ASSESSING THE MATHEMATICAL LITERACY OF A CROSS-SECTION OF IRISH THIRD LEVEL STUDENTS

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ABSTRACT

A mathematically literate workforce is seen as essential for the knowledge economy. Such economies rely on the production of sufficient numbers of graduates in Science and Technology. However, the number of students studying in these fields is decreasing, and Mathematics is often seen as a barrier. Moreover, any participation of citizens in society requires a certain level of mathematical literacy. Here, we aim to measure the mathematical literacy of a cross-section of first year third level students. We consider the correlation between mathematical literacy levels and Irish examination results, and investigate whether the Irish second level education system is achieving its goals. We also consider the attitudes of these students to Mathematics and the impact of these on their performance.

INTRODUCTION

The overall aim of education is to increase students' understanding and enable the transfer of their skills and knowledge across domains. However, numerous recent reports (Expert Group on Future Skills Needs 2003, State Examinations Commission 2005, Task Force on the Physical Sciences 2002) have expressed concern with the mathematical competences of Irish students and their adeptness to apply these in other contexts including science. In particular, many students seem to rely on rote-learning in Mathematics and display little ability to think independently. PISA 2003 measured the mathematical literacy of 15-year-olds across forty countries by assessing each individual's ability to analyse, reason and communicate mathematical ideas. The PISA 2003 study, which encouraged students to appreciate the role Mathematics plays in everyday life through the

formulation of, engagement with and solution of mathematical problems, ranked Ireland as twentieth.

In the study reported here, a PISA style test was administered to first year students (in the engineering, education and humanities departments) in three institutions across the Irish third level sector. The majority of these students would have been in the same cohort as the respondents to PISA 2003 and were 18- or 19-years-old at the time of the study. An analysis of the results aims to determine whether there is any improvement in students' literacy on completion of secondary schooling and whether the level of literacy achieved correlates to Irish state examination results obtained in Mathematics. It may be interesting to conduct a similar study in other countries. As far as we are aware, this has not yet been done. In Ireland, Corcoran (2005) used similar ideas to study the Mathematics subject knowledge of preservice primary teachers.

Furthermore, in the study reported here, information collected from the students on their experiences of and attitudes to Mathematics as they embark on third level education is reported.

THE IRISH EDUCATIONAL SYSTEM

The Irish Educational System has been credited as one of the main factors contributing to the recent success of the Irish Economy. One of the good attributes of the system is that 85% of students complete second level education (Department of Education and Science 2005). Irish students spend 5 or 6 years at second level. After three years they take an examination called the Junior Certificate (JC) and at the end of their time in secondary school they take an examination called the Leaving Certificate (LC). These are state examinations. The results of the LC examination determine entry into third level education. Because of third level entry requirements most students study English, Irish and Mathematics and four other subjects. In fact 96% of students who take the LC study Mathematics. There are three different levels of Mathematics at LC – they are Foundation Level (FL), Ordinary Level (OL) and Higher Level (HL). 11% of students take FL, 71% take OL and 18% take HL. Students who study Mathematics at FL are not eligible for entry to third level so we will not consider these students here. The OL syllabus covers Algebra, Geometry, Discrete Mathematics and Statistics, and elementary Differential Calculus. In addition, the HL syllabus covers further Differential Calculus, Integral Calculus and has optional modules on more advanced topics such as Groups (Department of Education and Science 2000). There are no coursework components in LC Mathematics so the final result depends totally on the students' performance on the June exam. It has been said (NCCA 2006) that the predictable nature of the LC Mathematics exams leads to teaching that emphasises procedure over conceptual understanding. One of the aims of this project was to examine this phenomenon.

In Ireland there are two main types of third level education: the University Sector (including Colleges of Education for future teachers) and the Institute of

Technology Sector. About 64% of Irish school-leavers go on to study at third level with 59% of these at University and 41% at Institutes of Technology (Department of Education and Science 2005).

IRELAND'S PERFORMANCE ON PISA 2003

It has recently been recognised that the future economic success and social well-being of countries is closely linked to the knowledge and skills of their populations. The OECD's Programme for International Student Assessment (PISA) keeps this in mind by designing assessment instruments to measure how well prepared students are to meet the challenges of the future, rather than how well they have mastered particular curricula. To this end, PISA 2003 placed an emphasis on the understanding of concepts, the mastery of processes and the students' abilities to function in a variety of real-life situations.

PISA scores are scaled to give a mean of 500 and a standard deviation of 100. In the Mathematics section of PISA 2003, Ireland achieved a mean score of 502.8 (with a standard deviation of 85.3): this was not significantly different from the overall mean. Male students achieved a higher mean score than female students and students taking Mathematics at HL achieved a mean score that was 94 points higher than the mean for OL students The correlation between Junior Certificate scores in 2003 and PISA 2003 scores was 0.75 (Cosgrove 2005).

TEST INSTRUMENT

In order to assess the mathematical literacy of first year students in Irish third level institutions and to identify their strengths and weaknesses, a PISA style test was administered in this study to students in three institutions across the third level sector. The participants are students of the engineering department at the Institute of Technology, Tralee (ITT), the humanities faculty in the National University of Ireland, Maynooth (NUIM), and the humanities or education faculties of St Patrick's College, Drumcondra (SPD).

The test administered used test items released from PISA 2000 and PISA 2003 in order to better facilitate a comparison of the results obtained with the results of these studies. It was comparable to a single cluster used in PISA 2003, containing 13 mathematical items to be completed in 30 minutes. The particular items chosen were spread across all six proficiency levels identified by PISA, in such a way as to keep the average difficulty of the test (551.69) in line with those of the PISA clusters (ranging from 536.75 to 568.03) (OECD 2004a). Moreover, the items were spread across the four content subdomains (Quantity (Q), Space & Shape (S&S), Change & Relationships (C&R) and Uncertainty (U)), and the three broad clusters of competencies (Reproduction, Connections and Reflection) identified by PISA. Full details are given in the table in the appendix.

In the PISA system, each question is assigned a difficulty level. Using Item Response theory and these difficulty levels the raw scores (0-13) were converted to a score on the PISA scale (OECD 2004b). In what follows, we will refer to this score as the PISA score.

QUESTIONNAIRE

After they had completed the PISA test, the students were asked to fill in a questionnaire. The purpose of the survey was to gather information about the students. The questions could be grouped into five categories: personal, second level experience & examination results, third level experience, study habits, attitude to Mathematics. In the latter category, we asked various questions including how confident the students felt in approaching the subject and whether mathematical ability can be increased. The survey was anonymous so that students would be more likely to give frank answers, and was administered at the beginning of the second semester when the students had some experience of third level.

PROFILE OF STUDENT GROUPS

While all of the participants in this study were enrolled as first-year students of third level institutes, the three different groups of students involved displayed quite different characteristics.

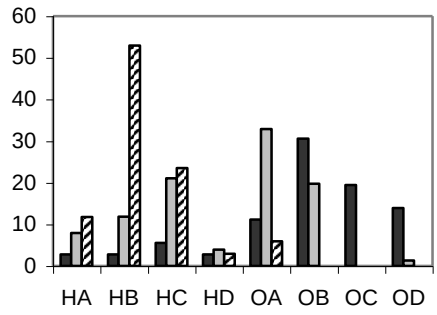
The ITT sample consisted of 36 students who study Mathematics as a compulsory subject in their Civil Engineering course. Only 4 of the students are female. Of the 36 students, 25% completed their Leaving Certificate prior to 2004, and 22% of them were over 25 years of age. In contrast, only 3 of the 34 pre-service primary teachers and humanities students in SPD are male. They have chosen to study Mathematics as one of their academic subjects, and only one had completed LC prior to 2004. The 76 participating students from NUIM have chosen Mathematical Studies as a subject for the first year of their B.A. degree. This group consists of 19 males and 57 females. 11 of the group (14%) completed the LC prior to 2004 and 9 of these (12%) were over 25. Moreover, the LC profiles of the groups, shown overleaf in **Table 1**, are very different. The vast majority of the students in ITT completed Ordinary Level Mathematics or did not sit the LC exams. On the other hand, only 2 of the 34 SPD students sat OL Mathematics. The majority of NUIM students obtained a B in OL or above.

RESULTS

1. Overall Results

In the group as a whole the mean score on the PISA test was 608.93 with a standard deviation of 86.66. Recall that the Irish mean in PISA 2003 was 502.8 with a standard deviation of 85.3. So with the benefit of 3 years extra schooling the

Table 1: Leaving Certificate Profiles of Groups



Leaving Certificate Grades (in percentages)

	ITT	NUIM	SPD
HA	1	6	4
HB	1	9	18
HC	2	16	8
HD	1	3	1
OA	4	25	2
OB	11	15	
OC	7		
OD	5	1	
Miss.	4	1	1
Total	36	76	34

Frequencies

group has increased its mean by about one standard deviation. This is in line with the TIMSS study, where the mean score for 13-year-olds was about one standard deviation higher than the mean for 9-year-olds (Beaton et al 1996). However, it must be remembered that our sample is not representative of all 18- and 19-year-olds in Ireland. Our students have chosen to study Mathematics or Engineering at third level and half have studied HL Mathematics at school, and so one would expect them to be more mathematically literate than an average school-leaver.

2. Literacy Levels

In PISA 2003, the Mathematics scale was divided into six mathematical literacy levels to represent degrees of proficiency. Level 1 students succeed only on the most basic tasks whereas level 6 students are able to handle complex problems and have advanced reasoning skills. The table below shows the percentages of students in this study at each literacy level.

Table 2: Literacy Level

Literacy Level	1	2	3	4	5	6
Frequency	2	8	19	54	25	38
Percentage	1.4	5.5	13	37	17.1	26

In PISA 2003, 15-year-old students with a literacy level of 2 or below were said to be unsuitable for further education (OECD 2004a). Thus, serious questions would have to be asked of the 7% of this sample (8% of ITT, 6.5% of NUIM and 6% of SPD students) remaining at level 2 or below.

Only 26% of this sample has displayed evidence of the advanced mathematical thinking and ability to conceptualise or generalise information characterised by a level 6 proficiency and the students performing at this level are distributed across institutions, regardless of the nature of the Mathematics courses

they now undertake. Of more concern, perhaps, is the large number of students (almost 20% of the entire sample) performing at level 3 or below.

These results highlight gaps in the skills sets of the students examined, students who could be expected to be more mathematically literate than the average student at the end of secondary schooling, and spark concerns for the effective functioning of a society more and more dependent on the knowledge economy. Remembering that many of these students aim to become teachers only serves to exacerbate these concerns.

3. PISA Scores and the Leaving Certificate

We were interested to see whether the PISA test and the Irish Leaving Certificate measured similar skills. In PISA, students with good problem solving skills who can apply Mathematics to real world problems score highly. These are the kind of students who are needed in order to ensure the continued growth of the Irish economy. However, it has been said that the Irish mathematical education system rewards rote-learning at the expense of understanding (NCCA 2006).

LC HL and OL grades are awarded points on a 0-100 scale with HA1 receiving 100 points and OA1 receiving 60 points (CAO 2007). For our group, the correlation between LC points in Mathematics and PISA scores is 0.418. However, we can see from **Figure 1** overleaf that the LC score is not a good predictor of the PISA score. This may provide evidence that the Irish state examinations examine different skills from those considered in the PISA tests.

In our sample, exactly half the students took Higher Level Mathematics at LC and half took Ordinary Level. The mean PISA score for HL students was 637 and for OL students it was 585. It is to be expected that HL students would do better on average but it should be noted that the difference in the means is much less than the difference observed in PISA 2003 (for JC levels).

There were no significant differences between the mean scores of the ITT, NUIM or SPD groups. One might have expected, for instance, that the SPD group would have done significantly better than the ITT group since most of the SPD group had studied Mathematics at HL and most of the ITT group had studied at OL. Indeed the LC points mean of the ITT group is 39.4 (standard deviation 19.6) whereas the mean for the SPD group is 77.9 (standard deviation 11.4). The other major difference between these groups is that the ITT group is mostly male and the SPD group is mostly female. In the next section we investigate the effect gender has on PISA scores.

4. PISA Scores and Gender

There were 146 students surveyed in our sample and of these 92 were female and 54 were male. In our group as a whole, gender and PISA score are independent (Fisher Exact Test (FET), $p=0.247$) and a 95% confidence interval for the difference in mean scores of males and females is given by (-0.09504, 1.28587).

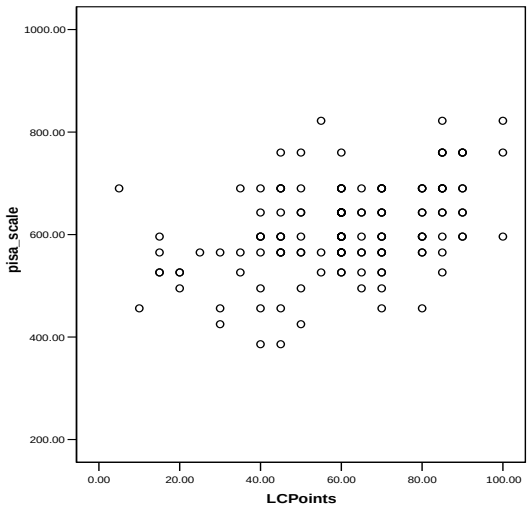


Figure 1: Scatterplot of PISA scores versus Leaving Certificate points.

This is in contrast to the Irish results in PISA 2003 (Cosgrove et al 2005). However, as we have seen previously, most of the female students had studied Mathematics at HL and most of the males had studied at OL. There is a significant difference between the LC points of the females and the males. The mean LC points for females is 65.3 and for males it is 51.5. Therefore studying gender differences in the group as a whole is problematical, so we grouped students with the same LC grade together.

We found that if we also grouped students into 'average' (600 or less on PISA test) and 'above average' (more than 600), then performance on PISA and gender are not independent when LC grade is kept fixed (Mantel-Haenszel test, p=0.003). A 95% confidence interval for the common odds ratio is (0.059, 0.541). So given a male and a female student with the same LC grade, the male is about twice as likely as the female to have scored more than 600 on the PISA test. PISA scores and LC points (in Mathematics) are not independent for females (FET, p=0.000) with higher LC points students attaining higher scores on PISA. However, this is not the case for males (FET, p=0.329) with some males with low points scoring well on PISA. The results seem to suggest that males do better on PISA than on LC Mathematics. This again may be an indication that PISA and LC measure different skills.

5. PISA Scores and Confidence

The students were asked to rate their level of confidence in approaching Mathematics on a five point scale with 5 being very confident. We were interested to see if confidence and high scores on PISA or on LC were related.

If we again group the students using their LC grade we find that PISA scores and confidence levels are independent. If we consider HL students as a

group then PISA score and confidence level are not independent (p=0.013, FET) with students with higher confidence levels doing better on the PISA test. For OL students PISA scores and confidence levels are independent (p=0.591, FET).

Not surprisingly, LC level and confidence level are not independent (p=0.000, Chi squared test), with HL students being more confident than OL students. 61.4% of HL students said they were confident or very confident but only 27.1% of OL students did. It might be interesting to investigate whether confidence (or lack of it) influences a student's decision to study at HL.

Grouping students into 'confident', i.e. 4 or 5 on the scale, or 'non-confident', i.e. 1, 2, or 3 on the confidence scale gives that 43% of students are confident and 57% are not. We found that confidence and gender are not independent when LC grade is kept fixed (p=0.002). In general, males are more likely to be confident than females. For example, at OA1 grade, 85.7% of males rated themselves as confident but only 25% of females did.

6. Innate Mathematical Ability & Motivational Factors

Students were asked if they believed it was possible to improve their natural mathematical ability: 78.8% said yes, 11.6% said no and the remainder were unsure or didn't comment. The responses of male and female, and confident and non-confident students followed a similar pattern. Dweck (1986) maintains that a student's theory of intelligence (as to whether intelligence is fixed or malleable) and confidence in his/her present ability combine to influence the student's behaviour when presented with an unfamiliar task. This could go some way to explain the relatively poor PISA scores of female students with high Leaving Certificate grades who nonetheless may display little confidence in their own abilities to tackle new problems and concepts. This is a question that requires further investigation.

The questionnaire distributed also investigated how interesting, enjoyable and difficult students had found and were finding second and third level Mathematics respectively. The PISA scores and literacy levels attained by the students were found to be independent of the levels of enjoyment, interest and difficulty experienced by students at second level. Their PISA scores and literacy levels were also independent of the enjoyment and interest in third level Mathematics reported by students but were not independent of the difficulty of Mathematics at this level (p=0.00 for PISA scores and p=0.09 for literacy level on a FET). It may be that the students who find third level difficult perform less well on PISA-type questions because they lack independent thinking skills.

7. Procedural Skills versus Conceptual Understanding

In the questionnaire, the students were asked to comment on the statement that an emphasis is placed on procedural skills rather than conceptual understanding of Mathematics in Irish schools. 62% of the respondents agreed, 8% disagreed and the remainder were unsure or didn't comment.

Some said that there was not enough time to ensure understanding (4 students), or that understanding was not necessary in second level (3). Six students said that the lack of emphasis on conceptual understanding made the transition to third level difficult. Moreover, when asked what was the difference between second and third level Mathematics, 54.6% of students said more depth was required at third level. Not only that, but when asked what they liked about third level Mathematics, the most popular answer among students was the depth and understanding required (27.7%). Such student feeling is in agreement with the findings of Boaler and Greeno (2000). In their interviews with high school students, they found that traditional didactic Mathematics classrooms alienate many learners. The students involved in their study who were planning not to continue with Mathematics were those who wanted the opportunity to think for themselves and understand the procedures they encountered.

The Chief Examiners Report on the performance of students in LC Mathematics states that their weaknesses "relate to inadequate understanding of mathematical concepts and a consequent inability to apply familiar techniques in anything but the most familiar of contexts and presentations" (State Examinations Commission 2005). The National Council for Curriculum Assessment (NCCA 2006) has noted that the Irish examination system has contributed to increased emphasis on recall and the application of routine procedures. The results of this study (both the questionnaire responses and students' test scores) echo and reinforce these findings.

CONCLUSIONS

The results of this study show the level of mathematical literacy attained by students has improved with further schooling but not improved as much as would be expected or desired. There was also evidence that the Leaving Certificate examination is measuring different skills than that of PISA and questions must be raised about how well the current delivery of the Leaving Certificate curriculum is achieving its aim "to help to provide them [the students] with the mathematical knowledge, skills and understanding needed for life and work" (Department of Education and Science 2000). Moreover, the performance of the students on the PISA style test used in this study was even more disappointing given that more than half of the students involved had experienced the recently revised (2000) Junior Certificate curriculum. It places renewed emphasis on 'Mathematics for life' and efforts were made in its implementation to facilitate the improved mathematical understanding of students and to move away from previous rote learning habits. Not only that but this study found evidence to suggest that secondary school students themselves would support such a move.

In conclusion, the results do not bode well for a society that relies on the mathematical literacy of its members, and in particular, its workforce.

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APPENDIX

Description of the PISA items used

Item ID	Item Name	Subdomain	Competency	Level	Difficulty
M124Q01	Walking	C&R	Reproduction	5	611
M124Q03T	Walking	C&R	Connections	6	723
M438Q01	Exports	U	Reproduction	2	427
M438Q02	Exports	U	Connections	4	565
M509Q01	Earthquake	U	Reflection	4	557
M555Q02T	Number Cubes	S&S	Connections	3	503
M136Q03	Apples	C&R	Reflection	6	723
M037Q02	Farms	S&S	Connections	3	524
M413Q01	Exchange Rate	Q	Reproduction	1	406
M413Q02	Exchange Rate	Q	Reproduction	2	439
M413Q03T	Exchange Rate	Q	Reflection	4	586
M266Q01T	Carpenter	S&S	Connections	6	687
M547Q01T	Staircase	S&S	Reproduction	2	421