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Dynamic inventory sharing, ordering, and pricing strategies for perishable foods to maximize profit and minimize waste

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ABSTRACT

Effective management of perishable food products is essential for grocery retailers to balance profitability and waste reduction. This study addresses the challenge of selling perishable food products with varying ages across two branches, incorporating consumer behavior and demand shifts between old and new products. A bi-objective infinite horizon dynamic programming model is developed to optimize centralized pricing, ordering, and inventory sharing decisions, aiming to maximize profit and minimize waste. Numerical analysis demonstrates that inventory sharing effectively balances stock levels and reduces food waste. Findings indicate that prioritizing waste reduction leads to higher price discounts and increased inventory sharing, while prioritizing profit maximization results in selling newer products and reducing inventory sharing. Sensitivity analyses highlight the importance of market segmentation and price differentiation strategies. These insights provide valuable guidance for retailers in refining inventory and pricing decisions, adapting to regulatory pressures, and improving overall supply chain performance.

1. Introduction

Perishable food products like fruits, vegetables, meat, dairy, and bakery items deteriorate over time, leading to food waste and disposal costs. Perishable food products constitute up to 40 % of a grocery chain's revenue and play a significant role in attracting consumers and enhancing loyalty (Buck & Minvielle, 2014). Approximately 930 million tons of food is wasted annually, with 13 % from grocery retailers (UN, 2021). In the retail industry, food waste represents a significant cost due to low margins and high operating expenses (Teller et al., 2018), accounting for 0.1 % to 2.5 % of a retailer's monthly turnover (Horoś & Ruppenthal, 2021). Excess inventory can cause food waste and costs, bringing environmental, social, and economic implications. Waste minimization can be achieved by ordering fewer products, but it causes lost sales; therefore, stock-out becomes another key issue for retailers. It not only causes a revenue loss for a single item, since consumers generally shop for the whole basket. Therefore, if consumers encounter stock-out several times they switch to a competitor and the retailer loses consumers permanently (Saghiri et al., 2023).

Therefore, in the perishable food industry retailers may face the following two concerns: profit maximization and waste minimization. Most grocery retailers sell single food products with different ages (expiration dates) simultaneously, in these cases FIFO sales system to reduce waste might be suboptimal (Önal et al., 2015). FIFO rule is common if customers cannot distinguish old from new due to 'best by' label or unavailable expiration date, or when sellers strategically manipulate search costs by positioning older inventory to be depleted first (Li et al., 2009). However, in practice, a product age-sensitive consumer's willingness to buy an old product will decrease as the age of a product and customers choose products on LIFO rule. However, if retailers discount the sale price of the old products, price-sensitive consumers may purchase them. Also, existing studies show that with inventory sharing retailer can balance inventories across different branches and hence can reduce food waste at some branches while reducing shortages at others (Li et al., 2021). Therefore, to overcome these two concerns inventory and pricing decisions must be given properly. For instance, retailers like Tesco, Walmart, and Kroger apply price markdowns for older products to achieve zero food waste under Goal 12 – Responsible Consumption and Production. Inventory sharing improves profitability and reduces system inventory, benefiting both supply and demand sides (Zhao et al., 2021). Transshipment can be one-way or two-way. One-way transshipment, used in omnichannel retailing

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by companies like Orvis and CompUSA, involves moving inventory from physical stores to online orders when online warehouses lack stock (Liang et al., 2014). Two-way transshipment, which allows inventory movement in both directions, provides flexibility in managing stock levels across channels and locations (Liang et al., 2014). This paper focuses on two-way transshipment.

Inventory sharing decisions can be centralized or decentralized. Large firms with multiple branches generally make centralized decisions (Davis et al., 2022). Centralized ordering benefits risk-averse managers and reduces risks associated with demand uncertainty (Yang et al., 2021). This paper focuses on completely centralized decision-making in order to maximize profit and minimize waste of the firm as a whole.

Managing perishable food products in retail is a constant challenge because these items have short shelf lives, demand is unpredictable, and customer preferences vary. Retailers must carefully balance the goals of profit maximization and waste reduction. Most existing studies on managing perishable goods focus on inventory control using simple rules like first-in-first-out (FIFO) or last-in-first-out (LIFO). However, they often overlook the more complex consumer behaviors seen in real-world situations. These behaviors include price sensitivity, stockout-driven substitutions, and varying demand for products of different ages. In addition, growing regulations and social pressure to reduce food waste make it essential to find new strategies that can balance both profitability and sustainability in retail operations.

In this paper, we address a retailer's problem of selling a single type of perishable food product with varying ages across two branches. We explore demand shifts (stockout-based substitutions) between old and new products based on prices and consumer behavior. Unlike most studies, we incorporate these behaviors into a bi-objective dynamic programming model to optimize discounted prices, sale prices, order quantities, and quantity of shared inventory between two branches and its direction. We investigate the impact of inventory sharing on optimal dynamic pricing and inventory strategies and its effectiveness in reducing food waste and maximizing profits. The "curse of dimensionality" makes it difficult to determine an optimal inventory policy for perishable inventory systems, even in single-location cases (Nahmias, 1975; Zhang et al., 2023). Therefore, we aim to observe structural characteristics of inventory and pricing policies. We also aim to quantify the tradeoff between profit and waste for companies, providing insights that can inform managerial decisions.

The remainder of the paper is organized as follows: Section 2 reviews relevant literature on perishable inventory management and dynamic pricing. Section 3 presents the problem formulation and model development. Section 4 introduces the infinite horizon average cost dynamic programming model. Section 5 provides numerical results to analyze optimal decisions and conducts sensitivity analysis to assess the impact of different model parameters. Finally, Section 6 concludes the paper, summarizing key findings and suggesting directions for future research.

2. Literature review

This paper is mostly related to two streams of literature: (1) dynamic inventory control and pricing for perishable products, (2) inventory sharing and transshipment.

Nahmias (1982) reviews ordering policies for perishable products under deterministic and stochastic demand scenarios for products with fixed and random lifetime of single and multiple products. Raafat (1991) expands on this by reviewing continuously deteriorating mathematical inventory models. Subsequently, Goyal and Giri (2001) provide a detailed review of the classification of products with deterioration and necessary management policies. Bakker et al. (2012) present a literature review specifically on perishable product inventory control, following Goyal and Giri's (2001) review. Elmaghraby and Keskinocak (2003) focus on dynamic pricing within inventory considerations. Karaesmen et al. (2011) provide an overview of supply chain management research for perishable products, including inventory management literature.

van Zyl (1964) explores optimal inventory strategies for perishable products with a two-period lifetime and periodic review systems, initially focusing on shortage and ordering costs. They extend their study to longer lifetimes, developing heuristics due to the complexity of finding optimal solutions. Nahmias (1975) addresses optimal ordering policies for perishable products with multiple period lifetimes, considering ordering, holding, shortage, and outdating costs, but the high complexity makes optimal results unattainable. Pourmohammad-Zia et al. (2021) explore dynamic pricing and ordering strategies within a two-level food supply chain, comparing decentralized and centralized scenarios, and developing an analytical solution verified through experiments. Keskin et al. (2022), using data from a leading supermarket, address a retailer's joint pricing and inventory ordering for perishable products over a finite time horizon, considering lost sales. Li et al. (2009) study joint pricing and inventory control decisions, finding optimal solutions for perishable products with a two-period lifetime and developing heuristics for products with m-period lifetimes. Li et al. (2015) focus on maximizing total discounted profit through joint dynamic pricing and inventory control in a stochastic inventory system for perishable products, modeled with continuous-time stochastic differential equations and random disturbances. Vahdani and Sazvar (2022) examine dynamic pricing and inventory control for perishable products, highlighting how social learning from consumer reviews enhances profitability, and waste reduction under Expiration Date-Based Pricing. However, none of these papers consider selling different ages of products simultaneously, sales through multiple branches, or a bi-objective dynamic programming model that optimize both profit maximization and waste minimization.

For perishable products, it is common to have various product ages available simultaneously, requiring more complex issuing rules than FIFO or LIFO. This allows interchangeability of items with different ages or shelf-lives (Karaesmen et al., 2011). Pierskalla and Roach (1972) propose that demand exists for any age and can be met from corresponding or younger stock. Ferguson and Koenigsberg (2007) analyze joint pricing and inventory control in a two-period scenario, considering competition between old and new products. Deniz et al. (2010) focus on inventory management for a perishable product with a two-period lifetime, considering age-based demand differentiation. Chao et al. (2015) develop approximation algorithms for perishable inventory systems over a finite horizon, focusing on the FIFO issuing policy, which may be reasonable for blood products but suboptimal for food retailers. Zhou et al. (2023) develop a two-period dynamic pricing and inventory model for perishable products with partial lost sales, optimizing order quantities and retail prices under uncertain demand. Yavuz and Kaya (2024) develop Deep Reinforcement Learning algorithms to optimize dynamic pricing and inventory management for perishable products with price- and age-dependent stochastic demand, allowing the simultaneous sale of products with different ages for items with a lifetime of more than two periods. Moshtagh et al. (2025) model a dynamic inventory-pricing system for perishable products with multiple freshness levels as a Markov decision process (MDP), optimizing pricing and production decisions to maximize firm profitability. None of these studies develop a bi-objective infinite horizon dynamic programming model that simultaneously minimizes waste and maximizes profit, nor do they examine inventory sharing decisions in a two-location setting.

Different from the above studies, our research also relates to the expanding stream on inventory sharing and transshipment. Paterson et al. (2011) review lateral transshipment (LT) policies, categorizing studies into reactive and proactive transshipments based on timing. Reactive transshipment occurs after demand realization to address stockouts, while proactive transshipment balances inventory among locations periodically. Gross (1963) pioneered proactive two-location transshipment, expanded to multi-location models by Karmarkar and Patel (1977). Krishnan and Rao (1965) initiated reactive transshipment studies, extended to multi-location, multi-period stochastic models by Robinson (1990). Paterson et al. (2012) enhance reactive strategies by integrating a proactive component with a continuous review policy (R, Q), optimizing transshipment quantities and source selection during shortages. Lee et al. (2024) develop a hybrid deep reinforcement learning approach to optimize proactive transshipment decisions and order quantities for perishable products in an online–offline channel system, aiming to maximize average profit. The literature also distinguishes between centralized and decentralized decision-making. Zhou and Wang (2023) study centralized ordering and proactive transshipment, while Dong and Rudi (2004), followed by Zhang (2005), examine transshipment impacts on independent manufacturers and retailers in a supply chain.

In our study, we consider a retailer deciding inventory sharing quantity at the beginning of each period before demand realization in a two-location setting. Our approach integrates dynamic inventory and pricing strategies for perishable products, capturing real-world complexities and providing insights into optimal inventory sharing and pricing decisions. While much literature covers single-location inventory management for perishables, fewer studies address inventory sharing decisions. Zhang et al. (2023) focus on platelet inventory management in a two-location hospital system, using inventory sharing to reduce waste. Unlike their FIFO-based approach, our research emphasizes optimal pricing, ordering, and transshipment strategies for perishable foods.

Li et al. (2021) examine proactive transshipment decisions for perishable food products under the LIFO rule, focusing on benefits and costs. They determine order and transshipment quantities, as well as the unexpired inventory to carry over and clearance sale items. For products with lifetimes of up to three periods, they derive the optimal policy using dynamic programming. For longer lifetimes, they compute an optimal heuristic policy with an approximated formulation. Unlike our study, they do not investigate pricing of old and new products or emphasize waste reduction. Wei et al. (2022) propose a lateral transshipment policy for perishable inventory management, integrating replenishment and recycling decisions in a stochastic dynamic programming framework. Unlike their work, our study includes centralized pricing and ordering decisions, while also considering the different ages of inventory.

Most literature on inventory management of perishable food products focuses on FIFO or LIFO rules. However, consumer behavior is more complex. In scenarios where products of different ages are sold with price markdowns on older products, price-sensitive or environmentally conscious consumers may choose the older product. If the desired product is unavailable, consumers may switch to purchasing the alternative (new or old) product. Our research differs by focusing on these consumer behaviors, capturing the complexities of decision-making and providing insights that reflect real-world retail dynamics. Table 1 provides a comparative analysis of our study against key research on perishable product management, focusing on critical decision-making aspects. Unlike prior studies, our research uniquely integrates dynamic pricing, ordering, and inventory sharing decisions while also considering the simultaneous sale of products with different ages. Additionally, our bi-objective dynamic programming approach explicitly models the trade-off between profit maximization and waste reduction, incorporating demand shifts.

Our paper contributes to the current literature on perishable inventory management in several important ways:

a. The model incorporates consumer preferences, including stockout-driven substitutions and consumer preferences for products of different ages. It also emphasizes the role of price differentiation between new and old products, demonstrating that strategic markdowns improve profitability while reducing waste.
b. Introducing inventory sharing between two branches enhances stock balancing, reducing waste while optimizing pricing and inventory decisions. Findings show that strategically allocating older products to locations where they are more likely to sell mitigates food waste. Additionally, market segmentation, considering differences in how consumers value product freshness, can further refine inventory strategies, helping retailers effectively target different customer groups.
c. A bi-objective dynamic programming model using the weighted-sum method is developed to assess the trade-off between profit maximization and waste reduction. By selecting appropriate weight values, retailers can adapt strategies to regulatory changes, market conditions, and sustainability goals while ensuring long-term profitability.

By addressing these elements, our research provides practical insights into balancing profit and sustainability, offering a framework that aligns with modern retail dynamics.

3. Problem formulation and model

We consider a retailer's problem of selling a single type of perishable food product with different ages simultaneously through two branches, enabling demand shifts based on prices and consumer behaviors. The product has a two-period lifetime: if sold in the same period as produced, it is "new" (subscript 0), and if sold later, it is "old" (subscript 1). We develop a bi-objective infinite horizon dynamic programming model for pricing, ordering, and shared inventory decisions of perishable products. The system's states at every point in time are defined by the quantities of old products in branches A and B, denoted as (q_1^A, q_1^B), respectively. Branches are represented by the superscript $m = A, B$. All notations are summarized in Table 2.

Sequence of events: (1) Firm decide the sale price of the new product then (2) At the beginning of each period, the firm checks the quantity of old product at hand in branch A and B (q_1^A, q_1^B) and decides the order quantity of new products, q_0^m, and the sale price of old

Table 1
Comparison of the Literature.

Articles	Dynamic Pricing Decision	Dynamic Ordering Decision	Dynamic Inventory Sharing Decision	Sell Different Aged Products Simultaneously	Bi-objective	Demand Shift
This study	✓	✓	✓	✓	✓	✓
Chao et al. (2015)		✓		✓		
Keskin et al. (2022)	✓	✓				
Lee et al. (2024)		✓	✓	✓		
Li et al. (2009)	✓	✓				
Li et al. (2015)	✓					
Li et al. (2021)		✓	✓	✓		
Moshtagh et al. (2025)	✓			✓		
Pourmohammad-Zia et al. (2021)	✓					
Vahdani and Sazvar (2022)	✓	✓				
Wei et al. (2022)		✓	✓			
Yavuz and Kaya (2024)	✓	✓		✓		
Zhang et al. (2023)			✓			

Table 2
Notations.

Parameters	Definition
N^m	Market size of product in branch m
c	Ordering cost per product
h	Inventory holding cost
α_{ij}^m	Stockout-based substitution probability of consumer from type i product to type j product in branch m
F	Fixed cost of inventory sharing
v	Consumer's valuation
$G(.)$	Distribution function of consumer's valuation
$g(.)$	Density function of consumer's valuation
δ^m	Rate of decrease in the valuation of the old product $0 \leq \delta < 1$
p_0	Sale price of the new product
θ_i	Probability of a consumer to purchase product i
State of the System	**Definition**
q_1^m	Quantity of old product at hand in branch m
Decision variables	**Definition**
q_0^m	Order quantity for branch m
p_1^m	Sale price of the old product for branch m
q_s	Shared inventory $q_s \in$ [- $q_1{}^A$, $q_1{}^B$]*

* if $q_s < 0$ then Branch A shares its inventory with Branch B, if $q_s > 0$ vice versa.

products, p_1^m. We assume that the lead time is zero. Branches also make the inventory sharing quantity $q_s \in [-q_1^A, q_1^B]$ decision from one branch to the other. (3) Consumers arrive and make purchasing decision based on their utility and demand for each branch is observed. (4) At the end of each period, unsold and unshared old products become waste and unsold new products are carried to the next period as old products.

For products of different ages, developing consumer choice models is crucial (Chen et al., 2014). Therefore, we use utility-based demand models similar to Tirole (1988) and Transchel (2017). In our model, v represents the consumer's valuation for the new product and θ_i is the expected ratio of consumers who purchase product *i.* G(.) and g(.) are the distribution function and density function of v, respectively. The old product's price, p_1^m, is always lower than or equal to the new product's price, p_0. δ^m denotes the decrease in consumer valuation as the product ages in branch m. δ can vary across branches due to differences in consumer preferences based on demographics and location. In low-income areas, consumers are more price-sensitive and prioritize value over freshness. In rural areas with fewer grocery options, access is prioritized over freshness, while in areas with strong environmentalism, reducing food waste is valued more. In these cases, δ would be higher. Conversely, in high-income neighborhoods, urban areas with many alternatives, or places with weak environmentalism, δ would be lower as freshness is more valued.

We let $u_0 = v - p_0$ and $u_1 = \delta^m v - p_1{}^m$ demonstrate the utility function of consumers when they buy the new product and the old product, respectively. A consumer buys the new product if and only if $u_0 \geq 0$ *and* $u_0 \geq u_1$, and buys the old product if and only if $u_1 \geq 0$ *and* $u_0 \leq u_1$. Note that a consumer prefers the new product over the old one if $u_0 \geq u_1 \Rightarrow v - p_0 \geq \delta v - p_1{}^m \Rightarrow v \geq \frac{p_0 - p_1{}^m}{(1-\delta^m)}$.

Proposition 1. states the possible situations for the consumer's purchasing decision depending on the sale prices and the consumer's valuation. These situations occur based on the different values of the prices. All proofs are given in the Appendix.

Proposition 1**.** The purchase probabilities of the consumers under the given prices are as follows:

$$\text{if } p_0 < \frac{p_0 - p_1^m}{(1-\delta^m)} \tag{1}$$

Consumers will buy new products with the probability of $\theta_0 = 1 - G\left(\frac{p_0 - p_1^m}{1-\delta^m}\right)$

Consumers will buy old products with the probability of $\theta_1 = G\left(\frac{p_0 - p_1^m}{1-\delta^m}\right) - G\left(\frac{p_1^m}{\delta^m}\right)$

Consumers will not buy anything with the probability of $\theta_2 = G\left(\frac{p_1^m}{\delta^m}\right)$

$$\text{if } \frac{p_0 - p_1^m}{(1-\delta^m)} \leq p_0 \tag{2}$$

Consumers will buy new products with the probability of $\theta_0 = 1 - G(p_0)$

Consumers will not buy anything with the probability of $\theta_2 = G(p_0)$

i. There will be no sales in all other cases.

For *situation i,* if the old product's price is low enough compared to the new product price ($p_1^m < \delta^m p_0$), consumers with a higher valuation of the product prefer to buy the new product, while customers who value the product a little lower buy the old product, and customers who value the product lowest buy nothing. For *situation ii,* since the old product's price is too high ($p_1^m \geq \delta^m p_0$) none of the customers will buy the old product. High valuation customers prefer to buy the new product while low valuation consumers prefer not to buy anything.

Similar to Kaya (2010) and Transchel (2017), if the demand for new products exceeds the available quantity q_0^A (q_0^B) and consumer net utility derived from the old product is positive, then a proportion $\alpha_{01}{}^A(\alpha_{01}{}^B)$ of the excess demand will shift to purchasing the old products. Likewise, if the demand for old products surpasses the available quantity q_1^A, (q_1^B) and there is positive net utility for the new products, then a portion $\alpha_{10}{}^A$, ($\alpha_{10}{}^B$) of the excess demand for old products will buy new products. Similar to Transchel (2017), we consider endogenous demand shift rates (stockout-based substitution rates).

Proposition 2. *The stockout-based substitution probabilities of consumers from the new to the old products* ($\alpha_{01}{}^m$) *and from the old to the new products* ($\alpha_{10}{}^m$) *are given below depending on the distribution function of consumer valuation G*(.), *and prices of old and new products*:

$$\text{if} p_0 < \frac{p_0 - p_1^m}{1-\delta^m} \tag{3}$$

$$\alpha_{01}{}^m = 1$$

$$\alpha_{10}{}^m = \left[G\left(\frac{p_0 - p_1^m}{1-\delta^m}\right) - G(p_0)\right] \Big/ \left[G\left(\frac{p_0 - p_1^m}{1-\delta^m}\right) - G\left(\frac{p_1^m}{\delta^m}\right)\right]$$

$$\text{if } \frac{p_0 - p_1^m}{(1-\delta^m)} \leq p_0 \tag{4}$$

$$\alpha_{01}{}^m = \left[1 - G\left(\frac{p_1^m}{\delta^m}\right)\right] \Big/ [1 - G(p_0)]$$

$$\alpha_{10}{}^m = 0$$

i. There will be no demand shift in all other cases.

4. Infinite horizon average cost dynamic programming model

In this model, we consider a single product of different ages sold in two branches of a grocery retailer and develop a bi-objective dynamic programming model. We use the weighted-sum method to maximize the grocery retailer's average profit and minimize the average waste. w is the weight factor for the profit function, while $1 - w$ represents the weight factor for the waste. This method is a way to explore the trade-off between the two objectives. Using the weighted-sum method allows retailers to prioritize one objective over the other, depending on regulatory requirements or market pressures. The weighted-sum method was selected for its flexibility and ability to generate solutions that align with real-world decision-making in retail environments.

We develop an infinite horizon average cost dynamic programming formulation. In the model, we consider an infinite horizon case and state of the system can be denoted as (q_1^A, q_1^B). The decision of the order amount of the new product, q_0^A, q_0^B, old product's sale price, p_1^A, p_1^B, and quantity of shared inventory, $q_s \in [-q_1^A, q_1^B]$, are given based on the (q_1^A, q_1^B). Our model also determines the direction of inventory sharing; if it gets negative (positive) value then inventory sharing is from branch A (B) to branch B(A). We let F denote the fixed inventory sharing cost. Also, for both branches decisions are centralized and given by the retailer. $V(q_1{}^A, q_1{}^B)$ denotes the relative value function in the dynamic programming model.

In our model, we let N^m denote the market size of the product in branch *m*. d_i^m, is the possible demand value of product *i* in branch *m* and $N^m - d_1^m - d_0^m$ is the number of consumers buy nothing which is represented as d_2^m $(\sum d_i{}^m = N^m)$. We assume that d_i^m follows a multinomial distribution with parameters, N^m and $v_i{}^m : d_0{}^m\ Mult(N^m,\ v_0{}^m)$, and $d_1{}^m\ Mult(N^m, v_1{}^m)$. We denote the demand shift from the new product to the old product as $d_{01}{}^m$ and from the old product to the new product as $d_{10}{}^m$, both of which follow a binomial distribution as follows: $d_{01}{}^m\ B((d_0{}^m - q_0{}^m)^+, \alpha_0{}^m)$, and $d_{10}{}^m\ B((d_1{}^m - q_1{}^m)^+, \alpha_{10}{}^m)$. Hence, the total demand of new and old products for branch *m* are given as $d_0{}^m + d_{10}{}^m$, $d_1{}^m + d_{01}{}^m$.

λ is the weighted-sum objective function of our problem. We use the Bellman equation, which provides a recursive framework for optimizing this objective function by balancing profit maximization and waste minimization. The Bellman equation evaluates the costs and revenues of the current period while considering the impact of current decisions on future outcomes. It is used in the dynamic programming formulation to determine the optimal sale price of the old product, the order quantities for branches A and B, and the quantity of shared inventory. The Bellman equation for our problem is presented below:

maxλ

s.*t*.

$$\lambda + V(q_1{}^A, q_1{}^B) = \max_{p_1{}^m, q_0{}^m, q_s} w(-c(q_0{}^A + q_0{}^B) - Fmin(1, |q_s|)) + E\Big[w\big(p_1{}^A min(d_1{}^A + d_{01}{}^A, q_1{}^A + q_s) + p_1{}^B min(d_1{}^B + d_{01}{}^B, q_1{}^B - q_s) + p_0\big(min(d_0{}^A + d_{10}{}^A, q_0{}^A) + min(d_0{}^B + d_{10}{}^B, q_0{}^B)\big) - h\big([q_0{}^A - (d_0{}^A + d_{10}{}^A)]^+ + [q_0{}^B - (d_0{}^B + d_{10}{}^B)]^+\big)\big) - (1-w)\big([q_1{}^A - (d_1{}^A + d_{01}{}^A) + q_s]^+ + [q_1{}^B - (d_1{}^B + d_{01}{}^B) - q_s]^+\big) + V\big([q_0{}^A - (d_0{}^A + d_{10}{}^A)]^+, [q_0{}^B - (d_0{}^B + d_{10}{}^B)]^+\big)\Big] \quad (5)$$

In Equation (5), $c(q_0{}^A + q_0{}^B)$ is the total ordering cost of new product in both branches. q_s represents the inventory sharing amount between two branches. At the beginning of each period, Branch A (Branch B) decides the quantity of inventory that is going to be shared with Branch B (Branch A), where the maximum amount of q_s can be $q_1{}^A(q_1{}^B)$. If $q_s < 0 (q_s > 0)$ then Branch A (Branch B) shares its inventory with Branch B (Branch A). The fixed cost of inventory sharing is denoted as $Fmin(1, |q_s|)$, if any of the branch shares its inventory, then a fixed cost F will occur. We do not allow backlogging; therefore, the amount of sales is the minimum of its demand or the quantity at hand after inventory sharing. The revenues from selling the old and new products are represented as follows: $p_1{}^A min\big(d_1{}^A + d_{01}{}^A, q_1{}^A + q_s\big) + p_1{}^B min\big(d_1{}^B + d_{01}{}^B, q_1{}^B - q_s\big)$, and $p_0\Big(min\big(d_0{}^A + d_{10}{}^A, q_0{}^A\big) + min\big(d_0{}^B + d_{10}{}^B, q_0{}^B\big)\Big)$. If the quantity of the new product exceeds its demand, then the excessive quantity will be moved to further period with the inventory holding cost, *h*. This situation is represented in the model as $h\Big(\big[q_0{}^A - d_0{}^A\big]^+ + \big[q_0{}^B - d_0{}^B\big]^+\Big)$. The unsold and unshared products at the end of their lifetime will be wasted and $\big[q_1{}^A - \big(d_1{}^A + d_{01}{}^A\big) + q_s\big]^+ + \big[q_1{}^B - \big(d_1{}^B + d_{01}{}^B\big) - q_s\big]^+$ represents the quantity of waste. However, excessive quantity of the new product will be the new state of the system, and it will be updated as $\big[q_0{}^A - (d_0{}^A + d_{10}{}^A)\big]^+, \big[q_0{}^B - (d_0{}^B + d_{10}{}^B)\big]^+$. Therefore, future relative value function will be $V\Big(\big[q_0{}^A - (d_0{}^A + d_{10}{}^A)\big]^+, \big[q_0{}^B - (d_0{}^B + d_{10}{}^B)\big]^+\Big)$. The dynamic programming formulation given in Equation (5) aims to maximize the weighted function of total profit and amount of waste, with weights *w* and 1-*w*. For the decision variables in Equation (5), the amount of shared inventory needs to satisfy the condition, $-q_1{}^A < q_s < q_1{}^B$ since more than the available amount at hand cannot be shared. In addition, $q_0{}^m \geq 0$ since the order quantities cannot be negative, and $0 \leq p_1{}^m \leq p_0$ since the price of the old products need to be nonnegative, and at most equal to the price of the new product.

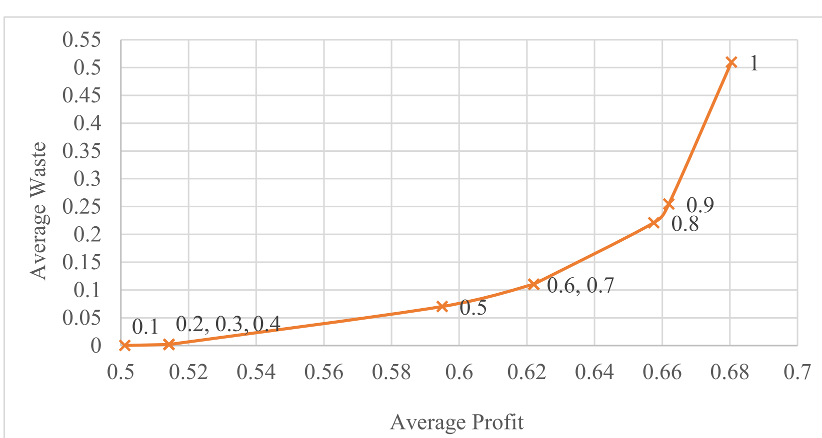

Fig. 1. Pareto Frontier.

Additionally, we develop dynamic programming formulations separately to calculate the average profit, λ_P, and the average waste, λ_W, These formulations utilize the optimal decision variables $(p_1^{m^*}, q_0^{m^*} and\ q_s^{\ *})$ obtained by solving Equation (5). The dynamic programming models for the average profit and the average waste are given in Equations (6) and (7), respectively. In Equation (6), we focus only on the average profit function and formulate the dynamic programming equation considering the profit related terms, while in Equation (7), we evaluate the average waste amount focusing on the waste related terms, as given in Equation (5).

$$\lambda_P + V_P(q_1^{\ A}, q_1^{\ B}) = -c(q_0^{A^*} + q_0^{B^*}) - Fmin(1, |q_s^{\ *}|) + E\begin{bmatrix} p_1^{A^*} min(d_1^{\ A} + d_{01}^{\ A}, q_1^{\ A} + q_s^{\ *}) + p_1^{B^*} min(d_1^{\ B} + d_{01}^{\ B}, q_0^{B^*} - q_s^{\ *}) \\ +p_0(min(d_0^{\ A} + d_{10}^{\ A}, q_0^{A^*}) + min(d_0^{\ B} + d_{10}^{\ B}, q_0^{B^*})) \\ -h([q_0^{A^*} - d_0^{\ A}]^+ + [q_0^{B^*} - d_0^{\ B}]^+) \\ +V_P([q_0^{A^*} - (d_0^{\ A} + d_{10}^{\ A})]^+, [q_0^{B^*} - (d_0^{\ B} + d_{10}^{\ B})]^+) \end{bmatrix} \tag{6}$$

$$\lambda_W + V_W(q_1^{\ A}, q_1^{\ B}) = E\begin{bmatrix} [q_1^{\ A} - (d_1^{\ A} + d_{01}^{\ A}) + q_s^{\ *}]^+ + [q_1^{\ B} - (d_1^{\ B} + d_{01}^{\ B}) - q_s^{\ *}]^+ \\ +V_W([q_0^{A^*} - (d_0^{\ A} + d_{10}^{\ A})]^+, [q_0^{B^*} - (d_0^{\ B} + d_{10}^{\ B})]^+) \end{bmatrix} \tag{7}$$

5. Numerical results

We provide detailed numerical results and managerial insights for the analyzed problem. We analyze the optimal order amount, the old product's sale price decisions for both branches and the optimal quantity of shared inventory and its direction which maximize the objective function in Equation (5), and we conduct a sensitivity analysis. In our numerical studies, we consider an artisan and/or diet-specific bakery product, such as sourdough bread, gluten-free croissant, vegan cake, which has a two-day shelf life. The parameters, system state arrays, and decision variables in our model are selected based on a combination of empirical insights from the literature and industry standards. For example, market size is modeled following the approaches of Li et al. (2021), ensuring that our model accurately represents real-world retail conditions. In our base case, we use $N = 3$ which is a typical scenario for a small to medium-sized bakery section in a grocery store since the demand for a specific product is smaller. We let $p_0 = 0.55$, $h = 0.002$, $c = 0.2$ and $F = 0.2$, and $\delta^m = 0.6$. We suppose that consumer valuation follows the distribution $G(v) = 1 - \frac{(\overline{v}-v)^b}{\overline{v}^b}$ where $b > 0$ and with support of $[0, \overline{v}]$. This distribution is commonly used to model consumer preference, as seen in Debo et al. (2005); Pan and Honhon (2012); Transchel (2017). The uniform distribution commonly utilized in market segmentation literature (Dong & Wu, 2019; Jerath et al., 2010; Shen et al., 2022) is a special case of this distribution, achieved by setting $b = 1$ (Debo et al., 2005). For our analysis, we use $b = 1$, such that v follows a uniform distribution between $[0, \overline{v}]$. In our numerical analysis we assume $\overline{v} = 1$. To analyze the tradeoff between the objective functions of profit and waste, we use varying weight parameters between 0 and 1, with 0.1 increments, $w = (0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1)$.

The numerical results were obtained using a 3.40 GHz Intel i7-13700KF server with 32 GB of 5200 MHz DDR5 RAM. To solve the problem and determine the optimal solutions, we use the relative value iteration algorithm for our infinite horizon average cost dynamic programming model, as outlined in Bertsekas (2005).

In our formulation, at least one of the states (for example the state $q_1^{\ A} = 0, q_1^{\ B} = 0$) is visited with positive probability at least once within the first n stages for some integer $n > 0$, for all initial states and policies. This satisfies Assumption 7.4.1 in Bertsekas (2005). As stated in Proposition 7.4.1 in Bertsekas (2005), the optimal average cost λ^* is the same for all initial states along with some vector V^* in our formulations, meets Bellman's Equation (5) in our model. Then, the relative value iteration algorithm for the average cost per stage formulations, as described in Bertsekas (2005, Chapter 7, pg.430–432), yields the optimal solution. In this algorithm, we let $V_k(q_1^{\ A}, q_1^{\ B}) = J_k(q_1^{\ A}, q_1^{\ B}) - J_k(r, r)$ where r is the reference state (we use $r = 0$), $J_k(q_1^{\ A}, q_1^{\ B})$ is the optimal k-stage cost for $k = 1, 2, \ldots$, and can be computed through the recursion below for our model.

$$\begin{aligned} J_{k+1}(q_1^{\ A}, q_1^{\ B}) = \max_{p_1^{\ m}, q_0^{\ m}, q_s} & w(-c(q_0^{\ A} + q_0^{\ B}) - Fmin(1, |q_s|)) + E[w(p_1^{\ A} min(d_1^{\ A} + d_{01}^{\ A}, q_1^{\ A} + q_s) + p_1^{\ B} min(d_1^{\ B} + d_{01}^{\ B}, q_1^{\ B} - q_s) + p_0(min(d_0^{\ A} + d_{10}^{\ A}, q_0^{\ A}) \\ & + min(d_0^{\ B} + d_{10}^{\ B}, q_0^{\ B})) - h([q_0^{\ A} - (d_0^{\ A} + d_{10}^{\ A})]^+ + [q_0^{\ B} - (d_0^{\ B} + d_{10}^{\ B})]^+)) - (1-w)([q_1^{\ A} - (d_1^{\ A} + d_{01}^{\ A}) + q_s]^+ \\ & + [q_1^{\ B} - (d_1^{\ B} + d_{01}^{\ B}) - q_s]^+) + V([q_0^{\ A} - (d_0^{\ A} + d_{10}^{\ A})]^+, [q_0^{\ B} - (d_0^{\ B} + d_{10}^{\ B})]^+)] \end{aligned} \tag{8}$$

We use the following variant of the relative value iteration algorithm, which guarantees the convergence under Assumption 7.4.1, as stated in Bertsekas (2005), for any scalar τ, $0 < \tau < 1$.

$$
\begin{aligned}
V_{k+1}({q_1}^A,{q_1}^B) = {} & (1-\tau)V_k({q_1}^A,{q_1}^B) + \max_{{p_1}^m,{q_0}^m,q_s} E\Big[w\Big(-c({q_0}^A+{q_0}^B) - Fmin(1,|q_s|) + {p_1}^A min({d_1}^A+{d_{01}}^A,{q_1}^A+q_s) + {p_1}^B min({d_1}^B+{d_{01}}^B,{q_1}^B-q_s) \\
& + p_0\big(min({d_0}^A+{d_{10}}^A,{q_0}^A) + min({d_0}^B+{d_{10}}^B,{q_0}^B)\big) - h\big([{q_0}^A-({d_0}^A+{d_{10}}^A)]^+ + [{q_0}^B-({d_0}^B+{d_{10}}^B)]^+\big)\Big) - (1 \\
& - w)\big([{q_1}^A-({d_1}^A+{d_{01}}^A)+q_s]^+ + [{q_1}^B-({d_1}^B+{d_{01}}^B)-q_s]^+\big) + V\big([{q_0}^A-({d_0}^A+{d_{10}}^A)]^+,[{q_0}^B-({d_0}^B+{d_{10}}^B)]^+\big)\Big] - \max_{{p_1}^m,{q_0}^m,q_s} E\Big[w\Big(\\
& - c({q_0}^A+{q_0}^B) - Fmin(1,|q_s|) + {p_1}^A min({d_1}^A+{d_{01}}^A,r+q_s) + {p_1}^B min({d_1}^B+{d_{01}}^B,r-q_s) + p_0\big(min({d_0}^A+{d_{10}}^A,{q_0}^A) + min({d_0}^B \\
& + {d_{10}}^B,{q_0}^B)\big) - h\big([{q_0}^A-({d_0}^A+{d_{10}}^A)]^+ + [{q_0}^B-({d_0}^B+{d_{10}}^B)]^+\big)\Big) - (1-w)\big([r-({d_1}^A+{d_{01}}^A)+q_s]^+ \\
& + [r-({d_1}^B+{d_{01}}^B)-q_s]^+\big) + V\big([{q_0}^A-({d_0}^A+{d_{10}}^A)]^+,[{q_0}^B-({d_0}^B+{d_{10}}^B)]^+\big)\Big]
\end{aligned} \tag{9}
$$

We observe that even the single-period price-setting newsvendor model is not necessarily quasi-concave in terms of price and inventory decisions. Due to this complexity, an analytical solution or structured analytical properties of the optimal solutions cannot be determined. We evaluate the optimal solutions by examining the results from the numerical experiments, where we solve the stochastic dynamic programming formulations using the relative value iteration algorithm as described above. In this analysis, due to the non-concavity issues, we use a grid-search approach to determine the optimal values of the decision variables. In our models, old product sale prices range from 0 to 0.55 ($p_1^m \le p_0$), with 0.05 increments, and shared inventory quantity, $q_s \in$ [-q_1^A, q_1^B]. q_0^m ranges between 0 and N^m. We conduct an exhaustive grid search within these ranges to find the optimal decision variables in the relative value iteration algorithm, and overcome the non-concavity issues in the model through this approach.

For our model, in the algorithm $V_{k+1}({q_1}^A,{q_1}^B)$ is calculated for all $({q_1}^A,{q_1}^B)$ values on the left-hand side of the Bellman's equation by using the $V_k({q_1}^A,{q_1}^B)$ on the right-hand side for all $k = 0,1,2,\ldots$ until all $V_k({q_1}^A,{q_1}^B)$ values converge to some vector V, which is explained in detail by Bertsekas (2005), where $V_k({q_1}^A,{q_1}^B) = 0$ *and* $V_k(0,0) = 0$.

5.1. Model results

Table A.1 presents the optimal decisions for order quantity, pricing, and inventory sharing, along with the corresponding average waste, profit, and computational time required to obtain the results (total CPU time: 28.7 h). Table A.1 indicates that there is price differentiation between new and old products in both branches ($p_1^m < p_0$) for all w values. When waste minimization is more important (w is lower), retailer always shares inventory aiming to minimize waste by ensuring that old products are distributed evenly between branches, if one of the branch has 3 old products and other has less than or equal to 1 old product.

For example, when *(q_1^A, q_1^B)*=(3,0) or *(q_1^A, q_1^B)*=(0,3), the branch with three old products shares one unit with the branch that has none, ensuring a more balanced distribution. Alternatively, the branch with three old products shares two units, and the sharing branch orders one new product. This strategy results in one branch holding two old products, while the other has one old and one new product before consumer demand is realized. This highlights how inventory sharing not only helps separate products by age between branches but also ensures that old products are sold before they perish. In these cases, after sharing, both branches end up with two old products each, effectively balancing inventory across locations. This strategy helps meet expected demand while minimizing waste, ensuring that products are sold before expiration. A similar balancing effect is observed at w = 0.1, where inventory sharing occurs when initial stock levels are (0,2) and (2,0).

However, when both branches have relatively low inventory levels e.g $({q_1}^A,{q_1}^B) = (0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1)$ the retailer decides not to share inventory. Instead, they prioritize ordering new products, particularly in branches with lower or no old products.

When inventory sharing occurs, the direction of transfer follows a threshold-based policy. When inventory of old product at hand at any branch reaches a threshold level, $\overline{q_1}$, it is sufficient to compare the q_1^A and q_1^B to determine the direction of inventory. That is, $q_s > 0$, *if* $q_1^B = \overline{q_1}$ *and* $q_1^B - q_1^A > 1$; $q_s < 0$, *if* $q_1^A = \overline{q_1}$ *and* $q_1^A - q_1^B > 1$; $q_s = 0$, *if* q_1^B, $q_1^A < \overline{q_1}$. For example, based on our numerical results, when $w = 0.2$ or *0.3,* we find $\overline{q_1}$=*N*. For lower w values, $w = [0,0.1]$, $\overline{q_1}$ is also lower because when waste is the only objective or much more important, retailer tries to avoid overstocking and shares the inventory even though branches have fewer inventory of old product at hand.

For high w values, retailer decides not to share inventory between branches to avoid the inventory sharing cost. Instead, retailer orders new products if one of the branches has less than two old products at hand ($e.g$ $(0,0)$, $(2,0)$, $(1,2)$, $(3,1)$ $\cdots$) and ensures each branch maintains a minimum level of 2 inventory items. Which means q_0^m can be determined based on the current old product inventory levels for each branch and this strategy can be formulated as follows: $q_0^m = \max(2 - q_1^m, 0)$. Maintaining a minimum inventory level of 2 by reordering new products as needed benefits the retailer to maximize the sales potential. This strategy ensures sufficient stock is available to meet demand without leading to overstocking, which could increase carrying costs or result in waste.

For $w \in [0.2, 0.9]$; when inventory of old product at hand in both branches is higher retailer neither shares old inventory nor orders new product. In these cases (e.g., (2,2) or (3,2)), the decision is also affected by the potential demand shifts. With a maximum demand of 3, such inventory levels are sufficient to meet consumer demand, accounting for those who might prefer new products but are willing to purchase old ones. This strategy not only helps retailers minimize waste by focusing on selling older products but also maximizes profit from their sales while avoiding inventory sharing, holding and ordering costs.

From Table A.1 we can observe that the decision of p_1^m in branch m depends on the old product inventory level at hand after inventory sharing, that is; $(q_1^m + q_s)$. When $(q_1^m + q_s)$ surpasses a threshold, the retailer sets a too low p_1^m according to $p_1^m \le p_0 + \delta - 1$, resulting in zero initial demand for the new product in branch m, in order to decrease waste as much as possible, especially when w is smaller (i.e. waste is more important). However, since demand shift is permitted, any excess demand from the old product can be shifted to the new product.

If we let T^m denote the threshold level in branch m;

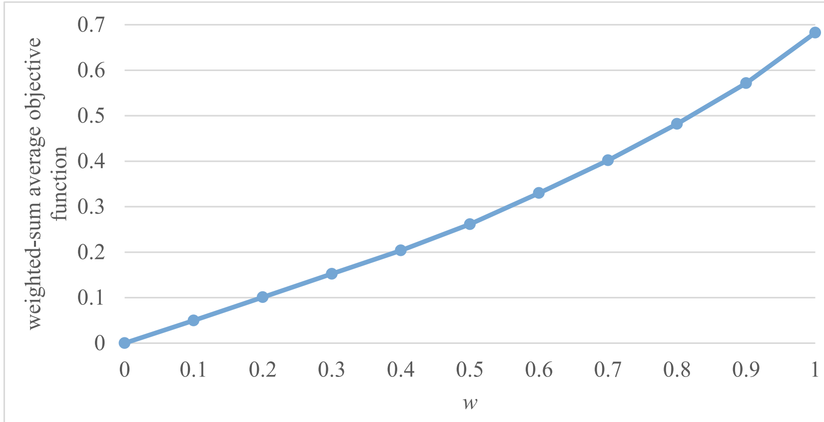

Fig. 2. Weighted-sum objective function.

- In branch A, if $(q_1^A + q_s) \in [0, T^A]$ then retailer charges moderate p_1^A, $p_0 + \delta^A - 1 < p_1^A < \delta^A p_0$, allowing consumers to buy the old product, the new product, or leave without purchasing.
- In branch B, if $(q_1^B - q_s) \in [0, T^B]$ then retailer charges moderate p_1^B, $p_0 + \delta^B - 1 < p_1^B < \delta^B p_0$, allowing consumers to buy the old product, the new product, or leave without purchasing.
- In branch A, if $(q_1^A + q_s) \in [T^A, 3]$, the retailer sets very low p_1^A, $p_1 < p_0 + \delta^A - 1$ resulting in consumers not purchasing new products.
- In branch B, if $(q_1^B - q_s) \in [T^B, 3]$ the retailer sets very low p_1^B, $p_1 < p_0 + \delta^B - 1$ resulting in consumers not purchasing new products.

These thresholds are lower for the lower w values. For example, when $w \in [0.1, 0.4]$, $T^A = T^B = 0$; when $w = 0.5$, $T^A = T^B = 1$; when $w \in [0.6, 0.7]$, $T^A = T^B = 2$; and $w \in [0.8, 1]$, $T^A = T^B = 3$. Choosing moderate prices for old products attracts consumers without undercutting new product revenue. Increasing w results in higher profit but also higher waste, highlighting the need to balance economic and sustainability goals. For example, comparing the values $w = 0.7$ and $w = 0.8$ in Table A.1, shows a moderate increase in λ_P and a dramatic rise in λ_W. This dramatic shift, along with other changes, is clearly reflected in the Pareto frontier shown in Fig. 1. In the figure, the date labels show the w values, starting from $w = 0.1$. The figure demonstrates how an appropriate w value helps profit maximization and waste reduction, allowing retailers to adjust their strategies based on regulatory changes and market pressures. Lower w suits markets with strict waste regulations, while higher w is better for competitive markets with thin profit margins.

From Fig. 1, it is evident that with strategic planning, waste can be significantly reduced without a substantial loss in profit. This insight is particularly relevant for policymakers designing food waste regulations, as they can leverage an appropriate weight value (penalty) to encourage sustainable retail practices. For example, setting $w = 0.7$ results in a 78 % reduction in average waste (from 0.51 to 0.11), while average profit decreases by 9 % (from 0.68 to 0.62) compared to $w = 1$. These findings underscore the practical applicability of our model in guiding regulatory decisions and helping retailers achieve an optimal balance between economic performance and sustainability goals.

Fig. 2 shows the optimum weighted-sum average objective values across different weight values (w). As expected, the optimum objective value increases as w increases. This occurs because higher w values assign greater importance to profit maximization within the objective function, leading to strategic decisions that emphasize revenue generation and inventory efficiency over waste reduction. Conversely, lower w values shift the focus toward minimizing waste, which may result in slightly lower overall objective values but better align with sustainability goals and regulatory constraints.

Table 3
Sensitivity Results of Decision Variables when $w = 0.5$.

	λ	λ_P	λ_W	λ **Difference %**
Base Case	0.2624	0.59492	0.0702	–
$p_0 = 0.44$	0.244	0.50642	0.0184	6.988 % ↓
$p_0 = 0.66$	0.2633	0.53817	0.0116	0.351 % ↑
$c = 0.16$	0.3208	0.71182	0.0702	22.277 % ↑
$c = 0.24$	0.2178	0.46807	0.0325	16.99 % ↓
$h = 0.0016$	0.2626	0.59535	0.0702	0.082 % ↑
$h = 0.0024$	0.2621	0.59449	0.0702	0.082 % ↓
$\delta^A = 0.48$	0.2594	0.55899	0.0403	1.145 % ↓
$\delta^A = 0.72$	0.2733	0.60855	0.0619	4.185 % ↑

5.2. Sensitivity analysis

In this subsection, we examine the results of our models under different parameter settings to understand how these parameters influence the overall system outcomes. We address a retail problem where the importance of minimizing waste and maximizing profits is equal ($w = 0.5$). Table A.2 presents the sensitivity results, and Table 3 shows the effects of the parameters on decision variables.

When $p_0 = 0.44$, the demand for new products increases and consumers will be more willing to buy new products. Therefore, retailer sets relatively lower p_1^m for specific q_1^m values. Hence, this strategy increases the attractiveness of old products to consumers and helps retailer to clear inventory to minimize the waste. As a result, optimal λ_W decreases from 0.0702 to 0.0184. However, lower p_0 leads to a decrease in profit per unit sold. Despite potentially higher volumes of sales due to lower prices, the reduced margin results in an overall lower λ_P. Therefore, optimal λ_P also decreases from 0.594922 to 0.506417. Therefore, weighted-sum objective function (λ) decreases by 6.988 %.

When $p_0 = 0.66$ the demand for new products decreases and consumers will be more willing to buy old products. Therefore, retailer orders less new products and sets relatively higher p_1^m for specific q_1^m values. This strategy benefits the retailer to decrease λ_W from 0.0702 to 0.0116; however, λ_P also decreases from 0.594922 to 0.538172. Since decrease in λ_P is not as significant as the decrease in λ_W, λ increases by 0.351 %.

When the cost of ordering new products decreases ($c = 0.16$), the retailer orders more new products and does not share inventory. Hence, λ_W remains the same, λ_P increases; therefore, λ increases by 22.277 %. This strategy enables the retailer to refresh its inventory with new products, focus on their sales, and gain profit since their price is higher. When ordering cost increase ($c = 0.24$), the retailerorders fewer new products and share inventory for specific q_1^m values. Hence, λ_W decreases but also, λ_P; therefore, λ increases by 16.99 %. This strategy helps the retailer avoid high costs and instead focus on the sale of old products to enhance cost efficiency and contribute to sustainability by reducing waste. p_1^{m} also changes for particular states of the system based on $(q_1^{\mathrm{A}} + q_s)$ and $(q_1^{\mathrm{B}} - q_s)$ since inventory decisions have changed.

$\pm$ 20 % changes in inventory holding cost (h) does not affect the optimal decisions and λ_W. However, it affects λ_P directly; therefore, when $h = 0.0024$, λ decreases by 0.082 % and when $h = 0.0016$, λ increases by 0.082 %.

A lower δ^{A} indicates a market where consumers value freshness more, have higher income levels, or have more shopping alternatives. Setting lower p_1^{m} in Branch A when $\delta^{\mathrm{A}} = 0.48$ encourages purchases, enhancing the attractiveness of older products and ensuring they are sold to reduce waste and generate revenue. When there are no old products at the beginning of the period $((q_1^A, q_1^B) = (0,0), (0,1), (0,2))$, ordering fewer new products helps optimize inventory levels. When δ^{A} is high, indicating less sensitivity to product age, higher price sensitivity, or environmental concerns, the demand for older products increases. The retailer sets higher sale prices for older products in Branch A when $\delta^{\mathrm{A}} = 0.72$. Similar insights are gained for $\pm$ 20 % changes in δ^{B}. In general, when δ is high, consumers show less sensitivity to product freshness, making the purchasing decision less dependent on whether a product is new or old. As δ becomes higher, the model aligns closer with the case where perishability and aging does not impact demand and all products are perceived as identical. In such cases, the retailer optimally sets higher prices for older products, and it is observed that both the average profit and the average waste increase.

6. Conclusion

We develop a bi-objective dynamic programming model to address the complex problem faced by grocery retailers in managing perishable food products across multiple branches. Our model optimizes pricing, ordering, and inventory sharing decisions while balancing two critical objectives: maximizing profit and minimizing food waste. The numerical experiments reveal several key insights that can be directly applied to real-world retail operations.

The use of a bi-objective approach, applying the weighted-sum method, allowed us to explore the trade-offs between profit and waste reduction which provides critical insights for managers. For instance, when retailers forced to prioritize waste minimization, they adapt their strategies to comply with regulations. France's 2016 legislation prohibits supermarkets from discarding unsold food, while nine U.S. states have banned commercial waste generators from landfilling food waste, aiming for waste reduction (Anglou et al., 2024). In response, retailers share more inventory between branches, and lower prices on older products to sell them more quickly. Conversely, when profit maximization is more important, retailers focus on revenue generation through avoid sharing inventory, focusing instead on the sale of newer, more profitable items while maintaining an acceptable level of waste reduction. This dynamic highlights how businesses can tailor their strategies to varying market and regulatory conditions.

Inventory sharing between branches is an effective strategy for balancing stock levels and reducing food waste, especially when one branch holds excess inventory of older products. Sharing inventory ensures that old products are distributed to locations where they are more likely to sell, rather than being wasted. However, the decision to share inventory must be carefully balanced against the associated logistical costs. Retailers should conduct regular cost-benefit analyses to determine when inventory sharing is more advantageous than simply ordering new products for individual branches.

Sensitivity analyses on discount rates in consumer utility from old products show the importance of market segmentation. Retailers should consider market segmentation strategies recognizing differences in consumer valuation of product freshness and adjust pricing and inventory strategies accordingly. When δ is high, indicating consumers place a smaller discount on older products, retailers gain more profit while reducing waste. Collaboration between retailers and government to educate consumers on the environmental impact of food waste can support retailers' efforts by highlighting how purchasing older products contributes to sustainability goals.

Price differentiation between new and old products consistently benefits retailers by appealing to different customer segments and reducing waste. Price reductions can encourage consumers to purchase perishable foods nearing expiration when the discount sufficiently offsets perceived drawbacks (Chang et al., 2024). Dynamic pricing ensures older products attract price-sensitive consumers while newer products maintain premium pricing, maximizing revenue and minimizing unsold goods. From a managerial perspective, the threshold-based pricing policy identified in the model provides practical guidance for setting prices for older products. By adjusting prices based on inventory levels, retailers can ensure that products nearing expiration are sold before they become waste, thereby minimizing losses.

Regulatory bodies and non-governmental organizations should establish comprehensive guidelines and implementation standards to promote a sustainable food economy (Aka and Buyukdag, 2021). This study is critical for retailers, policymakers, and researchers as it offers actionable insights into how grocery chains can balance profit and sustainability in managing perishable goods.

- For retailers, it provides strategies to reduce waste while maximizing revenue. Additionally, retailers can use our model as a decision-support tool to optimize pricing, ordering, and inventory sharing strategies, enabling them to maximize profits while minimizing waste.
- For policymakers, especially those advocating for food waste reduction, the findings can guide legislation. Also, our findings offer quantitative evidence that food waste reduction policies can be effectively designed using structured inventory-sharing and pricing mechanisms
- For researchers, the model serves as a foundation for further study in dynamic inventory control, consumer behavior modeling, and transshipment policies for perishables and sustainability.

To reinforce the practical implications of our findings, we summarize the key takeaways that highlight the impact of inventory sharing, pricing strategies, and regulatory considerations in perishable inventory management:

- When waste reduction is prioritized, retailers share inventory and discount older products. Investing in inventory sharing can help retailers align with food waste policies. When profit maximization is prioritized, retailers may prefer to sell newer products and limit inventory transfers.
- Governments aiming to reduce waste could encourage collaborative inventory strategies through incentives or regulations. Our results quantify the benefits of inventory sharing and price differentiation under different settings, and also provide quantitative results to help determine the optimal values of incentive values to be used in regulatory mechanisms.
- Sharing inventory between branches is a valuable approach to balancing stock levels and minimizing food waste. Price differentiation between new and old products increases profitability while reducing waste by shifting demand toward older products nearing expiration. We quantify the benefits of these strategies under different settings.
- Regulatory-driven waste reduction policies can be effective. Our model shows that an appropriate waste reduction weight can cut waste by 80 % while reducing profit by only 10 %.
- Market segmentation influences pricing and inventory decisions. Retailers should adjust pricing strategies based on consumer

preferences for product freshness and collaborate with policymakers to shape sustainable purchasing behavior.

By integrating these findings, we address practical, managerial, and policy-relevant challenges in perishable inventory management. Future research can extend this study in several ways. Firstly, due to the curse of dimensionality, determining the optimal inventory policy for perishable products is complex even when sale prices are fixed (Nahmias, 1982; Chen et al. 2014). Considering larger problems will cause high computational time, novel solution algorithms or heuristics using approaches like approximate dynamic programming, deep learning, or reinforcement learning could be developed. Secondly, with omnichannel retailing becoming significant in the grocery sector, our study could expand to this context. Also, we can consider reactive transshipments to examine how optimal policies may differ. Finally, the detailed analysis of the system including incentive or penalty mechanisms can be an extension of this study in the future.

CRediT authorship contribution statement

Melda Hasiloglu-Ciftciler: Writing – original draft, Software, Resources, Methodology, Investigation, Formal analysis, Conceptualization. **Onur Kaya:** Writing – review & editing, Validation, Supervision, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Table A.1
Optimal Decisions.

$w = 0$		(0,0)	(0,1)	(0,2)	(0,3)	(1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)	(3,0)	(3,1)	(3,2)	(3,3)	Profit	Waste
	p_1^A	–	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
	p_1^B	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
	q_0^A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	q_0^B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	q_s	0	0	1	1	−1	0	0	1	−1	−1	0	0	−2	−1	−1	0		
$w =$	p_1^A	–	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
0.1	p_1^B	–	0.05	0.05	0.05	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
	q_0^A	1	1	1	0	1	1	1	0	1	0	0	0	0	0	0	0	0.501	0.0002
	q_0^B	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0		
	q_s	0	0	1	2	0	0	0	1	−1	0	0	0	−1	−1	0	0		
$w =$	p_1^A	–	–	–	0.05	0.1	0.1	0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
0.2	p_1^B	–	0.1	0.05	0.1	–	0.1	0.05	0.05	–	0.1	0.05	0.05	0.1	0.05	0.05	0.05		
	q_0^A	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0.514	0.002
	q_0^B	1	1	0	1	1	1	0	0	1	1	0	0	1	0	0	0		
	q_s	0	0	0	2	0	0	0	1	0	0	0	0	−1	−1	0	0		
$w =$	p_1^A	–	–	–	0.05	0.1	0.1	0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
0.3	p_1^B	–	0.1	0.05	0.1	–	0.1	0.05	0.05	–	0.1	0.05	0.05	0.1	0.05	0.05	0.05		
	q_0^A	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0.514	0.002
	q_0^B	1	1	0	1	1	1	0	0	1	1	0	0	1	0	0	0		
	q_s	0	0	0	2	0	0	0	1	0	0	0	0	−1	−1	0	0		
$w =$	p_1^A	–	–	–	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.05	0.05	0.05		
0.4	p_1^B	–	0.1	0.1	0.1	–	0.1	0.1	0.05	–	0.1	0.1	0.05	0.1	0.1	0.1	0.05		
	q_0^A	1	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0.514	0.002
	q_0^B	1	1	0	1	1	1	0	0	1	1	0	0	1	1	0	0		
	q_s	0	0	0	2	0	0	0	0	0	0	0	0	−1	0	0	0		
$w =$	p_1^A	–	–	–	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.05	0.05	0.05		
0.5	p_1^B	–	0.2	0.1	0.1	–	0.2	0.1	0.05	–	0.2	0.1	0.05	0.1	0.2	0.1	0.05		
	q_0^A	2	2	2	1	1	1	1	1	0	0	0	0	1	0	0	0	0.595	0.070
	q_0^B	2	1	0	0	2	1	0	0	2	1	0	0	0	1	0	0		
	q_s	0	0	0	1	0	0	0	0	0	0	0	0	−2	0	0	0		
$w =$	p_1^A	–	–	–	–	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.05	0.05	0.05	0.05		
0.6	p_1^B	–	0.2	0.2	0.05	–	0.2	0.2	0.05	–	0.2	0.2	0.05	–	0.2	0.2	0.05		
	q_0^A	2	2	2	2	1	1	1	1	0	0	0	0	0	0	0	0	0.622	0.110
	q_0^B	2	1	0	0	2	1	0	0	2	1	0	0	2	1	0	0		
	q_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
$w =$	p_1^A	–	–	–	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.1		
0.7	p_1^B	–	0.2	0.2	0.2	–	0.2	0.2	0.1	–	0.2	0.2	0.1	0.2	0.2	0.2	0.1		
	q_0^A	2	2	2	0	1	1	1	1	0	0	0	0	0	0	0	0	0.622	0.110
	q_0^B	2	1	0	1	2	1	0	0	2	1	0	0	1	1	0	0		
	q_s	0	0	0	2	0	0	0	0	0	0	0	0	−1	0	0	0		
$w =$	p_1^A	–	–	–	–	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2		
0.8	p_1^B	–	0.25	0.25	0.2	–	0.25	0.25	0.2	–	0.25	0.25	0.2	–	0.25	0.25	0.2		
	q_0^A	2	2	2	2	1	1	1	1	0	0	0	0	0	0	0	0	0.658	0.221
	q_0^B	2	1	0	0	2	1	0	0	2	1	0	0	2	1	0	0		
	q_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
$w =$	p_1^A	–	–	–	–	0.25	0.25	0.25	0.25	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25		
0.9	p_1^B	–	0.25	0.3	0.25	–	0.25	0.3	0.25	–	0.25	0.3	0.25	–	0.25	0.3	0.25		
	q_0^A	2	2	2	2	1	1	1	1	0	0	0	0	0	0	0	0	0.662	0.254
	q_0^B	2	1	0	0	2	1	0	0	2	1	0	0	2	1	0	0		

(*continued on next page*)

Table A.1 (*continued*)

	q_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
$w = 1$	p_1^A	–	–	–	–	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3		
	p_1^B	–	0.3	0.3	0.3	–	0.3	0.3	0.3	–	0.3	0.3	0.3	–	0.3	0.3	0.3		
	q_0^A	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	0.680	0.510
	q_0^B	2	1	1	1	2	1	1	1	2	1	1	1	2	1	1	1		
	q_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		

Table A.2
Sensitivity Results when $w = 0.5$.

		(0,0)	(0,1)	(0,2)	(0,3)	(1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)	(3,0)	(3,1)	(3,2)	(3,3)
	p_1^A	–	–	–	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.05	0.05	0.05
Base Case	p_1^B	–	0.2	0.1	0.1	–	0.2	0.1	0.05	–	0.2	0.1	0.05	0.1	0.2	0.1	0.05
	q_0^A	2	2	2	1	1	1	1	1	0	0	0	0	1	0	0	0
	q_0^B	2	1	0	0	2	1	0	0	2	1	0	0	0	1	0	0
	q_s	0	0	0	1	0	0	0	0	0	0	0	0	−2	0	0	0
$p_0 = 0.44$	p_1^A	–	–	–	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.05	0.05	0.05
	p_1^B	–	0.1	0.1	0.1	–	0.1	0.1	0.05	–	0.1	0.1	0.05	0.1	0.1	0.1	0.05
	q_0^A	2	2	2	0	1	1	1	1	0	0	0	0	0	0	0	0
	q_0^B	2	1	0	1	2	1	0	0	2	1	0	0	1	1	0	0
	q_s	0	0	0	2	0	0	0	0	0	0	0	0	−1	0	0	0
$p_0 = 0.66$	p_1^A	–	–	–	0.1	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.05	0.05	0.05
	p_1^B	–	0.15	0.1	0.15	–	0.15	0.1	0.05	–	0.15	0.1	0.05	0.15	0.15	0.1	0.05
	q_0^A	1	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0
	q_0^B	1	1	0	1	1	1	0	0	1	1	0	0	1	1	0	0
	q_s	0	0	0	2	0	0	0	0	0	0	0	0	−1	0	0	0
$c = 0.16$	p_1^A	–	–	–	–	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.05	0.05	0.05	0.05
	p_1^B	–	0.2	0.1	0.05	–	0.2	0.1	0.05	–	0.2	0.1	0.05	–	0.2	0.1	0.05
	q_0^A	2	2	2	2	1	1	1	1	0	0	0	0	0	0	0	0
	q_0^B	2	1	0	0	2	1	0	0	2	1	0	0	2	1	0	0
	q_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$c = 0.24$	p_1^A	–	–	–	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.05	0.05	0.05
	p_1^B	–	0.2	0.1	0.2	–	0.2	0.1	0.05	–	0.2	0.1	0.05	0.2	0.2	0.1	0.05
	q_0^A	1	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0
	q_0^B	1	1	0	1	1	1	0	0	1	1	0	0	1	1	0	0
	q_s	0	0	0	2	0	0	0	0	0	0	0	0	−1	0	0	0
$h = 0.0016$	p_1^A	–	–	–	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.05	0.05	0.05
	p_1^B	–	0.2	0.1	0.2	–	0.2	0.1	0.05	–	0.2	0.1	0.05	0.1	0.2	0.1	0.05
	q_0^A	2	2	2	0	1	1	1	1	0	0	0	0	0	0	0	0
	q_0^B	2	1	0	1	2	1	0	0	2	1	0	0	1	1	0	0
	q_s	0	0	0	2	0	0	0	0	0	0	0	0	−1	0	0	0
$h = 0.0024$	p_1^A	–	–	–	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.05	0.05	0.05
	p_1^B	–	0.2	0.1	0.2	–	0.2	0.1	0.05	–	0.2	0.1	0.05	0.2	0.2	0.1	0.05
	q_0^A	2	2	2	0	1	1	1	1	0	0	0	0	0	0	0	0
	q_0^B	2	1	0	1	2	1	0	0	2	1	0	0	1	1	0	0
	q_s	0	0	0	2	0	0	0	0	0	0	0	0	−1	0	0	0
$\delta^A = 0.48$	p_1^A	–	–	–	0.1	0.1	0.1	0.1	0.1	0.05	0.05	0.05	0.05	0.1	0.05	0.05	0.05
	p_1^B	–	0.2	0.1	0.1	–	0.2	0.1	0.05	–	0.2	0.1	0.05	0.1	0.2	0.1	0.05
	q_0^A	1	1	1	1	1	1	1	1	0	0	0	0	1	0	0	0
	q_0^B	2	1	0	0	2	1	0	0	2	1	0	0	0	1	0	0
	q_s	0	0	0	1	0	0	0	0	0	0	0	0	−2	0	0	0
$\delta^A = 0.72$	p_1^A	–	–	–	0.25	0.25	0.25	0.25	0.25	0.15	0.15	0.15	0.15	0.25	0.05	0.05	0.05
	p_1^B	–	0.2	0.1	0.1	–	0.2	0.1	0.05	–	0.2	0.1	0.05	0.1	0.2	0.1	0.05
	q_0^A	2	2	2	1	1	1	1	1	0	0	0	0	1	0	0	0
	q_0^B	2	1	0	0	2	1	0	0	2	1	0	0	0	1	0	0
	q_s	0	0	0	1	0	0	0	0	0	0	0	0	−2	0	0	0

Proof of Proposition 1

Following the same reasoning as in Transchel (2017), consumer will buy the new product if the utility is

$$u_1{}^m \geq 0 \Longrightarrow v - p_0 > 0 \tag{A.1}$$

and

$$u_0 \geq u_1{}^m \Longrightarrow v - p_0 > \delta^m v - p_1{}^m \tag{A.2}$$

If A.1 and A.2 are adjusted

$$v > p_0 \text{ and } v > \frac{p_0 - p_1{}^m}{1 - \delta^m} \tag{A.3}$$

So, when we consider the inequalities in A.3, two cases occur depending on the values of p_0 and p_1.

$$Case i : p_0 < \frac{p_0 - {p_1}^m}{1-\delta^m} \Longrightarrow Consumer\ buys\ the\ new\ product\ if\ v > \frac{p_0 - {p_1}^m}{1-\delta^m}$$

$$Case ii : p_0 > \frac{p_0 - {p_1}^m}{1-\delta^m} \Longrightarrow Consumer\ buys\ the\ new\ product\ if\ v > p_0$$

Consumer will buy the old product if the utility is

$$ {u_1}^m \geq 0 \Longrightarrow \delta^m v - {p_1}^m > 0 \tag{A.4}$$

and

$$u_0 < {u_1}^m \Longrightarrow v - p_0 < \delta^m v - {p_1}^m \tag{A.5}$$

If A.4 and A.5 are adjusted

$v > \frac{{p_1}^m}{\delta^m}$ and $v < \frac{p_0 - {p_1}^m}{1-\delta^m}$ (A.6).

So, consumers will buy the old product if their valuation is $\frac{{p_1}^m}{\delta} < v < \frac{p_0 - {p_1}^m}{1-\delta}$

$$Case i : p_0 < \frac{p_0 - {p_1}^m}{1-\delta^m} \Longrightarrow p_0 > \frac{{p_1}^m}{\delta^m} \Longrightarrow Consumer\ buys\ the\ old\ product\ if\ \frac{{p_1}^m}{\delta^m} < v < \frac{p_0 - {p_1}^m}{1-\delta^m}$$

$$Case ii : p_0 > \frac{p_0 - {p_1}^m}{1-\delta^m} \Longrightarrow p_0 < \frac{{p_1}^m}{\delta^m} \Longrightarrow \frac{{p_1}^m}{\delta^m} > \frac{p_0 - {p_1}^m}{1-\delta^m} \Longrightarrow No\ solution\ for\ the\ condition\ in\ (A.6)$$

Consumer will not buy anything if the utility is:

$u_0 < 0$ and $.{u_1}^m < 0$

If A.4 and A.5 are adjusted

$$v < p_0\ and\ v < \frac{{p_1}^m}{\delta^m} \tag{A.7}$$

So, when we consider the inequalities in A.3 and A.7 two cases occur depending on the values of p_0 and ${p_1}^m$.

$$Case i : p_0 < \frac{p_0 - {p_1}^m}{1-\delta^m} \Longrightarrow p_0 > \frac{{p_1}^m}{\delta^m} \Longrightarrow Consumer\ buys\ nothing\ if\ v < \frac{{p_1}^m}{\delta^m}$$

$$Case ii : p_0 > \frac{p_0 - {p_1}^m}{1-\delta^m} \Longrightarrow p_0 < \frac{{p_1}^m}{\delta^m} \Longrightarrow Consumer\ buys\ nothing\ if\ v < p_0$$

Hence,

$$Case i : p_0 \leq \frac{p_0 - {p_1}^m}{(1-\delta^m)} \Longrightarrow A\ consumer\ will\ buy\ the\ new\ product\ if\ his\ /her\ valuation\ v > \frac{p_0 - {p_1}^m}{(1-\delta^m)} buys\ the\ old\ product\ if\ \frac{{p_1}^m}{\delta^m} < v < \frac{p_0 - p_1}{1-\delta^m}, and\ buys\ nothing\ if\ v < \frac{{p_1}^m}{\delta^m}.$$

$$Case ii : \frac{p_0 - {p_1}^m}{(1-\delta^m)} < p_0 \Longrightarrow A\ consumer\ will\ buy\ the\ new\ product\ if\ his/her\ valuation\ v > p_0, and\ buys\ nothing\ if\ v < p_0.$$

Proof of Proposition 2

Similar to Transchel (2017), considering the valuations of the consumers and the purchase probabilities given in Proposition 1,

if$p_0 \leq \frac{p_0 - {p_1}^m}{(1-\delta^m)}$i) then the valuation of the consumers who prefer to buy the new products satisfy the condition $v > \frac{p_0 - {p_1}^m}{(1-\delta^m)}$ due to Proposition 1. Since $v > \frac{p_0 - {p_1}^m}{(1-\delta^m)} > p_0 > \frac{{p_1}^m}{\delta^m} \Longrightarrow \delta v - {p_1}^m > 0$ and thus, all of these consumers would prefer to buy the old product if the new one is not available, leading to ${\alpha_{01}}^m = 1$.

When $p_0 < \frac{p_0 - {p_1}^m}{(1-\delta^m)}$ then the valuation of consumers who prefer to buy the old products satisfy the condition $\frac{{p_1}^m}{\delta^m} < v < \frac{p_0 - {p_1}^m}{(1-\delta^m)}$ due to Proposition 1. If the old products are not available, consumers with the valuation $v > p_0$ would buy the new products. Thus, conditional on the condition $\frac{{p_1}^m}{\delta^m} < v < \frac{p_0 - {p_1}^m}{(1-\delta^m)}$, the probability of a consumer's valuation to satisfy $v > p_0$ is equal to

$$\alpha_{10} = \left[G\left(\frac{p_0 - {p_1}^m}{(1-\delta^m)}\right) - G(p_0) \right] / \left[G\left(\frac{p_0 - {p_1}^m}{(1-\delta^m)}\right) - G\left(\frac{{p_1}^m}{\delta^m}\right) \right]$$

if $p_0 > \frac{p_0 - {p_1}^m}{(1-\delta^m)}$ii) then the valuation of the consumers who prefer to buy the new products satisfy the condition $v > p_0$ due to Proposition 1. If the new products are not available, consumers with the valuation $v > \frac{{p_1}^m}{\delta^m}$ would buy the old products. Thus, conditional on the condition $p_0 < v$, the probability of a consumer's valuation to satisfy $v > \frac{{p_1}^m}{\delta^m}$ is equal to

$$ {\alpha_{01}}^m = \left[1 - G\left(\frac{{p_1}^m}{\delta^m}\right) \right] / [1 - G(p_0)]$$

In all other cases, there will be no demand shift since there will either be no sales in those cases, or the valuations of the consumers do not satisfy the purchase conditions.

Data availability

No data was used for the research described in the article.

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