Irish Educational Studies

ISSN: 0332-3315 (Print) 1747-4965 (Online) Journal homepage: www.tandfonline.com/journals/ries20

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To cite this article: Máire Ní Ríordáin, Aoibhinn Ni Shuilleabhain, Mark Prendergast & Patrick Johnson (2023) Irish pre-service mathematics teachers' knowledge of curriculum-aligned content, Irish Educational Studies, 42:1, 1-20, DOI: 10.1080/03323315.2021.1899030

To link to this article: https://doi.org/10.1080/03323315.2021.1899030

Published online: 05 Apr 2021.

Irish Educational Studies, 2023
Vol. 42, No. 1, 1–20, https://doi.org/10.1080/03323315.2021.1899030

Irish pre-service mathematics teachers' knowledge of curriculum-aligned content

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(*Received 28 August 2020; accepted 2 February 2021*)

This study critically examines the mathematics knowledge of pre-service post-primary mathematics teachers (N = 85) on commencing their Professional Master of Education (Level 9) initial teacher education (ITE) programme across four institutions in Ireland. Given the nature of a consecutive approach to mathematics teacher education, pre-service teachers enter their ITE programme from a variety of undergraduate degree backgrounds and it is largely accepted that their mathematical content knowledge does not change during their teacher qualification programme. This study utilises a paper-and-pencil test to assess participants' cognitive and conceptual proficiency with curriculum-aligned mathematical content. Between group comparisons in relation to participants' undergraduate studies and cognitive and conceptual proficiency are also examined. The data suggest that pre-service teachers demonstrate a strong proficiency with Junior Cycle curriculum-aligned content and poor proficiency with Senior Cycle content, irrespective of the curriculum strand. In addition, significant mean differences exist between participants who had undertaken undergraduate degree studies in physics and mathematics in comparison to other degree programmes. Such investigations are essential given the increased focused on developing highly qualified teachers and policy revisions on ITE programmes. This research alerts us to issues within ITE programmes that may be addressed prior to mathematics teachers' induction into the profession.

Keywords: Pre-service teachers; mathematical knowledge; post-primary; curriculum-aligned content; consecutive teacher education

Introduction

It is well documented that a solid knowledge of mathematics is important for all learners, as it can contribute to increased outcomes in relation to job prospects and dealing with everyday life (e.g. Gurria 2014). A key focus of the Irish government is on developing a highly educated population with a strong proficiency in

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STEM related subjects, inclusive of mathematics (The STEM Education Review Group 2016). Henderson and Rodrigues (2008) observe the need for higher teacher capability in order to meet the need to increase children's mathematical proficiency. In particular, research has demonstrated the essential function of a teacher's mathematical knowledge base for undertaking key teaching and learning roles such as lesson preparation, facilitating classroom discussion, creating purposeful learning opportunities, and establishing connections between topics (e.g. Baumert et al. 2010; Hill, Rowan, and Ball 2005). Cognisant of this we examine the mathematical content knowledge (MCK) of pre-service mathematics teachers (N = 85) commencing their consecutive teacher education programmes (graduate pathway) in an Irish context involving four universities. For the purposes of this study, MCK is concerned with 'an understanding of mathematics concepts anticipated to be taught' (Norton 2019, 530) and, accordingly, pre-service teachers' proficiency with curriculum-aligned content (Ní Ríordáin, Paolucci, and O' Dwyer 2017). In addition, we investigate MCK in relation to participants' undergraduate degree studies in order to examine if a relationship exists between the type of degree undertaken and mathematical proficiency.

This research work has been undertaken in a time of immense change in terms of mathematics and teacher education in Ireland. In 2012, a national reform of the post-primary mathematics curriculum was introduced which had considerable implications on classroom teaching and learning practices (Ní Shúilleabháin and Seery 2018) and associated implications on pre-service teachers' concerns around implementing the new curriculum (Johnson et al. 2019). Also in 2012 an international review panel, led by Prof. Pasi Sahlberg, was established to examine the provision of initial teacher education (ITE) in Ireland (Department of Education and Skills 2012). Arising from this review, it was recommended that the consecutive route (i.e. postgraduate route) would increase from 1 to 2 years and the Professional Master of Education (PME), a Level 9 Masters qualification, was thereby established. Consequently, students entering the PME to qualify as post-primary mathematics teachers will have undertaken degree level studies in mathematics or related areas, as outlined by the subject requirements to register with the Teaching Council of Ireland (Teaching Council 2013). It is largely accepted that ITE teachers' MCK does not change during their teacher qualification programme (Osborne 2013) and, in relation to the PME, the focus of the pre-service teachers' learning is primarily on the development of pedagogical practices and professional experiences in the classroom.

Lowrie and Jorgensen (2016, 205) question the 'The 'silencing' of content knowledge' and the 'PCK fever' that has emerged in relation to researching teacher knowledge. They emphasise that most significant studies relating to pre-service mathematics teachers' MCK are dated and have been overtaken by studies focused on PCK (pedagogical content knowledge). Given the lack of recent research examining MCK relating to pre-service teachers, in particular those pursuing the postgraduate route, and the focus of postgraduate ITE programmes on pedagogical aspects of teaching, the purpose of this research is to examine PME mathematics students' MCK on commencement of their studies. As affirmed by Shulman (1986, 5), 'Although knowledge of the theories and methods of teaching is important, it plays a decidedly secondary role' (to content knowledge). Such investigations are therefore essential given the increased focus on developing highly qualified teachers and examining MCK can alert us to issues within ITE programmes that could be addressed prior to induction into the teaching profession (Lowrie and Jorgensen 2016).

Mathematical knowledge for teaching and pre-service teachers

The last 15 years have witnessed much research conceptualising and assessing the kinds of mathematical knowledge that teachers draw upon or need to acquire for effective teaching (Oldham and Prendergast 2019). While MCK and PCK may be assumed to represent conceptually separate forms of knowledge, it has been determined that the two can be merged to form a single body of domain-specific knowledge for teaching (Krauss, Baumert, and Blum 2008). This single body of knowledge has been typically phrased as mathematical knowledge for teaching (MKT), which includes both subject matter and pedagogical considerations (Ball, Thames, and Phelps 2008; Hill, Rowan, and Ball 2005). Examples of such knowledge include: using different approaches to explain mathematical concepts to students, interpreting students' answers, using various representations, and selecting examples that help develop students' understanding (Clivaz and Ní Shúilleabháin 2019; Hill, Rowan, and Ball 2005). Many studies have found that teachers' MKT is essential to the improvement of mathematics teaching and has been linked with improvements in students' achievement (Baumert et al. 2010; Hill, Rowan, and Ball 2005). However, despite such obvious importance, research studies have shown evidence of inadequate knowledge of mathematics for teaching amongst teachers (Ma 1999). Many teachers exhibit a procedural or rule-based understanding of mathematics and this is mirrored in their instruction (Kulm 2008). There are various reasons cited for this in the literature. Thanheiser et al. (2013) suggest that there are inadequacies in teachers' knowledge of mathematics when they graduate from their ITE programmes and many lack conceptual understanding of the mathematics they will be required to teach (O'Meara, Fitzmaurice, and Johnson 2017). For example, Slattery and Fitzmaurice (2014) carried out a study at an Irish university to measure pre-service post-primary mathematics teachers' conceptual understanding of fraction division. The results showed that participants – who were near the end of their degree programme – had a fragmented understanding of the fraction concept and were unable to explain the invert and multiply rule. They relied on a series of 'rules without reason' to answer the questions posed. Thus, when teaching, these pre-service teachers would have to rely on a series of learned procedural steps, as they did not have the conceptual understanding necessary to teach for understanding (O'Meara, Fitzmaurice, and Johnson 2017). As determined by Shulman, 'the teacher need not only understand *that* something is so, the teacher must further understand *why* it is so' (Shulman 1986, 9). In their research with German post-primary mathematics teachers, Baumert et al. (2010) determine that limited subject matter knowledge can have detrimental effects on a teachers' PCK and, consequently, negative effects on instructional quality and student progress. Furthermore, they find that these differences in teachers' MCK persist across an entire teaching career.

Despite such concerns, and as noted in the Introduction, it is largely accepted that ITE programmes do not positively alter pre-service teachers' subject matter knowledge (Osborne 2013). In light of this, Baumert et al. (2010) suggest that ITE programmes should increase the attention given to teachers' subject matter knowledge and more specifically achieve a balance between MCK and PCK. However, Baumert et al. also point out that ITE programmes for mathematics teachers should not be identical to programmes provided for students majoring in mathematics. While students entering the PME will have undertaken degree level studies in mathematics or a related mathematical area, many may feel that the mathematical content that they have studied at third level has little or no relevance to the content that they will be teaching once they qualify (Prendergast et al. 2014). For example, in a 1992 published study involving Greek pre-service mathematics teachers, Toumasis (1992) calls for a greater connection linking the content from their degree programmes and the content which they will be teaching in schools, as one is often far removed from the other. Of course, mathematics teachers need to possess knowledge of the curriculum content at a much deeper level of understanding than their students (Krauss, Baumert, and Blum 2008), but connections and overlaps should also made with their own knowledge which has been constructed from previous learning experiences (Bryan 1999). Deng (2018) interestingly re-conceptualises PCK through an examination of a teacher's understanding of content within an institutional curriculum. Such a conceptualisation requires focusing on teaching from a theory of content perspective and accordingly places the development of teachers' content knowledge as core to process. As noted by Artzt et al. (2012), affording pre-service teachers the opportunity to re-examine post-primary school mathematics content from an advanced perspective may be an important element in preparing them to teach mathematics meaningfully. A deeper understanding of mathematical concepts may enable teachers to access a wider collection of strategies for explaining and illustrating mathematical content to their students (Ma 1999).

However, changing such long-held practices and structures in ITE presents a significant challenge to mathematics teacher educators globally. While there have been a number of Irish studies focusing on specific mathematical knowledge of pre-service teachers' in topics such as fractions, ratio and functions (respectively Slattery and Fitzmaurice 2014; Costello, Stafford, and Oldham 2018; Oldham and Prendergast 2019), there has been no research undertaken to document the subject matter knowledge of pre-service teachers on a broader scale. This present study looks to examine the MCK of pre-service post-primary mathematics teachers across a range of curriculum topics as they commence their ITE programmes and will provide evidence of issues in relation to subject matter knowledge that may be addressed during the two year PME programme.

Pre-service mathematics teacher education in Ireland

In 2011 the Teaching Council, the professional standards body for the teaching profession in Ireland, completed a major review and re-conceptualisation of teacher education (Teaching Council 2011a). In conjunction with this review, the international review panel established in 2012 (noted in the Introduction) recommended the development of a postgraduate level 9 qualification for those pursuing a career in teaching via the consecutive model, in line with international best practice (Department of Education and Skills 2012). In Ireland, the concurrent model of ITE comprises of a Bachelor or Masters of Education degree, while the consecutive model involves a primary degree followed by a level 9[1] Professional Master of Education (PME).

The PME programme is a 2-year, full-time, consecutive postgraduate ITE course designed to qualify graduates as teachers. Primary teachers are qualified to teach all curriculum areas, whereas post-primary teachers qualify (typically) in two subject areas, although this depends on the graduates meeting the specific subject content requirements outlined by the Teaching Council (Teaching Council 2013). The PME combines school placements with university lectures and tutorials, which develop student teachers' requisite professional knowledge, skills, understanding, and competences. The aim of the programme is to ensure that graduates acquire the extensive and complex integrated knowledge base that is necessary for effective teaching at their required level. The core modules on the post-primary PME programme address the foundation disciplines of education: assessment, inclusive education, history and policy, philosophy, sociology, and psychology. The programme also provides pedagogy related to the subject(s) specialisms of the student, as well as a significant component of school placement in both year 1 and year 2 and a research thesis that has to be completed by the end of year 2.

Mathematics specialism modules in the PME typically address material related to the pedagogical content knowledge and practices of pre-service teachers. Such material includes: knowledge of content and curriculum (e.g. lateral & vertical curriculum knowledge), knowledge of content and teaching (e.g. sequencing mathematical content) and knowledge of content and students (e.g. identifying students' difficulties or misconceptions from student responses). These subject specific modules typically exceed the minimum 5 ECTS credits required by the Teaching Council (Teaching Council 2011b), in order to include both Junior and Senior cycle content over the two-year PME course. In line with Osborne's (2013) findings on initial teacher education, subject matter knowledge is typically not addressed as part of these modules.

Methods

This study employed a quantitative approach (paper-and-pencil test) to examining the MCK of pre-service mathematics teachers on commencing their PME studies. Adopting such an approach allows for breadth in examining concepts relating to the curriculum aligned mathematical content, as well as depth in relation to strengths and weaknesses relating to this content. Adopting a quantitative approach also affords an opportunity to examine mathematics degree study backgrounds and correlation with performance on the paper-and-pencil test.

Participants

All participants had completed a degree programme, with interconnected mathematical content, and had secured a place on a PME programme in order to commence their ITE studies. Participants (N = 85) were recruited from four (of eight) Universities in Ireland offering the PME programme over the course of three intakes. Nationally, the intake into the PME route for mathematics graduates is low (Teaching Council 2015) and, accordingly, the number of PME mathematics pre-service teachers is low at approximately 70 per year (Prendergast et al. 2020). Examining the MCK of entrants into the participating PME programmes over three consecutive years allows for a more considerable sample for data analysis and to make appropriate inferences. A convenience sampling approach was utilised in the study, whereby all four authors had access to the participants through their involvement in the delivery of the PME programme in the participating universities. All participation was voluntary, consent was provided, and participants had the right to withdraw at any stage of the research process. In total, 85 pre-service teachers participated in the study examining their MCK as related to curriculum-aligned content. The sample consisted of 27 males and 58 females, with a median age of 23 (range 38). A diverse range of ages were present in the sample with the youngest aged 20 years and the oldest aged 58 years, highlighting the distinct intake of the consecutive ITE route.

Test instrument and data collection

Underpinning this work is the idea that MCK is related to 'an understanding of mathematics concepts anticipated to be taught' (Norton 2019, 530). Accordingly, a paper-and-pencil test, developed to examine Irish post-primary out-of-field mathematics teachers' knowledge of curriculum-aligned content (see Ní Ríordáin, Paolucci, and O' Dwyer 2017) was utilised in the study. The TEDS-M conceptual framework supported the development of item design for the paper-and-pencil test (Ní Ríordáin, Paolucci, and O' Dwyer 2017; Tatto et al. 2008). The TEDS-M study examined the professional knowledge of pre-service mathematics teachers across 21 countries (Schmidt et al. 2008). The TEDS-M framework encompasses professional knowledge as mathematics content knowledge and mathematics pedagogical knowledge. Our study is focused on pre-service mathematics teachers' mathematics content knowledge and the TEDS-M framework purports that this should be examined in relation to the content that teachers are required to teach. Therefore, items were developed that closely align with the Irish post-primary mathematics curriculum (Andrews 2011; Ball, Thames, and Phelps 2008; Krauss, Baumert, and Blum 2008). In addition, international comparisons can be undertaken given the alignment of the Irish curriculum with the PISA mathematical framework (Kirwan 2015; Merriman et al. 2014). Drawing on the principles of Kane (2006) and Delaney (2012), the items were validated for use with teachers teaching mathematics in the Irish context (see detailed explanation in Ní Ríordáin, Paolucci, and O' Dwyer 2017). Given that participants in this study had completed their undergraduate studies in mathematics and were pursuing their teacher qualification studies, it was deemed appropriate to utilise this paper-and-pencil test to assess MCK for the purposes of this study.

The Irish post-primary system is composed of two key cycles – Junior Cycle (Years 1–3, approx. age 12–15) at lower secondary and Senior Cycle (Years 5–6, approx. age 17–18) at upper secondary. A Transition Year (Year 4, approx. age 16) bridges both cycles but no formal curriculum exists for this year of study. As noted in the Introduction, a reform of the entire post-primary mathematics curriculum was initiated in 2008 and implemented over a phased basis from 2012. The new curriculum consists of five strands at both Junior and Senior Cycle (Statistics and Probability, Geometry and Trigonometry, Number, Algebra and Functions (Functions and Calculus at Senior Cycle)). Mathematics can be studied at Foundation (only at Senior Cycle), Ordinary and Higher level. At that time, the Junior Cycle culminated in a state examination knows as the Junior Certificate (JC) and Senior Cycle culminated in a state examination known as the Leaving Certificate (LC). The paper-and-pencil test utilised in this research was designed to assess concepts from all five strands and across both the JC and LC (Ní Ríordáin, Paolucci, and O' Dwyer 2017). Subsequent to the development of this test, a new Junior Cycle was developed and implemented on a phased approach, with the new subject specification for mathematics at Junior Cycle introduced in 2018 (Junior Cycle for Teachers 2018). Although structured differently to the previous specification, with a new Unifying strand and Algebra and Number combined into one strand, the mathematical content has remained the same. Therefore, the paper-and-pencil test is still appropriate to use in order to assess MCK of curriculum-aligned content in the Irish context.

The original paper-and-pencil test had 24 MCK items, 10 multiple-choice and 14 open-ended items. However, for the purpose of this study a reduced version, 17 MCK items (9 multiple-choice, 8 open-ended), was utilised due to time constraints in collecting the data. Figure 1 provides an example of a multiple-choice item and Figure 2 provides an example of an open-ended up (further examples available in Ní Ríordáin, Paolucci, and O' Dwyer 2017). Three of the multiple-choice items consisted of two sub-items, therefore participants were asked to complete 20 items in total. The paper-and-pencil test was administered to participants in their first mathematics pedagogy lecture of the term, on commencement of their PME programme. The test was closed book and no calculators were allowed. All those in attendance at the first lecture consented to participate in the study. The completed MCK paper-and-pencil tests were inputted into Statistical Package for the Social Sciences (Version 25) for analysis.

1. A class has 10 students. If at one time, 2 students are to be chosen from the class, and another time 8 students are to be chosen from the class, which of the following statements is true?

Check *one* box.

A. There are more ways to choose 2 students than 8 students from the class. ☐

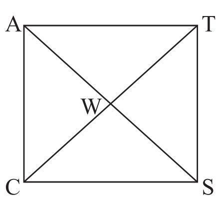
B. There are more ways to choose 8 students than 2 students from the class. ☐

C. The number of ways to choose 2 students equals the number of ways to choose 8 students. ☐

D. It is not possible to determine which selection has more possibilities. ☐

Figure 1. Example of a multiple-choice item.

8. CATS is a square with CT = $6\sqrt{2}$. Determine the measures of the following:



|CA| = ______________ ∠CAW = _______________

|CW| = ______________ ∠AWT = _______________

Figure 2. Example of an open-ended item.

Data analysis

The scoring of the MCK items on the paper-and-pencil test involved two different scoring processes. Ní Ríordáin, Paolucci, and O' Dwyer (2017) provides a very detailed description of the processes and the following paragraphs draw on that work. The first scoring process involved the calculation of a *cognitive score*, with each item scored based on the correctness of the answer. Depending on the nature of the item, open-ended items were scored between 0 (incorrect) and 3 (correct), with partially correct answers being scored a 1 or a 2. On the other hand, multiple choice items were scored either 0 (incorrect) or 1 (correct). An unanswered item was given a score of 0. The total possible cognitive score a pre-service teacher could get on the MCK test was 42. The cognitive score received by a pre-service teacher was divided by the total score (42) in order to express it as a percentage score. In addition, a cognitive proficiency rate for each individual item was determined by calculating the mean score for each item and then converting to a percentage (multiple-choice items) or the mean score of an item as a percentage of total possible score (3) of that item and then converted to a percentage (open-ended items).

The second scoring system employed examined conceptual errors made by participants on items answered. Each item was broken down into key concept(s) necessary for answering the question and linked to the post-primary mathematics curriculum. Based on a participant's solution to a given item, a *conceptual error* score was awarded in the following way: 0 – reflects complete/correct knowledge of a particular concept; 1 – incomplete or error in knowledge of a particular concept; or NA – no determination can be made regarding this concept. An NA was given if the item was not answered by a participant. An overall conceptual score was calculated for each individual participant and conveyed as an occurrence rate (percentage). NAs were not included in the calculation, so each participant's calculation was determined by dividing the total number of errors observed in a participant's answers by the total number of potential errors, dependent on how the participant answered the items. Given the number of conceptual errors associated with items on the paper-and-pencil test, as well as the variation in participants' approaches to solving an item, conceptual error occurrence rates were examined in terms of response rate and identification of strengths and weaknesses in relation to mathematical concepts.

Taking into consideration that the participants were entering their PME programme from a variety of undergraduate degree backgrounds, this provided an opportunity to examine if any relation exists between degree background and cognitive scores and conceptual error rates. Participants were grouped, based on their degree, as the following: B.A. inclusive of Mathematics; B.Sc. inclusive of Mathematics; Engineering; Business/Commerce; B.Sc. non-Mathematics Specific; or B.Sc. Mathematics and Physics. One-way between group analysis of variance were conducted with follow-up post-hoc tests (Tukey) where appropriate.

Findings

Cognitive proficiency with curriculum-aligned content

The first stage of the cognitive scoring analysis examined the overall scores for each pre-service teacher. This overall cognitive score provides a gauge of a pre-service's

Table 1. Overall cognitive scores.

	Overall cognitive scores (percentages)
M	40.2
SD	18.0
Min	4.8
Max	92.9

teacher's performance based on their responses to the test items. The cognitive score was calculated as a percentage out of a possible 42 marks. Table 1 provides a description of the overall scores for this group of pre-service teachers.

The first notable finding is the low mean score of only 40.2% with a standard deviation of 18.0. However, there is a range of 88.1 with a maximum scored of 92.9% achieved by one pre-service teacher. Three pre-service teachers received a cognitive score of less than 10% – on examination of their scripts it was clear that not much effort was put into completing the mathematical items.

The second stage of analysis examined the proficiency rates for each item on the test. An item's mean score is divided by its total possible score (1 or 3) in order to calculate the proficiency rate. Only 4 items (of 20) had a proficiency rate higher than 50%. Table 2 provides an overview of these four items. A proficiency rate of 50% or higher is considered a strength as it is aligned with an honours classification in grading and inline with Teaching Council expectations of qualified post-primary teachers (see Ní Ríordáin, Paolucci, and O' Dwyer 2017). It is important to note that all of these items are linked to Junior Cycle mathematics and that no strengths were recorded at Senior Cycle level. Also included in Table 2 are three items with a proficiency rate between 40% and 50% (pass standard). Only one of these items is at the Leaving Certificate level, all others are linked to the Junior Cycle curriculum. Likewise, test items with proficiency rates below 40% are deemed a fail as they fall below minimum standards expected in studies completed by qualified mathematics teachers. Thirteen items fell below the required 40% pass rate. Table 3 provides an overview of these items.

Overall, the pre-service mathematics teachers demonstrated poor proficiency with the Functions strand. All items had a proficiency rate of below 40%. Although these items relate to LC content, all but one is at the Ordinary level. The case is similar for the Statistics and Probability strand where participants demonstrated poor proficiency in five of the seven items connected to this strand, four of which are at the Ordinary level and related to representing data graphically, the other item relates to Counting. Pre-service teachers demonstrated, on average, a pass standard proficiency with the remaining two items: one related to JC Probability content and the other LC content identifying data sets in which the Mean and Median were equal. The data relating to the Statistics and Probability content suggests a poor knowledge of concepts overall, particularly items involving Senior Cycle and Higher level.

In general, no distinct pattern emerged from the data in relation to strengths and weaknesses relating to the other strands. For example, Item 13a (finding what comes next in a pattern) with the highest proficiency rate (91%, with a standard deviation of 27.5) involved content from Junior Cycle Algebra. Yet its related item, 13b (writing arithmetic expressions for terms in a sequence), only had a proficiency rate of

Table 2. Pre-service mathematics teacher strengths: curriculum-aligned content of items with a cognitive proficiency rate over 40%.

Topic	Item	Level: JC/LC[a]	Curriculum content strand	Proficiency rate (%)	*SD*
Making predictions about what comes next in a pattern	13a	JC	Algebra & Functions: Representing situations with tables, diagrams and graphs	91.0	27.5
Solving problems involving shopping: % discount; Performing calculations with percentages	12	JC	Number: Applied arithmetic	64.3	34.4
Properties and equations of a line – slope, *x*/*y* intercepts; Relationship between the slopes of parallel lines; Labelling axes w/ appropriate scales	9	JC	Geometry & Trigonometry: Coordinate Geometry	60.6	35.5
Properties of a Square; Applying Pythagoras's Theorem; Operations with Surds	8	JC	Geometry & Trigonometry: Synthetic Geometry; Trigonometry	58.3	34.5
Relationships between the number systems	11	JC	Number: Number Systems	46.3	37.8
Probability: Finding the probability of equally likely outcomes	2	JC	Statistics & Probability: Equally Likely Outcomes	46.0	49.7
Representing Data Graphically: Mean vs. Median	3	LC	Statistics & Probability: Representing Data Graphically	45.0	49.9

[a]JC and LC indicate Junior Certificate and Leaving Certificate – Ordinary level is assumed, unless otherwise noted.

38.7%, with a standard deviation of 39.8. The four items with the highest proficiency rates involve three different strands (Algebra, Number and Geometry), but all content is related to Junior Cycle. The items with the weakest proficiency rates involve Number and Algebra also, but at Senior Cycle. Therefore, overall, the data suggest that pre-service teachers demonstrate a stronger proficiency with Junior Cycle curriculum aligned content and poor proficiency with Senior Cycle content, irrespective of the curriculum strand.

Conceptual error occurrence rates

An analysis of pre-service mathematics teachers' overall conceptual scores establishes a relatively high occurrence rate of errors amongst the 85 participants (see Table 4). Overall, of the questions answered, there was a 45.8% mean occurrence rate for

Table 3. Pre-service mathematics teacher weaknesses: curriculum-aligned content of items with a cognitive proficiency rate below 40%.

Concept(s)	Item	Level: JC/LC[a]	Curriculum content strand	Proficiency rate (%)	*SD*
Writing an arithmetic expression for the terms in a sequence	13b	JC	Algebra & Functions: Generating arithmetic expressions from repeating patterns	38.7	39.8
Standard Deviation	4	LC	Statistics & Probability: Representing Data Graphically	35.0	47.8
Mean vs. Median	6	LC	Statistics & Probability: Representing Data Graphically	29	46.1
Count the number of ways to select r objects from n distinct objects	1	LC (HL)	Statistics & Probability: Counting	28.0	45.6
Recognize a bijective function and find its inverse	15a	LC (HL)	Functions: Functions	26.3	42.4
Differentiation	15b	LC	Functions: Calculus	25.3	32.5
Solve simultaneous equations with two variables and interpret results	16a	LC	Algebra: Solving equations	25.3	39.5
Graph of the function $f(x) = x^3$; Transformations $f(x) + a$; Graphs of inverse functions are reflections over $y = x$.	14	LC	Functions: Functions	25.3	23.8
Median	7	LC	Statistics & Probability: Representing Data Graphically	20.0	39.5
Interquartile Range	5	LC	Statistics & Probability: Representing Data Graphically	19.0	39.5
Associate derivatives with slopes and tangent lines	17	LC	Functions: Calculus	19.3	39.5
Relationships between number systems	10	JC	Number: Number Systems	18.0	38.6
Solve simultaneous equations with infinite number of solutions	16b	LC	Algebra: Solving equations	15.2	31.0

[a]JC and LC indicate Junior Certificate and Leaving Certificate – Ordinary level is assumed, unless otherwise noted.

Table 4. Overall conceptual scores.

	Occurrence rate for potential conceptual errors (percentages)
M	45.8
SD	17.6
Min	7.7
Max	77.8

conceptual errors or evidence of incomplete conceptual understanding. The implications of this are that more than two out of every five errors that could be made were made by these pre-service teachers. Unsurprisingly, there is a strong negative correlation between pre-service teachers' overall cognitive scores and their occurrence rate for conceptual errors (Pearson Correlation -.706, significant at the 0.01 level).

Table 5 provides an overview of all the items in the test, the concepts being examined within each item, the response rate to each item and the occurrence rate for potential conceptual errors. Given that there is variation in the response rates for individual items, it is difficult to undertake a comparison of occurrence rates on individual concepts. However, there are a number of items with a high response rate, with a high conceptual error occurrence rate, that can extend our understanding of the cognitive scores and proficiency rates outlined in the previous section. For example, items 3–7 relating to Statistics and Probability had a high response rate with a high occurrence rate (50%-78%) for conceptual errors relating to the identified concepts. In general, items around or below the average occurrence rate for conceptual errors and with a high response rate are associated with Junior Cycle content relating to Geometry and Trigonometry (Item number 8 and 9), Number (Item 10 (part 1 & 2), Item 11 (part 1 & 2) and Item 12) and Algebra (Item 13a).

It is interesting to note variation in occurrence rates within items with high response rates. For example, Item 10 examines Number Systems. Pre-service teachers demonstrated low occurrence rates for conceptual errors relating to Junior Cycle concepts of Natural and Rational Numbers, but an increase in conceptual errors is evident with higher order concepts such as Prime and Complex Numbers relating to Leaving Certificate content. This is consistent with findings relating to pre-service teachers' cognitive proficiency rates with curricular aligned content. These insights help identify the particular areas of weakness that may have caused participants to answer an item incorrectly. For example, Item 17 examines derivatives relating to Leaving Certificate Functions content (which had a low cognitive proficiency rate). There was a relatively high response rate to this item (70.6%) and the data demonstrates that pre-service teachers' primary conceptual error relates to understanding what a derivative represents. By profiling pre-service mathematics teachers' knowledge of content related items, it can help us address such misconceptions within our teacher education programmes.

Undergraduate degree and impact on performance

As described above, given the nature of a consecutive approach to mathematics teacher education, pre-service teachers enter their ITE programme from a variety

Table 5. Pre-service mathematics teachers: conceptual error rates for curriculum-aligned content of items.

Item #	Strand	Level LC	Level JC	Concepts required for item completion	Response rate (percentages)	Occurrence rate for potential conceptual errors (percentages)
1	1: S&*P*	LC		Counting: Count the number of ways of selecting *r* objects from *n* distinct objects	100	72.9
2	1: S&*P*		JC	Probability: Finding the probability of equally likely outcomes	94.1	51.3
3	1: S&*P*	LC		Representing Data Graphically: Mean vs. Median	90.6	51.4
4	1: S&*P*	LC		Representing Data Graphically: Standard deviation	87.1	59.5
5	1: S&*P*	LC		Representing Data Graphically: Interquartile range	82.4	77.1
6	1: S&*P*	LC		Representing Data Graphically: Mean vs. Median	85.9	65.8
7	1: S&*P*	LC		Representing Data Graphically: Median	81.2	73.9
8	2: G&T		JC	Properties of a Square	90.6	31.2
	2: G&T		JC	Applying Pythagoras's Theorem	90.6	44.2
			JC	Operations with surds	83.5	25.4
9	2: G&T		JC	Properties/Equations of a line – slope	90.6	42.9
	2: G&T		JC	*x*/*y*-intercept	90.6	45.5
	2: G&T	LC	JC	Relationship between the slopes of parallel lines	89.4	31.6
	2: G&T		JC	Coordinating the plane: labelling axes with appropriate scales	54.1	19.6
10	3: Num		JC	Number systems: Identifying Natural Numbers	95.3	12.3
	3: Num		JC	Number systems: Identifying Rational Numbers	95.3	22.2
	3: Num		JC	Identifying Prime Numbers	95.3	45.7
	3: Num	LC		Numbers systems: Complex Number is $a + bi$	95.3	80.2
11	3: Num		JC	Sets: Union	78.8	23.9
	3: Num		JC	Sets: Intersection	77.6	33.3
	3: Num		JC	Relationships between the number systems	77.6	53.0

(*Continued*)

Table 5. Continued.

Item #	Strand	Level LC	Level JC	Concepts required for item completion	Response rate (percentages)	Occurrence rate for potential conceptual errors (percentages)
12	3: Num		JC	Applied Arithmetic: Solving problems involving shopping – % discount, selling price	94.1	37.5
	3: Num		JC	Performing Calculations with percentages	92.9	24.1
	3: Num	LC	JC	Synthesis and Problem solving skills: Answering the question	90.6	31.2
13a	4: Alg		JC	Making predictions about what comes next in a pattern	90.6	3.9
13b	4: Alg		JC	Writing arithmetic expressions for terms in a sequence	67.1	59.6
14	5: Fun		JC	Graph of the function $f(x) = x^3$	67.1	63.2
	5: Fun	LC		Transformations $f(x) + a$	42.4	66.7
	5: Fun	LC		Graphs of inverse functions – reflection over $y = x$	30.6	84.6
15a	5: Fun	LC		Inverting functions	34.1	27.6
	5: Fun	LC		Properties of a function	32.9	25.0
	5: Fun	LC		Bijective functions	34.1	27.6
15b	5: Fun	LC		Differentiation	51.8	61.4
16a	4: Alg	LC		Simultaneous equations w/ no solution are parallel lines	40.0	50.0
16b	4: Alg		JC	Equations of parallel lines have the same slope	40.0	50.0
	4: Alg		JC	Simultaneous equations w/ infinite number of solutions are the same line (same equations)	25.9	63.6
17	5: Fun	LC		Derivative represents the slope of a curve	70.6	73.3
	5: Fun	LC		Curves with the same slope/derivative are parallel	70.6	38.3
	5: Fun	LC		Negative slope indicates that the curve is decreasing	70.6	35.0

Table 6. Undergraduate degree and performance.

Degree	*N*	Mean cognitive proficiency rate (percentage)	SD	Mean conceptual error rate (percentage)	SD
B.A. inclusive of Mathematics	47	41.7	14.8	46.0	18.0
B.Sc. inclusive of Mathematics	19	34.8	19.7	48.4	15.4
Engineering Degree	7	49.3	15.5	39.9	16.4
Business/Commerce	5	18.6	4.6	59.7	12.6
B.Sc. Non-Mathematics Specific	4	31.0	5.8	47.3	12.1
B.Sc. Mathematics & Physics	3	77.0	17.6	15.2	4.9

of undergraduate degree backgrounds. Table 6 provides an overview of the various groups' mean cognitive proficiency and mean conceptual error rates in relation to curriculum-aligned content.

A one-way between group analysis of variance was conducted to explore the impact of type of degree studies on cognitive proficiency rate and conceptual error rate. Participants were grouped according to their undergraduate degree studies (as in Table 6). There was a statistically significant difference at the $p < .05$ level in the cognitive proficiency rate for the groups: F (5, 79) = 6.535. The effect size, calculated using eta squared, was .29. The magnitude of the difference is large (greater than .14 as per Cohen 1988) suggesting that a significant portion of the difference in the cognitive proficiency rate between groups can be explained by their degree studies. Post-hoc comparisons using the Tukey HSD test indicated that the mean cognitive proficiency rate for the B.Sc. Mathematics and Physics group (M = 77.0, SD = 17.6) was significantly different from the BA inclusive of Mathematics (M = 41.7, SD = 14.8), B.Sc. inclusive of Mathematics (M = 34.8, SD = 19.7), Business/Commerce (M = 18.6, SD = 4.6) and B.Sc. Non-Mathematics Specific (M = 31.0, SD = 5.8) groups. In addition, the Tukey HSD test indicated that the mean cognitive proficiency rate for the Business/Commerce group (M = 18.6, SD = 4.6) was significantly different from the BA inclusive of Mathematics (M = 41.7, SD = 14.8) and Engineering (M = 49.3, SD = 15.5) groups. No significant difference was found between the other degree programmes in relation to cognitive proficiency rates.

Similarly, there was a statistically significant difference ($p < .05$) in the conceptual error rates for the groups: F (5, 79) = 2.999. The effect size, calculated using eta squared, was 0.16. The difference in the means is large (Cohen 1988) suggesting that a significant portion of the difference in the conceptual rate between groups can be explained by their degree studies. Post-hoc comparisons using the Tukey HSD test indicated that the mean conceptual rate for the B.Sc. Mathematics and Physics group (M = 15.2, SD = 4.9) was significantly different from the BA inclusive of Mathematics (M = 46.0, SD = 18.0), B.Sc. inclusive of Mathematics (M = 48.4, SD

= 15.4) and Business/Commerce (*M* = 59.7, *SD* = 12.6) groups. No significant difference was found between the other degree programmes in relation to conceptual error rates. This may be explained by the scoring system employed in that an NA was given when there was no evidence of mastery or error for a given concept, including unanswered items (0 was given in the cognitive scoring, see Ní Ríordáin, Paolucci, and O' Dwyer 2017).

Discussion and conclusion

This study examines the MCK of pre-service post-primary mathematics teachers on commencement of their PME ITE programmes in four institutions, across a broad range of mathematics curriculum topics. The findings of this research suggest that while these pre-service teachers may be well equipped to teach Junior Cycle mathematics content, they do not demonstrate the same proficiency with Senior Cycle content. Furthermore, there are strands of the curriculum (e.g. Statistics & Probability and Functions) where pre-service teachers demonstrate poor cognitive and conceptual knowledge. This research aligns with the findings of Ma (1999) and Slattery and Fitzmaurice (2014) that record shortcomings in pre-service primary teachers' knowledge of mathematics. Given the importance of teachers' adequate knowledge of content to develop learners' conceptual understanding and to positively impact on learner achievement (Baumert et al. 2010; Hill, Rowan, and Ball 2005; Kulm 2008), these findings may have broad implications for consecutive post-primary ITE programmes in Ireland.

As it is largely accepted that pre-service teachers' MCK does not change during their ITE (Osborne 2013), the results of this research suggest that, despite the rigorous content requirements of the Teaching Council (2013), pre-service teachers on consecutive programmes may require additional subject matter knowledge prior to their commencement of teaching. In line with recommendations from Baumert et al. (2010), ITE programmes should, perhaps, increase the attention given to pre-service teachers' content knowledge and explicitly align MCK and PCK in PME mathematics pedagogy modules. This may be particularly relevant to consider in keeping with recent policy objectives (The STEM Education Review Group 2016), since high levels of teacher PCK are not achievable without a high level of subject matter knowledge and both have significant impact on student achievement (Baumert et al. 2010). In these pedagogy modules, MCK materials should not mirror that of programmes for mathematics majors, but rather explicitly link mathematical content at a deeper level of understanding to that of the school curriculum (Artzt et al. 2012; Toumasis 1992). This may be particularly relevant in LC content and in strands such as Functions. Such strands, or topics, are often core components of undergraduate mathematics and mathematics-related qualifications, but may require revisiting for conceptual understanding related to the post-primary curriculum (Bryan 1999). On the basis of this evidence, any reduction of the current minimum subject specific pedagogy from 5 ECTS credits (Teaching Council 2011b, 2013) should be avoided. It may be of interest to repeat this research with additional cohorts of pre-service teachers who have studied the revised post-primary mathematics curriculum as part of their own post-primary education and may therefore be more familiar with the revised curriculum content, particularly that of Statistics and Probability, which was not compulsory content prior to 2012.

Specifically focusing on the consecutive ITE aspect of this research, the findings suggest statistically significant differences regarding the MCK and undergraduate qualification of pre-service teachers. Those who had graduated with a BSc. Mathematics and Physics had a mean score that contrasted greatly with those graduating with a Business/Commerce degree (77.0% versus 18.6% respectively). Given the necessity of high MCK on developing PCK and on positively impacting student achievement (Baumert et al. 2010; Hill, Rowan, and Ball 2005), such findings are worrying particularly considering the large cohort of out-of-field mathematics teachers already in the Irish post-primary system (Ní Ríordáin and Hannigan 2011). This research may point to the continued necessity of the high-quality content requirement for post-primary mathematics teaching currently in the Teaching Council guidelines (Teaching Council 2013).

This research provides the first record of pre-service post-primary teachers' MCK across a range of topics in Ireland. However, the research is limited by the fact that the paper-and-pencil questions were completed by participants during their lecture time, where they may not have been motivated to fully engage with an assessment of their subject matter knowledge. Furthermore, the research did not include all institutions providing PME programmes nor any concurrent ITE programmes. Given calls for a renewed focus on content knowledge in teacher education (Lowrie and Jorgensen 2016), further research should be conducted on both pre-service and in-service mathematics teachers' knowledge on a larger and longitudinal scale. Such research would be of particular relevance in informing any future ITE and teacher education policies. In the context of various ITE programmes underway across the country, it may be of particular interest to compare and contrast the MCK of pre-service teachers from consecutive and concurrent programmes. Additional research should be conducted to investigate whether, by virtue of teaching various JC and LC topics, pre-service teachers' knowledge improves over the duration of their PME programme. This may be unlikely, however, as many pre-service teachers are not provided by their placement schools with an opportunity to teach senior cycle classes during their ITE programme. Similarly, experiences for pre-service teachers would be inhomogeneous, as their classroom experiences of teaching the post-primary curriculum would be dependent on each school and placement context. Nevertheless, the strong association between MCK and classroom practices needs to be considered at all times and by assessing pre-service teachers' MCK it allows for the development of awareness of the challenges they may face (Norton 2019). As proposed by Deng (2018, 162) 'the development of pre-service teachers' content knowledge needs to be placed within the context of interpreting and enacting the institutional curriculum for teaching' and facilitate their development of a theory of content that can inform their curriculum thinking and pedagogical approaches in the classroom.

Note

1. A level 9 qualification in the Irish National Framework of Qualifications (NQF) is equivalent to a level 7 qualification in the European Qualifications Framework (EQF).

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