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Dynamic inventory control and pricing strategies for perishable products considering both profit and waste

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ABSTRACT

With the increasing sustainability considerations throughout the world, there is an increasing interest in the effective management of perishable products both in the industry and the academia. There is a need to control the inventories, as well as the prices of perishable products in order to increase the profits while minimizing the waste. In this study, we focus on a retailer who sells old and new perishable food products, enabling demand shifts between products based on their prices and consumer behaviors. A bi-objective dynamic programming model is developed to optimize the discounted price, sale price, and order quantity of perishable food products in order to maximize the retailer's profit and minimize food waste. We develop four static and dynamic pricing policies commonly practiced and quantify the advantages of dynamic pricing and price differentiation between old and new products in terms of both profit and waste. Our findings reveal that significant benefits can be obtained when the order quantity and the old product's sale price decisions are given in a dynamic manner by considering the available inventory at hand. Additionally, this research analyzes the results of various weight combinations for profit and waste in the objective function. The findings highlight the significance of waste and sustainability concerns, underline the tradeoff between profit and waste and provide insights to companies to achieve improvements in their system results.

1. Introduction

Food waste is a major problem which includes environmental, economic, social, and ethical implications (de los Mozos et al., 2020). Reducing food waste has the potential to lower greenhouse gas (GHG) emissions by up to 8 % of total global emissions, especially when considering the deforestation linked to food production (Creutzig et al, 2022). In 2022, approximately 1.05 billion tons of food—equivalent to 19 percent of the food available to consumers—were wasted across the retail, food service, and household sectors, averaging 132 kg per capita annually (United Nations Environment Programme, 2024). Meanwhile, in 2023, an estimated 713 to 757 million people, representing 8.9 to 9.4 percent of the global population, were undernourished (FAO et al., 2024).

Perishable products play a significant role in food waste due to their limited shelf life. These products can be characterized based on how their quality is perceived by consumers as they age. Similar to Ferguson and Koenigsberg (2007), our focus is on perishable products whose perceived quality deteriorates over time, and thus old products are valued less by consumers compared to new ones, requiring retailers to implement price differentiation strategies to sell both effectively. This is particularly relevant for perishable food items like vegetables, fruits, and dairy products, which deteriorate over time and eventually become obsolete if not sold within a certain period (Kaya and Bayer, 2020). The demand for perishable goods is strongly influenced by product freshness, which consumers often perceive through expiration dates (Li and Teng, 2018). As willingness to pay decreases throughout the shelf life of perishable food products, implementing discounting strategies for products nearing their expiration date can be an effective approach (Tsiros and Heilman, 2005). Therefore, it is crucial to understand how product freshness (quality) impacts a firm's operational and pricing decisions to optimize inventory management and maximize profitability (Ferguson and Koenigsberg, 2007). The study of Şen (2013) shows the benefits of dynamic pricing applications on the profitability of perishable products by affecting demand. Jing and Chao (2021) emphasize the importance of production planning and inventory management for perishable products.

Retailers have a pivotal effect on food waste. Poor demand

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forecasting, inefficient management and excess stocks cause food waste at the retailer level (Lemaire and Limbourg, 2019). It is estimated that in the EU, 5 % (5 million tons) of total food waste originates from retail and wholesale sectors (Stenmarck et al., 2016). Retailers have a strong incentive to reduce food waste, as it results in financial losses and affects their already narrow profit margins (Cicatiello et al.,2017). A study conducted in Italy revealed that a large retail store wasted 70.6 tons of food annually, valued at nearly €170,000, with the majority consisting of bakery items and fresh produce (Cicatiello et al., 2017).

Beyond the financial implications, food waste has significant environmental consequences, including greenhouse gas emissions from decomposing organic waste in landfills. To address this issue, potential measures include encouraging the sale of food products nearing their expiration date, promoting imperfect food items, enhancing inventory management practices, and facilitating food donations (Eičaitė et al., 2022). Effective inventory control in food supply chains is essential for reducing waste, as it ensures operational efficiency (Pourmohammad-Zia et al., 2021). If the inventory and pricing decisions are not made properly, it causes a high amount of waste and decreases retailers' profits. Effective ordering and pricing strategies for perishable products, including determining optimal replenishment quantities and discounting prices, are critical for maximizing profitability (Chew et al., 2014; Fadda, 2024).

Pricing strategies are essential tools for managing perishable products and reducing waste. Constant pricing cannot effectively align demand with the quality of the remaining inventory (Chen et al., 2018). Many of the world's largest retailers focus on reducing in-store food waste, aiming to achieve Goal 12: Responsible Consumption and Production, one of the 17 Sustainable Development Goals (SDGs) set by the United Nations in 2015. Retailers such as Tesco, Kroger, and Walmart sell products of different ages simultaneously. However, when these products are sold at the same price, customers may prefer the newer products over the older ones, leading to waste of the older inventory. By decreasing the prices of older products, retailers can appeal to price-sensitive customers and encourage their purchase, thereby reducing waste. To address this, many supermarkets employ dynamic pricing strategies to better match supply with demand and minimize spoilage of perishable products that deteriorate over time (Chen et al., 2018). Sanders (2024) also finds that encouraging grocery chains to adopt dynamic pricing strategies could be a more effective approach for regulators aiming to reduce grocery-store waste.

For example, Tesco implemented a multi-stage Clearance Pricing Optimisation system (Kolev et al., 2023). By systematically reducing prices on items nearing expiry, Tesco achieved a 5 % reduction in fresh food waste while simultaneously increasing revenue from reduced-to-clear items (Kolev et al., 2023). Kroger (2024) has set goals of achieving zero hunger and zero waste. To support these objectives, the company employs various strategies, including offering price reductions on perishable foods as their expiration dates approach, making them more affordable and encouraging faster sales. Walmart (2025) reduced operational food loss and waste by 21 % by the end of 2023 compared to a 2016 baseline, while continuing its commitment to a 50 % reduction by 2030. Through initiatives like "Imperfect but Good" in Chile and Mexico, Walmart offers discounted prices on food nearing expiration or with aesthetic imperfections to help reduce food waste.

In this study, we consider a retailer who sells old and new products simultaneously, allowing a demand shift between these products depending on their prices and consumer behaviors. The product under consideration has a two-period lifetime. Products are considered "new" when sold during the period they are produced. Unsold new products deteriorate at the end of the period and transition to "old" in the following period. Old products experience a reduction in perceived quality, resulting in lower consumer valuations. At the end of the period, unsold old products become obsolete and are wasted.

In the retail sector, stockout-based substitution is a common phenomenon where customers substitute their preferred product with an alternative when the preferred product is unavailable. Instead of leaving the store and resulting in lost sales, customers may switch to a different product that is of lower, equal, or even higher quality, as long as the alternative is within their acceptable price range. Ignoring this substitution behavior can lead to significant supply–demand mismatches and adversely impact a retailer's profitability (Transchel, 2017).

Different than most of the literature, we include these behaviors in our model. For example, if consumers' demand for the new (old) products is higher than the available quantity at hand, the excess demand shifts to old (new) products. Our study aims to determine when old and new products should be sold together, and at what prices, and when it would be optimal to sell only one type of these products. To achieve this, we develop a bi-objective dynamic programming model to find the optimal discounted price, sale price and order quantity of the perishable food product to maximize the retailer's profit and minimize food waste. Furthermore, we aim to quantify the tradeoff between profit and waste, offering insights not only for companies but also for policymakers. These findings can help governments design effective incentives or penalties to address food waste in the retail sector.

The remainder of this paper is structured as follows: Section 2 presents a review of the relevant literature, highlighting key contributions and positioning this study within the existing body of work. Section 3 defines the problem, outlines the assumptions underlying the model, and provides the notations used throughout the paper. In Section 4, we introduce the Infinite Horizon Average Cost Dynamic Programming Model. Section 5 discusses the numerical results, including the main findings, sensitivity analyses, and the no-demand-shift case. Finally, Section 6 concludes the paper by summarizing the key insights and suggesting potential directions for future research.

2. Literature Review

Our study investigates joint ordering and pricing decisions for a perishable product with a bi-objective approach, aiming to maximize profit and minimize waste. Accordingly, the literature review focuses on three key areas: inventory control models for perishable products, studies addressing the joint optimization of pricing and inventory decisions for perishables, and the application of multi-objective methods in similar contexts. Perishable products, as defined by Karaesmen et al. (2011), are those that age over time and eventually outdate, requiring strategic inventory and pricing decisions to mitigate waste and optimize profitability. Several studies have explored dynamic inventory management and pricing strategies for perishable products. Key reviews include Nahmias (1982), Karaesmen et al. (2011), and Bakker et al. (2012), which focus on inventory control for perishables, and Elmaghraby and Keskinocak (2003), which address dynamic pricing in inventory management.

Effective inventory control for perishable products is critical for reducing waste and maximizing operational efficiency, making it a key focus area for researchers. Parlar (1985) extends the classical newsvendor problem by introducing a generalized model for perishable products with a two-period lifetime. The study incorporates substitution behavior by assuming that a fixed proportion of unmet demand for old (new) products shifts to new (old) products. This model focuses on determining the optimal ordering policy over an infinite horizon, accounting for stochastic demand and the substitutability of products that perish within two periods. Similarly, Deniz et al. (2010) develop a discrete-time supply chain model for perishable goods with age-differentiated demand. Their study evaluates the performance of heuristic replenishment policies and substitution rules, providing insights into the effectiveness of these strategies for managing perishable inventory.

Minner and Transchel (2010) develop a periodic-review inventory control method to determine dynamic order quantities for perishable food products with limited shelf life, positive lead time, and FIFO/LIFO issuing policies. Coelho and Laporte (2014) examine optimal joint replenishment and inventory decisions for perishable products, optimizing delivery routes and inventory control using a branch-and-cut algorithm. Gioia et al. (2023) adapt a discrete choice model to capture consumer heterogeneity and tradeoffs between price and quality. They use simulation-based optimization to develop ordering rules, aiming to maximize long-term average profit under a lost sales assumption. Chen et al. (2021) explore inventory control for perishables in a periodic-review system with multiple demand classes and freshness requirements. Using an adaptive approximation approach, they minimize total discounted costs, achieving near-optimal performance and outperforming existing heuristics.

While the reviewed studies offer valuable insights into inventory control for perishables, our paper stands out by jointly addressing dynamic pricing and ordering decisions within a bi-objective framework that balances profit maximization and waste minimization.

The following literature focuses on inventory control and pricing strategies for perishable products, highlighting various approaches to optimize profitability and manage product lifecycles effectively. Chintapalli (2015) examines price discounting and inventory management for perishable goods with a two-period lifetime under a periodic review framework. Using a price-dependent linear demand function to maximize profit, the study, like our model, assumes that old and new products are sold simultaneously. Ferguson and Koenigsberg (2007) study a firm's inventory and pricing decisions for a perishable product in the presence of new and old product competition. Chua et al. (2017) investigate optimal discounting and periodic replenishment policies for perishables over a finite horizon using dynamic programming. Fan et al. (2020) develop a dynamic pricing model and a heuristic replenishment policy for multi-batch perishable products, aiming to maximize retailer profit. Duan et al. (2018) explore joint dynamic pricing, production, and inventory decisions for perishables over a short selling season under stochastic demand.

Kaya and Polat (2017), Kaya and Ghahroodi (2018), and Kaya and Bayer (2020) examine inventory control and pricing for perishable products. Unlike these studies, our paper introduces a bi-objective dynamic programming model that considers both profit maximization and waste minimization, analyzing the tradeoffs between them in detail. While Kaya and Polat (2017) use a deterministic approach, Kaya and Ghahroodi (2018) and Kaya and Bayer (2020) explore periodically and continuously reviewed systems, respectively. However, these studies do not allow simultaneous sales of old and new products at different prices. This paper extends their work by incorporating price differentiation for old and new products and models consumer behavior based on price and valuation. Additionally, it considers demand shifts when inventory is insufficient. The paper presents four models with static and dynamic pricing, comparing results to provide managerial insights into sustainability, waste, and profit.

Similar to our work, Fadda et al. (2024) investigate joint discounting and replenishment for perishable products using a linear discrete choice model. They compare a range of policies that combine constant ordering and base-stock approaches with simple discounting strategies, optimizing their parameters through a simulation-based framework. Their findings align with ours, highlighting the effectiveness of age-based discounting in managing inventory and reducing waste. While Fadda et al. (2024) focus on heuristic methods, our study adopts a bi-objective dynamic programming model to explicitly address the trade-offs between profit and waste.

Studies addressing optimal pricing and/or ordering for perishable products with lifetimes exceeding two periods often use heuristics or approximation methods due to solution complexity. Li et al. (2009) explore joint pricing and inventory control for two-period perishable products, extending heuristics to longer shelf lives. Chew et al. (2009) develop a discrete-time dynamic programming model for optimal pricing and inventory in two-period lifetimes, introducing heuristics for longer lifetimes. Chew et al. (2014) focus on multi-period perishables with price-dependent demand, using stochastic dynamic programming and heuristics to maximize profit. Li et al. (2016) propose heuristics for joint replenishment and clearance sales in multi-period lifetimes. Li et al. (2022) examine LIFO-based transshipment policies for perishable goods, deriving optimal strategies for up to three periods and heuristics for longer lifetimes. While these studies provide valuable insights into managing perishable inventory, they do not simultaneously address the dual objectives of profit maximization and waste minimization within a bi-objective framework, nor do they incorporate dynamic strategies that explicitly account for demand shifts between products of different ages.

Several studies in the perishable products literature address waste reduction while aiming to maximize profit. Sanders (2024) evaluates dynamic pricing and organic waste landfill bans as solutions to grocery food waste, using a structural econometric model. The study shows that dynamic pricing reduces waste by 21 % while improving profits and consumer surplus, whereas waste bans achieve only a 4 % reduction in waste while decreasing both. The retailer's decision-making is modeled as a Markov decision process optimizing prices and inventories. Li et al. (2012) explore joint pricing and inventory control for perishable food products, aiming to maximize retailer profit while considering waste costs. Chen et al. (2014) address joint pricing and inventory control for perishable products, where the retailer decides order quantity and pricing for products of different ages in a periodic-review system. They incorporate waste costs and aim to optimize pricing, ordering, and disposal policies to maximize total expected profit. Azadi et al. (2019) propose a two-stage stochastic optimization model for supplier selection, replenishment scheduling for a periodic-review inventory system, and pricing reductions, aiming to maximize profit and minimize waste for perishables. Vahdani and Sazvar (2022) address an online retailer's coordinated dynamic pricing and inventory control under social learning, using a case study from an Iranian online supermarket to analyze the impact on profit and waste. Kayikci et al. (2022) propose a real-time IoT sensor-driven dynamic pricing strategy to enhance profit by reducing perishable food waste.

Some studies, like ours, adopt a bi-objective approach to perishable inventory management. Abbasian et al. (2023) develop a bi-objective model for a resilient and sustainable perishable food supply network, integrating location, inventory, and routing decisions. Their approach incorporates dynamic pricing to mitigate disruptions while minimizing cost and CO_2 emissions. They proposed a hybrid solution method to solve the mixed-integer nonlinear problem. Pilati et al. (2024) propose a bi-objective stochastic optimization model for inventory control focusing on minimizing inventory logistics emissions and costs for perishable products. Their study utilizes the Pareto-based optimization approach as the basis for their multi-objective optimization method. Where our study uses a weighted-sum method to balance profit maximization and waste minimization, explicitly modeling demand shifts and price differentiation for old and new products. As detailed in the Introduction section, minimizing food waste is crucial for both economic efficiency and sustainability, as it reduces financial losses and mitigates greenhouse gas emissions.

Adenso-Díaz et al. (2017), explore the impact of dynamic pricing on revenue and waste reduction across various scenarios. They find that dynamic pricing effectively reduces waste, though its impact on revenue is highly scenario-dependent. Unlike Adenso-Díaz et al. (2017), who use a parametric approach to assess trade-offs between revenue and waste without directly optimizing the bi-objective function, our study employs a weighted-sum bi-objective optimization to explicitly compute Pareto-efficient solutions for profit and waste. However, unlike our study, these papers assume that new and old products are not sold simultaneously.

Table 1 compares this study with key papers on dynamic programming for pricing and/or ordering decisions in perishable product management. This study uniquely examines joint pricing and ordering decisions, incorporates a bi-objective model, explicitly models demand shifts, and addresses both profit maximization and waste minimization under stochastic demand.

This study focuses on a retailer's problem who sells both old and new

Table 1
Comparison of dynamic programming approaches for pricing and/or inventory decisions in perishable product management.

Articles	Infinite or Finite Horizon	Price Discount	Ordering Decision	Demand Function	Sell Different Aged Product Simultaneously	Bi-objective	Demand Shift
This Paper	infinite	✓	✓	stochastic	✓	✓	stockout-based
Chintapalli (2015)	infinite	✓	✓	stochastic	✓		
Chua et al. (2017)	finite	✓	✓	stochastic	✓		
Chew et al. (2014)	finite	✓	✓	stochastic	✓		price-based
Adenso-Díaz et al. (2017)	finite	✓		deterministic		✓	
Li et al. 2012	infinite	✓	✓	stochastic			
Chen et al. (2014)	both	✓	✓	stochastic	✓		
Vahdani and Sazvar (2022)	finite		✓	stochastic			

perishable products simultaneously, considering that each product has a two-period lifetime and allowing a demand shift between products of different ages. Table 1 highlights that different from the literature, we do not only focus on the profit of the company, but also consider the generated amount of waste as a result of the decisions. We analyze the tradeoff between the profit and the waste and quantify the changes in profit to decrease the waste. The objective is to determine optimal sale prices and order quantities that maximize the retailer's profit while minimizing food waste. The main contributions of this study to the literature are as follows:

1. We develop four different pricing strategies and determine the optimal dynamic pricing and ordering strategies for perishable products under stochastic demand, considering both the profit and waste as the retailer's objectives.
2. We allow old and new products to be sold at the same time with different prices and model the consumer purchase behavior between old and new products, considering demand shift between them depending on their prices.
3. We develop a bi-objective dynamic programming model using the weighted-sum method to analyze the results for different combinations of the weights for the profit and the waste.
4. We quantify the changes in profit as a result of the changes in the importance of waste for the company.
5. We compare the results of four different static and dynamic pricing policies commonly used in reality and quantify the benefits of dynamic pricing and price differentiation between old and new products, in terms of both profit and waste.

3. Problem Definition

In this study, similar to Chintapalli (2015), we consider that the product has a two-period lifetime, which can be interpreted as two days, two weeks, two months, or any comparable timeframe depending on the specific product. From the day the product is produced until the end of the first period (the first half of its lifetime), it is considered 'new.' Products in the second half of their lifetime are classified as 'old'. Unsold old products at the end of the period are considered obsolete and become waste. To represent this, we define the binary parameter where $i = 0$ denotes new products and $i = 1$ denotes old products. We assume a lead time of zero, meaning that ordered products are delivered immediately with their full shelf life remaining. We develop a dynamic pricing and ordering model for perishable food products and at every point in time, the state of the system denotes the quantity of old products at hand, which is represented as q_1.

The objective of our models is to maximize the retailer's total profit and minimizing waste by jointly determining the optimal order quantities and pricing strategies for new and old products. At the beginning of each period, the retailer decides the order quantity of new products, q_0, and the pricing strategies for old and new products. At the end of each period, the retailer carries the remaining new products to the next period with the inventory holding cost, h, and unsold old products become waste. Moreover, c is the unit ordering cost per product.

It is important to differentiate perishable products based on their age, as products of different ages can attract various market segments (Chew et al., 2014). By implementing price differentiation based on product age, businesses can boost their profitability (Chew et al., 2014) and sustainability performance. Therefore, we develop four pricing and ordering strategy models as given in Table 2. In Model 1, the price of the old product (p_1) and the price of the new product (p_0) are static and their optimal values are determined by our model. The order quantity is dynamic in all models and their optimal values are found by using dynamic programming. In Model 2, we consider that p_0 is static and we use dynamic programming to find optimal dynamic values for p_1 and q_0. Under Model 3, we consider that the values of p_1 and p_0 are dynamic and the same (no price differentiation), and we use dynamic programming to find the optimal dynamic values of p and q_0. In Model 4, all p_1, p_0 and q_0 values are found dynamically.

For products of different ages, it is important to model the consumer choice models (Chen at al 2014). Therefore, we use consumer utility-based demand models, similar to Tirole (1988) and Transchel (2017). We let v define the consumer's valuation and θ_i define the probability of consumers to purchase product i. G(.) and g(.) are the cumulative distribution function and density function of v, respectively. δ denotes the rate of decrease in the valuation of the product when its age gets older. Also, old product's price, p_1, is assumed to be always less than or equal to the new product's price, p_0. All notations are summarized in Table 3.

We let $u_0 = v - p_0$ and $u_1 = \delta v - p_1$ denote the utility function of consumers when they buy the new product and the old product, respectively. A consumer buys the new product if and only if $u_0 \geq 0 and u_0 \geq u_1$, and buys the old product if and only if $u_1 \geq 0 and u_0 \leq u_1$. Note that a consumer prefers the new product over the old one if $u_0 \geq u_1 \Rightarrow v - p_0 \geq \delta v - p_1 \Rightarrow v \geq \frac{p0 - p1}{(1-\delta)}$.

Proposition 1 states the possible situations for the consumer's purchasing decision depending on the sale prices and the consumer's valuation. These situations occur based on the different values of the prices. Proposition 1 is derived by adjusting the notations from Transchel (2017) to align with those used in this study. We establish a direct correspondence between Proposition 1 in Transchel (2017) and our Proposition 1. Readers interested in the detailed proof can refer to the proof of Proposition 1 in Transchel (2017).

Proposition 1. *The purchase probabilities of the consumers under the*

Table 2
Four pricing and ordering strategy models.

	p_0	p_1	q_0
Model 1	static	static	dynamic
Model 2	static	dynamic	dynamic
Model 3	dynamic ($p_0 = p_1$)	dynamic ($p_0 = p_1$)	dynamic
Model 4	dynamic	dynamic	dynamic

Table 3
Notations.

Parameters	Definition
N	Market size of the product
c	Ordering cost per product
h	Inventory holding cost
s	Waste cost
α_{ii}	Stockout-based substitution probability of consumer from type i product to type j product
v	Consumer's valuation
G(.)	Cumulative distribution function of consumer's valuation
g(.)	Density function of consumer's valuation
δ	Rate of decrease in the valuation of the old product, $0 \leq \delta < 1$
θ_i	Probability of a consumer to purchase product i, $i = 0, 1$
θ_2	Probability of a consumer purchasing nothing
u_i	Utility function of consumers when product i is purchased
State of the System	***Definition***
q_1	Quantity of old product at hand
Decision variables	***Definition***
q_0	Order amount of new product
p_0	New product's sale price
p_1	Discounted price (Old product's sale price)

given prices are as follows:

$$if\,\mathrm{p}_0 < \frac{\mathrm{p}_0\text{-}\ \mathrm{p}_1}{(1-\delta)} \tag{1}$$

Consumers will buy new products with the probability of $\theta_0 = 1 - G\left(\frac{\mathrm{p0\text{-}p1}}{1\text{-}\delta}\right)$

Consumers will buy old products with the probability of $.\theta_1 = G\left(\frac{\mathrm{p0\text{-}p1}}{1\text{-}\delta}\right) - G(\frac{\mathrm{p1}}{\delta})$

Consumers will not buy anything with the probability of $\theta_2 = G(\frac{\mathrm{p1}}{\delta})$

$$if\,\frac{\mathrm{p}_0\text{-}\mathrm{p}_1}{1\text{-}\delta} \leq \mathrm{p}_0 \tag{2}$$

Consumers will buy new products with the probability of $\theta_0 = 1 - G(\mathrm{p}_0)$

Consumers will not buy anything with the probability of $.\theta_2 = G(\mathrm{p}_0)$

i There will be no sales in all other cases.

For *situation i,* if the old product's price is low enough compared to the new product price ($p_1 < \delta p_0$), consumers with a higher valuation of the product prefer to buy the new product, while consumers who value the product a little lower buy the old product, and consumers who value the product lowest buy nothing. For *situation ii,* since the old product's price is too high ($p_1 \geq \delta p_0$) none of the consumers will buy the old product. High valuation consumers prefer to buy the new product while low valuation consumers prefer not to buy anything.

Similar to Kaya (2010) and Transchel (2017), if the demand for the new products exceeds the quantity of new products at hand, q_0, but consumer's net utility from the old product is positive then α_{01} of excess demand consumers buy old products. Similarly, if the demand for the old products exceeds the quantity of old products at hand, q_1, but consumer's net utility from the new product is positive then α_{10} portion of excess demand consumers for the old products buy new products. Similar to Transchel (2017), we consider endogenous demand shift rates (stockout-based substitution rates).

In our model, we let N denote the total market size for the product. D_i is the demand of product i, such that demand of the new product and the old product are D_0 and D_1, respectively. N-D_0-D_1 of consumers buy nothing and is represented as D_2 ($\sum D_i = N$). Note that the demand values D_i, follow a multinomial distribution with parameters, N and θ_i : $D_0, D_1, D_2\ Mult(N, \theta_0, \theta_1, \theta_2)$. We denote the demand shift from the new product to the old product as D_{01} and from the old product to the new product as D_{10}, both of which follow a binomial distribution as follows: $D_{01}\ B((D_0 - q_0)^+, \alpha_{01})$, and $D_{10}\ B((D_1 - q_1)^+, \alpha_{10})$. Hence, the total demand of new and old products are given as $D_0 + D_{10}$, and $D_1 + D_{01}$, respectively.

Similarly, Proposition 2 corresponds to Proposition 2 in Transchel (2017). By adjusting the notations in Transchel (2017) to match ours, we derive the conditions for our Proposition 2. Readers seeking the detailed proof are directed to the proof of Proposition 2 in Transchel (2017).

Proposition 2. *The stockout-based substitution probabilities of consumers from the new to the old products (α_{01}) and from the old to the new products (α_{10}) are given below depending on the distribution function of consumer valuation G(.), and prices of old and new products:*

$$if\,\mathrm{p}_0 < \frac{\mathrm{p}_0\text{-}\mathrm{p}_1}{1\text{-}\delta} \tag{3}$$

$$\alpha_{01} = 1$$

$$\alpha_{10} = \left[G\left(\frac{\mathrm{p}_0\text{-}\mathrm{p}_1}{1\text{-}\delta}\right) - G(\mathrm{p}_0)\right] \Big/ \left[G\left(\frac{\mathrm{p}_0\text{-}\mathrm{p}_1}{1\text{-}\delta}\right) - G\left(\frac{\mathrm{p}_1}{\delta}\right)\right]$$

$$if\,\frac{\mathrm{p}_0\text{-}\mathrm{p}_1}{1\text{-}\delta} \leq \mathrm{p}_0 \tag{4}$$

$$\alpha_{01} = \left[1 - G\left(\frac{\mathrm{p}_1}{\delta}\right)\right] \Big/ [1 - G(\mathrm{p}_0)]$$

$$\alpha_{10} = 0$$

ii There will be no demand shift in all other cases.

For *situation i,* if the old product's price is low enough compared to the new product price ($p_1 < \delta p_0$), the substitution probability from the new product to the old product (α_{01}) is equal to 1, meaning all consumers who cannot purchase the new product due to stockout will prefer the old product. The substitution probability from the old product to the new product (α_{10}) depends on the distribution of consumer valuations and is given by the specified formula. For *situation ii,* since the old product's price is too high ($p_1 \geq \delta p_0$) and the initial demand for the old product is zero, substitution probability from the old product to the new product (α_{10}) is equal to 0. However, if there is an excess demand for new products substitution probability from the new product to the old product is α_{01}.

4. Infinite horizon average cost dynamic programming Model

We consider a single product that is decoupled into two products by age and develop a bi-objective dynamic programming model. We use the weighted-sum method in order to maximize the average profit and minimize the average waste. ***w*** is the weight factor for the profit function where *(1-w)* is the weight factor for the waste. We develop an infinite horizon average cost dynamic programming formulation. In the model, we consider an infinite horizon case and the state of the system is denoted by the number of old products at hand *(q_1)*. According to the state of the system, the decisions of the order amount of the new product, q_0, the old product's sale price, p_1, and/or the new product's sale price, p_0 are given. *$V(q_1)$* denotes the relative value function for state q_1 in the dynamic programming model. It represents the differential cost between a given state and a reference state.

We note that, even though we consider a system in which old and new products are allowed to be sold at the same time, our model results will provide whether it is optimal to sell both types of products at the same time at different prices, or to sell only one type of these products. For example, if the optimal value of q_0 turns out to be 0 in the optimal solution, it means that only old products are to be sold at that period, and no new product is ordered. Similarly, if it turns out that $p_1 = p_0$ in the optimal solution, it means that only new products will be sold due to consumer choices, and old products can only be sold if the new ones are depleted.

4.1. Model 1: Static p_0 and p_1, dynamic q_0

In this section, we consider a model in which p_0 and p_1 are static variables which means that sale prices do not change based on the state of the system (q_1), while the optimal order quantities, q_0, are dynamically decided. We use exhaustive search to find the optimal p_0 and p_1, by running the dynamic programming model for the possible values of sale prices and chose the optimal values which optimize our objective function. In other words, the retailer decides the optimal sale prices in advance and do not offer any discounts based on the old product inventory, and at the beginning of each period decides the order quantity in a dynamic manner, based on the quantity of the old product at hand, q_1.

$\lambda(p_0, p_1)$ is the weighted-sum objective function of the problem for given values of p_0 and p_1, where the Bellman's equation for the dynamic programming formulation of our problem to determine the optimal order quantities is given below. For clarity, $[\,\cdot\,]^+$ denotes the positive part of the argument (i.e., max(0, ·)), and E[·] represents the expected value of the random variable within the brackets.

$$\underset{p_0, p_1}{\text{Max}}\lambda(p_0, p_1)$$

s.t.

$$\lambda(p_0,p_1)+V(q_1)=\max_{q_0}E\left[\begin{array}{c} w\left(\begin{array}{c}-cq_0+p_1 min(D_1+D_{01},q_1)\\ +p_0 min(D_0+D_{10},q_0)-h[q_0-(D_0+D_{10})]^+\end{array}\right)\\ -(1-w)s[q_1-(D_1+D_{01})]^+ + V([q_0-(D_0+D_{10})]^+)\end{array}\right] \tag{5}$$

In the above equation, $-cq_0$ is the total ordering cost of the new product. Since we do not allow backlogging, the amount of sales is the minimum of the demand or the quantity at hand, which is denoted as $min(D_1 + D_{01}, q_1)$ for the old product. In here, $D_1 + D_{01}$ represent the old product's total demand, D_1 is demand of old product and D_{01} demonstrates the demand shift from new product to old product. $p_0 min(D_0 + D_{10}, q_0)$ denotes the revenue from the sale of new products. If the quantity of the new product exceeds its demand, then the excessive quantity will be moved to the next period with the inventory holding cost, h, which is represented in the model as $h[q_0 - (D_0 + D_{10})]^+$. The old products which are not sold at the end of their lifetime, will be wasted and $[q_1 - (D_1 + D_{01})]^+$ represents the waste quantity, and s denotes the cost of this waste. The excessive quantity of the new product will be the new state of the system, and it will be updated as $[q_0 - (D_0 + D_{10})]^+$. Therefore, the future relative value function will be $V([q_0 - (D_0 + D_{10})]^+)$.

Observe that the dynamic programming formulation stated above aims to maximize the weighted function of the total profit and the amount of waste. We also develop dynamic programming formulations to determine the resulting values of average profit, λ_P, and the average waste, λ_W, separately. We use the optimal decision variables ($p_0{}^*, p_1{}^*$ $and q_0{}^*$) obtained by solving Eq. (5). The dynamic programming models for the average profit and the average waste are given in Eqs. (6) and (7), respectively.

$$\lambda_P + V_P(q_1) = E\left[\begin{array}{c}-cq_0{}^* + p_1{}^* min(D_1 + D_{01}, q_1) + p_0{}^* min(D_0 + D_{10}, q_0{}^*)\\ -h[q_0{}^* - (D_0 + D_{10})]^+ + V_P([q_0{}^* - (D_0 + D_{10})]^+)\end{array}\right] \tag{6}$$

$$\lambda_W + V_W(q_1) = E[s[q_1 - (D_1 + D_{01})]^+ + V_W([q_0{}^* - (D_0 + D_{10})]^+)] \tag{7}$$

$V_P(q_1)$ is the relative value function for profit and $-cq_0{}^* + p_1{}^* min(D_1 + D_{01}, q_1) + p_0{}^* min(D_0 + D_{10}, q_0{}^*)$ denotes the profit for the current period. Remaining quantity of the new product will be the new state of the system in the next period, and it will be updated as $[q_0{}^* - (D_0 + D_{10})]^+$. Therefore, future relative value function will be $V([q_0{}^* - (D_0 + D_{10})]^+)$.

$V_W(q_1)$ is relative value function for waste and $[q_1 - (D_1 + D_{01})]^+$ denotes the quantity of waste. The old products that are not sold during the period will reach the end of their lifetime, and thus cannot be sold anymore and will be wasted.

4.2. Model 2: Static p_0, dynamic p_1 and q_0

In this model, we consider that p_0 is a static variable which means it does not change based on the state of the system, while p_1 and q_0 are dynamically decided. Like Model 1, we use exhaustive search to find the optimal p_0 by running the dynamic programming model for all possible values of it and chose the optimal values which optimize our objective function. In other words, the retailer sets a static p_0 for the new product and decides p_1 and q_0 at the beginning of each period by considering q_1.

$\lambda(p_0)$ is the weighted-sum objective function of the problem for given values of p_0, where the Bellman's equation for the dynamic programming formulation of our problem to determine the optimal order quantities and old products' sale prices is given below:

$$\underset{p_0}{\text{Max}}\lambda(p_0)$$

s.t.

$$\lambda(p_0)+V(q_1)=\max_{p_1,q_0}E\left[\begin{array}{c} w\left(\begin{array}{c}-cq_0+p_1 min(D_1+D_{01},q_1)\\ +p_0 min(D_0+D_{10},q_0)-h[q_0-(D_0+D_{10})]^+\end{array}\right)\\ -(1-w)s[q_1-(D_1+D_{01})]^+ + V([q_0-(D_0+D_{10})]^+)\end{array}\right] \tag{8}$$

Terms in Bellman's Eq. (8) are similar to those in (5). Like Model 1, we also develop dynamic programming formulations to determine the resulting values of average profit, λ_P, and the average waste, λ_W, separately. We use the optimal decision variables ($p_0{}^*, p_1{}^* and q_0{}^*$) obtained by solving Eq. (8).

4.3. Model 3: Dynamic ($p_0 = p_1$) and q_0

In Model 3, we assume that old and new products' prices are equal which means that there will be no discount for old products and the retailer charges the same sale price for old and new products. The retailer gives the decision of q_0 and the products price, p, dynamically at the beginning of each period based on the state of the system, q_1. In this scenario, since $p_1 = p_0 = p$ *Proposition 1 (ii)* will occur and none of the consumers prefer to buy old product. In other words, there will be no demand for old product because consumers will not be willing to buy the old product when its price is the same as new product. However, in this scenario we let the demand shift between old and new products. Hence, even though $p_1 = p_0 = p$ the old product can still be sold because α_{01} may be positive, as indicated by *Proposition 2(ii)*, while α_{10} will be equal to 0. λ is the weighted-sum objective function of the problem and the Bellman's for the dynamic programming formulation of our problem to determine the optimal order quantities and sale price is given below:

Maxλ

s.t.

$$\lambda + V(q_1) = \max_{p,q_0} E\left[\begin{array}{c} w\left(\begin{array}{c} -cq_0 + p\min(D_1 + D_{01}, q_1) \\ + p\min(D_0 + D_{10}, q_0) - h[q_0 - (D_0 + D_{10})]^+ \end{array}\right) \\ -(1-w)s[q_1 - (D_1 + D_{01})]^+ + V([q_0 - (D_0 + D_{10})]^+) \end{array}\right] \tag{9}$$

Terms in Bellman's Eq. (9) are similar to those in (5). Similar to Model 1, we also develop dynamic programming formulations to determine the resulting values of average profit, λ_P, and the average waste, λ_W, separately. We use the optimal decision variables ($p^* and q_0{}^*$) obtained by solving Eq. (9).

4.4. Model 4: Dynamic p_1, p_0 and q_0

In Model 4, retailer decides the order amount of new product, q_0, old product's sale price, p_1, and new product's sale price, p_0, based on the state of the system, q_1.

λ is the weighted-sum objective function of the problem and the Bellman's for the dynamic programming formulation of our problem to determine the optimal order quantities, and old and new products sale price is given below:

Maxλ

s.t.

$$\lambda + V(q_1) = \max_{p_0,p_1,q_0} E\left[\begin{array}{c} w\left(\begin{array}{c} -cq_0 + p_1\min(D_1 + D_{01}, q_1) \\ + p_0\min(D_0 + D_{10}, q_0) - h[q_0 - (D_0 + D_{10})]^+ \end{array}\right) \\ -(1-w)s[q_1 - (D_1 + D_{01})]^+ + V([q_0 - (D_0 + D_{10})]^+) \end{array}\right] \tag{10}$$

Terms in Bellman's Eq. (10) are similar to those in (5).

Similar to Model 1, we also develop dynamic programming formulations to determine the resulting values of average profit, λ_P, and the average waste, λ_W, separately. We use the optimal decision variables ($p_0{}^*, p_1{}^* and q_0{}^*$) obtained by solving Eq. (10).

5. Numerical Results

We provide detailed numerical results and managerial insights for the analyzed problem in this section. For each model, we analyze the optimal order amount and the price decisions which maximize the stated objective function in the previous sections, and we compare the results of the models. In our numerical studies we consider a bakery product (bagel, donut, croissant or cake etc.), as an example which has a two-day lifetime. In our base case, we use $N = 10$ which is a typical scenario for a small to medium-sized bakery section in a grocery store since the demand for a specific product is smaller. We let $c = 0.2$, $h = 0.002$, $s = 1$ and $\delta = 0.6$. Please note that, unit measures for costs and prices are product-dependent and can be scaled accordingly. We suppose that consumer valuation follows the distribution $G(v) = 1 - \frac{(\bar{v}-v)^b}{\bar{v}^b}$ where $b > 0$ and with support of $[0, \bar{v}]$. This is a commonly used distribution to model the consumer preference; see, for example, Debo et al (2005), Pan and Honhon (2012) and Transchel (2017). The uniform distribution commonly utilized in market segmentation literature (Jerath et al., 2010; Dong and Wu, 2019; Shen et al., 2022) is a special case of this distribution, achieved by setting the $b = 1$ (Debo et al., 2005). We use $b = 1$, such that v follows a uniform distribution between $[0, \bar{v}]$ (we assume $\bar{v} = 1$ in our numerical analysis). To analyze the tradeoff between the objective functions of profit and waste, we use varying weight parameters between 0 and 1, with 0.1 increments, $w = (0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1)$. Also, since $\bar{v} = 1$, we let the possible sale prices vary between 0 and 1, with 0.05 increments.

Numerical results are obtained by using 3.40 GHz Intel i7-13700KF server with 32 GB 5200 MHz DDR5 RAM. To solve the problem and find the optimal solutions, we use the relative value iteration algorithm for our infinite horizon average cost dynamic programming model, as explained in Bertsekas (2005). We use the Python programming language to implement the algorithm and obtain our numerical results.

In all our formulations, at least one of the states (for example the state $q_1 = 0$) is visited with positive probability at least once within the first m stages for some integer $m > 0$, for all initial states and for all policies. Thus, Assumption 7.4.1 in Bertsekas (2005) is satisfied and as stated in Proposition 7.4.1 in Bertsekas (2005), the optimal average cost λ^* is the same for all initial states and together with some vector V^* in our formulations, satisfies Bellman's Eqs. (5), (8)–(10) in our models. Then, the relative value iteration algorithm for the average cost per stage formulations, as explained in Bertsekas (2005, Chapter 7, pg.430–432), provides the optimal solution. In this algorithm, we let $V_k(q_1) = J_k(q_1) - J_k(r)$ where r is the reference state (we use $r = 0$), $J_k(q_1)$ is the optimal k-stage cost for $k = 1,2,\ldots$, and can be calculated through the recursion below for Model 1. We note that similar recursions can be used for Models 2, 3 and 4.

$$J_{k+1}(q_1) = \max_{q_0} E\left[\begin{array}{c} w\left(\begin{array}{c} -cq_0 + p_1\min(d_1 + D_{01}, q_1) \\ + p_0\min(D_0 + D_{10}, q_0) - h[q_0 - (D_0 + D_{10})]^+ \end{array}\right) \\ -(1-w)s[q_1 - (D_1 + D_{01})]^+ + J_k([q_0 - (D_0 + D_{10})]^+) \end{array}\right] \tag{11}$$

We use the following variant of the relative value iteration algorithm, which guarantees the convergence under Assumption 7.4.1, as

Table 4
Optimal decisions of Model 1.

	p_0	p_1		$q_1 = 0$	$q_1 = 1$	$q_1 = 2$	$q_1 = 3$	$q_1 = 4$	$q_1 = 5$	$q_1 = 6$	$q_1 = 7$	$q_1 = 8$	$q_1 = 9$	$q_1 = 10$	**profit**	**waste**
$w = 0$	0.05	0.05	q_0	0	0	0	0	0	0	0	0	0	0	0	0	0
$w = 0.1$	0.55	0.35	q_0	4	1	0	0	0	0	0	0	0	0	0	1.085	0.011
$w = 0.2$	0.55	0.35	q_0	4	1	0	0	0	0	0	0	0	0	0	1.085	0.011
$w = 0.3$	0.55	0.35	q_0	4	2	1	0	0	0	0	0	0	0	0	1.144	0.029
$w = 0.4$	0.55	0.35	q_0	4	2	1	0	0	0	0	0	0	0	0	1.144	0.029
$w = 0.5$	0.55	0.35	q_0	5	3	1	0	0	0	0	0	0	0	0	1.208	0.084
$w = 0.6$	0.55	0.35	q_0	5	3	2	1	0	0	0	0	0	0	0	1.248	0.129
$w = 0.7$	0.55	0.35	q_0	5	3	2	1	0	0	0	0	0	0	0	1.248	0.129
$w = 0.8$	0.55	0.35	q_0	5	4	3	2	1	0	0	0	0	0	0	1.299	0.265
$w = 0.9$	0.55	0.35	q_0	5	4	3	3	2	1	0	0	0	0	0	1.305	0.310
$w = 1$	0.6	0.4	q_0	5	4	4	3	3	3	3	3	3	3	3	1.330	0.636

stated in Bertsekas (2005), for any scalar τ, $0 < \tau < 1$.

$$V_{k+1}(q_1) = (1-\tau)V_k(q_1) + \max_{q_0} E\left[\begin{array}{c} w\left(\begin{array}{c} -cq_0 + p_1 \min(D_1 + D_{01}, q_1) \\ +p_0 \min(D_0 + D_{10}, q_0) - h[q_0 - (D_0 + D_{10})]^+ \end{array} \right) \\ -(1-w)s[q_1 - (D_1 + D_{01})]^+ + \tau V_k([q_0 - (D_0 + D_{10})]^+) \end{array} \right] - \max_{q_0} E\left[\begin{array}{c} w\left(\begin{array}{c} -cq_0 + p_1 \min(D_1 + D_{01}, r) \\ +p_0 \min(D_0 + D_{10}, q_0) - h[q_0 - (D_0 + D_{10})]^+ \end{array} \right) \\ -(1-w)s[r - (D_1 + D_{01})]^+ + \tau V_k([q_0 - (D_0 + D_{10})]^+) \end{array} \right] \tag{12}$$

We note that even the single-period price-setting Newsvendor model is not necessarily quasi-concave in price and inventory decisions and because of this issue, an analytical solution or the properties of the optimal solutions could not be found in a structured analytical manner. We analyze the optimal solutions using the results of the numerical experiments where we solve the stochastic dynamic programming formulations using the relative value iteration algorithm as explained above. In this analysis, due to non-concavity issues, we employ a grid search to determine the optimal values of the decision variables, which inherently makes the results grid-dependent. However, similar discrete optimization approaches are used in practice. For example, Kolev et al. (2023) optimize price reductions by selecting from a set of predefined reductions to reduce waste and maximize profit across all Tesco stores in the UK. In our models, the prices, p_0 and p_1 ($p_1 \leq p_0$), range between 0 and 1, and we use 0.05 increments between 0 and 1 for possible prices. Similarly, the optimal order quantity, q_0, lies between 0 and N. Thus, we use an exhaustive grid search for the optimal decision variables in these ranges in the relative value iteration algorithm and overcome the non-concavity issues in the model through this approach.

For each of the four models, in the algorithm $V_{k+1}(q_1)$ is calculated for all (q_1) values on the left-hand side of the above equations by using the $V_k(q_1)$ on the right-hand side for all $k = 0, 1, 2, \ldots$ until all $V_k(q_1)$ values converge to some vector V, which is explained in detail by Bertsekas (2005), where $V_0(q_1) = 0$ and $V_k(0) = 0$.

5.1. Results of Model 1

Table 4 presents the optimal price and order quantity decisions under the state of the system (quantity of old product) and the different weight factors. CPU time to obtain all results that are given in Table 4 is in total 4.1 h. We find that the optimal order quantity decreases monotonically in the quantity of the old products at hand, and no new product is ordered when there are two objectives $w \in (0, 0.9]$ and q_1 is

Table 5
Optimal decisions of Model 2.

			$q_1 = 0$	$q_1 = 1$	$q_1 = 2$	$q_1 = 3$	$q_1 = 4$	$q_1 = 5$	$q_1 = 6$	$q_1 = 7$	$q_1 = 8$	$q_1 = 9$	$q_1 = 10$	**profit**	**waste**
$w = 0$	$p_0 = 0.05$	p_1	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0
		q_0	0	0	0	0	0	0	0	0	0	0	0		
$w = 0.1$	$p_0 = 0.6$	p_1	–	0.25	0.2	0.2	0.15	0.1	0.1	0.05	0.05	0.05	0.05	1.247	0.001
		q_0	5	4	3	3	2	2	1	1	1	0	0		
$w = 0.2$	$p_0 = 0.6$	p_1	–	0.25	0.25	0.2	0.2	0.15	0.1	0.05	0.05	0.05	0.05	1.269	0.005
		q_0	5	4	3	3	2	2	1	1	1	0	0		
$w = 0.3$	$p_0 = 0.6$	p_1	–	0.25	0.25	0.2	0.2	0.15	0.15	0.1	0.05	0.05	0.05	1.269	0.005
		q_0	5	4	3	3	2	2	1	1	1	0	0		
$w = 0.4$	$p_0 = 0.6$	p_1	–	0.25	0.25	0.25	0.2	0.15	0.15	0.1	0.05	0.05	0.05	1.283	0.014
		q_0	5	4	3	3	2	2	1	1	1	0	0		
$w = 0.5$	$p_0 = 0.6$	p_1	–	0.3	0.25	0.25	0.2	0.2	0.15	0.1	0.1	0.05	0.05	1.297	0.025
		q_0	5	4	3	3	2	1	1	1	0	0	0		
$w = 0.6$	$p_0 = 0.6$	p_1	–	0.3	0.25	0.25	0.25	0.2	0.2	0.15	0.1	0.05	0.05	1.301	0.031
		q_0	5	4	3	3	2	1	1	1	1	0	0		
$w = 0.7$	$p_0 = 0.6$	p_1	–	0.3	0.3	0.25	0.25	0.25	0.2	0.2	0.15	0.1	0.1	1.319	0.065
		q_0	5	4	3	3	2	1	1	0	0	0	0		
$w = 0.8$	$p_0 = 0.6$	p_1	–	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.2	0.2	0.15	1.330	0.106
		q_0	5	4	3	3	2	2	1	0	0	0	0		
$w = 0.9$	$p_0 = 0.6$	p_1	–	0.3	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	1.339	0.152
		q_0	5	4	4	3	2	2	1	1	1	0	0		
$w = 1$	$p_0 = 0.6$	p_1	–	0.35	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.347	0.405
		q_0	5	4	4	3	3	2	2	2	2	2	2		

high enough, in order to be able to sell the old ones at hand and decrease the waste. In those cases, only old products are sold in the system. However, when $w = 1$ the only objective is maximizing the profit and new products are ordered even for extreme values of q_1. In this model, the difference in the optimal new and old products' sale prices is low (*Proposition 1 (ii)*), and the company focuses on the sale of new products. In this model, only new products are sold due to consumer choices, and old products can only be sold if the new ones are depleted.

In addition, as we can see from Table 4, as w increases, the cost of waste becomes less important, and the optimal prices tend to increase. Table 4 indicates how average profit and average waste changes based on w. As expected, profit and waste increases as w increases. In the extreme case, when $w = 0$ (i.e. the objective is purely minimizing waste), since profit is not in the objective function, the optimal solution is to close the business in order to eliminate waste, and the model tends to order nothing and waste is decreased to 0. In the other extreme, when $w = 1$ (i.e. profit is the only term in the objective function), the company can generate an average profit of 1.330 by generating an average waste of 0.636. As the companies become more environmentally conscious (or forced by the governments through incentive or penalty mechanisms), and not only consider the profit, but also put some weight on the amount of waste they generate, they can decrease their waste at the cost of some decrease in their profits. It is observed that the waste can be decreased by almost 87 %, from 0.636 to 0.084, without compromising from the profit that much (profit decreases by around 9 % from 1.33 to 1.208), when w is set to 0.5. Another choice of $w = 0.7$ leads to about 80 % decrease in waste, as opposed to only 6 % decrease in profit. Depending on the importance of waste, sustainability and the environmental concerns, companies can choose a suitable level of w for themselves, and improve their system results significantly as a whole.

5.2. Results of Model 2

In this section, a static price for the optimal p_0 is determined, while optimal p_1 and q_0 is set in a dynamic manner and changes based on the changing value of q_1. Therefore, computational complexity grows and CPU time to obtain all results that are given in Table 5 is in total 7.4 h which is higher than Model 1. From Table 5, it can be observed that as q_1 increases, both p_1 and q_0 decrease monotonically. This strategy helps manage the inventory balance and minimize potential waste and maximize profit by encouraging sales of old products before they expire.

Similar to Model 1, Table 5 indicates that in Model 2 the optimal value for p_0 is consistent across all w values; however, it is higher, i.e., p_0*=0.6. This can be explained as follows, when p_1 is dynamic, retailer can set a higher p_0 and then decide p_1 based on the quantity of the old product at hand, q_1. Hence, higher profit and lower waste can be obtained. However, when p_1 is static, it cannot be adjusted based on q_1; therefore, in order to not cause waste, retailer sets lower p_0 from the very beginning. However, this causes lower profit.

From Table 5 we can observe that the decision of p_1 depends on q_1. When q_1 surpasses a threshold, the retailer employs a discount strategy by setting p_1 according to $p_1 < p_0 + \delta$-1, resulting in zero initial demand for the new product, in order to decrease waste as much as possible, especially when w is smaller (i.e. waste is more important). However, since demand shift is permitted, any excess demand from the old product can be shifted to the new product. In Table 5 we observe a threshold pricing policy. Let T denote the threshold level.

- If $q_1 \in [0, T]$ then retailer charges moderate p_1, $p_0 + \delta - 1 \leq p_1 < \delta p_0$, such that consumers can give the following three decisions; whether to buy the old product, the new product or leave the system without any purchase. Yellow highlighted cells in Table 5 indicate this policy.
- If $q_1 \in (T, 10]$, retailer's best strategy is to set too low p_1, $p_1 < p_0 + \delta - 1$ and none of the consumers will purchase new products. Grey highlighted cells in Table 5 indicate this policy.

These thresholds are lower for the lower w values. Which means that even though the retailer has more of the old products at hand, optimal p_1 is higher when profit maximization is more important. Also, from Table 5 it can be seen that the optimal order quantity, q_0*, decreases when w is lower. This is realistic because when the waste minimization becomes more important retailer tends to decrease optimal q_0 to avoid overstocking and focuses on selling old products at hand by decreasing the optimal p_1.

From Table 5, we can observe how average profit and average waste change based on w. Similar to Model 1, as w increases waste and profit both increase. For example, when we compare the results of $w = 0.3$ and $w = 0.5$, it can be seen that average profit decreases moderately; however, average waste decreases dramatically. Therefore, a suitable choice of w becomes critical for managers to balance the profit and waste of the company.

Table 6
Optimal decisions of Model 3.

		$q_1 = 0$	$q_1 = 1$	$q_1 = 2$	$q_1 = 3$	$q_1 = 4$	$q_1 = 5$	$q_1 = 6$	$q_1 = 7$	$q_1 = 8$	$q_1 = 9$	$q_1 = 10$	**profit**	**waste**
$w = 0$	p	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0
	q_0	0	0	0	0	0	0	0	0	0	0	0		
$w = 0.1$	p	0.55	0.3	0.3	0.25	0.2	0.15	0.1	0.05	0.05	0.05	0.05	1.050	0.003
	q_0	4	2	0	0	0	0	0	0	0	0	0		
$w = 0.2$	p	0.55	0.35	0.3	0.25	0.2	0.15	0.1	0.1	0.05	0.05	0.05	1.074	0.007
	q_0	4	2	1	0	0	0	0	0	0	0	0		
$w = 0.3$	p	0.6	0.4	0.3	0.3	0.25	0.2	0.15	0.1	0.05	0.05	0.05	1.105	0.018
	q_0	4	2	1	0	0	0	0	0	0	0	0		
$w = 0.4$	p	0.6	0.4	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0.05	0.05	1.140	0.038
	q_0	4	3	1	0	0	0	0	0	0	0	0		
$w = 0.5$	p	0.6	0.4	0.35	0.3	0.25	0.25	0.2	0.15	0.1	0.05	0.05	1.150	0.047
	q_0	4	3	2	0	0	0	0	0	0	0	0		
$w = 0.6$	p	0.6	0.45	0.35	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0.05	1.172	0.074
	q_0	4	3	2	0	0	0	0	0	0	0	0		
$w = 0.7$	p	0.6	0.45	0.4	0.35	0.3	0.25	0.25	0.2	0.15	0.1	0.1	1.203	0.129
	q_0	4	4	2	1	0	0	0	0	0	0	0		
$w = 0.8$	p	0.6	0.5	0.4	0.35	0.35	0.3	0.25	0.25	0.2	0.2	0.2	1.224	0.190
	q_0	4	4	3	1	0	0	0	0	0	0	0		
$w = 0.9$	p	0.55	0.5	0.45	0.4	0.35	0.3	0.3	0.25	0.25	0.25	0.25	1.256	0.395
	q_0	5	5	4	2	1	0	0	0	0	0	0		
$w = 1$	p	0.6	0.55	0.55	0.55	0.55	0.35	0.3	0.3	0.3	0.3	0.3	1.278	0.987
	q_0	5	5	5	5	5	0	0	0	0	0	0		

Table 7
Optimal decisions of Model 4.

		$q_1=0$	$q_1=1$	$q_1=2$	$q_1=3$	$q_1=4$	$q_1=5$	$q_1=6$	$q_1=7$	$q_1=8$	$q_1=9$	$q_1=10$	**profit**	**waste**
	p_0	0.05	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0	0
$w=0$	p_1	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
	q_0	0	0	0	0	0	0	0	0	0	0	0		
	p_0	0.55	0.6	0.55	0.6	0.6	0.6	0.65	0.6	0.55	0.6	0.6	1.259	0.002
$w=0.1$	p_1	–	0.25	0.2	0.2	0.15	0.1	0.1	0.05	0.05	0.05	0.05		
	q_0	5	4	4	3	2	2	1	1	1	0	0		
	p_0	0.55	0.6	0.55	0.6	0.6	0.6	0.65	0.6	0.55	0.6	0.6	1.261	0.002
$w=0.2$	p_1	–	0.25	0.2	0.2	0.2	0.15	0.1	0.05	0.05	0.05	0.05		
	q_0	5	4	4	3	2	2	1	1	1	0	0		
	p_0	0.55	0.6	0.6	0.55	0.6	0.6	0.6	0.6	0.55	0.6	0.6	1.275	0.006
$w=0.3$	p_1	–	0.25	0.25	0.2	0.2	0.15	0.15	0.1	0.05	0.05	0.05		
	q_0	5	4	3	3	2	2	1	1	1	0	0		
	p_0	0.55	0.6	0.6	0.55	0.6	0.6	0.65	0.6	0.55	0.6	0.6	1.275	0.006
$w=0.4$	p_1	–	0.25	0.25	0.2	0.2	0.15	0.15	0.1	0.05	0.05	0.05		
	q_0	5	4	3	3	2	2	1	1	1	0	0		
	p_0	0.6	0.6	0.6	0.6	0.6	0.65	0.65	0.6	0.4	0.6	0.6	1.297	0.025
$w=0.5$	p_1	–	0.3	0.25	0.25	0.2	0.2	0.15	0.1	0.1	0.05	0.05		
	q_0	5	4	3	3	2	1	1	1	0	0	0		
	p_0	0.6	0.6	0.55	0.6	0.6	0.65	0.6	0.6	0.55	0.5	0.5	1.3138	0.0471
$w=0.6$	p_1	–	0.3	0.25	0.25	0.25	0.2	0.2	0.15	0.1	0.05	0.05		
	q_0	5	4	4	3	2	1	1	1	1	0	0		
	p_0	0.6	0.6	0.55	0.6	0.6	0.55	0.6	0.55	0.4	0.45	0.45	1.314	0.0477
$w=0.7$	p_1	–	0.3	0.25	0.25	0.25	0.2	0.2	0.2	0.15	0.1	0.1		
	q_0	5	4	4	3	2	2	1	0	0	0	0		
	p_0	0.6	0.6	0.6	0.55	0.6	0.6	0.6	0.55	0.55	0.4	0.4	1.324	0.078
$w=0.8$	p_1	–	0.3	0.3	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.15		
	q_0	5	4	4	3	2	2	1	1	1	0	0		
	p_0	0.6	0.6	0.6	0.6	0.6	0.55	0.6	0.6	0.6	0.5	0.5	1.339	0.150
$w=0.9$	p_1	–	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	0.25		
	q_0	5	4	4	3	2	2	1	1	1	0	0		
	p_0	0.6	0.6	0.55	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	1.347	0.297
$w=1$	p_1	–	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3		
	q_0	5	4	4	3	3	2	2	2	2	2	2		

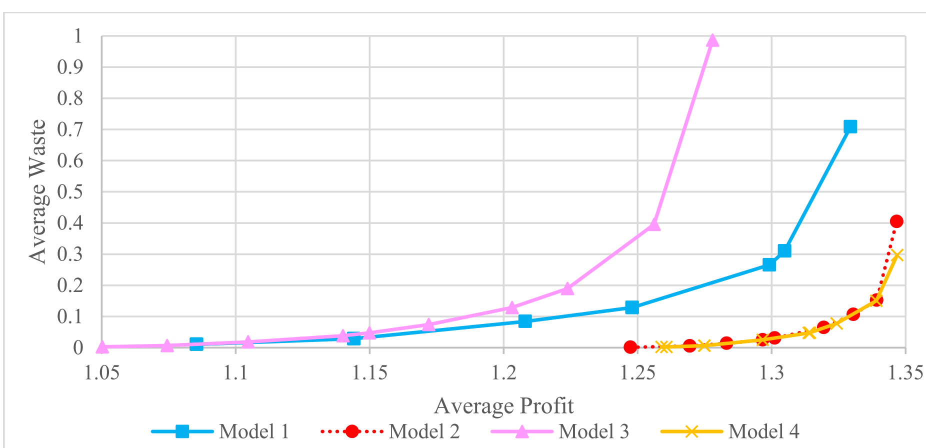

Fig. 1. Comparison of pareto frontiers of all models.

5.3. *Results of Model 3*

Table 6 represents the optimal price and order quantity decisions under the state of the system (quantity of old product) and the different weight factors. CPU time to obtain all results that are given in Table 6 is in total 1.6 h.

Since $p_0 = p_1 = p$ because of *Proposition 1(ii)*, none of the consumers would like to pay the same price to old products as new products and the old product's initial demand will be zero. However, since we allow the demand shift, the old products can be sold when the new products are depleted and their waste can only be prevented then. Because of this reason, as observed in Table 6, a smaller number of new products are ordered as the amount of old products at hand increases. Consistent with the literature, we find that the optimal order quantity and the optimal sale price decrease monotonically in the quantity of old products. In addition, if the waste becomes more important (as w decreases), the prices decrease more and a smaller number of new products are ordered. Table 6 also shows that average profit and average waste increase as w increases. When the results of $w = 0.1$ and $w = 0.4$ are compared, average profit decreases by 8 %, while waste decreases by 93 %. Therefore, a huge decrease in waste can be achieved by compromising comparably a smaller amount in profit.

5.4. *Results of Model 4*

In this model, retailer offers a discount for the old products, and

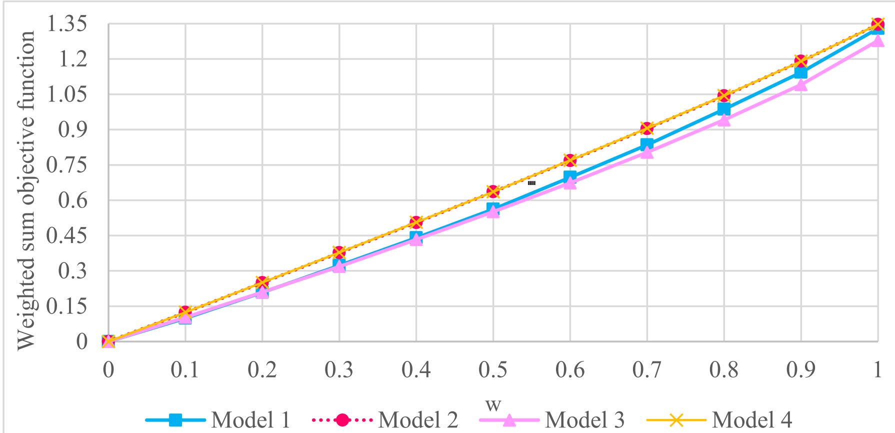

Fig. 2. Comparison of the weighted-sum objective functions of all models.

Table 8
Sensitivity results for Model 2 with $w = 0.5$.

	λ	λ_P	λ_W	λ Difference %
Base Case	0.6359	1.2966	0.0249	–
$\boldsymbol{\delta = 0.48}$	0.6238	1.2642	0.0165	1.892 % ↓
$\boldsymbol{\delta = 0.72}$	0.6566	1.3279	0.0147	3.254 % ↑
$\boldsymbol{c = 0.16}$	0.7216	1.4712	0.0280	13.481 % ↑
$\boldsymbol{c = 0.24}$	0.5543	1.1271	0.0185	12.825 % ↓
$\boldsymbol{h = 0.0016}$	0.6361	1.2970	0.0249	0.031 % ↑
$\boldsymbol{h = 0.0024}$	0.6357	1.2962	0.0249	0.031 % ↓

decide the sale prices of both the old and new products and the order quantity based on the inventory of old products at hand. CPU time to obtain all results that are given in Table 7 is in total 7.8 h.

Similar to Model 2, we observe the same threshold pricing policy as stated in Section 5.2. These thresholds are lower for the lower w values. Which means that when waste minimization is more important (w is lower) the difference in the optimal new and old products' sale prices ($p_0{}^* - p_1{}^*$) increases. Hence, old product's demand increases and waste can be prevented.

As we can see from Table 7, q0 monotonically decrease (to zero) with q_1, because when q_1 is high enough to meet the old product's demand retailer does not need to order new product and instead try to sell the inventory. Hence, for the lower w values, retailer can prevent waste and for the higher w values, can avoid paying h and c.

Different than previous models, in Model 4 both sale prices are dynamic which let retailers to adjust sale prices according to old product at hand, q_1, and gain more profit and cause less waste. As it can be seen in Table 7, similar to previous models average profit and average waste increase as w increases. We also can observe that average profit decreases moderately, whereas average waste decreases dramatically with changing w. For example, when the results of $w = 0.7$ and $w = 0.9$ are compared, average profit decrease by only about 2 %, while average waste decreases by about 68 %. This again explains the substance of determining the relative importance of the objective functions.

5.5. Comparison of all models

In this section, we compare the pareto frontiers of the models. Fig. 1 (starting from $w = 0.1$) demonstrates the comparison of pareto frontier for four DS Models, where we can observe that Model 2 and Model 4 provide better Pareto solutions, mainly due to the dynamic and free choice of p_1, that is different than p_0. For all models, it is observed that significant savings can be obtained in waste, with only a small decrease in profit. Furthermore, our findings in Fig. 2 show that the optimum weighted-sum objective value in Model 4 exceeds that of the other three models. However, there is minor difference between the optimum weighted-sum objective values of Model 4 and Model 2. From these results, we can conclude that when there is demand shift between old and new products, pricing old and new products differently in a dynamic manner can provide significant savings for the retailer. Therefore, we can conclude that the retailer optimizes both the average profit and average waste when the sale prices and order quantity decisions are given in a dynamic manner at the beginning of each period by considering the old product's inventory.

5.6. Sensitivity analysis for Model 2

In this section, we analyze the results of our models for varying

Table 9
Sensitivity results of decision variables for Model 2 with $w = 0.5$.

	p_0		$q_1 = 0$	$q_1 = 1$	$q_1 = 2$	$q_1 = 3$	$q_1 = 4$	$q_1 = 5$	$q_1 = 6$	$q_1 = 7$	$q_1 = 8$	$q_1 = 9$	$q_1 = 10$
Base Case	0.6	p_1	0.35	0.3	0.25	0.25	0.2	0.2	0.15	0.1	0.1	0.05	0.05
		q_0	5	4	3	3	2	1	1	1	0	0	0
$\boldsymbol{\delta = 0.48}$	0.6	p_1	0.2	0.2	0.2	0.15	0.15	0.1	0.1	0.1	0.05	0.05	0.05
		q_0	5	4	3	3	2	2	1	1	1	0	0
$\boldsymbol{\delta = 0.72}$	0.6	p_1	0.25	0.35	0.35	0.3	0.3	0.25	0.2	0.15	0.1	0.05	0.05
		q_0	5	4	3	3	2	2	1	1	1	0	0
$\boldsymbol{c = 0.16}$	0.6	p_1	0.3	0.25	0.25	0.2	0.2	0.2	0.1	0.1	0.1	0.05	0.05
		q_0	5	5	4	3	3	2	2	1	1	0	0
$\boldsymbol{c = 0.24}$	0.6	p_1	0.35	0.3	0.25	0.25	0.2	0.2	0.15	0.1	0.1	0.05	0.05
		q_0	4	4	3	3	2	1	1	1	0	0	0
$\boldsymbol{h = 0.0016}$	0.6	p_1	0.3	0.3	0.25	0.25	0.2	0.2	0.15	0.1	0.1	0.05	0.05
		q_0	5	4	3	3	2	1	1	1	0	0	0
$\boldsymbol{h = 0.0024}$	0.6	p_1	0.35	0.3	0.25	0.25	0.2	0.2	0.15	0.1	0.1	0.05	0.05
		q_0	5	4	3	3	2	1	1	1	0	0	0

Table 10
Optimal decisions of Model 2 when $v \sim$ Triangular (0,1,0.5).

			$q_1 = 0$	$q_1 = 1$	$q_1 = 2$	$q_1 = 3$	$q_1 = 4$	$q_1 = 5$	$q_1 = 6$	$q_1 = 7$	$q_1 = 8$	$q_1 = 9$	$q_1 = 10$	profit	waste
$w = 0$	$p_0 = 0.05$	p_1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0
		q_0	0	0	0	0	0	0	0	0	0	0	0		
$w = 0.1$	$p_0 = 0.45$	p_1	0.15	0.2	0.2	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	1.2429	0.001
		q_0	6	6	5	4	4	3	2	2	0	0	0		
$w = 0.2$	$p_0 = 0.45$	p_1	0.15	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1	0.1	0.1	1.2506	0.003
		q_0	6	6	5	4	4	3	2	2	1	0	0		
$w = 0.3$	$p_0 = 0.45$	p_1	0.15	0.2	0.2	0.2	0.15	0.15	0.15	0.15	0.1	0.1	0.1	1.2506	0.003
		q_0	6	6	5	4	4	3	2	1	1	0	0		
$w = 0.4$	$p_0 = 0.45$	p_1	0.15	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.15	0.1	0.1	1.2526	0.004
		q_0	6	6	5	4	3	3	2	1	0	0	0		
$w = 0.5$	$p_0 = 0.45$	p_1	0.15	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.15	0.1	0.1	1.2526	0.004
		q_0	6	6	5	4	3	3	2	1	0	0	0		
$w = 0.6$	$p_0 = 0.45$	p_1	0.15	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.1	1.2538	0.005
		q_0	6	6	5	4	4	2	1	0	0	0	0		
$w = 0.7$	$p_0 = 0.45$	p_1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.15	0.15	1.2538	0.006
		q_0	6	6	5	4	4	3	2	1	0	0	0		
$w = 0.8$	$p_0 = 0.45$	p_1	0.15	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.15	1.2538	0.006
		q_0	6	6	5	4	4	3	2	1	0	0	0		
$w = 0.9$	$p_0 = 0.45$	p_1	0.15	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	1.2541	0.007
		q_0	6	6	5	5	4	3	2	2	1	0	0		
$w = 1$	$p_0 = 0.45$	p_1	0.15	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	1.2615	0.389
		q_0	7	6	5	5	4	3	3	2	2	2	2		

parameter values in order to observe the effects of the parameters on the system results. We provide the sensitivity results of only Model 2 for the sake of brevity, but we note that the sensitivity results for the other models are also similar to the results presented in this section. We consider the retailer's problem in which the importance of waste minimization and profit maximizations are equal ($w = 0.5$). Table 8 shows the sensitivity results and Table 9 represents the effects of parameters on decision variables.

δ is the rate of decrease in the consumer's valuation of the product when its age gets older. If δ is low ($\delta = 0.48$) then consumer's valuation for old product decreases and they will not be willing to purchase old products and waste may occur. Therefore, retailer's best strategy is to set lower p_1 to increase the old product's expected demand and order higher quantity of new product. This strategy benefits the retailer to decrease optimal average waste (λ_W) from 0.025 to 0.016; however, optimal average profit (λ_P) also decreases. Therefore, weighted-sum objective function (λ) decreases by 1.892 %. However, when δ is higher ($\delta = 0.72$), meaning that when decrease in the consumer's old product valuation is not so significant, retailer's optimal strategy is setting higher p_1 and ordering more new products for particular q_1 values. This strategy decreases λ_W, λ_P and increases the λ by 3.254 %.

Retailer's optimal sale price decision is not affected by + 20 % changes in ordering cost (c). However, when c is lower ($c = 0.16$), retailer's best strategy is to set lower p_0 ($p_0 = 0.55$), lower p_1 for particular q_1 values and order more new products. Hence, λ_W decreases, λ_P increases; therefore, weighted-sum objective function (λ) increases by 13.481 %. For higher c values ($c = 0.24$), retailer order less new products which decreases λ_W but also λ_P; therefore, λ decreases by 12.825 %. ± 20 % changes in inventory holding cost (h) does not affect the optimal decisions and λ_W. However, it affects λ_P directly; therefore, when $h = 0.0024$, λ decreases by 0.031 % and when $h = 0.0016$, λ increases by 0.031 %.

5.6.1. Triangular distribution for Model 2

In our numerical experiments, the utility function is assumed to be uniformly distributed but the distribution of the valuations of consumers might be expected to be more centered around the middle rather than being uniform. Thus, in this section, we use a triangular distribution with lower bound 0, upper bound 1, and most likely value 0.5, In other words, we analyze the results of Model 2 when v *Triangular*$(0, 1, 0.5)$ and compare them with the previously stated results. Table 10 presents the optimal price and order quantity decisions under the state of the

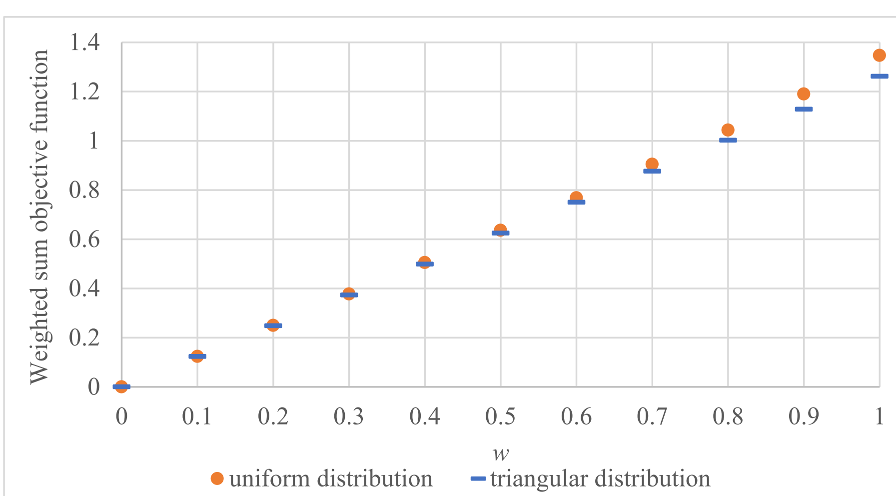

Fig. 3. Comparison of Weighted-Sum Objective Function Values of Model 2 for Uniform Distribution and Triangular distribution.

Table 11
Optimal decisions of Model 2 when $N = 20$.

			q_1=0	q_1=1	q_1=2	q_1=3	q_1=4	q_1=5	q_1=6	q_1=7	q_1=8	q_1=9	q_1=10	q_1=11	q_1=12	q_1=13	q_1=14	q_1=15	q_1=16	q_1=17	q_1=18	q_1=19	q_1=20	profit	waste
$w = 0$	p_0=0.05	p_1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.000	0.000
		q_0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
$w = 0.1$	p_0=0.6	p_1	–	0.3	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.15	0.15	0.1	0.1	0.1	0.05	0.05	0.05	0.05	0.05	0.05	2.756	0.001
		q_0	9	8	8	7	6	6	5	5	4	3	3	3	3	2	2	2	1	1	0	0	0		
$w = 0.2$	p_0=0.6	p_1	–	0.3	0.3	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.15	0.15	0.1	0.1	0.05	0.05	0.05	0.05	0.05	0.05	2.7758	0.0059
		q_0	9	8	8	7	6	6	5	5	4	3	3	3	2	2	2	2	1	1	0	0	0		
$w = 0.3$	p_0= 0.6	p_1	–	0.3	0.3	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1	0.05	0.05	0.05	0.05	0.05	2.7763	0.0061
		q_0	9	8	8	7	7	6	5	5	4	3	3	3	2	2	2	1	2	1	0	0	0		
$w = 0.4$	p_0=0.6	p_1	–	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.15	0.15	0.1	0.1	0.1	0.05	0.05	0.05	0.05	2.7956	0.0177
		q_0	9	8	8	7	7	6	5	4	4	3	3	2	2	2	2	1	1	1	0	0	0		
$w = 0.5$	p_0=0.6	p_1	–	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.15	0.15	0.1	0.1	0.05	0.05	0.05	0.05	2.7958	0.0178
		q_0	9	8	8	7	7	6	5	5	4	3	3	2	2	2	1	1	1	1	0	0	0		
$w = 0.6$	p_0=0.6	p_1	–	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.15	0.15	0.1	0.1	0.05	0.05	0.05	2.808	0.032
		q_0	9	8	8	7	6	6	5	5	4	3	3	2	2	1	1	1	1	0	0	0	0		
$w = 0.7$	p_0=0.6	p_1	–	0.3	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1	0.1	2.821	0.060
		q_0	9	9	8	7	7	6	5	5	4	3	2	2	2	1	0	1	0	0	0	0	0		
$w = 0.8$	p_0=0.6	p_1	–	0.3	0.3	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.2	0.2	0.2	2.824	0.070
		q_0	9	9	8	7	7	6	5	5	4	4	3	2	1	0	0	0	0	0	0	0	0		
$w = 0.9$	p_0=0.6	p_1	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	2.825	0.077
		q_0	9	9	8	7	7	6	6	5	4	4	3	3	2	2	2	1	1	1	1	1	1		
$w = 1$	p_0=0.6	p_1	0.35	0.35	0.35	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	2.843	0.457
		q_0	9	9	8	8	7	6	6	6	5	5	5	5	5	5	5	5	5	5	5	5	5		

system (quantity of old product) and the different weight factors. Average waste and profit for different weight factors can also be seen from the table.

From Table 10, similar to uniform distribution results, it can be observed that as q_1 increases, both p_1 and q_0 decrease monotonically, also p_0* is consistent across all w values. However, the value of p_0* is lower compared to uniform distribution, i.e. p_0*=0.45 which is expected since consumer's valuation is highly located around 0.5. Similar to uniform distribution, optimal p_1* monotonically decreases with q_1. However, when retailer has two objectives, i.e. $w \in [0.1, 0.9]$, the retailer always charges a moderate p_1, $p_0 + \delta - 1 < p_1 < \delta p_0$, allowing consumers to make one of three decisions: to buy the old product, the new product, or to leave the system without making a purchase. Also, behavior of q_0 is similar to uniform distribution; however, the value of q_0 is higher for certain q_1 values under the triangular distribution. Since, sale prices are lower compared to uniform distribution both the average waste and average profit are also lower. However, as it can be seen from Fig. 3, weighted-sum objective function is lower for all w values under triangular distribution.

5.6.2. *Results of Model 2 when $N = 20$*

In this subsection we run our numerical analysis for Model 2 for a larger size of the problem where $N = 20$, and thus the possible demand values and the state space of the model, q_1, ranges between 0 and 20. Table 11 shows the optimal price and order quantity decisions under the state of the system and the different weight factors. The main difference between the cases when $N = 10$ and $N = 20$ arises in the run time of the algorithm due to the increase in the size of the state space and the possible demand values. Recall that the CPU time to obtain all results in Table 5 is in total 7.4 h, whereas the CPU time to obtain all results in Table 11 turns out to be around 140 h. Due to the curse of dimensionality, the run time of the algorithm increases exponentially. However, we note that similar structural insights are obtained for Model 2 when $N = 20$ compared with the case $N = 10$. For example, as q_1 increases, both p_1 and q_0 decrease monotonically, also p_0* is consistent across all w values, i.e., p_0*=0.6. From Table 11 we also observe that decision of p_1 depends on q_1 in a similar manner to $N = 10$ case. The same threshold policy can also be observed: if $q_1 \in [0, T]$ then retailer charges moderate p_1, $p_0 + \delta - 1 \le p_1 < \delta p_0$, such that consumers can give the three decisions (this policy highlighted as yellow in Table 11); if $q_1 \in (T, 20]$, retailer's best strategy is to set too low p_1, $p_1 < p_0 + \delta - 1$ and none of the consumers will purchase new products (this policy highlighted as grey in Table 11). The behavior of q_0*, average profit and average waste based on w is also similar to $N = 10$ case. For example, q_0* decreases when w is lower, and average waste and profit both increase with w. When the results of $w = 0.3$ and $w = 0.5$ are compared, average profit decreases by 0.7 %, while waste decreases by 66 %. Thus, a significant reduction in waste can be achieved by compromising a relatively smaller amount of profit.

5.6.3. *Results of Model 2 when $s = 0.1$*

In our numerical experiments, we initially set $s = 1$ to represent not only the direct monetary cost but also the broader implications of food waste. These include the opportunity cost of unsold goods, and the significant environmental impact associated with food waste disposal. The latter is particularly relevant considering recent policy developments. As of 2023, nine U.S. states have enacted organic waste bans for grocery retailers, also known as mandatory recycling programs (Sanders, 2024). These regulations aim to reduce landfill waste, mitigate methane emissions from decomposing organic waste, and encourage the rescue of food suitable for consumption.

However, the cost of waste may be lower in some contexts, particularly when penalties are minimal or when there is a salvage value associated with transforming the product for alternative use. To explore this scenario, we reran our numerical analysis for Model 2 with $s = 0.1$. Table 12 shows the optimal price and order quantity decisions under the

Table 12
Optimal decisions of Model 2 when $s = 0.1$.

			$q_1 = 0$	$q_1 = 1$	$q_1 = 2$	$q_1 = 3$	$q_1 = 4$	$q_1 = 5$	$q_1 = 6$	$q_1 = 7$	$q_1 = 8$	$q_1 = 9$	$q_1 = 10$	**profit**	**waste**
$w = 0$	$p_0 = 0.05$	p_1	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0
		q_0	0	0	0	0	0	0	0	0	0	0	0		
$w = 0.1$	$p_0 = 0.6$	p_1	–	0.3	0.25	0.25	0.2	0.2	0.15	0.15	0.1	0.05	0.05	1.297	0.025
		q_0	5	4	3	3	2	1	1	1	0	0	0		
$w = 0.2$	$p_0 = 0.6$	p_1	–	0.3	0.3	0.25	0.25	0.25	0.2	0.2	0.15	0.15	0.1	1.319	0.065
		q_0	5	4	3	3	2	1	1	0	0	0	0		
$w = 0.3$	$p_0 = 0.6$	p_1	–	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.2	0.2	0.2	1.330	0.106
		q_0	5	4	3	3	2	2	1	0	0	0	0		
$w = 0.4$	$p_0 = 0.6$	p_1	–	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	0.25	1.3389	0.150
		q_0	5	4	4	3	2	2	1	1	0	0	0		
$w = 0.5$	$p_0 = 0.6$	p_1	–	0.3	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	0.25	1.3392	0.152
		q_0	5	4	4	3	2	2	1	1	1	1	1		
$w = 0.6$	$p_0 = 0.6$	p_1	–	0.3	0.3	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25	1.340	0.162
		q_0	5	4	4	3	3	2	2	1	1	1	1		
$w = 0.7$	$p_0 = 0.6$	p_1	–	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.345	0.253
		q_0	5	4	4	3	3	2	2	2	2	2	2		
$w = 0.8$	$p_0 = 0.6$	p_1	–	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.345	0.253
		q_0	5	4	4	3	3	2	2	2	2	2	2		
$w = 0.9$	$p_0 = 0.6$	p_1	–	0.35	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.347	0.405
		q_0	5	4	4	3	3	2	2	2	2	2	2		
$w = 1$	$p_0 = 0.6$	p_1	–	0.35	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.347	0.405
		q_0	5	4	4	3	3	2	2	2	2	2	2		

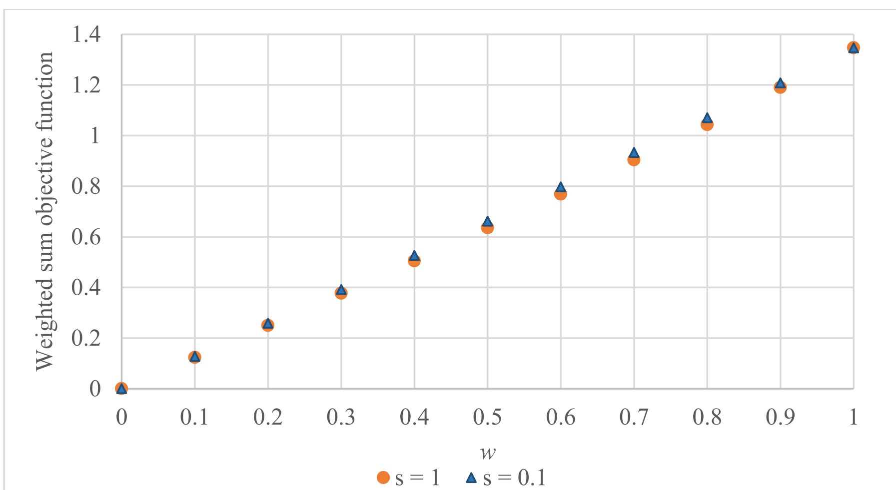

Fig. 4. Comparison of Weighted-Sum Objective Function Values of Model 2 for $s = 1$ and $s = 0.1$.

state of the system and the different weight factors. Average waste and profit for different weight factors can also be seen from the table. The total CPU time required to obtain all results presented in Table 12 is 7.33 h. The average waste shown in Table 12 is calculated by multiplying the average waste cost by 10, as $s = 0.1$.

From Table 12, similar to the results for $s = 1$, it can be observed that as q_1 increases, both p_1* and q_0* decrease monotonically. Additionally, p_0* remains consistent across all across all w values and is equal to 0.6, identical to the results for $s = 1$ results. Although the behavior of p_1* and q_0* for $s = 1$ is similar, the optimal values of p_1* and q_0* are higher for certain system states. This is expected, as the lower waste cost ($s = 0.1$) allows the retailer greater flexibility in deciding order quantities and the sale price of old products. The lower financial implications of waste incentivize the retailer to explore higher price points and order

Table 13
Optimal decisions of NoDs Model 1.

	p_0	p_1		$q_1 = 0$	$q_1 = 1$	$q_1 = 2$	$q_1 = 3$	$q_1 = 4$	$q_1 = 5$	$q_1 = 6$	$q_1 = 7$	$q_1 = 8$	$q_1 = 9$	$q_1 = 10$	**profit**	**waste**
$w = 0$	0.05	0.05	q_0	0	0	0	0	0	0	0	0	0	0	0	0	0
$w = 0.1$	0.55	0.25	q_0	2	2	2	2	2	2	2	2	2	2	2	0.607	0.010
$w = 0.2$	0.5	0.5	q_0	3	3	3	3	3	3	3	3	3	3	3	0.867	0.066
$w = 0.3$	0.5	0.5	q_0	3	3	3	3	3	3	3	3	3	3	3	0.867	0.066
$w = 0.4$	0.55	0.55	q_0	3	3	3	3	3	3	3	3	3	3	3	0.981	0.125
$w = 0.5$	0.55	0.55	q_0	3	3	3	3	3	3	3	3	3	3	3	0.981	0.125
$w = 0.6$	0.55	0.55	q_0	3	3	3	3	3	3	3	3	3	3	3	0.981	0.125
$w = 0.7$	0.55	0.55	q_0	4	4	4	4	4	4	4	4	4	4	4	1.184	0.391
$w = 0.8$	0.6	0.6	q_0	4	4	4	4	4	4	4	4	4	4	4	1.238	0.602
$w = 0.9$	0.6	0.6	q_0	4	4	4	4	4	4	4	4	4	4	4	1.238	0.602
w = 1	0.6	0.6	q_0	5	5	5	5	5	5	5	5	5	5	5	1.256	1.235

quantities, as the economic risk of unsold inventory is reduced.

Similar to the results for $s = 1$ a threshold policy can be observed. When $q_1 \in [0, T]$ then retailer charges moderate p_1, such that $p_0 + \delta - 1 \leq p_1 < \delta p_0$, enabling consumers to make three possible decisions: buying new products, old products, or neither. This policy is highlighted in yellow in Table 12. When $q_1 \in (T, 10]$, retailer's optimal strategy is to set too low p_1, such that $p_1 < p_0 + \delta - 1$ and none of the consumers will purchase new products. This policy is highlighted in grey in Table 12. Notably, the thresholds for these policies are higher compared to those observed for $s = 1$. This is because lower waste costs allow the retailer to adopt higher p_1.

When comparing waste and profit values for $s = 1$ and $s = 0.1$ it is observed that, as expected, higher profits are achieved when $s = 0.1$. However, this causes the increased waste. Additionally, as shown in Fig. 4, the weighted-sum objective function is higher for $s = 0.1$ indicating that the lower waste cost allows for greater flexibility in balancing profit maximization and waste minimization. However, all these analyses demonstrate that similar structural insights can be obtained when comparing the results for $s = 1$ with those for $s = 0.1$.

5.7. No demand shift (NoDS) models

In this section we discuss a special case in which there is no demand shift between new and old products *($\alpha_{01} = \alpha_{10} = 0$)*.

5.7.1. NoDS Model 1: Static p_0 and p_1, dynamic q_0

In this case, Table 13 shows that even though q_0 is dynamic, retailer's best strategy is to order the same quantity as q_1 changes. At first, this may seem counterintuitive because usually higher old inventory requires fewer (sometimes no) new products. However, in this case, since we do not allow demand shift, the amount of old inventory does not affect the demand for the new ones, and only the prices affect the demand. Since the prices are not dynamically changed based on the amount of inventory at hand, for given fixed prices, the problem about deciding the order quantity q_0 reduces to a two-period Newsvendor problem, in which unsold new products are not salvaged at the end of the first period but have a second opportunity to be sold as old products in the second period. The quantity of old products at hand does not affect the order quantity of the new products. CPU time to obtain all results that are given in Table 13 is in total 2.1 h.

Our numerical analysis reveals that when the cost of waste becomes less important (higher w values), the retailer's best strategy is to not apply discount and set the static prices $p_0 = p_1$. When waste minimization is more important ($w = 0.1$) retailer can benefit from setting lower p_1 than p_0. Hence, due to *Proposition 1 (i)* ($p_0 + \delta - 1 < p_1 < \delta p_0$,) consumers will choose whether to buy old or new product or leave the system based on their utilities, which can help retailer to gain profit and minimize the waste. However, as w increases and waste becomes less important, it is observed that p_1 is set equal to p_0, meaning that it is not profitable to sell old products at a lower price, and the company focuses only on selling the new products. In order to avoid cannibalization between the old and new products, the company prefers to only sell the new products in this case. More new products are ordered, and higher prices are charged. The demand for the old products will be zero and they all be dumped.

5.7.2. NoDS Model 2: Static p_0, dynamic p_1 and q_0

As it can be seen from Table 14, different than NoDs Model 1, since the decision of p_1 is given dynamically based on the quantity of the old products at hand *(q_1)*, retailer's best strategy is to apply discount as q_1 increases in order to prevent waste. The prices of old products tend to decrease as q_1 increases, and as w decreases, since waste becomes more important. The static price of the new products also tends to be lower for lower w. In addition, the order quantity q_0 monotonically decreases as q_1 increases. Therefore, in Model 2 average profit is higher and average waste is lower compared to Model 1, stating that dynamically adjusting the prices of old products can benefit the company significantly. CPU time to obtain all results that are given in Table 14 is in total 1.25 h.

5.7.3. NoDS Model 3: Dynamic ($p_0 = p_1$) and q_0

As shown in Table 15, since $p_0 = p_1 = p$ and since there is no demand shift between old and new products, the demand for the old products will be zero in this case. Thus, the quantity of the old products at hand (q_1) does not affect any of the decisions and the retailer's best strategy is to keep the prices and the order quantities constant, regardless of the state of the system. CPU time to obtain all results that are given in Table 15 is in total 0.059 h. The prices and the order quantities tend to decrease as w decreases (i.e. waste becomes more important).

5.7.4. NoDS Model 4: Dynamic p_0, p_1 and q_0

As we can see from Table 16, the retailer's best strategy changes based on the inventory of old products (q_1) and w. Results show that optimal prices are not always decreasing as q_1 increases, since q_0 changes, too, where q_0 is decreasing with respect to q_1. Similar to other

Table 14
Optimal decisions of NoDS Model 2.

			$q_1 = 0$	$q_1 = 1$	$q_1 = 2$	$q_1 = 3$	$q_1 = 4$	$q_1 = 5$	$q_1 = 6$	$q_1 = 7$	$q_1 = 8$	$q_1 = 9$	$q_1 = 10$	**profit**	**waste**
$w = 0$	$p_0 = 0.05$	p_1	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0
		q_0	0	0	0	0	0	0	0	0	0	0	0		
$w = 0.1$	$p_0 = 0.55$	p_1	–	0.25	0.2	0.2	0.1	0.1	0.1	0.05	0.05	0.05	0.05	1.066	0.004
		q_0	4	2	1	1	0	0	0	0	0	0	0		
$w = 0.2$	$p_0 = 0.55$	p_1	–	0.25	0.2	0.2	0.1	0.1	0.1	0.1	0.05	0.05	0.05	1.066	0.004
		q_0	4	2	1	1	0	0	0	0	0	0	0		
$w = 0.3$	$p_0 = 0.6$	p_1	–	0.3	0.25	0.25	0.2	0.2	0.15	0.1	0.05	0.05	0.05	1.110	0.017
		q_0	4	2	1	1	0	0	0	0	0	0	0		
$w = 0.4$	$p_0 = 0.6$	p_1	–	0.3	0.25	0.25	0.2	0.2	0.15	0.1	0.05	0.05	0.05	1.110	0.017
		q_0	4	2	1	1	0	0	0	0	0	0	0		
$w = 0.5$	$p_0 = 0.55$	p_1	–	0.3	0.25	0.25	0.2	0.2	0.1	0.1	0.1	0.05	0.05	1.174	0.069
		q_0	4	4	2	2	1	1	0	0	0	0	0		
$w = 0.6$	$p_0 = 0.55$	p_1	–	0.3	0.25	0.25	0.2	0.2	0.1	0.1	0.1	0.05	0.05	1.204	0.103
		q_0	5	4	2	2	1	1	0	0	0	0	0		
$w = 0.7$	$p_0 = 0.55$	p_1	–	0.3	0.25	0.25	0.2	0.2	0.2	0.1	0.1	0.1	0.1	1.210	0.115
		q_0	5	4	3	3	1	1	1	0	0	0	0		
$w = 0.8$	$p_0 = 0.55$	p_1	–	0.3	0.3	0.25	0.25	0.2	0.2	0.2	0.1	0.1	0.1	1.2578	0.250
		q_0	5	4	4	3	3	1	1	1	0	0	0		
$w = 0.9$	$p_0 = 0.55$	p_1	–	0.3	0.3	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	1.2582	0.252
		q_0	5	4	4	3	3	3	3	1	1	1	1		
$w = 1$	$p_0 = 0.6$	p_1	–	0.35	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.285	0.699
		q_0	5	5	5	3	3	3	3	3	3	3	3		

Table 15
Optimal decisions of NoDS Model 3.

		$q_1=0$	$q_1=1$	$q_1=2$	$q_1=3$	$q_1=4$	$q_1=5$	$q_1=6$	$q_1=7$	$q_1=8$	$q_1=9$	$q_1=10$	**profit**	**waste**
$w=0$	p	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0
	q_0	0	0	0	0	0	0	0	0	0	0	0		
$w=0.1$	p	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.594	0.012
	q_0	2	2	2	2	2	2	2	2	2	2	2		
$w=0.2$	p	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.867	0.066
	q_0	3	3	3	3	3	3	3	3	3	3	3		
$w=0.3$	p	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.867	0.066
	q_0	3	3	3	3	3	3	3	3	3	3	3		
$w=0.4$	p	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.981	0.125
	q_0	3	3	3	3	3	3	3	3	3	3	3		
$w=0.5$	p	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.981	0.125
	q_0	3	3	3	3	3	3	3	3	3	3	3		
$w=0.6$	p	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	1.184	0.391
	q_0	4	4	4	4	4	4	4	4	4	4	4		
$w=0.7$	p	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	1.184	0.391
	q_0	4	4	4	4	4	4	4	4	4	4	4		
$w=0.8$	p	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	1.238	0.602
	q_0	4	4	4	4	4	4	4	4	4	4	4		
$w=0.9$	p	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	1.238	0.602
	q_0	4	4	4	4	4	4	4	4	4	4	4		
$w=1$	p	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	1.256	1.235
	q_0	5	5	5	5	5	5	5	5	5	5	5		

Table 16
Optimal decisions of NoDS Model 4.

		$q_1=0$	$q_1=1$	$q_1=2$	$q_1=3$	$q_1=4$	$q_1=5$	$q_1=6$	$q_1=7$	$q_1=8$	$q_1=9$	$q_1=10$	**profit**	**waste**
	p_0	0.05	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0	0
$w=0$	p_1	–	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05		
	q_0	0	0	0	0	0	0	0	0	0	0	0		
	p_0	0.55	0.55	0.75	0.7	0.65	0.6	0.55	0.5	0.5	0.5	0.5	1.074	0.004
$w=0.1$	p_1	–	0.25	0.3	0.25	0.2	0.15	0.1	0.05	0.05	0.05	0.05		
	q_0	4	2	0	0	0	0	0	0	0	0	0		
	p_0	0.55	0.55	0.5	0.7	0.65	0.6	0.55	0.55	0.5	0.5	0.5	1.084	0.006
$w=0.2$	p_1	–	0.25	0.2	0.25	0.2	0.15	0.1	0.1	0.05	0.05	0.05		
	q_0	4	2	2	0	0	0	0	0	0	0	0		
	p_0	0.6	0.5	0.5	0.75	0.7	0.65	0.6	0.55	0.5	0.5	0.5	1.138	0.025
$w=0.3$	p_1	–	0.25	0.2	0.3	0.25	0.2	0.15	0.1	0.05	0.05	0.05		
	q_0	4	3	2	0	0	0	0	0	0	0	0		
	p_0	0.6	0.5	0.55	0.75	0.7	0.65	0.6	0.55	0.5	0.5	0.5	1.154	0.033
$w=0.4$	p_1	–	0.25	0.25	0.3	0.25	0.2	0.15	0.1	0.05	0.05	0.05		
	q_0	4	3	2	0	0	0	0	0	0	0	0		
	p_0	0.6	0.5	0.55	0.75	0.7	0.7	0.65	0.6	0.55	0.5	0.5	1.159	0.037
$w=0.5$	p_1	–	0.25	0.25	0.3	0.25	0.25	0.2	0.15	0.1	0.05	0.05		
	q_0	4	4	2	0	0	0	0	0	0	0	0		
	p_0	0.6	0.55	0.6	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.5	1.219	0.105
$w=0.6$	p_1	–	0.3	0.3	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0		
	q_0	4	4	3	0	0	0	0	0	0	0	0		
	p_0	0.6	0.55	0.6	0.55	0.75	0.7	0.7	0.65	0.6	0.55	0.55	1.229	0.123
$w=0.7$	p_1	–	0.3	0.3	0.25	0.3	0.25	0.25	0.2	0.15	0.1	0.1		
	q_0	4	4	3	3	0	0	0	0	0	0	0		
	p_0	0.55	0.55	0.6	0.6	0.8	0.75	0.7	0.7	0.65	0.65	0.6	1.245	0.175
$w=0.8$	p_1	–	0.3	0.3	0.3	0.35	0.3	0.25	0.25	0.2	0.2	0.15		
	q_0	5	4	3	3	0	0	0	0	0	0	0		
	p_0	0.55	0.6	0.55	0.6	0.6	0.75	0.75	0.7	0.7	0.7	0.7	1.282	0.380
$w=0.9$	p_1	–	0.35	0.3	0.3	0.3	0.3	0.3	0.25	0.25	0.25	0.25		
	q_0	5	4	4	3	3	0	0	0	0	0	0		
	p_0	0.6	0.6	0.55	0.65	0.6	0.6	0.6	0.75	0.75	0.75	0.75	1.290	0.559
$w=1$	p_1	–	0.35	0.3	0.35	0.3	0.3	0.3	0.3	0.3	0.3	0.3		
	q_0	5	5	4	3	3	3	3	0	0	0	0		

models average profit and average waste increase as w increases. However, in this no demand shift model waste is much lower compared to other NoDS models, because retailer's sale prices decisions are given dynamically based on q_1. Hence, even though there is no demand shift waste can be prevented, and profits can be improved significantly through dynamic pricing. CPU time to obtain all results that are given in Table 16 is in total 1.36 h.

6. Conclusion

Perishable food products such as fruits, vegetables, and dairy products, deteriorate over time; therefore, their demand decreases gradually which makes inventory management and pricing strategies important. Approximately one-third of edible food is wasted at the retail and consumer levels and it not only causes costs for businesses but also harms the environment. For example, some of the world's largest retailers i.e. Tesco, Walmart, Kroger etc. focus to reduce in-store food waste, aiming to achieve Sustainable Development Goals by offering price discounts on food that is close to its shelf life. Therefore, ordering and pricing strategies is critical not only for reducing waste but also for increasing profits.

This study focuses on a retailer's problem of selling old and new perishable food products to strategic consumers. We develop a bi-objective dynamic programming model to optimize the discounted price, sale price, and order quantity of perishable food products in order to maximize the retailer's profit and minimize food waste.

We allow demand shifts between products based on their prices and consumer behaviors. We model four static and dynamic pricing policies commonly used in practice, and investigate the benefits of dynamic pricing and price differentiation between old and new products in terms of both profit and waste. In addition, we use weighted-sum method for our bi-objective function and analyze the results for different combinations of the weights for the profit and the waste. Hence, we obtain the following findings:

- When the price difference between old and new products is high, the consumers prefer to buy the old products or buy nothing, when the difference is low they only buy the new products or nothing. However, in other conditions, they may buy the old products, new products or nothing. In addition, since we allow demand shift, old (new) product's total demand also depends on new (old) product's demand and quantity at hand.
- We observe a threshold pricing policy, hence, decisions of sale prices depend on the quantity of the old products at hand. We reveal that these thresholds are lower for lower weight values, which shows that even though the level of old products at hand is high, higher prices are charged for the old products when profit maximization is more important.
- Order quantity, q_0, decreases when waste minimization is more important (lower w). It is because retailer tends to order less to avoid overstocking and focuses on selling old products at hand by decreasing the optimal price of the old products.
- For our four different static and dynamic pricing strategies, we reveal the importance of choosing suitable weights in order to find the balance between minimizing waste and maximizing profit.
- After comparing four pricing strategies, we find that Model 2 and Model 4 are the better strategies in order to maximize average profit and minimize average waste. So, findings show that significant improvements can be obtained by dynamic decision making. Although Models 2 and 4 may incur higher computational costs than the other models in this study, they demonstrate superior performance in terms of reducing waste and maximizing profit. Moreover, in many practical settings, it is sufficient to solve these models only once (or infrequently) as long as market conditions remain relatively stable.

The results of our study can also have some implications for the governments and strategy builders, since it is observed that when companies are forced to put more weight on their wastes, they need to change their pricing and ordering decisions. Thus, incentive or penalty mechanisms can be designed in this perspective in order to align the decisions of the companies with the social objectives of the governments in order to decrease waste. We note that a detailed analysis of the system including incentive or penalty mechanisms can be an extension of this study in the future. In addition, this research has some limitations which also make various extensions possible for future studies. First, our model considers that there is only one branch of the particular retailer; however, in real life, retailers may have more than one branch, which shares the inventory based on their needs, aiming to maximize the centralized profit. Second, we assume that the product has a two-period lifetime. As an extension we may consider more than two-period of lifetime; however, it can be challenging to model and solve dynamically. Third, we may consider more real-world related constraints such as varying replenishment lead times, menu cost, and limited shelf space which could affect the retailer's optimal decisions. Finally, due to the curse of dimensionality, it is stated that determining the optimal inventory policy for perishable products is extremely complex even when the sale prices are fixed (Nahmias, 1982; Chen et al. 2014). Thus, we can obtain the optimal solutions for small and moderate sized problems in our study and report the run times of the algorithms for each model for the parameters used in this study. For larger sized problems, novel solution algorithms or heuristics that utilize different approaches such as approximate dynamic programming formulations, deep learning or reinforcement learning algorithms need to be developed. These issues can be incorporated into future studies.

CRediT authorship contribution statement

Melda Hasiloglu-Ciftciler: Writing – review & editing, Writing – original draft, Software, Methodology, Investigation, Conceptualization. **Onur Kaya:** Writing – review & editing, Validation, Supervision, Conceptualization.

Data availability

No data was used for the research described in the article.

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