"What you see and don't see, shapes what you do and don't do": noticing in first year mathematics lectures

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Abstract: We report on an analysis of 'decision points' that occurred during first year undergraduate calculus lectures. We analysed 135 accounts written by three lecturers concerning their own teaching; these accounts were written during a professional development project on employing the Discipline of Noticing (Mason, 2002). We classified the decision points in these accounts into eight categories. Furthermore, the triggers of these moments were identified and classified into seven categories; of these, the majority (58.2%) arose as a result of the lecturer monitoring either her own practice or the students' engagement.

Keywords: Teacher noticing, decision points, mathematics lecture, university mathematics.

Introduction

> Today I was happy with my 'performance' from a teacher-centred perspective as the lecture evolved: I felt I was coherent, explained and connected ideas well, used multiple representations of concepts and built on students' prior knowledge, However, I realised more than halfway through the class that my lecture was just that – very teacher-centred! I tried to rectify this but was not happy that my attempts were successful. (Lecturer C)

Lecturer C speaks about a tension inherent in large-group mathematics lectures between teacher-centered and student-centered methodologies, and this is probably familiar to anyone who has taught such a course. It also highlights the dilemma that faces a lecturer when it occurs to her mid-way through a lecture that she might deviate from her original plan. The change of plan can feel like a risky strategy, especially when working with very large groups of students; however research on mathematics teaching at school level indicates that rich learning opportunities can arise from decisions to change direction (Rowland and Zaskis, 2013). It is important then, that lecturers are aware of opportunities that present themselves, for the simple reason that the quote in the title, borrowed from Schoenfeld (2011, p. 228), suggests. It is also important that lecturers develop the skills to take advantage of these opportunities as they arise. In this paper, we explore the opportunities to make decisions that might arise in a mathematics lecture; we have named instances when a non-trivial choice between alternative courses of action could be taken 'decision points'.

The Discipline of Noticing developed by Mason (2002) consists of "a collection of practices which together can enhance sensitivity to notice opportunities to act freshly in the future" (p. 59). Mason notes that practitioners, of necessity, form habits in order to deal with issues that arise in their everyday practice. The practices described by Mason provide educators with techniques to assist them in developing the dual abilities to notice key events in the classroom, and secondly to have possible actions come to mind in-the-moment in order to respond to these in non-habitual ways.

Mason (2002, pp. 33-34) distinguishes between levels of noticing, from ordinary-noticing, through to marking, and finally recording. "Ordinary-noticing" is where a person's memory of something can be jogged if another person remarks upon it; "marking" is where someone has taken sufficient notice of something to "re-mark" upon it at a later stage; and, finally, "recording" is where one records or makes a note of something one has noticed, usually in writing. To do the latter, Mason (2002, p. 46) advocates the practice of writing "brief-but-vivid accounts". These are brief notes which give an "account-of" an incident, rather than an "account for" as Mason explains:

> To *account-for* something is to offer interpretation, explanation, value-judgement, justification, or criticism. To give an *account-of* is to describe or define something in terms that others who were present (or who might have been present) can recognize. (p. 41)

The data for this paper comes from a set of brief-but-vivid accounts written by the three authors on incidents that occurred during teaching over a two-year period when they took part in a project aimed at using the Discipline of Noticing to study their own teaching (see Breen, McCluskey, Meehan, O'Donovan & O'Shea (2014) for more details).

In this paper we will present an analysis of the accounts which relate to large-group teaching of first year modules; in particular we will endeavour to answer the questions: What types of decision points can occur during a mathematics lecture? What triggers these decision points?

Literature Review

Teacher noticing has been receiving attention in the research literature recently, see for example the research studies included in the book edited by Sherin, Jacobs & Philipp (2011). In presenting an overview of these studies, the editors observe that while the conceptualisation of teacher noticing may vary, it is generally considered as consisting of one or both of two main processes. The first process, "attending to particular events in an instructional setting" (p. 5), relates to where the teacher does (and does not) place her attention in the classroom setting. The second process they describe as "making sense of events in an instructional setting" (p. 5) and note that some researchers conceptualise this process only as "interpreting" (p. 9) what is noticed, while others view it as "both interpreting and deciding how to respond" (p. 9).

Instances when a choice presents itself during teaching are labeled "contingent moments" by Rowland, Huckstep, and Thwaites (2005) in their work on the Knowledge Quartet. At these moments teachers have to think on their feet and possibly deviate from the planned lesson. Rowland, Thwaites and Jared (2015) identified three types of triggers of contingent moments in their study of mathematics teaching: responding to student ideas; teacher insight; and, responding to the availability of tools and resources. Teachers may deal with the first trigger in one of three ways: ignore; acknowledge but put aside; acknowledge and incorporate (Rowland and Zaskis, 2013).

Schoenfeld (2010) has developed a theory to explain what influences the decisions an individual makes when carrying out a particular task. He has applied his theory in particular to decision-making in mathematics classrooms. He proposes that decisions a teacher makes while teaching are a function of her resources, orientations and goals. Although "resources" is a broad term, Schoenfeld classifies the knowledge that a teacher possesses as being a key component of the resources she

brings to the classroom. He uses "orientations" to encompasses one's "*dispositions, beliefs, values, tastes,* and *preferences"* (p. 29). And while the term "goal" is self-evident, Schoenfeld notes that an individual may pursue a particular goal "simply in the service of other goals" (p. 20).

Most of the research studies discussed above in relation to noticing, contingent moments, or decision-making in the classroom have been conducted at the school level. An exception is the work of Barton, Oates, Paterson, and Thomas (2015) and colleagues in New Zealand who use Schoenfeld's (2010) theory to discuss taped video excerpts from participating mathematicians' and mathematics educators' lectures as a means of engaging in professional development on teaching practice. However in terms of noticing and the occurrence of decision points in lectures, there is little research. Indeed to many it might seem like traditional lectures provide few opportunities for contingency. However McAlpine, Weston, Beauchamp, Wiseman and Beauchamp (1999) report on a study of monitoring of student cues by university lecturers. The lecturers in this study were found to attend to four types of cues: student written, student verbal, student non-verbal, student state. McAlpine et al. (1999, p. 117) posit that the lecturers had a *corridor of tolerance* for these cues and a decision to change practice was only taken when the cue lay outside of this corridor. In contrast to the work of both Schoenfeld (2010) and McAlpine et al. (1999), we focus here on the opportunities for decision-making that arise in lectures rather than the process of decision-making itself.

Methodology

The authors are lecturers of mathematics at three different universities in Ireland. Each has a doctorate in mathematics or applied mathematics and has a minimum of fifteen years' experience of teaching mathematics at the tertiary level. Between them they have taught mathematics classes from first year undergraduate through to postgraduate level, and have experience of teaching students in class sizes ranging from single figures up to a few hundred students. In 2010/11, along with two other colleagues, they embarked on a project aimed at reflecting on their teaching using the ideas and philosophy described by Mason (2002). As part of this process, over the course of two years, they engaged in writing brief-but-vivid accounts of incidents or moments that occurred in relation to their teaching. See Breen et al. (2014) for further details.

The accounts of all five members of the group were collected and a general inductive approach (Thomas, 2006) was used to identify themes in a sample of the accounts. We noticed that many accounts described instances where the lecturer was faced with a decision about what to do next, or, instances where an opportunity to make a decision that might change the course of the lecture or discussion was implicit. We labeled these moment *decision points* (DPs). In order to focus specifically on these moments, all accounts that did not specifically deal with lecturing were removed. The first and second authors (AOS and MM) independently analysed all the accounts to both identify and code the DPs. After some discussion they agreed on the identification of DPs and the codes assigned. In addition, AOS identified the triggers. Then all five members worked through all the accounts to confirm their agreement or express disagreement with the DPs and triggers identified and the codes assigned. By the end of this process, the group had reached a consensus. AOS then grouped the codes into categories, and the third author (SB) examined these for

consistency. In some of the accounts, the *action taken* by the lecturer as a result of a DP was recorded, however we will not discuss the identification or classification of these here.

In this paper we present findings on the DPs and triggers from accounts written by each of the three authors while lecturing a first year mathematics class. In total there were 135 accounts with 141 DPs identified, as some accounts contained more than one DP. These DPS were classified into 8 categories. In order to provide a context for the accounts, we note that Lecturer A taught Calculus to a group of approximately 200 students consisting of both mathematics and finance students in the first semester of both 2010/11 and 2011/12. Lecturer B taught mathematics to a group of over 200 first-year business students in the first semester of both 2011/12 and 2012/13, while Lecturer C taught Calculus to a first year class of approximately 50 mathematics students for the duration of the academic years 2010/11 and 2011/12. The format for each course consisted of either 2-3 lectures per week given by the lecturer, to which all students were required to attend.

While all three lecturers engaged in what might be considered *lecturing* - that is, the lecturer speaks to the whole class, and perhaps writes on a projector or board while the class is expected to remain silent - they also engaged in initiatives aimed at increasing student participation. These fall into two categories – *whole class question or discussion* and *class activity*. The former relates to where the lecturer asks the whole class a question or attempts to conduct a whole-class discussion, while by class activity we mean an activity that students are expected to engage in during class, usually in small groups. As a final part of the analysis MM classified each account containing a DP as occurring in either Setting 1 (S1) – lecturing; Setting 2 (S2) – whole-class question or discussion; and, Setting 3 (S3) – class activity. We now present the findings.

		Lecturer A			**Lecturer B**			**Lecturer C**			**Total**
	Decision Points	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	
DP1	How to engage students?	5	0	3	1	0	0	4	2	0	15
DP2	How to respond to students' questions, answers, or comments?	8	0	1	0	4	0	3	0	3	19
DP3	How to ask questions to gather information?	4	0	2	1	0	0	1	1	1	10
DP4	How to deal with disruption?	1	0	0	18	1	1	2	1	0	24
DP5	How to conduct class activity or discussion?	0	0	3	0	2	0	2	2	7	16
DP6	How to deal with students' mathematical difficulties?	4	0	2	0	17	0	1	7	5	36
DP7	What to do next in the lecture?	5	0	0	1	2	0	4	3	2	17
DP8	Other	1	0	0	2	1	0	0	0	0	4
	Total	28	0	11	23	27	1	17	16	18	141

Table 1: Decision Points by Lecturer and Setting

Findings

In Table 1 we present the categories of DP with frequency by lecturer and setting. Lecturer A mainly wrote accounts about S1 and S3, Lecturer B about S1 and S2, while Lecturer C wrote about all three. This is perhaps not surprising as she had a much smaller class. Over a quarter of all DPs identified belongs to the category "How to deal with students' mathematical difficulties?" (DP6, n=36, 25.5%). It is noteworthy that 31 of these occurred in Settings 2 and 3. The next largest category of DPs is "How to deal with disruption?" (DP4, n=24, 17.0%) with three-quarters of these attributed to Lecturer B in the lecture setting. The third largest category is "How to respond to students' questions, answers or comments?" (DP2, n=19, 13.4%) and while these DPs may be expected to occur in Settings 2 and 3, it is interesting to note that just under half are attributable to Lecturer A in the lecture setting. The category "What to do next in the lecture?" (DP7, n=17, 12.0%) contains DPs relating to opportunities for decisions that present themselves when moving from a whole-class question/discussion or class activity, back to the lecture setting. Four of the DPs did not seem to fit in any of the categories identified and were grouped as "other".

Each DP was found to have an associated trigger and in Table 2 we present the categories of triggers with frequency by lecturer and setting.

		Lecturer A			**Lecturer B**			**Lecturer C**			**Total**
	Triggers	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	
T1	Lecturer monitors aims/goals	3	0	1	1	0	0	2	1	3	11
T2	Lecturer monitors practice	6	0	1	2	1	0	3	1	0	14
T3	Lecturer monitors student nonverbal	9	0	0	0	5	1	8	4	0	27
T4	Lecturer monitors absence of student verbal	0	0	6	0	0	0	0	0	4	10
T5	Lecturer monitors disruptive behavior	1	0	0	18	0	0	0	1	0	20
T6	Student question or comment or answer	8	0	3	2	19	0	6	9	9	56
T7	Other	1	0	0	2	0	0	0	0	0	3
	Total	28	0	11	25	25	1	19	16	16	141

Table 2: Triggers by Lecturer and Setting

The first five triggers listed in Table 2 (T1-T5) are as a consequence of the lecturer monitoring her aims/goals for the class, her practice, what the students were (not) doing, which students were not answering questions or contributing, and student behaviour. These account for 58.2% of triggers identified. In terms of the triggers identified when the lecturer monitored students (T3-T5) there are some similarities between the cues identified in McAlpine et al. (1999) and those described in the accounts relating to these triggers. The category labeled "Student question or comment or answer" (TP6, n=56, 39.7%) contains student-initiated triggers and is the largest of the trigger categories. The "Other" category relates to triggers that are neither lecturer- nor student-initiated and relate to

issues such as a cold room or poor attendance due to bad weather. These findings are similar to those of Rowland et al. (2015) who in their study classified triggers of contingent moments as emanating from the teacher, the students, or resources and tools, with the latter category accounting for far fewer triggers than the first two. Exploring links between DPs and triggers, not surprisingly almost all DPs categorized as DP2 (18/19) and most of those as DP6 (29/36) arose from T6 triggers. Over half of DP3 and most of DP4 (20/24) resulted from T1 and T5 triggers respectively.

We now present some examples of accounts featuring DPs and associated triggers that occurred in the lecture setting. We note that the account by Lecturer C at the start of the paper is an example of a lecturer attempting to make her lecture more student-centred in order to engage students (DP1) as a consequence of monitoring her practice (T2). In the following account by Lecturer A, the trigger for the decision point about how to engage students (DP1) is the lecturer monitoring what the students are (not) doing (T3) while she is lecturing.

> I continued the introduction to limits today. I was doing a lot of talking and I realized that people weren't taking anything down. I tried writing more explanations on the pictures I was drawing, so hopefully it will make more sense when they look at it again. (Lecturer A)

Similarly the account by Lecturer C describes how noticing students' expressions (T3) during a lecture, prompts her to explain what mathematicians do in an attempt to engage the students (DP1):

> On noticing students exchanging glances when I asked them why they would attempt to solve a particular problem in a particular way, I was prompted to reiterate what "doing mathematics" (at this level) entails. (Lecturer C)

The next account illustrates how a student asking a question (T6) during a lecture results in a decision point for the lecturer about whether to review material already covered (DP2).

> I found the limit of $\sin(2x)/\sin(3x)$ as $x \to 0$ when a mature student, who asks lots of questions, asked why you couldn't just cancel the sin's and the x's to get 2/3. We proceeded to discuss the meaning of the term $\sin(2x)$ and the difference between multiplication and composition. I had spoken about this before but felt talking about it here was useful. (Lecturer A)

When a lecturer monitors student behaviour (T5) during a lecture, decision points about how to deal with disruptions (DP4) may arise as illustrated in the following account by Lecturer B:

> Shortly into the lecture I ask a group of four students to stop talking. Minutes later I tell two other students to stop talking. Some minutes later, I ask the first group to stop talking again. I look at the class – most of them are staring straight at me and not moving. I realize I am nagging and stopping the lecture for the sake of a few "talkers". As I continue to write and talk, I hear whispering coming from various parts of the theatre and my explanation falters. I decide I can't get any more annoyed. I put up a question and ask the class to work on it. (Lecturer B, T5, DP4)

Unlike the accounts so far which occurred in the lecture setting, the following account takes place during a class activity. During the activity the lecturer experiences a decision point concerning how best to respond to student questions (T6 and DP2). But on completion of the activity another decision point arises about what to do next in the lecture (DP7).

I put up an exercise for the students to work on. I remind them that the first part is revision – they have to find the profit function. As I walk around the theatre a student asks: "What are the fixed costs?" I remind him that this example is different to the one I did earlier in class. Another asks: "Do I multiply this function by q to get total revenue?" "No, you are given total revenue, you don't have to find it", I reply. "What is q?" another asks. I feel deflated – this is revision. I planned to finish the exercise today, but instead, show them how to get the profit function and complete the exercise in the next lecture. (Lecturer B)

Discussion and Conclusions

We wish to discuss three points in this section. The first concerns decision points that arise while lecturing. One might assume that in the traditional sense of a lecture, a lecturer delivers from a pre-prepared script and unless a student asks a question, the lecturer will not deviate from the script. However there is some evidence from the three lecturers' accounts on their practice that indicates that even while lecturing in the traditional sense, they monitor their aims for the class, their own practice, and how the student cohort are acting, and that this monitoring leads to the occurrence of many of the decision points. About one third of the triggers for decision points in our study were student initiated, (T6 in Table 2) in contrast to the findings of McAlpine et al. (1999), but in comparing one must be cognizant that our methodology differs from theirs. While we acknowledge that the fact that the three lecturers undertook a professional development project on using the Discipline of Noticing (Mason, 2002) to improve their practice may mean that they are not typical, it would be interesting to explore further what mathematics lecturers focus on while in a lecture as well as gathering more data to illuminate the relationship between triggers and decision points.

Secondly we observe from our findings that decision points relating to how to address students' mathematical difficulties usually emerged in the context of a whole class question or discussion or a class activity. This is perhaps not surprising but does highlight the importance of including such activities during a lecture in order to assess students' mathematical understanding. However it is also worth pointing out that over half of all the decision points identified in this study were as a consequence of such an activity, which may suggest an increased cognitive load for the lecturer who engages in such activities.

Our final point relates to the methodology used in this study. Our analysis of the settings seems to indicate the lecturers' individual preferences for pedagogical techniques and activities however we cannot use our data to draw conclusions about the proportion of class-time they each spent in the three settings, or make general claims about the relationship between the settings and the occurrence of decision points. This is because the lecturers in this study had complete autonomy over what incidents they chose to write about. They wrote about what mattered to them and many of the accounts relate to incidents where the lecturer felt unsure about what to do, or uncomfortable about a decision she had made. In any lecture, there may have been a multitude of more interesting moments worthy of noting, but either they did not notice them, or chose not to write about them. In this way there is a parallel with the findings of Barton et al. (2015). In choosing excerpts from a video-taped lecture for group discussion, the authors note that: "Counterintuitively, lecturers chose

parts in which they felt less comfortable" and in group discussions "frequently chose to focus the group's attention on interludes in the lecture when unexpected decisions were made" (p. 152).

We return to the quote from Schoenfeld (2011): "Noticing is consequential – what you see and don't see shapes what you do and don't do" (p. 228). We suggest that the use of the Discipline of Noticing (Mason 2002) can help lecturers to identify opportunities for making (possibly different) decisions in their lectures. Individually the process also highlighted for each of us different aspects of our practice that we wanted to work on – for Lecturer A it was how to make her lectures more student-centered, for Lecturer B the issue was how to deal with disruptive behavior, for Lecturer C, how to address mathematical difficulties effectively and sensitively. We also recommend that lecturers, as a professional development exercise, write a selection of brief-but-vivid accounts and discuss them in a group setting using Schoenfeld's theory (2011) to frame the discussion.

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