how much cake each person gets. If 16 friends share 8 cakes equally, each person gets $8 \div 16 = \frac{8}{16} = \frac{1}{2}$ of a cake each.

b) Provide strategies for reading and summarising contextual questions

We need to show our learners how to make sense of questions. It is thus worth regularly doing word or contextual problems as a whole class activity:

1. Read and ask the meaning of any words you don't know	Start by doing an initial reading of the problem and then ask learners if there are any words in the question that they do not understand. Clarify the meaning of those words.
2. Re-read and make sense of the problem in your own words/ diagrams	This is one of the most crucial steps. Getting learners to slow down and actually make sure they understand the questions before attempting a solution is important. There are many options for doing this, some suggestions are: • Think, pair, share: Ask learners to re-read and think about what the question means for a while on their own, then then divide them into pairs into pairs and explain their understanding of the question to each other and then finally share in a whole class discussion. • Draw diagrams to summarise the situation described in the question. It might be necessary for you to model this for learners initially.
3. Set-up and solve the problem	Once learners have made sense of the question they can set-up and solve the problem.
4. Check your answer makes sense in the context	Once learners have solved the problem, they should make sense of their answer within the context. Encourage learners to write a full sentence with the answer or have a class discussion where learners must explain their answer in context.

c) Build learners' vocabulary, paying attention to words that appear regularly in questions

It is worth making a conscious effort to build learners' vocabulary. As you work with learners on questions and check their understanding of particular words you will be able to start developing a vocabulary list for your class. You can then regularly return to the words that were unfamiliar to your learners to ensure they become an established part of their vocabulary.

For example, if while working on a question you determine that your learners do not know the word "descending" you can:

- Have a class discussion to clarify the word, give its meaning and examples of descending numbers and numbers that are not descending.
- In the next class, ask your learners something like, "Are the numbers 53; 12; 5 written in descending order? Explain why you say so?" or to ask them to give you an example of numbers in descending order.
- Return a week or so later to another question that requires them to put numbers in descending order.
- Continue returning to the word until all learners in the class are completely familiar with it.

In order to do this you need to keep a list of vocabulary words that you are working on with your class – it will help to have these written down. Some textbooks have a glossary list that might be a helpful starting point, but you will need to tailor it to your own class.

Some groups of words to consider checking your learners' understanding are:

a) Question instruction words, e.g. determine, evaluate, estimate, simplify, expand.
b) Everyday words used in mathematics, e.g. descending, ascending, variable, constant.
c) Word order and words that can confuse, e.g. 12 divided by 3 versus 3 divided into 12; take 43 away from 60 and the difference between 43 and 12.
d) Specialised mathematical vocabulary, e.g. vertex, binomial, perpendicular.

C.2.2. Language of algebra

Algebra itself is a language. We want learners to make meaning of algebraic expressions and equations, and to be able to translate back and forth between everyday language and algebraic language. In the TIMSS 2019 Grade 9 restricted use mathematics items we saw that only six percent of our learners (compared with 23 percent for the international group) could correctly translate the scenario described in **Item 21** below, into an equation. An issue here might be in the complexity of decoding the language in the text, but we know that even if learners understand the text, they can still have difficulty translating that into algebraic language.

Marius buys cell phones for x zeds each and sells them to make a profit. He determines his selling price for each phone, y zeds, by doubling the price he paid and subtracting 3 zeds.

Write an equation that shows y in terms of x.

Equation: ________________________

Although it is tempting to launch into applying the rules of algebra (e.g. only add like terms), it is vital that learners spend time reading, writing and interpreting algebraic expressions. There are many activities that allow learners to do this. Here are a few suggestions:

"I know your number" tricks

Get the class to do the following:

- Think of any number, double it, then add 5, then subtract your original number, then subtract 4, then subtract your original number again.
- Once they have done it tell them that you know that, no matter what number they started with, their answer is 1.
- Then explore with them how they can use algebra to show that.
- Here we have $2x+5-x-4-x=1$

Write an expression

1. A book costs x rand, a pencil costs y rand and an eraser costs z rand. Write an expression for the total cost (in rands) of:

a) 5 books
b) 4 pencils
c) 3 books and 2 pencils
d) 3 pencils and 2 erasers
e) p books
f) q pencils
g) 3 books and m pencils
h) m pencils and n erasers

2. A book costs x rand, a pencil costs y rand and an eraser costs z rand.
What will each of these expressions give?

a) $7x$
b) $3x+2y$
c) $x+3y+2z$
d) $ax+by$

3. In a game park there are x lions, write an expression for the number of each type of animal if we know:

a) There are 3 more leopards than there are lions;
b) There are 2 fewer cheetahs than there are lions;
c) There are 5 times as many giraffes as there are lions;
d) There are half as many rhinos than there are lions;
e) There are 2 times as many impalas as there are giraffes;
f) There are 5 fewer warthogs than there are impalas.

4a) In a game park there are x lions. There are $\frac{x}{2}$ elephants in the park.
Write a sentence (like those in question 3 above) about the number of elephants in the park.
b) In a game park there are x lions. There are $x + 20$ hippos in the park.
Write a sentence (like those in question 3 above) about the number of hippos in the park.

The Wits Maths Connect Secondary project has created a booklet on introductory algebra that is free to download. It can be downloaded from https://www.witsmathsconnectsecondary.co.za/resources. It contains tasks of varying difficulty requiring learners to match algebraic expressions and those in words. Two examples of varying difficulty are shown below:

Simpler version

In the table below the letter g represents any number.
e.g. The verbal expression "*a number increased by 2*" is written as $g + 2$ but it could also be written as $2 + g$. Match the columns. There may be more than one correct answer for some options!

	Verbal expression
1.	8 add a number
2.	A number multiplied by 8
3.	8 subtract a number
4.	A number divided by 8
5.	A number decreased by 8

	Algebraic expression
A	$8 + g$
B	$8g$
C	$g + 8$
D	$g - 8$
E	$g(8)$
F	$8 \div g$
G	$8 - g$
H	$g \div 8$

A *verbal expression* is written in words.
e.g. Add 3 to a number.
An *algebraic expression* uses symbols for operations (+; −;×;÷) and variables to replace "a number".
e.g. $x + 3$
So here we have replaced the words "a number" with x and we have used the symbol + in place of "add".

More difficult version

In the table below the letter m represents any number. Match the columns. There may be more than one correct answer for some options!

Verbal expression	
e.g. The product of a number and 5 is then increased by 2	
1.	Add 4 to the product of a number and 5
2.	Subtract 4 from the product of a number and 5
3.	Add a number to the product of that number and 5
4.	Subtract a number from the product of that number and 5
5.	Add a number to the product of that number and negative 5

Algebraic expression	
e.g. $5m + 2$	
A	$5m + m$
B	$-4 + 5m$
C	$5m - 4$
D	$m - 5m$
E	$5m - m$
F	$-5m + m$
G	$5m + 4$

C.2.3. Expressing generality

Expressing generality is at the heart of algebra. However, we saw in the TIMSS restricted use items that our learners struggled with this. Only five percent of Grade 9 learners managed to get **Item 1** (below) completely correct (compared to the international average of 15%), with a further 12 percent getting it partially correct (compared to the international average of 16%).

If a is an integer, are these statements true for **all** values of a?

Shade one circle for each statement.

Statement	True	False
$a^2 = 2a$	Ⓐ	Ⓑ
$a + 2 = 2 - (-a)$	Ⓐ	Ⓑ
$a - 2 = -2 + a$	Ⓐ	Ⓑ
$\frac{a+3}{2} = a + \frac{3}{2}$	Ⓐ	Ⓑ
$\frac{a \times 3}{2} = a \times \frac{3}{2}$	Ⓐ	Ⓑ

This kind of work on generalisation should be something that we work on with learners when they are working with numbers in primary school and early high school. For example, the properties of 0 and 1 are included as part of the work on whole numbers in the Senior Phase CAPS. Learners could explore how 0 and 1 behave in different calculations.

We have found it useful to ask learners to give meaning to the calculations by putting them in story situations. For example 0 ÷ 5 can be thought of as the number of sweets each learner will get if we have no sweets and 5 learners who want to share them. After doing some exploring with numbers, we can then ask learners to generalise by using an example like this one:

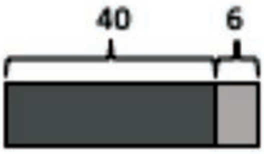
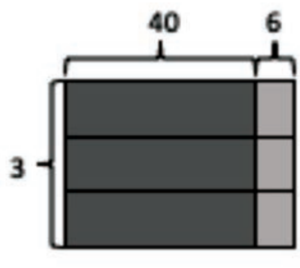
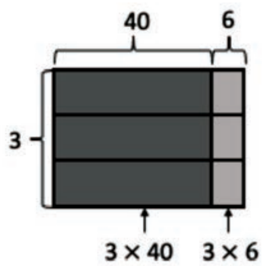
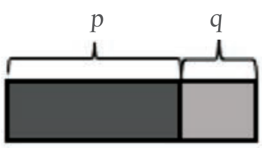
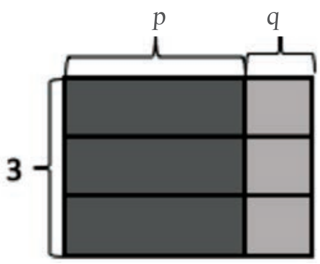
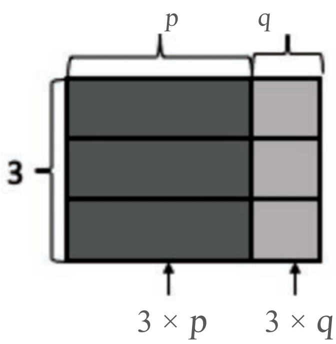
Match the expressions on the left with those on the right where x is any number (except 0):

$x \times 0$
$\frac{x}{x}$
$\frac{x}{1}$
$0 \times x$
$0 + x$
$1 \times x$
$\frac{0}{x}$
$\frac{x}{0}$
$\frac{1}{x}$

0
1
x
undefined
None of the above

Similarly, one can explore the distributive law through use of diagrammatic representations and generalise from specific examples. It is very helpful if learners have been exposed to these kinds of numeric examples and representations during their primary school years. It is something we need to pay more attention to in our curriculum so that primary school educators are aware that the work they do with numbers is not only intended to get learners to complete calculations, but also as a preparation for algebra. Discussion between primary and secondary school colleagues is important if we want to build a coherent whole and a pathway from number into algebra.

Here is an example that illustrates how we might leverage off number work going into algebra:
Study the picture explanation below and then answer the questions that follow.

We can think of 46 as $40 + 6$	If we have 3 × 46 we have $3 \times (40 + 6)$	And then we can see $3 \times (40 + 6) = 3 \times 40 + 3 \times 6$
		
If this is $p + q$	Then this is $3 \times (p + q)$ which we can write as $3(p + q)$	And then we can see that $3 \times (p + q) = 3 \times p + 3 \times q$ $3(p + q) = 3p + 3q$
		

a) Draw similar pictures to illustrate that: $4 \times (30 + 7) = 4 \times 30 + 4 \times 7$
$4(a + b) = 4a + 4b$

b) Is it true that: $4 + (30 + 7) = 4 + 30 + 4 + 7$?
$4 + (a + b) = 4 + a + 4 + b$?

A further example that illustrates how we might leverage off number work going into algebra would be to explore the differing effects of adding/subtracting and multiplying/dividing numerators and denominators when working with fractions using numbers and then ending with an expression of generality.

For example:

Which of the following is true?

$\frac{x+2}{2} = x$ and $\frac{\overset{2}{x \times 2}}{2} = x$

Each year the diagnostic report released by the Department of Basic Education (DBE) on the mathematics matriculation examinations results[9] indicates that candidates' basic algebraic skills (those that should have been acquired in previous grades) are poor and a hinderance to their performance. As such it can be very beneficial to regularly return to these basic algebraic skills.

An easy way to do this is to make a list of the typical misconceptions in algebra you see your learners display. For example, misconceptions like:

$a + b = ab$ or
$a + 2a = 2a^2$ or
Compare $a+a+a$ with $a \times a \times a$.

Once you have addressed the misconceptions with your learners, they can become part of your question bank that you repeatedly return to.

One way to quickly reinforce these basics skills is to present a set of three questions on the board as a quick starter to a lesson. Tell learners they must simplify them or write "cannot be simplified" if they cannot be simplified.

$x+x =$
$x+2x =$
$x+y =$

Give learners a minute to write their answers in large print on a piece of paper and when the minute is up, ask them to hold up their piece of paper for you to see.

You will get an immediate snapshot of whether your learners have got it right or not.

You can then give feedback and adapt the questions for the next day to be more difficult or more of the same, depending on how your class performed.

If you keep it simple and quick it will not eat into the body of your lesson, but will be a constant reinforcement of basic skills.

[9] See the National Senior Certificate examination diagnostic reports available at: https://www.education.gov.za/Resources/Reports.aspx

C.3. Data handling and probability

The TIMSS content area of data and probability contributes 20 percent to the test items, with data contributing 15 percent and probability 5 percent. It focuses on learners being able to extract meaning from visual displays and being familiar with how the statistics' underlying data distributions relate to the shape of the graphs. They need to be able to read and interpret data to solve problems; identify appropriate ways to collect, organise and represent data; calculate and interpret statistics (mean, median, mode and range); and be able to discuss the effect of spread and outliers. In probability they are expected to be able to determine the theoretical probability and estimate the empirical probability of simple and compound events.

Although this is similar to the overall suggested content of data and probability in CAPS in the Senior Phase, data handling and probability are often taught in the last term of the year and some of the more sophisticated parts of these topics might not have been covered in CAPS by the time learners wrote the TIMSS assessment. We see that overall learner performance on data and probability was low and that most topics were not taught before the TIMSS assessment was taken (Table 7). Reddy et al. (2022) report that the content of only 54 percent of the items was covered in classes before the assessment was written. The restricted use items were a better match with CAPS, however the performance in these items remained very low.

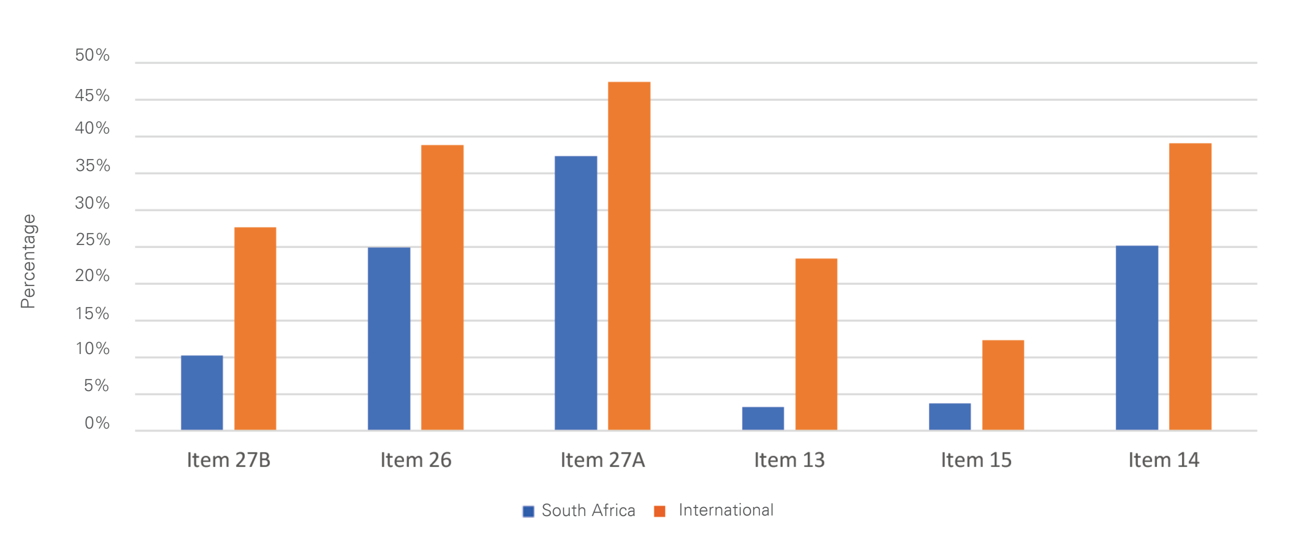
Table 7: Data and probability in TIMSS Grade 9 overall and restricted use items

Data and probability questions in TIMSS				Restricted use Algebra questions		
No. of items	Percentage match between TIMSS and CAPS	Mathematics scale score (SE) (difference from overall score)	Average percent correct	No. of items	Percentage match between restricted use items and CAPS	Average percent correct
39	54%	370 (2.4)* (-19 points)	19%	6	83%	17%

*Statistically significant difference from overall mean.

Figure 5 shows the performance of South African learners on these items. For all items, the performance of South African learners was substantially below that of their international counterparts.

Figure 5: Grade 9 performance on restricted use data and probability items



The key theme that arises from learners' work on data is the importance of sense-making alongside procedures. There were three questions relating to the mean. We look at two of these questions here to illustrate this. The first of these items asked learners to calculate a mean of four numbers, and 37 percent of our learners could do this correctly (compared with the international average of 47%). However, one third of learners (33%) chose option A which gave the mean of 12, 13, 11 and 13 seconds as 13 seconds. A large proportion of learners could not immediately recognise that if none of your data points are above 13 and some of them are below 13, the mean cannot be 13. This suggests a lack of understanding of the word 'mean'.

A relay team for a 400 m race has 4 runners. They took 12 seconds, 13 seconds, 11 seconds, and 13 seconds, respectively, to complete their legs of the race.

A. What is the mean time it takes the runners to complete their legs?

Ⓐ 13,0 sec.

Ⓑ 12,5 sec.

Ⓒ 12,25 sec.

Ⓓ 11,5 sec.

The second part of the question in this item reinforced the notion that learners did not have a good sense of what the mean is, with only ten percent of learners able to correctly answer this question (compared with the international average of 28%). Almost half the South African learners chose option D which they likely got by saying that if 2 runners improved their times by 2 seconds, the sum of the team members' times would have improved by 4 seconds.

In the next race, 2 of the runners each improved their times by 2 seconds, and the other 2 had the same times as before. By how many seconds did the team's mean running time improve?

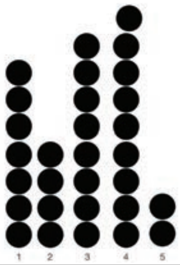
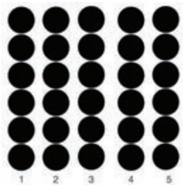
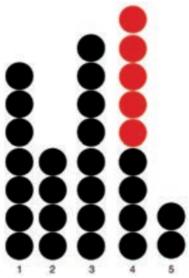
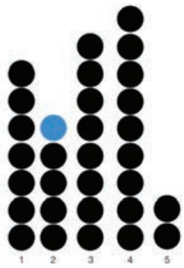
Ⓐ **0 sec.**

Ⓑ **1 sec.**

Ⓒ **2 sec.**

Ⓓ **4 sec.**

When working with statistics it is helpful to give learners a strong visual sense of the concept 'the mean'. One of the ways to do this is shown below. You can use bottle tops or some other physical manipulative to show this as it gives a good idea that what you are doing is putting everything together and then distributing it between the five people.

1. Set up a scenario where you have 5 people and each of them gets a certain number of bottle tops (shown below). Person 4 has a the most bottle tops whereas person 5 only has two.	2. You can illustrate the mean by putting all the bottle tops together and then sharing them out equally between the 5 people.
	
3. Once learners have that idea of the mean, set up the original distribution again. Then ask if person 4 had actually only started out with 4 bottle tops – illustrate that those 5 red circles were not originally there. How would it affect the mean? The learners should be able to see that each of the five towers would end up with 1 less coin if they were put together and distributed fairly.	4. Set up the original distribution again. Now ask if person 2 had started off with one more bottle top, how would that affect the mean? The learners should be able to see that the one extra blue circle would need to be shared between the 5 towers and so would need to be cut up – so it would not have much effect on the mean.
	

This concrete example can be followed up with more abstract examples. For example, give a scenario where someone writes five mathematics tests, each out of 10 marks, and their mathematics mark is the mean of all five marks.

Ask questions like:

- If the learner got 7, 6, 5, 6 marks for the first four tests, what mark would she need for the fifth test to ensure her mean mark remains at 6 marks?
- If the learner got 7, 6, 5, 6 marks for the first four tests, is it possible for her to end up with a mean mark of 7 for the five tests? If yes, what would she need for the fifth test in order to do this?

One can also get learners to think about means without doing calculations, for example:

This is the number of goals scored by three netball teams in their last six matches. Without doing any calculations, put the teams' means in ascending order.

Is the difference between the means small or large? Explain why you say so.

Team A: 5; 8; 12; 13; 16; 18
Team B: 5; 8; 11; 13; 17; 20
Team C: 4; 9; 12; 13; 16; 100

The following video clip shows how we can introduce the idea of the mean graphically.

Video clip 5: Data handling
https://youtu.be/IkI7EzZBCPQ

It can also be useful to get learners to not only calculate the mean, median, mode and range, but to also come up with sets of numbers that will produce particular values for these.

An example of this is taken from the work of Don Steward (donsteward.blogspot.com):

averages and range (positive integers)

1) 3 numbers: mean = 3 mode = 2	2) 3 numbers: mean = 7 mode = 10	3) 3 numbers: mean = 8 median = 10 range = 8	4) 3 numbers: mean = 6 median = 7 range = 11	5) 3 numbers: mode = 7 median = 7 mean = 6	6) 3 numbers: mean = 13 range = 8 find three sets
7) 4 numbers: mean = 4 mode = 1 median = 2	8) 4 numbers: mean = 9 mode = 6 median = 7	9) 4 numbers: mean = 6 median = 6½ range = 11	10) 4 numbers: mean = 7½ mode = 6 median = 7	11) 4 numbers: mean = 10 range = 12 mode = 13	12) 4 numbers: mean = 8 range = 8 median = 7 find three sets
13) 4 numbers: mean = 4 range = 6 median = 3 find two sets	14) 5 numbers: range = 5 mean = 6 median = 7 mode = 8	15) 5 numbers: range = 9 mean = 4 mode = 3 find two sets	16) 5 numbers: range = 6 mean = 4 mode = 2 find two sets	17) 5 numbers: range = 5 mean = 4 median = 3 find three sets	18) 5 numbers: range = 10 mean = 7 mode = 7 find three sets
19) 5 numbers: range = 5 mean = 5 mode = 5 median = 5 find two sets	20) 5 numbers: range = 10 mean = 10 mode = 10 median = 10 find three sets	21) 5 numbers: range = 10 mean = 4 mode = 1 median = 2	22) 6 numbers: range = 10 mean = 4 mode = 1 median = 2	23) range = 10 mean = 4 median = 2 mode = 1 a) 4 numbers b) 5 numbers c) 6 numbers	24) 5 numbers: 2, 5, n, 2n, 5n mean = 2 × median – 1 find n

C.4. Multiplicative reasoning

Multiplicative situations are situations that are underpinned by a multiplication/division-focused structure. These situations require multiplicative reasoning to thinking about how quantities in the problem situation are related to each other. Although multiplicative reasoning is not a defined content area in either TIMSS or CAPS, we chose to consider the cluster of items related to this type of reasoning as we feel multiplicative reasoning is a vital skill learners need to bring from primary school into high school, as it underpins a lot of their mathematical work in high school.

The Grade 9 restricted use mathematics questions in TIMSS 2019 that are underpinned by multiplicative reasoning encompass work with fractions, rate, ratio, gradient, mean, enlargement and products. Some of these have already been discussed in other sections (the questions on mean have been discussed in the data handling section). In this section we will focus on illustrating the importance of multiplicative reasoning and looking at ways we can link the work from primary school into high school.

This video clip provides a short introduction to the key ideas of multiplication and division that we have highlighted through our analysis of the TIMSS restricted use items, and includes attention to how key representations and language can connect between topics across the Phases to strengthen learning.

Video clip 6: Multiplicative reasoning
https://youtu.be/NNVQgXix3bQ

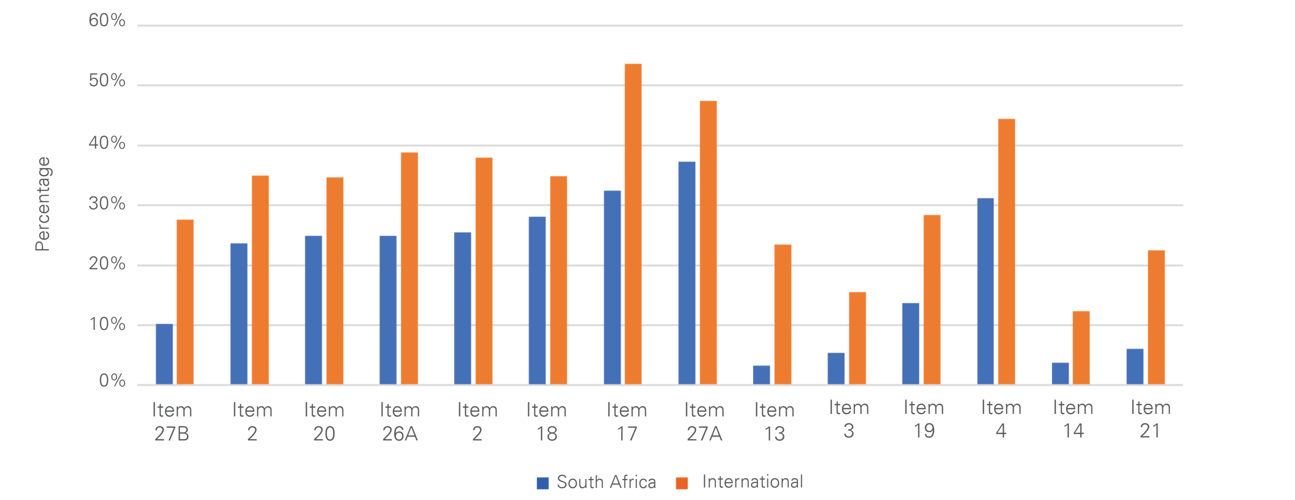
Table 8 shows the number of TIMSS restricted items that incorporate multiplicative reasoning, the match between the TIMSS items and CAPS, and South African learners' performance on these items.

Table 8: TIMSS Grade 9 restricted use items related to multiplicative reasoning

Restricted use Multiplicative reasoning questions in TIMSS		
No. of items	Percentage match between TIMSS restricted use items and CAPS	Average percent correct
14	93%	19%

Figure 6 shows the performance of South African learners on the multiplicative reasoning items, where performance on five of the 14 restricted use items was around or below ten percent correct. For all items, the performance of South African learners was substantially below that of their international counterparts.

Figure 6: Grade 9 performance on restricted use multiplicative reasoning items



Learners at the start of primary school often use additive reasoning exclusively. During their primary school years, we want learners to develop the power to reason multiplicatively as well. However, we see many learners struggle to make the transition from working with addition/subtraction to working with multiplication/division.

In one of the TIMSS restricted use items, **Item 18**, we see clear evidence that many of our Grade 9 learners were still thinking additively.

Nozazi wants to enlarge this photo keeping the same proportion between height and width.

height = 20 cm

width = 10 cm

She wants her new photo to have a width of 25 cm. What will be the height of her new photo?

Ⓐ 50 cm

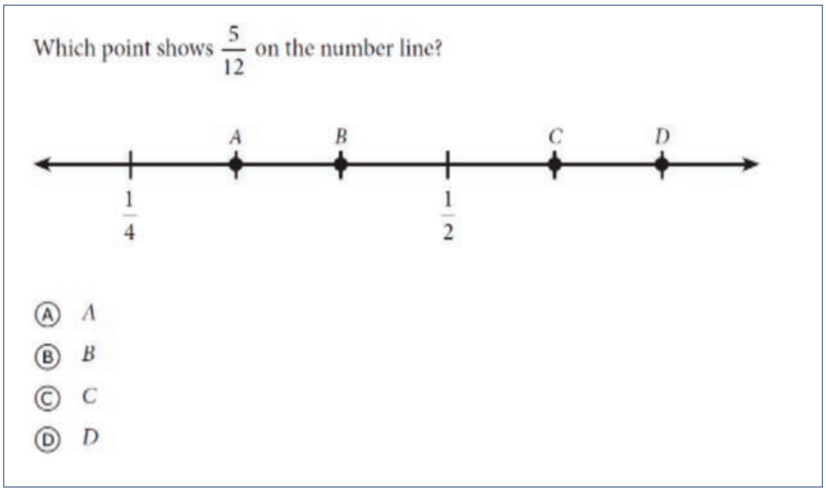
Ⓑ 45 cm

Ⓒ 40 cm

Ⓓ 35 cm

Here 42 percent of the learners chose the answer D (35 cm), produced by saying the width increased by 15 cm so the height should be 20 cm + 15 cm = 35 cm. A further 22 percent chose 45 cm which could be the result of adding 25 cm to the 20 cm. This suggests that a large portion of the learners were thinking using additive rather than multiplicative reasoning.

Learners' understanding of fractions also relies on multiplicative reasoning. For example, learners should be able to reason that as 5 is just a little smaller than half of 12, $\frac{5}{12}$ will lie just below $\frac{1}{2}$ on the number line. **Item 2**, shown below, required learners to identify the point representing $\frac{5}{12}$ on the number line. What we saw in learners' performance on this item was that only about a quarter of the learners chose the correct answer, with 40 percent choosing option D. Although it is hard to tell why they might have chosen option D, one possibility is that 5 and 12 are both 'big' numbers, leading to a selection of the farthest point along the line.



Learners faced similar levels of difficulty when working with ratio, which we saw in **Item 17**, discussed earlier, where only 32 percent of learners could identify a ratio equivalent to 3:2 with almost half the learners choosing the ratio 2:3.

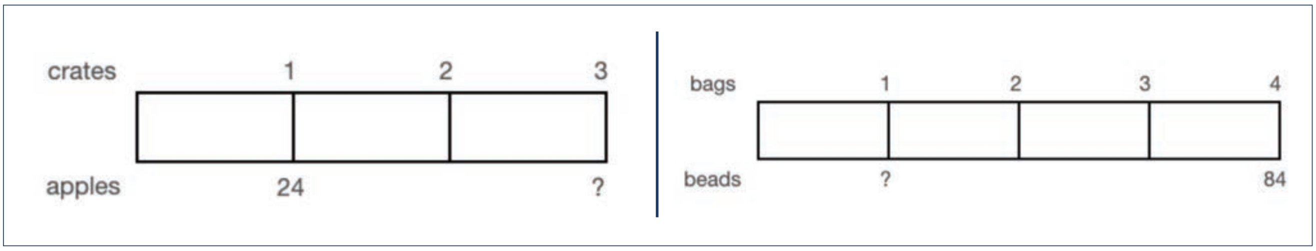
Poor multiplicative reasoning can lead to learners having difficulties working with rates, ratios, fractions, decimals and percentages. These are important tools in everyday life as well as being important foundational ideas in both the Mathematics and Mathematics Literacy curricula. It is clear from the Grade 9 learners' performance on the restricted use items linked to multiplicative reasoning that we cannot assume that Grade 9 learners have acquired strong multiplicative reasoning in primary school. It is thus important that we pay attention to these ideas in the Senior Phase and create connections from primary school through high school.

In the Foundation Phase and Intermediate Phase, bar diagrams are useful representations for multiplicative situations, and these can be carried through into the Senior Phase.

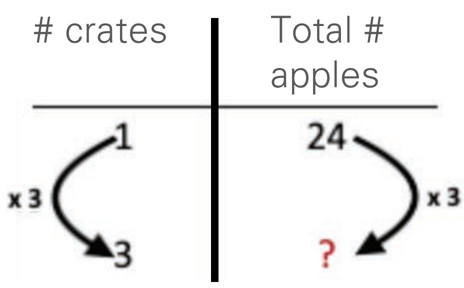
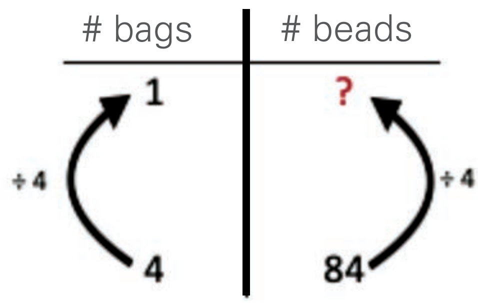
In the Foundation and Intermediate Phase, you might see examples like the following:

1. Every crate has 24 apples in it. How many apples are there in 3 crates?
2. I have 84 beads that I need to share equally between 4 bags. How many beads will be in each bag?

These can be modelled using bar diagrams:

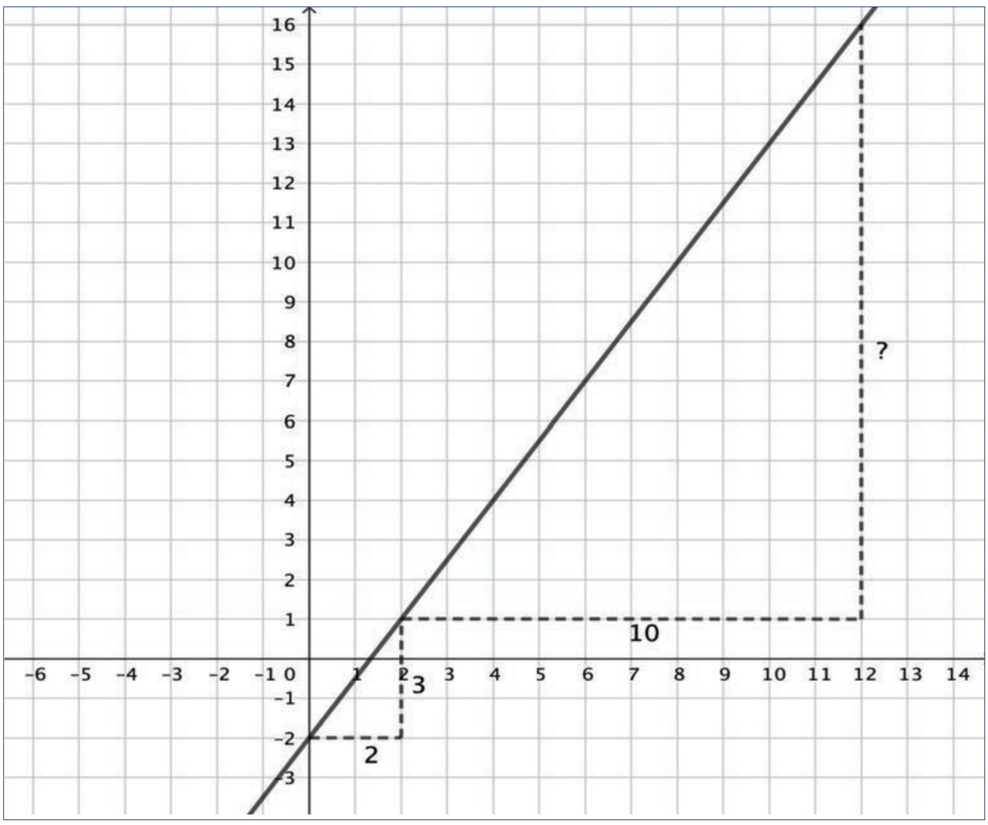


They can also be modelled slightly more abstractly using clue boards:

3 crates containing 24 apples each	84 beads shared equally between 4 bags
	

These ideas can then pull through into working with ratio and rate questions in the Senior Phase. For example, if we know the ratio of boys to girls is 2:3 and are asked how many girls there are if there are 10 boys, we can set up a clue board to solve the problem:

This same idea underpins the notion of a slope. If a straight-line graph has a slope of $\frac{3}{2}$ this means for every 2 units you move horizontally you move 3 units vertically. So if you move 10 units horizontally you will need to move 15 units vertically:



Common across all of these topics are the language expressions like: "For every…….of these, we have……..of those".

The Foundation Phase examples of this idea shared above involve sentences like:

- For every 1 crate, we have 24 apples
- For every 4 bags, we have 84 beads

In the Senior Phase examples, this language is adapted in the ratio situation above, for example, to:

- For every 2 boys in the class, we have 3 girls

In the slope situation, the language becomes:

- For every 2 horizontal units we travel, we travel 3 vertical units

An important part of successful work with multiplicative reasoning involves being able to recognise when a situation has a multiplicative structure.

If the phrasing of:

For every …………of these, we have…………..of those

can be applied to the situation. This tells us that multiplication/division can be used to solve the problem rather than addition/subtraction.

C.5. Other items

There were some items that did not fit into the categories above and did not warrant a separate section, but where learner responses signal useful implications for the classroom. We discuss them here under headings related to the implications for the classroom.

C.5.1. Provide opportunities for problem-solving and quite simple content areas for this

One of the restricted use items (**Item 19**) required learners only to use their understanding of place value and multiplication to solve the problem. Only 14 percent of South African learners (in comparison with 28% of the international group) could correctly answer this.

Write each of the digits 1, 2, 3, and 4 in a box below to make the smallest product. Each digit may only be used once.

☐ ☐ X ☐ ☐

These kinds of problems are powerful for use in the classroom for a number of reasons:

- Firstly, we need to encourage learners to be prepared to tackle non-routine, problem-solving questions, and those set in a context of fairly simple mathematics are ones we can get all our learners to engage in.
- Secondly, these kinds of problems can give learners a lot of practice on basic mathematics in ways that are not just simply drill and practice.

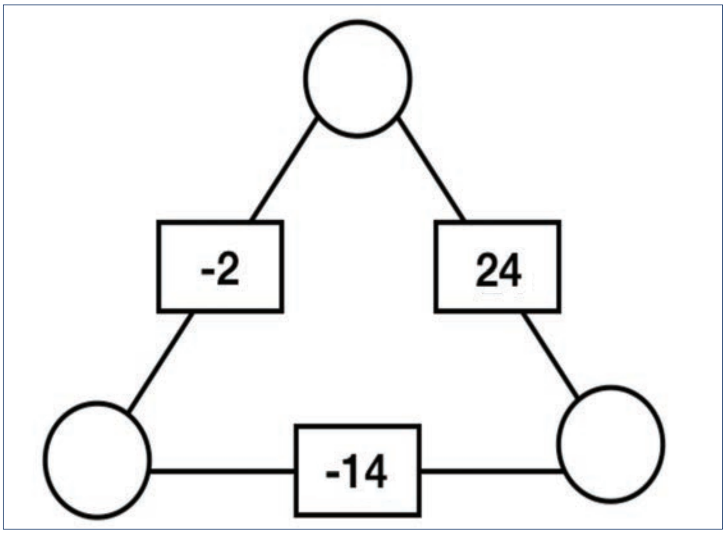
A British mathematics educator, Colin Foster, has put together a number of activities that help learners practice basic mathematics skills in a problem-solving context. We give an example of one of these involving fractions below (Foster, 2014). More can be found on his website at http://www.mathematicaletudes.com.

Add together as many of the six fractions below as you like (i.e. you can choose just two of them or all six of them or any other number of them to add together) to get an answer as close to 1 as possible.

$\frac{1}{6}$ $\frac{1}{25}$ $\frac{3}{5}$ $\frac{3}{20}$ $\frac{4}{15}$ $\frac{5}{8}$

This kind of question gives learners a lot of practice in adding fractions whilst at the same time engaging them in some strategic thinking.

Simple puzzles like arithmagons can fulfil a similar role. In the example below, learners need to figure out which numbers go in the circles using the information that the sum of the number of the two circles at the end of each line segment must equal the total in the rectangle in the middle of that line segment. This gives learners practice working with negative numbers, while at the same time doing some problem-solving.



Further ideas about using arithmagons in teaching and examples of arithmagon puzzles using different types of numbers and even algebra can be found on the following websites:

- http://www.mrbartonmaths.com/teachers/rich-tasks/arithmagons.html
- https://nrich.maths.org/2670

C.5.2. Basic work with negative numbers still poses difficulties for Grade 9 learners

There were two questions that involved negative numbers. One (**Item 16**) was a very basic question that only 25 percent of our learners (compared to the international average of 49%) could correctly answer.

> **On Thursday, the lowest temperature in City X was 6 °C and the lowest temperature in City Y was –3 °C. What was the difference between the lowest temperatures in the cities?**
>
> **Answer: ________________°C**

The second (**Item 13**) was a question that involved calculating the mean of a set of numbers that involved a negative number. Here only 3 percent of our learners (compared to the international average of 23%) gave the correct answer.

Sophie recorded the temperature (°C) at the same time each day for 5 days:

–2, 1, 3, 2, 3

What is the mean of these 5 temperatures?

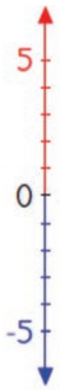
Answer: ________________°C

We do not have data on the typical errors that learners made here so it is not clear what aspects of work with negative numbers learners are struggling with. However, it is cause for concern that more than halfway through their Grade 9 year, learners were not comfortable working with negative numbers.

We suggest two possible areas for work in the classroom:

a) Number line work and number sense

The number line can be a useful tool in improving learners' understanding. We have found that learners often need to start with quite basic work on the number line where they can actually count steps. For example, start with:

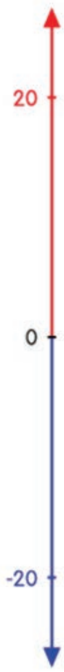


Make a dot at 3 and label the point A.
How far down from A to 0?

Make a dot at – 4 and label the point B.
How far down from 0 to B?

How far down from A to B?

Then move on to similar questions, but where learners can't just count down in ones:

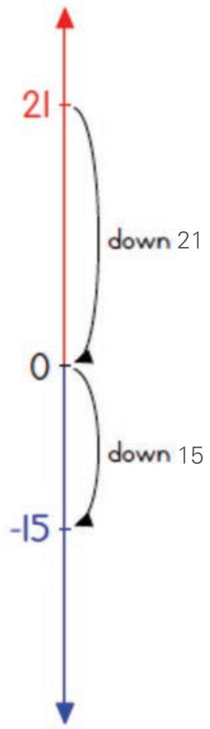


Make a dot at 10 (approximately) and label this Point A.
How far from A down to 0?

Make a dot at – 6 (approximately) and label this Point B.
How far from 0 down to B?

How far from A down to B?

This work can then be extended to using the number lines to do calculations:



To calculate 21 – 36 we are subtracting, so we're going down.

From 21 down to 0 is 21 steps.
But we need to go down 36 steps,
so we still need to go down 15 steps more
so we will end at – 15.

Exercises like the ones above can be used to help learners build a mental image of a number line involving negative numbers. Learners can then use these ideas to think about whether the answers to calculations will be positive or negative. For example:

Which of the following calculations will have a positive answer? Which will have a negative answer?
Why do you say so?

4732 – 6583
6583 – 4732
-4732 – 6538
-6538 – 4732
-4732 + 6583

This video clip illustrates working with integers on number lines.

Video clip 7: Integers
https://youtu.be/Fu7D7BsJdlw

b) Practice exercises for fluency

Learners should understand integers thoroughly in Grade 8, but we have found it useful to regularly get learners to return to practice working with adding, subtracting, multiplying and dividing integers with reasonably small numbers that we would want them to be able to do quickly in their heads.

Puzzles like arithmagons (discussed above) can be useful for doing this. Alternatively quick mental maths quizzes as a starter activity can be used. Or if learners have cellphones they can download the Four Minute Flamenco app from the Google Play Store for free and this will give them practice with basic calculations involving integers, fractions and decimals in a gamified environment.

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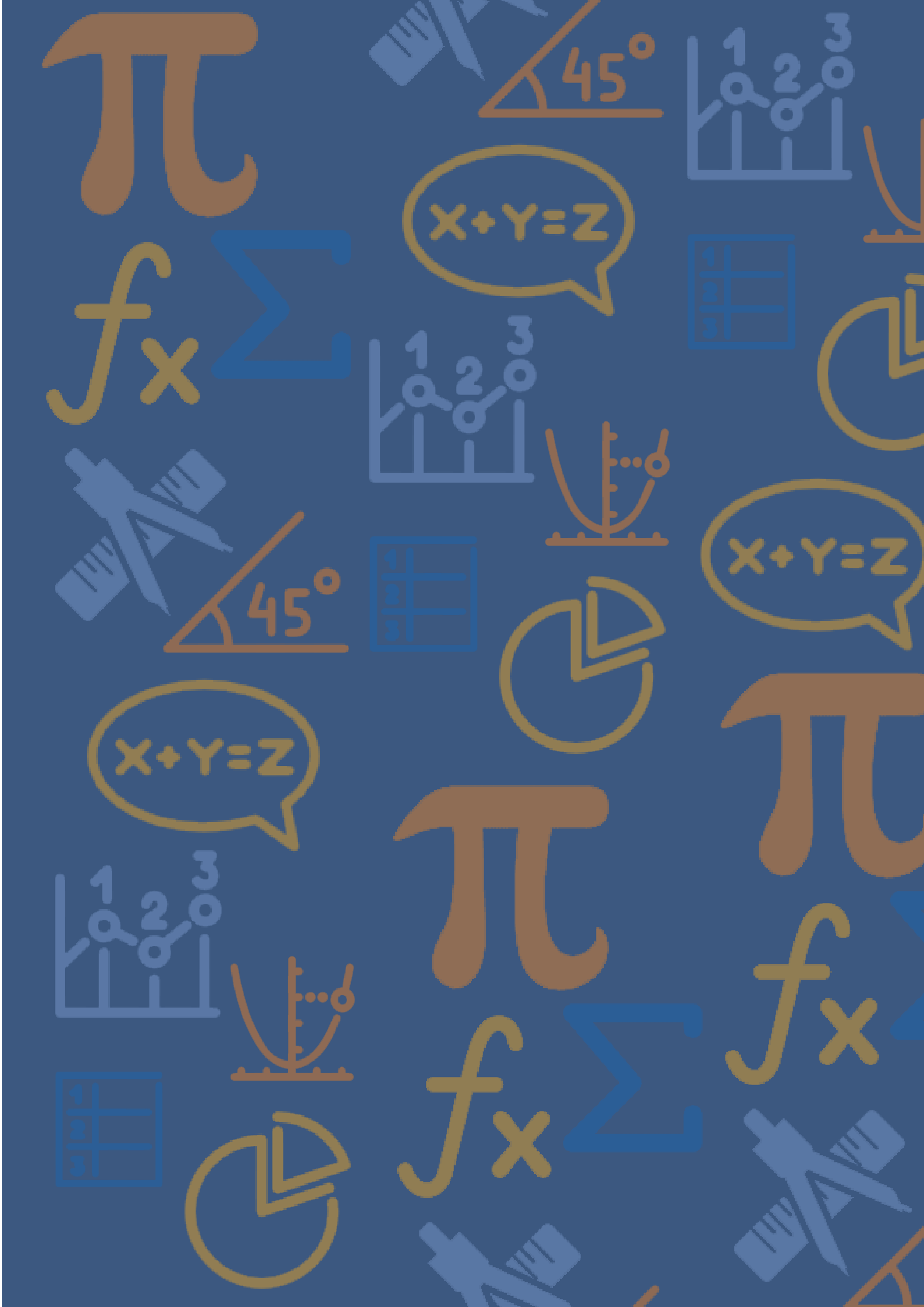
X+Y=Z
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