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# An introduction to TWG19: Mathematics teaching and teacher practice(s)

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*Teaching is everywhere, yet much of what is involved in teaching remains hidden, with comprehensive theories lacking. These challenges serve as the backdrop for research on mathematics teaching and the work of Thematic Working Group 19. To make progress, the group has used four domains to organize and consider research on teaching. This paper reviews the contributions and issues that arose in the group at CERME13. We elaborate on how the domains stimulated discussions of the meaning of teaching across papers, and we provide reflections and implications for future work.*

## Introduction

Thematic Working Group 19 (TWG19) engages scholars who study mathematics teaching and its practice. As a working group, we face a fundamental challenge. Teaching's ubiquity and invisibility lead to it being paradoxically taken as obvious and transparent on the one hand (Lortie, 1975) and overlooked and unconsidered on the other (Ghousseini & Sleep, 2011; Lewis, 2006). This fundamental challenge became evident as TWG19 took shape after the division of a working group that addressed the combination of teacher characteristics, teaching, and teacher education. One way the challenge was felt was that papers seemed to stand on their own, as lone entities scattered across a haphazard landscape. Participants found it hard to bring together in conversation studies that on the surface appeared related. For instance, one year we had six papers all on instruments for evaluating teaching, yet they seemed incommensurate, like apples and oranges. The reasons stemmed from the taken-for-granted and opaque character of teaching. As a result of and adding to this challenge, different studies grew out of different contexts, with different purposes, leveraging different theoretical and conceptual foundations, and deploying different methods.

To address these challenges and support our collective investigation of teaching, we adopted three strategies (Mosvold et al., 2022). One is the establishment of a repository of data that is shared with participants of TWG19. Shared data serves as a boundary object allowing communication across different studies. Second, we press ourselves and each other to define more explicitly what we mean by teaching in our studies. Looking at our own work and published research in the field, we have found that teaching, even when it is a central concept, is often left undefined. Attempts to infer implicit meaning reveal troubling ambiguity and inconsistency, and raise concerns about the interpretation of questions and claims and the appropriateness of theory and method. Third, we use an analytic frame for considering teaching to help organise and gauge our collective work. Fundamental to teaching is responsibility for interactions among teachers and students around

mathematical content, a way of thinking about teaching sometimes referred to as the didactic triangle. Increasingly, scholars have argued that to understand teaching, this simple didactic triangle must be expanded to include consideration of an array of social, cultural, and material factors, or systems, that shape and are shaped by what teachers and students do (Goodchild & Sriraman, 2012). While teaching involves all of these, efforts to study teaching tend to foreground one aspect as they attempt to keep in mind this fullness. Without prescribing or meaning to constrain research on teaching, we use four domains or foci:

1. Consideration of *mathematics* and the central endeavour of extending the subject to students.
2. Becoming acquainted with, relating to, and responding to *students* as people and learners.
3. Organisation and *enactment* of design, interaction, and discourse of teaching and learning.
4. Attending to broader social, cultural, and political *issues* that matter for teaching and learning, including imperatives of social justice.

We use these to organise the collection of papers of the group and to probe and survey the work.

### **Focus and contribution of papers**

A total of 18 papers and 7 posters were accepted for presentation at CERME13 in TWG19, and there were 29 registered participants from 14 different countries in the group. The sessions were organised to stimulate discussions about the meaning of teaching in papers and the analytic lens of the four domains. We thus organised the papers in groups according to the domains and structured our sessions by domain. In addition, seven posters were submitted to TWG19, and we organised a session where the posters were presented and discussed.

At previous CERMEs, few papers have focused on students in teaching. At CERME12, for instance, there were no papers in the *students* domain (Mosvold et al., 2022). It was thus interesting to see that five papers studied teaching with an emphasis on becoming acquainted with, relating to, or responding to students as people and learners. Below, we elaborate on and discuss contributions of the papers in each of the four domains that were highlighted. As we did in sessions at CERME13, we use the domains and the meaning of teaching to consider and relate papers in the group.

### **Mathematics papers**

Two papers highlight considerations of the mathematics. In one of the papers, Barbosa and Vale study characteristics of pre-service teachers' algebraic thinking. Analysing observations and participants' written work on tasks, they found that the pre-service teachers used various generalisation strategies but had difficulties with certain ones. Their aim is to inform the mathematical education of teachers and they conclude with recommendations for what mathematics teachers need to learn in order to recognize their students' algebraic thinking. In another paper, Çakır and Akkoc study socio-mathematical norms related to explanations in a classroom of gifted and talented students. They found that answers without explanations or justifications were not considered sufficient and examined the many why-questions the teacher asked in this classroom.

Although both studies emphasise mathematics as it matters inside the work of teaching, they approach this in different ways, with different underlying conceptions of mathematics and of teaching. Barbosa and Vale consider mathematics in more cognitive terms, highlighting the algebraic thinking of pre-

service teachers, whereas Çakır and Akkoc consider mathematics as a practice that is negotiated among participants in the mathematics classroom. Their conceptions of teaching also differ. Barbosa and Vale seem to conceptualise teaching as work that demands strong knowledge of content. This knowledge is considered a basis for understanding and supporting students' thinking, and they analyse the knowledge held by pre-service teachers because it is key to teaching. In contrast, Çakır and Akkoc consider teaching as a social and cultural activity that involves negotiation of sociomathematical norms, and they analyse the negotiation of such norms in a classroom of gifted and talented students. Whereas Barbosa and Vale focus on the algebraic thinking of pre-service teachers, which might help them attend to the algebraic thinking of students in teaching, Çakır and Akkoc focus on sociomathematical norms that support explanation and justification and how these might be established in classrooms.

### **Students papers**

Five papers attend in particular to students. Whereas this domain potentially includes studies that attend to students' identities or their active role in co-constructing what happens in classrooms, the five papers submitted to TWG19 in this conference focused on students' mathematical thinking. A common emphasis of these papers is that students' thinking has consequences for their learning of mathematics, and thereby also for teaching. However, there are differences among the papers in how students' thinking is viewed in relation to teaching. In their three papers, Aksoy and Bostan, Gössinger and Götz, and Pekkan and Kayitmaz focus on ways to grasp student thinking in teaching. The paper of Aksoy and Bostan and the paper of Pekkan and Kayitmaz emphasise attending to, interpreting, and responding to students' mathematical thinking. Pekkan and Kayitmaz attend in particular to students' unconventional thinking through spontaneous noticing, and they explore the acts of responding to students' non-conventional thinking of fractions. In contrast, Gössinger and Götz attend to individual mental models, which they study through imaginary dialogues. The last two papers in this group emphasise how choices about activities and tasks have consequences for students' opportunities to develop their thinking. Nic Mhuiri and Neary focus on students' current levels of thinking by investigating their pathways through a unit on long division, whereas Vissariou and Desli study the promotion of children's procedural thinking in problem solving through metacognition.

Across these papers, student thinking is considered in terms of individual and normative mental models, meaning-making processes, unconventional thinking, current and future forms of thinking, and metacognition. The different ways of writing about students' thinking give a sense that there are differences in how one sees teaching in relation to student thinking – as something that should replace *or* develop students' current thinking. Examples of these different approaches are Pekkan and Kayitmaz, who talk about eliminating or getting over unconventional thinking, and Nic Mhuiri and Neary, who focus on developing it into more conventional mathematical thinking.

The purpose of teaching was not only implicitly addressed in relation to student thinking, but it was also explicitly addressed in a wider perspective by Nic Mhuiri and Neary who referred to Biesta and Stengel's (2016) three domains of educational purpose: qualification, subjectification, and socialisation. In their paper, they give examples of how teaching with subjectification in mind can

contribute to students' agency and mathematical authority and how the purpose of socialisation can lead to particular ways of working in the classroom.

### **Enactment papers**

The largest group of papers addressed enactment. In past years, the majority of papers in this domain have used discourse as a theoretical lens to examine interactions in classrooms, from which implications for teaching are drawn. Eidissen's paper is in this vein, examining the influence of mini-whiteboards on opportunities for reflective communication. While it sets out to study effects on communication, it instead finds effects on opportunities, exposing the need to understand what it would take for teaching to make good on those opportunities. In general, though, papers at this conference started with this goal of examining directly the work of teaching. In doing so, both the work of teaching and the instructional interactions for which teaching is responsible were considered from more diverse perspectives.

The paper by Bostan and colleagues considers aspects of teaching similar to several of the papers in the *student* domain above, but with an emphasis on what is involved in the doing of responding to students' mathematical thinking. Zhang and Stylianides use a Reformed Teaching Observation Protocol (RTOP) to determine the extent of student-centred instruction in a Chinese school reform effort. The contrast between these two papers and the papers in the *student* domain demonstrates our use of the domains to note subtle differences in the study of teaching that matter for a fuller understanding. Every paper is concerned at some level with all four domains, but it is helpful to notice which is foregrounded and how they bump and slide into each other.

Other papers bring other lenses. Choutou uses communities of practice and boundary crossing to examine connections and tensions between emerging practices in mathematics and visual art teaching. She states that teaching is what takes place in a classroom related to epistemological content, classroom artefacts, teachers' actions, and students' learning strategies. She then uses these analytic distinctions to analyse tensions in emerging practices of integrative teaching. In contrast, McIvor and colleagues examine the improvisational character of teaching and explore it as a teachable skill. Although they do not say whether improvisation is a comprehensive conception of teaching or what conception of teaching they see it as fitting into, the paper leads our working group to ponder such questions, along with ones about how such a concept fits or does not fit into other conceptions used in the group.

The other three papers lean in different directions and surface different concerns for the study of teaching. Mosvold foregrounds issues of method in his study of teacher questioning across groups of students and instructional setting. In other words, he studies teaching by holding the teacher and the lesson content steady while students and instructional setting are systematically changed. Managing the complex variables of teaching enables him to note significant differences in questioning specific to differences in students and setting. Whereas Mosvold zooms in on a small slice of teaching, Baldry zooms out to a more comprehensive picture of teaching. She narrates the use of the Orchestration of Mathematics Framework (OMF), which was developed to capture elements of teachers' practice that shape the mathematics made available to students, to examine a 17-minute video of a Norwegian mathematics class from the shared data, the same one analysed by Eidissen and by Hoover and

Phillips. Her goal is to elaborate on the underlying theory of the framework in ways that will support both future use and conversation across frameworks.

The last paper in this domain raises concerns about being overly analytic in conceptualising and studying teaching. Toor and Mgomgelo conduct a phenomenological study of what it is like for teachers to experience teachable moments and how moment-to-moment teaching of mathematics is a complex balancing act between knowledge, emotions, and actions. They argue that teaching involves humanising the subject for learners and that its subjectivity is fundamental. The question they raise for scholars of teaching then is one of how we add teachers' experience to analytic approaches to understand teaching.

### **Issues papers**

Finally, three papers attend to the broader social, cultural, and political issues that matter for mathematics teaching and learning, albeit with different emphases and perspectives. One paper focuses on listening and listening being, in order to recognize and relieve harm in mathematics teaching (Hoover & Phillips). Another paper explores the issue of creating space for socio-mathematical agency (Kelly et al.), whereas a third paper investigates the perspectives around transition year mathematics in Ireland (Maher & Bray).

Maher and Bray are concerned with how teaching can 'prepare' students. In their adapted Delphi study, they reach beyond a traditional focus on academic preparedness, to also consider social, psychological, and skills-based preparedness. In their attention to the possibilities of the structure of the Irish transition-year context, they foreground how policy interacts with the social and cultural contexts that inform teaching. In this case, the ambitious policy environment appears to conflict with embedded social and cultural practice.

The other two papers explicitly attend to the imperatives of social justice. Hoover and Phillips focus on mathematics teaching as a practice that routinely traumatises students, but with the potential to disrupt patterns of harm. Kelly and colleagues highlight the positive potential of mathematics teaching in their participatory action research project, which aimed to develop students' socio-mathematical agency – the ability to use mathematics effectively to argue for social change. In this work, they draw on Biesta (2015) to argue the importance of teacher judgement in planning and enacting mathematics teaching. The participatory action research approach of these authors contrasts with the more conceptual approach adopted by Hoover and Phillips, who employ the notion of work of teaching to unpack what is logically entailed in teaching, using empirical analysis to inform and test conceptual proposals. In this case, they analysed video excerpts from the shared dataset to identify patterns of harm and their disruption, developing a coding system based on listening being/otherwise.

These three papers offer different views on teaching and, in their many differences, highlight the ways in which social, cultural, and political issues matter for teaching and learning. Although research methods vary considerably, two papers (Hoover & Phillips; Maher & Bray) produce frameworks that might be taken up by other researchers.

## **Reflections and implications**

The three strategies — establishing shared data, using the four domains to organise and survey our work, and articulating what is meant by teaching — have supported our collective efforts. First and foremost, they have allowed us to feel as though we are in conversation about teaching across our different studies, and they have been useful tools for interrogating studies and relating them, one to another. While we are working to address ongoing difficulties, we remain excited about the support and insight they have been providing, and we hope, over time, to be able to see and name a growing understanding of teaching as evidenced by our collective work.

### **Shared data**

When the idea of providing shared datasets for TWG19 first materialised after CERME10, there was some excitement. Previous initiatives like this in our field seemed promising (e.g., Schoenfeld, 2008), and we thought it likely that the initiative would provide common ground for discussions in our working group. Enabling common use and reuse of good data sets made sense in several ways. With scholars applying different theoretical and analytical lenses to the same data sets, we found that different aspects of teaching became visible, and this prompted new discussions about how we conceptualise teaching in our studies. Using shared and accessible data sets made analyses more transparent, invited others into discussion of the analyses, and allowed for discussion across different studies. In addition, the availability of shared data sets led to new and productive collaborations.

At the same time, as many know from experience, creating and sharing good video data of teaching can be challenging. The practical challenges of getting the necessary consents, and the navigating of ethical and juridical terrain around data security and sharing, which differ across countries and institutions, can be daunting. We have been able to gather and house the data in ways that responsibly manage confidentiality and appropriate use, but permissions for data collection are often written narrowly, restricting access and use in ways that do not allow for sharing, however responsibly it is managed.

A second challenge is to decide on the kind of data that can be productively reused. Although anonymous lesson transcripts are more straightforward to share, good video data are richer and seem to provide better opportunities for reuse. Yet not all video data is equally suitable for reuse. Beyond the technical aspects of audio and video quality, videos that focus on important mathematics and display examples of core slices of teaching seem more useful to share and reuse. Language and culture might also constitute a limitation for the usability of videos. To date, non-English language videos have been accompanied by English language transcriptions, and efforts have been made to share details about the classroom and lesson context.

A third challenge is that having usable shared data requires ongoing investment. In addition to needing to gather and maintain the data, scholars' use of the shared data requires additional, often unfamiliar work. Most scholars are engaged in their own data. Using the shared data to develop a paper submission often requires stepping out of current work, into a different workspace, to conduct an isolated study. Further developing such papers for publication can be difficult. As organisers of the working group, we have found it necessary to steadily remind and encourage participants to make use of the shared data and to consider collaborating in the work. Our investment has led to several

papers, a few new collaborations, and rewarding engagement in the working group. Still, the payoff is modest, and investment is required.

Finally, although we have seen examples of new collaborations around shared data sets, we are still searching for better ways to facilitate collaboration around shared data sets in TWG19, and in particular to develop opportunities also for younger researchers to productively engage in collaborations around shared data. In discussions at CERME13, it was suggested that the group should engage in more explicit attention to different frameworks and models of teaching, and we envisage that the shared data sets might provide a basis for such discussions. We have invited participants to consider this option, and we continue to encourage collaborations across groups, age, and experience in this endeavour.

### **Using the domains to organise our work**

When the domains were first introduced in the call for papers for CERME12, we were wondering if they would work as a way of organising papers in the group. Before, we had organised papers inductively by looking at the content and focus of papers, with the result that grouping of papers changed from conference to conference. Now, after having used the domains in CERME12 and CERME13, we already see how they might be useful as a way of tracing themes and variation over time. We have, for instance, discussed in the past how the students seem to be missing in the papers of TWG19, but this year we could see how this domain had more papers than before. Using constant domains as an analytic lens across conferences might help us see patterns and development both across and within groups of papers over time.

Another compelling usefulness of the domains is that they help us talk about teaching across papers. Before, there were times when all we could see was diversity and variation across papers. Now, we have experienced how the domains can help us see what is common across papers. For instance, although the two papers in the *mathematics* domain this year had different underlying conceptions of teaching and different conceptions of mathematics, it was still meaningful to discuss them as papers that both emphasised the mathematics in teaching. The domains have thus been useful as a way of seeing how research on mathematics teaching attends to the complexity of teaching by emphasising certain aspects of teaching, while still attending to other aspects.

### **Articulating conceptualisations of teaching**

Our press to be more explicit about the meaning of teaching was prominent at CERME13. First, simply asking the question of ourselves and each other has been eye-opening. Often, we have found that while we thought our papers and others have said what they mean by teaching, when we look, we see that nothing is explicitly said or what is said is unclear or inadequate. Further, when we try to say what we mean, we find it more difficult than we thought. Compounding this, we find one thing said at one point in a paper and another at another point, or we find ourselves questioning the match between how teaching is defined and the conceptual tools, methods, and claims in studies. As we have asked the question, papers in the working group have begun to say more about what is meant and to take up the implications of meaning in the work. Several of the papers provided frameworks or models of teaching. This has led to noticing connections among studies and to using connections and contrasts to consider deeper relationships among studies. It has also led to an awareness of the



need to attend to the purposes of studies, which gives more definite form and rationale for conceptions of teaching.

Discussions in the working group have spawned questions about where and how we see mathematics, students, teachers, and social, cultural, and material factors, or systems in the different conceptions and frameworks being offered. Biesta and Stengel's (2016) distinctions among the purposes of qualification, subjectification, and socialisation was considered as one framework for considering these issues; the mathematical preparedness quadrant that was developed by Maher and Bray was another. Both of these frameworks can be seen as connected to the didactical triangle (e.g., Cohen et al., 2003; Jaworski, 1994), where teaching is seen as interactions among teachers and students around some content, and these interactions are again influenced by the environments in which they occur. Our four domains, which were developed from the elements of the didactical triangle, thus constitute yet another framework for considering teaching. None of these frameworks capture all that is involved in teaching, but each provides perspectives that are useful to better understand the complex work of teaching mathematics. In the future, we wonder if more explicit attention to frameworks of teaching in our discussions might help us continue to move forward in our mutual efforts to study mathematics teaching.

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