Physics Education

PAPER • **OPEN ACCESS**

Students' scaling of axes when constructing qualitative graphs that represent a physics scenario

To cite this article: Orlaith Condon *et al* 2024 *Phys. Educ.* **59** 025001

OPEN ACCESS

PAPER

Phys. Educ. **59** (2024) 025001 (9pp) iopscience.org/ped

Students' scaling of axes when constructing qualitative graphs that represent a physics scenario

Orlaith Condon[1,2], T J Kelly[3], Stephen R Power[2] and Paul van Kampen[1,2,*]

[1] Centre for the Advancement of STEM Teaching and Learning, Dublin City University, Dublin, Ireland
[2] School of Physical Sciences, Dublin City University, Dublin, Ireland
[3] Department of Computing and Applied Physics, Atlantic Technological University, Galway, Ireland

E-mail: paul.van.kampen@dcu.ie

CrossMark

Abstract
We have studied how first-year university science students construct graphs based on hypothetical qualitative physics scenarios. We gave students a questionnaire that asked them to complete two Cartesian graphs in one of three different scenarios (a ball rolling down a track, a beaker being filled with water, the resistance between different points on a metal bar) given as a written piece of text accompanied by a diagram of a hypothetical experiment that included three evenly spaced points on the set-up. Two of the three points were also indicated on the position axis of the partially drawn graph. We found that students can find it hard to translate equal spatial intervals in the experiment to a line graph. We found that most students either did not explain why they put the third point on the graph where they did, or did not plot the point at all. Some students drew unequal intervals on the position axis to indicate unequal time or resistance intervals. The difficulties became

* Author to whom any correspondence should be addressed.

more prevalent as the levels of abstraction increased. Our findings suggest that constructing a scale on a qualitative graph requires significant mental effort from the students.

Keywords: graph construction, physics, axis, scaling, qualitative graph

1. Introduction

Students are introduced to graphs from a young age. In Ireland, for example, Cartesian graphs feature in both the primary school (in 6th class, typical ages 11–12) and secondary school curricula (National Council for Curriculum Assessment 1999, 2015, 2018). It is therefore not surprising that graphing has been studied from the earliest days of science and mathematics education research; see for example the overviews by Leinhardt *et al* (1990) and Glazer (2011). While there is a plethora of research on students' interpretations of graphs and there are quite a number of studies of students constructing quantitative (numerical) graphs, student construction of qualitative (non-numerical) Cartesian graphs has remained under-researched.

Research on students' interpretation of qualitative graphs includes early work by Kerslake (1977), who found that secondary students struggled to interpret qualitative distance-time graphs, with graph-as-picture interpretation being more common than correct interpretation, and many students not answering the question at all. In their Test Of Graphing in Science test, McKenzie and Padilla (1986) asked students, among other things, to interpret qualitative graphs of height of plant vs pot size and the amount of cleared land vs time. Bell and Janvier (1981) reported on difficulties students encountered when drawing height vs volume graphs for bottles with variable cross-sectional areas. McDermott *et al* (1987) found that students struggled to connect qualitative graphs and physics. Beichner (1994) included a number of questions that asked students to interpret qualitative position-time, velocity-time, and acceleration-time graphs in his Test of Understanding Graphs in Kinematics.

Tairab and Khalaf Al-Naqbi (2004) investigated students' responses to questions that dealt with both interpretation and construction of graphs. However, the construction aspect related solely to quantitative data: students were given tabulated data and asked to use it to construct a Cartesian graph. This is a common theme: we found that the majority of the literature on graph construction deals with quantitative data—see e.g. Kerslake (1977), Karplus (1979), Wavering (1985, 1989), and more recently Teuscher and Reys (2012) and Johnson (2015).

By contrast, Mevarech and Kramarsky (1997) asked young students to construct graphs (not necessarily Cartesian graphs) representing four different qualitative statements that relate success on tests to the amount of time spent preparing for the tests. Hattikudur *et al* (2012) studied students' ability to construct graphs from a written description of both a quantitative and a qualitative nature and compared their competency at both. Their focus was on graphs showing a linear relationship, and specifically on students' understanding of the y-intercept and slope of the graphs. Planinic *et al* (2013) found that first-year university students who have developed an understanding of the slope of a graph in mathematics are generally able to transfer it to physics and other contexts. Rodriguez *et al* (2020) found that students apply appropriate and productive reasoning to qualitative graphs in mathematics and chemistry contexts separately, but need more support in combining reasoning to use mathematics to model chemistry situations. Van den Eynde *et al* (2019) found that students were more successful in translating between graphs and equations in mathematics contexts than in physics contexts. Stefanel (2019) reported on secondary students' qualitative graph construction (with pre-drawn axes) based on experiments with motion sensors. He developed a typography of graphs reflecting students' focus on experimental noise and various mental models.

In this study we investigate what students attend to when drawing graphs. We asked students about a hypothetical experimental setup comprising a written and pictorial representation of equally spaced spatial intervals, and investigated whether and how students drew these intervals on a Cartesian graph without being explicitly prompted to do so. This study focuses on qualitative scenarios, where students are provided with no numerical data, but rather a written overview of a physical scenario. We asked students to complete a partially drawn graph that represents this scenario, which depending on the detail may be represented by a rectilinear or a curvilinear graph, and to then explain why they have chosen to complete their graph in that way. Figure 1 shows one of the questions pertaining to a beaker being filled with water, and a graphical representation of the variation of water level with time. One of the key aspects of the graphs we were interested in is how the students represented equal distances (*a*–*b* and *b*–*c* in figure 1) in the hypothetical setup on their graphs. We refer to the distances *a*–*b* and *b*–*c* as intervals. In this paper we detail research on one aspect of qualitative graph construction: that of the linearity of the axis scale.

2. Methods

The students involved in this study were first year undergraduate students, who were non-physics majors enrolled in a physics module. Initially, 328 students enrolled in a semester 1 module were split into five groups. We gave two of these groups questions related to water filling a beaker (figures 1 and 2), and the other three questions about a ball rolling along a track (figures 3 and 4). Later, 147 students enrolled in a semester 2 module were split into three groups, and were given questions about the resistance of a wire. We gave each group a different set of questions as part of an end-of-semester exam.

The questions for the track and beaker groups were designed to be as analogous as possible. For example, the step-down track question of figure 4(i) is equivalent to the half-cylinder beaker of figure 2(i); the upward-sloped track of figure 4(iii) is equivalent to the cone-in-beaker of figure 2(ii). (We were not able to create a matching group for the downward sloped track for logistical reasons.) The questions were designed to be of a similar standard of difficulty, which allows us to get a sense of to what extent context affects the students' responses. Following preliminary analysis of the track and beaker questions, we designed the wire questions shown in figures 5 and 6. The subject area was partly dictated by the content of the semester 2 module (optics and electric circuits).

2.1. Beaker questions

We gave two of the five groups questions involving water flowing into a beaker at a constant rate. In Question 1 we gave students the hypothetical setup of figure 1 and asked them to continue the accompanying graph of water level vs. time. We also asked students to explain why they had chosen to draw their graphs that way. The accompanying text for Q1 of the beaker questions is shown as an example:

> *A student fills a beaker using a constant stream of water. There is a solid cylinder inside the beaker. The student has marked three levels (a, b, and c) evenly spaced along the beaker. She starts her stopwatch when the water reaches point a, at the bottom of the beaker.*
>
> *After she has completed her measurements, she draws a graph showing how the water level changes with time. The part of her graph that represents the change between levels a and b is shown.*
>
> *Complete the graph so that it represents how the water level changes with time between levels a and c.*

In Question 2 we gave each group a different hypothetical beaker setup. We asked students to complete a second water level vs. time graph. These beakers are shown in figure 2.

In all questions concerning beakers, an accompanying piece of text explicitly stated that water flowed into the beaker at a constant rate; that points *a, b*, and *c* were evenly spaced on the beaker; and that the stopwatch was started when the water reached level *a*. As before, we asked the students to explain why they drew the graph as they did.

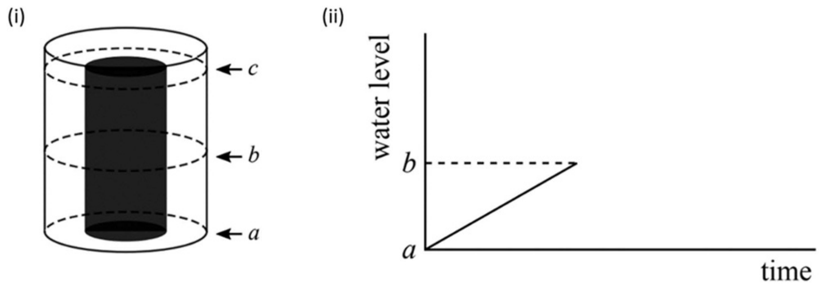

Figure 1. (i) Hypothetical beaker setup set-up used in Question 1. (ii) Water level vs. time graph used in beaker questions.

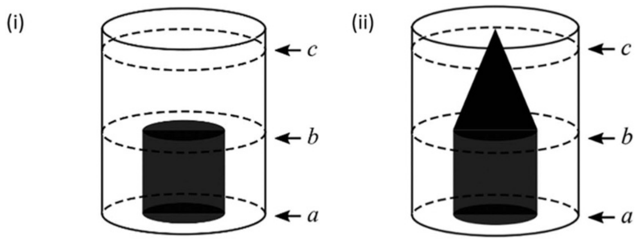

Figure 2. Different hypothetical beaker setups used in Question 2. (i) Half-cylinder. (ii) Cone.

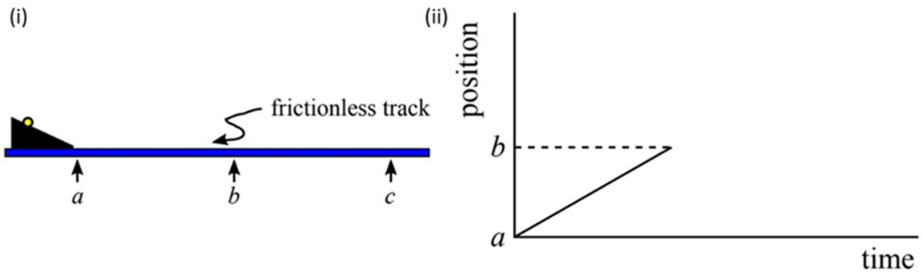

Figure 3. (i) Track setup used in Question 1. (ii) Position-time graph used in track questions.

2.2. Track questions

The other three groups were given two questions involving a ball rolling down a frictionless track and asked to graph position versus time. In Question 1 we gave students the hypothetical experimental setup shown in figure 3 and asked them to complete the accompanying graph. As with the beaker questions, we asked the students to explain why they had chosen to draw their graphs that way.

In Question 2 we gave each group a different hypothetical track. Again we asked students to complete a position-time graph showing the motion of the ball. These tracks are shown in figure 4.

In all cases with the track, there was an accompanying paragraph of text which explicitly stated that the track was frictionless; that points *a, b*, and *c* were evenly spaced; and that the stopwatch was started when the ball reached point *a*. As before, the students were

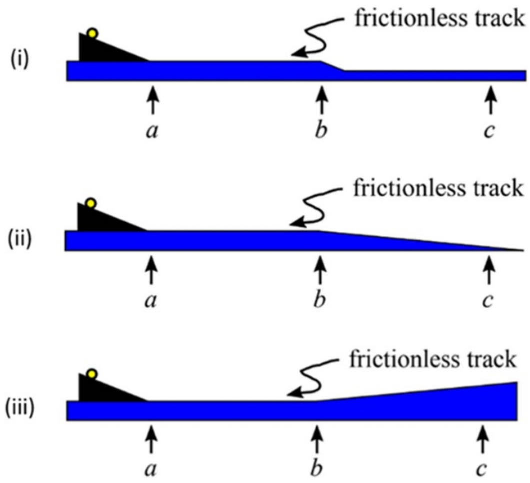

Figure 4. Different hypothetical tracks used in Question 2. (i) Step-down track. (ii) Downward sloped track. (iii) Upward sloped track.

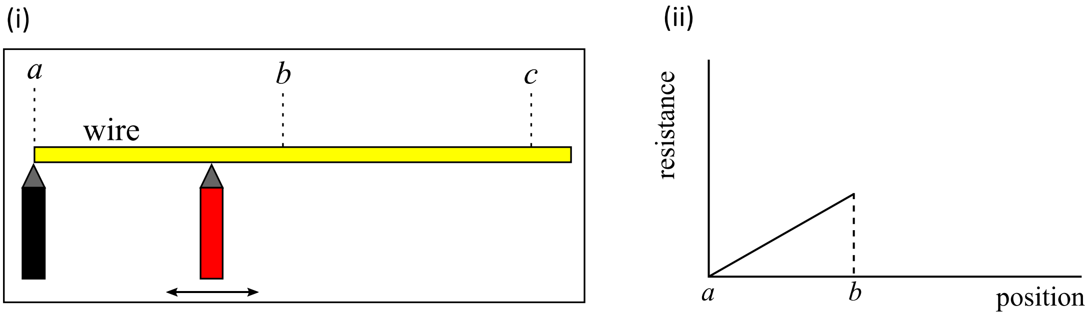

Figure 5. (i) Wire setup used in Question 1. (ii) Resistance vs. position graph used in the wire questions.

asked to explain why they drew the graph as they did.

2.3. Wire questions

The wire questions differed from the track and beaker questions. Here, ideally, all students would draw the same graph. Each group was given the same scenario presented differently. Two of the three groups were given the diagram of figure 5(i), with a text explaining that the resistance between the leads was measured as the red lead moved along the wire, while the third group was given essentially the same diagram but oriented vertically. The horizontal wire and vertical wire groups were given a brief description of the scenario, which explained that the red lead is moved further from the black lead and resistance is measured at each new location. One of the two horizontal wire groups was given an in-depth explanation that explicitly stated that the resistance increased by a constant amount for every cm the red lead moved. All three groups were told that *a, b*, and *c* were evenly spaced on the wire.

In Question 2 we gave the students a different hypothetical setup where the wire was thinner between points *b* and *c* (figure 6) with the same orientation and an explanation with the same level of detail as in Question 1. All three groups were asked to complete a new resistance vs. position graph. The less detailed description stated that the wire was thinner after *b*; the detailed description stated that the resistance increased by a constant amount with every cm between *b* and *c*, and that this constant amount was greater than between *a* and *b*.

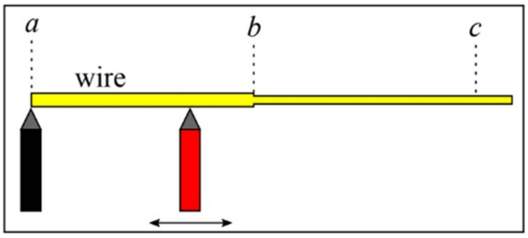

Figure 6. Hypothetical horizontal wire used in Question 2.

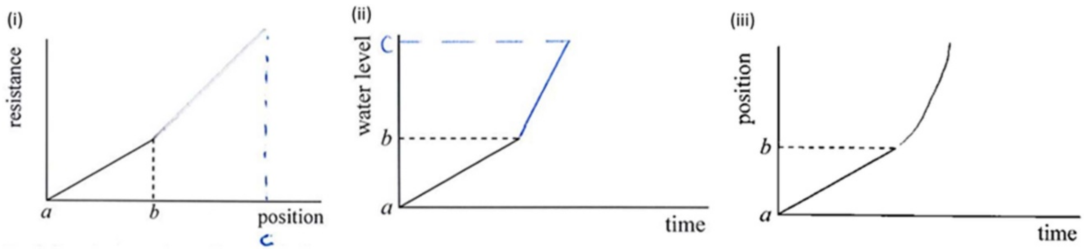

Figure 7. Sample student responses to Question 2 coded as (i) same interval, (ii) different interval, (iii) no interval.

We chose these new styles of question for the wire groups after carrying out a preliminary analysis of the beaker and track groups. A large difference was seen between the track and beaker groups in relation to whether or not they drew the interval, i.e. plotted point *c* on the position axis (see section 3). We hypothesised that this difference could be due to vertical alignment of points *a, b*, and *c* in the beaker questions, which could make it easier to position them on a vertical axis. For this reason we included the two different orientations of the wire, to see if the orientation had any effect. In the track and beaker groups we saw that some students misunderstood the physics and therefore drew an incorrect graph. To mitigate this problem we included the detailed horizontal wire group to see if when they were explicitly told how resistance changed, they were better able to plot it.

We are aware that the beaker and track scenarios are more easily visualised than the wire questions. We also want to draw attention to the fact that in the beaker and track questions time is the independent variable and position is the dependent variable; in the wire questions, position is the independent variable and the more abstract quantity resistance is the dependent variable. The cognitive load on students is likely significantly greater in the wire questions.

2.4. Data analysis

For the purpose of this study, we focused solely on the intervals students drew and their stated reasons for doing so. When analysing the graphs that students had drawn, we found that we could capture the different responses in just three categories. We named these: same interval, different interval, and no interval. Three typical responses, from each category, are shown in figure 7. We further differentiated the written explanations by presence or absence of an explanation for their choice.

3. Results

We initially treated all eight groups separately. However, upon examining our results, we saw no statistically relevant differences (by chi-squared testing) within groups of the same type, i.e. beakers, tracks, and wires. This was of particular interest in relation to the wire questions, as we had presented the same problem in three different ways but saw no statistically significant differences, which suggests that neither the orientation nor the level of detail in the explanation play a part in whether or not students pay attention to the scale of the axis.

The absence of statistically significant differences allowed us to aggregate the responses for each of the three types. Tables 1 and 2 summarise

Table 1. Categorisation of responses to Question 1.

	Same interval			Different interval	No interval
	Explanation	*No explanation*	Total	Total	Total
Beakers ($N = 137$)	*33% (46)*	*56% (77)*	89% (123)	5% (7)	5% (7)
Tracks ($N = 187$)	*10% (19)*	*58% (110)*	68% (129)	13% (25)	17% (33)
Wires ($N = 145$)	*7% (10)*	*47% (68)*	54% (78)	7% (10)	39% (57)

Table 2. Categorisation of responses to Question 2.

	Same interval	Different interval	No interval
Beakers ($N = 137$)	67% (92)	15% (21)	17% (24)
Tracks ($N = 187$)	59% (112)	19% (36)	21% (39)
Wires ($N = 145$)	40% (59)	13% (19)	46% (67)

the results for Questions 1 and 2 respectively for each of these aggregated groups. Depending on the context, between 54% and 95% of students represented point *c* on the graph. The percentage of students who explained why they drew uneven intervals or no intervals was typically less than 2%. The only time we saw a significant number of explanations was in the responses to Question 1 that comprised evenly spaced intervals. We have attributed the smaller fraction of students explaining their choice of interval in response to Question 2 to students feeling that it was not necessary to repeat the explanation.

As stated above, very few students wrote explanations when they drew unequal intervals. Their explanations were typically not easy to understand, but all point to a desire to reflect the different rate of change in position, water level, or resistance. Samples responses:

> same interval: *If the 3 points were evenly spaced, the graph would continue along the same line producing a linear graph* (taken from horizontal wire Q2)

> different interval: *Distance between a and b is less than distance between b and c as shape takes up less space meaning the water must take up more space which will take a longer time* (taken from cone-in-beaker Q2)

Not surprisingly, we have not found a quote to explain why students drew no interval.

4. Conclusions and implications for teaching and further research

This research shows that context plays an important role in how students construct axes on qualitative graphs. We were surprised at the large fraction of students who did not represent point *c* on the graph, or represented it incorrectly. The context dependence strongly suggests that this is more than simply a matter of unfamiliarity with qualitative questions, even though it is likely that most students have only ever been asked to draw graphs with a linear, numerical, labelled scale provided, or to construct quantitative graphs using graph paper and tabulated data, which ensures that they pay some attention to scaling the axis appropriately. If we order the contexts of beaker, track, and wire as representing a progression from most everyday/least abstract to most abstract, a clear trend emerges. Even in response to Question 1, almost 95% of students included point *c* on their graphs, and almost 90% spaced points *a, b*, and *c* evenly on their axis in the beaker context. In the track context, corresponding to a physics situation that students had encountered in the lab, these numbers drop to around 85% and 70%, respectively; in the wire context, the most abstract physics context that students had not encountered in the lab, these numbers drop to 60% and 55%. We found the same ranking for Question 2, in which a change in set-up occurred at point *b*, but with significantly increased numbers of students omitting point *c* or drawing unequal spacings on the axis.

Considering the students' explanations, there were practically no students who mentioned point *c* without marking it on the graph. Students were more likely to explain why they had drawn the interval in the beaker and track groups, often including a phrase like 'in the same time [interval]…'. One possible reason that so few people drew an interval in the wire questions is the absence of a time axis. In the beaker and track groups students tended to comment on how long it would take to reach point *c*, but as the wire graphs represented the change of resistance with position, they were not able to relate the graph to the length of a time interval.

Our analysis has revealed a hitherto unreported way in which students can find line graphs difficult to construct. Students are rarely if ever required to construct graphs like these, so the questions offer an opportunity to probe the depth of students' understanding of line graphs. We had not anticipated any difficulties with translating equal spatial intervals in an experiment to a graph, and interpret their occurrence to an incomplete understanding of what linear axis scaling conveys. It appears that with increasing levels of abstraction students are less likely to attend to all aspects of constructing an axis (e.g. by not including point *c*) and somewhat more likely to construct a non-linear scale to reflect a change in the rate of change of a variable. It may be the case that interpreting the more abstract contexts requires some students to use too much working memory to attend to point *c*, which they can do in the simpler context of a beaker containing a solid cylinder as in figure 1(i). If true, this would suggest that constructing a scale on a qualitative graph requires significant mental effort from the students, and that they do not employ an internalised procedure. Our hypothesis is corroborated by the surprisingly high percentage of students, around 1 in 6, who drew unevenly spaced points on the graphs. It is intriguing that in the wire questions neither the orientation of the wire nor the level of detail provided in the question influenced whether or not students paid attention to the scale of the axis. We will obtain more detailed insight into students' reasoning through interviews, and will investigate how to use our findings in the design of a teaching-learning sequence that addresses the issues in the near future.

Data availability statement

The data used in this study is available from the authors upon reasonable request. This research project was carried out in accordance with the principles outlined in the IoP ethical policy. Informed consent was obtained from the students to participate in the study, and for the results to be published. The institutional ethical approval number is DCU/2022/185.

ORCID iDs

Orlaith Condon https://orcid.org/0000-0002-4802-6416
T J Kelly https://orcid.org/0000-0001-8576-6846
Stephen R Power https://orcid.org/0000-0003-4566-628X
Paul van Kampen https://orcid.org/0000-0003-3043-8110

Received 25 July 2023, in final form 9 November 2023
Accepted for publication 13 December 2023
https://doi.org/10.1088/1361-6552/ad157e

References

Beichner R J 1994 Testing student interpretation of kinematics graphs *Am. J. Phys.* **62** 750–62

Bell A and Janvier C 1981 The interpretation of graphs representing situations *Learn. Math.* **2** 34–42 (available at: www.jstor.org/stable/40240746)

Glazer N 2011 Challenges with graph interpretation: a review of the literature *Stud. Sci. Educ.* **47** 183–210

Hattikudur S, Prather R W, Asquith P, Alibali M W, Knuth E J and Nathan M 2012 Constructing graphical representations: middle schoolers' intuitions and developing knowledge about slope and y-intercept *Sch. Sci. Math.* **112** 230–40

Johnson H L 2015 Together yet separate: students' associating amounts of change in quantities involved in rate of change *Educ. Stud. Math.* **89** 89–110

Karplus R 1979 Continuous functions: students' viewpoints *Eur. J. Sci. Educ.* **1** 397–415

Kerslake D 1977 The understanding of graphs *Math. Sch.* **6** 22–25 (available at: www.jstor.org/stable/30212405)

Leinhardt G, Zaslavsky O and Stein M K 1990 Functions, graphs, and graphing: tasks, learning, and teaching *Rev. Educ. Res.* **60** 1–64

McDermott L C, Rosenquist M L and van Zee E H 1987 Student difficulties in connecting graphs and physics *Am. J. Phys.* **55** 503–13

McKenzie D and Padilla M 1986 The construction and validation of the testing of graphing in science (TOGS) *J. Res. Sci. Teach.* **23** 571–9

Mevarech Z R and Kramarsky B 1997 From verbal descriptions to graphic representations: stability and change in students' alternative conceptions *Educ. Stud. Math.* **32** 229–63

National Council for Curriculum Assessment 1999 *Primary School Curriculum: Mathematics* (The Stationary Office) (available at: https://curriculumonline.ie/getmedia/c4a88a62-7818-4bb2-bb18-4c4ad37bc255/PSEC_Introduction-to-Primary-Curriculum_Eng.pdf)

National Council for Curriculum Assessment 2015 *Junior Cycle Science Curriculum Specification* (available at: www.curriculumonline.ie/Junior-cycle/Junior-Cycle-Subjects/Science)

National Council for Curriculum Assessment 2018 *Junior Cycle Mathematics Curriculum Specification* (available at: www.curriculumonline.ie/Junior-cycle/Junior-Cycle-Subjects/Mathematics)

Planinic M, Ivanjek L, Susac A and Milin-Sipus Z 2013 Comparison of university students' understanding of graphs in different contexts *Phys. Rev. ST Phys. Educ. Res.* **9** 020103

Rodriguez J-M G, Harrison A R and Becker N M 2020 Analysing students' construction of graphical models: how does reaction rate change over time? *J. Chem. Educ.* **97** 3948–56

Stefanel A 2019 Graph in physics education: from representation to conceptual understanding *Mathematics in Physics Education* ed G Pospiech, M Michelini and B S Eylon (Springer) pp 195–231

Tairab H H and Khalaf Al-Naqbi A K 2004 How do secondary school science students interpret and construct scientific graphs? *J. Biol. Educ.* **38** 127–32

Teuscher D and Reys R E 2012 Rate of change: AP calculus students' understandings and misconceptions after completing different curricular paths *Sch. Sci. Math.* **112** 359–76

van den Eynde S, van Kampen P, van Dooren W and de Cock M 2019 Translating between graphs and equations: the influence of context, direction of translation, and function type *Phys. Rev. Phys. Educ. Res.* **15** 020113

Wavering M J 1985 The logical reasoning necessary to make line graphs *Paper Presented at the Annual Meeting of the National Association for Research in Science Teaching* (French Lick Springs) (available at: https://files.eric.ed.gov/fulltext/ED254409.pdf)

Wavering M J 1989 Logical reasoning necessary to make line graphs *J. Res. Sci. Teach.* **26** 373–9