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Knowledge, disposition and the capacity for instruction: An optional mathematics course for preservice teachers

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Abstract
Effective initial teacher education will develop preservice teachers' disposition towards mathematics and the teaching of mathematics, alongside their mathematical and pedagogical content knowledge. This paper reports on a case study of eight students who participated in an optional mathematics course for preservice primary teachers. Analysis of focus group interviews indicates that students' dispositions to mathematics and mathematics teaching were influenced by prior negative experiences. Significantly, engagement in the mathematics course contributed both to their self-assessments of their mathematical knowledge, and their professed self-efficacy for inclusive mathematics teaching for conceptual understanding. In this paper, characteristics of the optional mathematics course are identified and implications for initial teacher education are discussed.

Keywords Mathematical knowledge · Initial teacher education · Disposition · Instruction · Preservice teachers

Introduction

Enacting reform-oriented pedagogy can represent a challenge for novice teachers, particularly for those with low levels of content knowledge and/or negative dispositions to mathematics. Gaps in mathematical content knowledge impact preservice teachers' ability to enact high-quality teaching (Thanheiser, 2018). Arising from poor content knowledge or previous negative experiences, these teachers may exhibit high-levels of mathematics anxiety and low self-efficacy, both of which have been shown to impact on how mathematics is taught and how students come to view mathematics (Gresham & Burleigh, 2019). However, the research literature provides evidence that initial teacher education, when appropriately designed and enacted, can impact outcomes for preservice teachers. For example, there is evidence of positive changes to student teachers' attitudes (Jong & Hodges, 2015)

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and mathematics anxiety, mathematics self-efficacy, and mathematics teaching efficacy beliefs (Gresham & Burleigh, 2019) arising from reform-oriented mathematics methods courses. Also, appropriately designed courses have been shown to have a significant effect on the content knowledge demonstrated by graduates up to 4 years after graduation (Hiebert et al., 2019). Despite this research base, the content that courses should cover and how exactly they should be designed is still discussed (Hill, 2010; Thanheiser, 2018).

In this study, which occurred in the context of a preservice mathematics education module in Dublin City University, we investigate the role of an optional short mathematics course in developing students' knowledge, disposition, and the instructional practices they intend to enact in classrooms. This paper answers the call for continued research on the effectiveness of particular teaching approaches within initial mathematics teacher education courses (e.g., Gresham & Burleigh, 2019; Jong & Hodges, 2015) and contributes to discussion of practice within initial teacher education; specifically, we consider how to support student teachers with low mathematics knowledge and/or negative dispositions within the time constraints of initial teacher education programmes.

Literature review

Developing the capacity for mathematics instruction

The primary purpose of teacher education programmes is to develop capacity for effective instruction. Baumert et al. (2010) argue that gaps in preservice teachers' content knowledge can limit the development of their pedagogical content knowledge but conceptualisations of teacher instructional competence tend to include aspects other than knowledge alone. For example, the influential model of professional competence proposed by Blömeke et al. (2008) includes professional beliefs and personal characteristics in addition to professional knowledge. Similarly, the important Teacher Education and Development Study in Mathematics (TEDS-M), recognises the importance of both cognitive and affective aspects of teacher education (Tatto et al., 2008). The seminal articulation of the interwoven strands of mathematical proficiency by Kilpatrick et al. (2001) also presented analogous strands of teaching for mathematical proficiency. Jacobson (2017) has reimagined these strands of teaching for mathematical proficiency as composed of *knowledge* (knowing that and knowing how), *disposition* (beliefs, attitudes and affect) and *instruction* (knowledge and disposition in action), see Fig. 1. Jacobson describes *knowledge* as conceptual understanding of the core knowledge required in the practice of teaching. Teacher *disposition* involves affective elements and is discussed more fully below. Finally, *instruction* is understood to involve fluency in carrying out basic instructional routines; strategic competence in planning effective instruction and solving problems that arise during instruction; and adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices (Jacobson, 2017).

Jacobson (2017) claims that knowledge and disposition are interdependent and develop together in relation to teaching activity or instruction. He suggests that there is a reciprocal relationship between a teacher's competence in particular teaching practices and their mathematics-related knowledge and disposition—"An individual's history of instructional activity supports and constrains what a teacher knows and believes both about herself or himself as a teacher and about the nature of teaching mathematics" (Jacobson, 2017, p. 15). This is particularly noteworthy for preservice teachers with low levels of mathematics

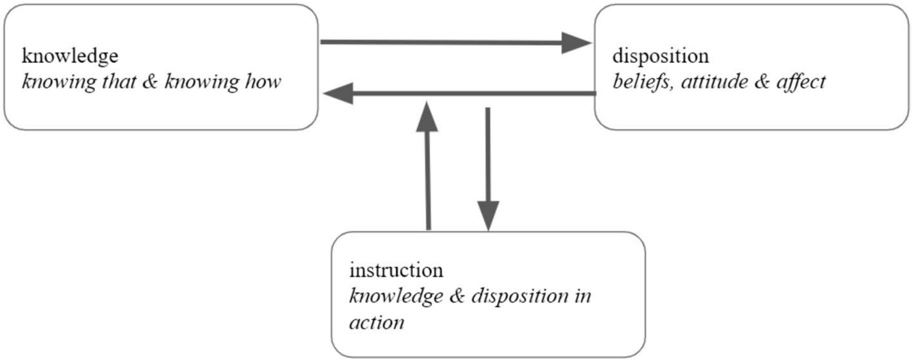

Fig. 1 Reciprocal relationships between categories of teaching proficiency (Jacobson, 2017)

knowledge and/or negative dispositions toward the subject. Jacobson's contention is supported by existing research which investigates links between knowledge and various aspects of disposition such as beliefs or attitudes (see Jong & Hodges, 2015). It has been noted that while preservice teachers may be able to successfully complete mathematical procedures, they sometimes lack conceptual understanding of the underlying mathematics (Thanheiser, 2009, 2018). Procedural (or instrumental) understanding of mathematics can lead to a view of mathematics as disconnected sets of rules and is unlikely to support the development of productive dispositions towards mathematics (Thanheiser, 2018). Procedural understanding and negative disposition are likely to constrain the types of instruction teachers enact. It has been suggested that the nature of preservice teachers' understanding of mathematics is also likely to have implications for what content they teach and how they teach it (Thanheiser, 2018). For this reason, developing capacity for instruction must involve both developing mathematical knowledge for teaching and the development of productive disposition.

In the following sections, we review briefly existing research on mathematical knowledge for teaching and on disposition before considering the implications for teacher education.

Mathematical knowledge for teaching

Developing mathematical knowledge for teaching (MKT) is a key aim of teacher education programmes (Baumert, 2010; Venkat & Spaull, 2015). There is a long history of research on MKT with the seminal work of Ball et al. (2008) identifying six different kinds of MKT. While the complexity of attempting to trace causal relationships in educational settings is recognised, in general it is acknowledged that robust MKT underpins teaching which is effective for students (Hiebert, 2019). While Baumert et al. (2010) found that teachers' pedagogical content knowledge was empirically distinguishable from their mathematics content knowledge, and that it was a better predictor of student progress than content knowledge, more recent research has found little theoretical or empirical basis for the distinctions in MKT proposed by Ball et al. (Copur-Gencturk et al., 2019).

While measurement of the (sub)constructs can be problematic, Ball et al.'s conceptualization of MKT, particularly the idea of specialized content knowledge, is useful for guiding our work as teacher educators. While specialized content knowledge is 'strictly mathematical', rather than related to knowledge of students or teaching, it is also essential for understanding and addressing mathematical issues which may arise when teaching (Ball et al., 2008). Hiebert et al. (2019) maintain that specialized content knowledge involves unpacking or 'decompressing' mathematical concepts, for example, understanding the concepts underlying conventional arithmetic. They suggest that because specialized content knowledge is not associated with teaching or students, it is possible to develop and study this within initial teacher education (p. 26). Preservice teachers need time, and multiple opportunities, to develop specialized content knowledge, but questions about how much content it is possible to address within time-bound initial teacher education programmes have long been raised (Hill, 2010).

Disposition

Teacher disposition has become a growing focus of research and policy (Jacobson & Kilpatrick, 2015). Cooke (2015) notes that disposition is often categorised as positive or negative but should instead be considered as a continuum or a measure to describe a person's willingness to use mathematics. A common thread in the diverse literature is the view that dispositions can be understood as a predisposition or tendency to act in a particular manner.

Jacobsen and Kilpatrick (2015) propose that productive disposition for *teaching* mathematics is part of the collection of affective traits of mathematics teachers (p. 403) and describe it as "mathematics teachers' malleable orientation toward—and concomitant beliefs, attitudes, and emotions about—their own professional growth, the subject of mathematics, and its teaching and learning that influences their own and their students' successful mathematics learning" (p. 402). Of relevance to the participants in this study, is existing research findings on mathematics anxiety and efficacy beliefs. Mizala et al. (2015) contend that mathematics anxiety impacts teachers' capacity to develop inclusive learning classrooms. Gresham and Burleigh (2019) note that preservice teachers with negative views about mathematics may deliberately avoid mathematics and lack confidence in their own teaching capacity. Their empirical work also found that reform-oriented mathematics methods courses can reduce mathematics anxiety and positively impact preservice teachers' attitudes and efficacy beliefs. While Hoy and Spero (2005) reported that self efficacy increased throughout initial teacher education before becoming more resistant to change, Clark et al. (2015) documented a continued development of efficacy beliefs in the first year of teaching. In the Irish context, Hourigan and Leavy's (2022) study of preservice primary teachers found medium levels of personal mathematics teaching efficacy. Participants were also found to have limited conceptions of what mathematics teaching involves. Jacobsen and Kilpatrick (2015) argue that disposition is 'necessarily entangled' with teachers' knowledge and instructional practice. For this reason, attention should be given to disposition within teacher education programmes, in the same way that attention is given to knowledge and skills (c.f., Cooke, 2015; Gresham & Burleigh, 2019).

Developing knowledge, disposition and instruction within initial teacher education

In relation to developing mathematical knowledge for teaching, Hill (2010) suggests that initial teacher education should develop specialized content knowledge in addition to common content knowledge. Developing specialized content knowledge may involve participation in activities that simulate teachers' work, e.g., unpacking mathematical procedures and generating a variety of solution strategies (Hiebert et al., 2019). Robust knowledge of this type might improve preservice teachers' perceptions of their own ability to enact effective instruction (see Morris et al., 2017), but the nature of preservice teachers' experience as learners of mathematics during initial teacher education is also critical for developing productive dispositions toward mathematics teaching in ways which will impact on their capacity for instruction (Gresham & Burleigh, 2019).

The influence of the 'apprenticeship of observation' (Lortie, 1975) has long been recognised. Jong and Hodges (2015, p.111) note that preservice teachers' attitudes, conceived as a component of productive disposition, become a part of the apprenticeship of observation, framing how they make meaning of their mathematical experiences within initial teacher education. They also demonstrate evidence of positive changes to attitudes arising from participation in mathematics methods courses aligned with reform recommendations. While various explanations for the effects of university-based initial teacher education are possible, mastery experiences in mathematics may influence mathematics self-efficacy (Bandura, 1997). In addition, vicarious experiences where one observes a model performing the required actions (Bandura, 1997) have been theorised as a source of teacher efficacy. For example, Yurekli et al., (2020a) investigated the sources of preservice teachers' self-efficacy in the context of a mathematics teaching methods course. This work highlighted a prominent role for vicarious experiences such as observation of lecturer and engagement in group work. Preservice teachers, with limited experiences of teaching mathematics, are likely to draw on related experiences, such as their experiences of learning mathematics, when making judgments about their own capacity to teach (Bandura, 1997). We suggest that the nature of mathematics learning experiences within initial teacher education are particularly important for preservice teachers who have had prior negative experiences as learners of mathematics and/or self-identify as poor at mathematics.

In summary, the research literature suggests that (i) specialized content knowledge should be developed in initial teacher education, and (ii) initial teacher education should aim to develop disposition alongside knowledge. However, research to date has not sufficiently grappled with the entangled nature of knowledge, disposition and preservice teachers' experiences as mathematics learners (prior to and during teacher education). We suggest that this is particularly important for preservice teachers with low confidence/competence in mathematics.

Research questions

This study examines: *How do participants' perspectives on teaching mathematics change following engagement with a short course of content knowledge for teaching mathematics?* A number of sub-questions allow us to consider participants' experiences as learners and as (future) teachers:

Preservice teachers as learners:

RQ1. What is the preservice teachers' (a) previous experience of learning mathematics and (b) self-perception of understanding mathematical concepts on completion of the mathematics support seminars?

RQ2. From the experience of participating in the mathematics support seminars, what did participants perceive that they learned about the teaching and learning of mathematics?

Preservice teachers as future teachers:

RQ3. Do participants perceive that participation in the mathematics support seminars influenced their competence and confidence in teaching mathematics? If so, what elements do they identify as significant in causing any changes?

RQ4. How do the participants plan to teach mathematics, and have their stated intentions changed as a result of participation in the mathematics support seminars?

Methods

Background

The participants in this study were drawn from one cohort of a two-year postgraduate programme of initial teacher education (ITE) in Dublin City University, Ireland, where the authors teach mathematics education. Graduates of this ITE programme are qualified to teach the whole primary age range in Ireland. This spans 8 years from when children are approximately 4–12 years old. Primary teachers in Ireland are generalist teachers, teaching 12 different subjects in total, and mathematics education is just one of a number of discipline-focused modules undertaken in ITE. Immediately prior to their commencement of the second year of the ITE programme, all students were invited to attend mathematics support seminars. There were five seminars in total which were optional, additional to the timetabled lectures and were designed to support the students' content knowledge for mathematics, with a focus on Number. Due to the Covid-19 pandemic, each hour-long seminar was conducted fully online using Zoom. The tutor was an experienced primary school teacher and initial teacher educator.

Intervention

As detailed on Table 1, the content covered in the five mathematics support seminars focused on unpacking and modelling traditional algorithms for multi digit multiplication and division and operations with fractions. Multiple methods were encouraged and discussed. In order to develop specialised content knowledge, activities included exploring the mathematics behind different solution strategies to number problems, checking the validity of various strategies, and making connections across multiple representations for central mathematical concepts and procedures (Hiebert et al., 2019).

In these seminars, the tutor aimed for conceptual understanding by giving explicit attention to mathematical concepts. While explicit attention to concepts may arise from various teaching approaches (Stein et al., 2017), of most relevance here are the teaching actions associated with explicit public attention to mathematical connections as discussed in Yurekli et al., (2020b). The teaching strategies listed below were employed across the seminars to promote explicit attention to concepts in ways that foreground sense-making.

- Asking students to use two different strategies for solving the same problem (employed in seminars 1 – 4)
- Students draw connections between numbers/symbols and other mathematical representations (employed in seminars 1, 2, 4 and 5)
- Demonstrating how different solution strategies to the same problem are similar to or different from each other to illustrate or make a generalization (employed in seminars 1 – 4)
- Students explore how two different strategies can lead to the same answer (employed in seminars 1 – 4)

Table 1 Overview of the content of the Mathematics Support Seminars

Topic	Content Covered	Sample Tasks
Place Value, Addition and Subtraction	Adding doubles, near doubles. Solving subtraction questions using constant difference. Comparing informal methods with traditional algorithms. Modelling of different approaches on empty number line.	Reach 100 task (University of Cambridge, n.d.). Given a grid, insert four different digits that give four two-digit numbers which add to a total of 100. Example: With the digits 5, 2, 1, 9, the following attempt can be made. 52 + 19 + 51 + 29 = 151
Multiplication	Use digital manipulatives and/or diagrams to model commutative, associative, distributive properties. Identification of inverse relationships between multiplication and division facts. Exploration of arrays and multiplication grids	Can you visualise a 3-d shape made of 24 cubes? Find ‘shortcuts’ for calculations (mental calculations and representations of same) 7 x 92 *Use of doubles* (92 x 2) x 2 x 2 - 92 *Known facts* (92 x 5) +(92 x 2) *Distributive Law* (7 x 90) + (7 x 2) (7 x 100) – (7 x 8) *Relationship between operations* 9 x 25 (9 x 100) ÷ 4
Division	“Think Multiplication”- rephrasing division problems as multiplication questions. Chunking (see Schultz, 2018) Low-level chunking adds small subtotals of the divisor to make the dividend (or subtracts these from the dividend to try to get to 0). High-level chunking relies on multiplicative reasoning and uses larger subtotals to add/subtract larger ‘chunks’.	Short division problems and questions from Multiplication and Division Pick n Mix 1 (New Zealand Ministry for Education, n.d) 75 ÷ 25 *Rephrasing as multiplication* 25 x __ = 75 *Low-level chunking* 25 + 25 + 25 = 75 75 – 25 – 25 – 25 = 0 *High-level chunking* 2376 ÷ 16 2376 -1600 (100 x 16) 776 - 320 (20 x 16)

- Facilitating students’ connection of ideas to arrive at their own explanations of a general mathematical principle (seminars 2 – 5)

Table 1 (continued)

		456 -320 (20 x 16) 136 - 80 (5 x 40) 56 -48 (3 x 16) 8 Answer 100+20+20+5+3 = 148 (r8)
Fractions	Use meaningful problem context to explore multiplying a fraction by a whole number. Use digital manipulatives and/or diagrams to model fraction operations - Linear Model - Set Model Use manipulative to model multiplying a fraction by a fraction - Paper folding - Fraction Wall	6 x ¾ ¾ x 6 ¾ x ½
Fractions and Decimals	Continued work on fractions as above. Decimal place value Estimation Strategies	True or false: 10 x 2.8 = 2.80 6 x 4.8 = 24.48 0.8 x 0.3 = 2.4

Note. All seminars took place over Zoom.

In addition, student problem-solving and discussion of strategies was employed in all seminars to create opportunities to struggle. Opportunities to struggle are understood as students' efforts "to make sense of mathematics, to figure something out that is not immediately apparent" (Stein et al., 2017, p. 2). We considered that creating opportunities for these under-confident participants to have mastery experiences (Bandura, 1997) would be impactful. While explicit tutor instruction was a feature of the seminars, it generally occurred after participants had opportunities to think about the mathematics themselves. The tutor used engaging problem contexts and a range of digital manipulatives including BrainingCamp (n.d), and GeoGebra (n.d) activities. Students were encouraged to select from appropriate resources to solve problems; they could use digital resources or pen and paper to model mathematical ideas. Breakout rooms were used throughout the seminars for students to discuss and make sense of each other's solutions strategies. In this way, the key teaching strategies listed above were enacted. This approach to teacher development which emphasizes opportunities for explicit attention to concepts and opportunities to struggle has also been used in large scale professional development efforts (Hughes et al., 2023).

For context, we include details of students' other relevant modules. The obligatory second-year mathematics education module, taught by the first author, ran concurrently with the support seminars. In this module, six of the ten two-hours seminars addressed Number. The teaching approach was similar to the support seminars, but time was also spent exploring theoretical ideas about curriculum expectations and recommended pedagogies. The obligatory mathematics education module in the first-year of the programme, was taught by the second author and followed a similar approach. Just one of the ten two-hours seminars in that module addressed Number directly. The chief difference between the obligatory modules and the support seminars was that the support seminars focused solely on the students as learners of mathematics (rather than teachers of mathematics). Additionally, while the first-year module took place in standard face-to-face format of classes of 30 students, both the mathematics support seminars and the second-year module took place online via Zoom. There were 60 students in the Zoom class for the obligatory module. Of these, 24 indicated their interest in attending mathematics support seminars. Two series of mathematics support seminars were organised, with each series accommodating 12 students.

Data collection

All students who attended the support seminars were invited to participate in focus group interviews which took place two weeks after the intervention ended. Eight of the 24 participants engaged in the interviews, all of whom were female. This is a 'total population sampling' purposive approach (Etikan et al., 2016) and as such sought to tell the full picture of the students' experience. However, as not all seminar participants volunteered for the interviews, the findings cannot be generalized beyond the sample of interview participants.

Focus groups were more suitable than individual interviews in this study as students were giving feedback on the teaching they received and may have felt vulnerable doing so in a one-to-one setting. The focus groups were conducted by the second author. This was deemed appropriate as she had no part in the support seminars or in the teaching or grading of the obligatory second-year mathematics module. We acknowledge that this approach does not allow us to distinguish between the impact of the concurrent standard module and the mathematics support seminars intervention. Attempting to do so would require collecting data on the whole student cohort and making comparisons between those who participated in the mathematics support seminars and non-participants. Unfortunately, this was not feasible.

An interview schedule was developed to guide the focus group discussion. The schedule structure aligned with Hennink and Leavy's (2014) 'hourglass design', where introductory questions build rapport and begin to direct thinking to key topics before questions addressing issues central to the research are discussed. Finally, closing questions signal that the focus group interview is coming to a close. Our introductory questions probed participants' experiences of the mathematics support seminars, e.g., *what did you feel you learned from the mathematics support seminars? Which session did you find most useful? Why?* These were followed by open questions, designed to generate in-depth discussion and participation by all members of each group. Firstly, participants were asked to consider the following teacher actions and asked to describe when and if such approaches were used by the mathematics support seminar tutor and how they worked for their mathematics learning; Demonstrating, Explaining, Questions, Figuring things out, Representations, Connections. Secondly, participants were asked: *Did the approaches align with or contrast with your previous experience of learning mathematics? In what way?* An openness to participants' ideas was signalled throughout and the interviewer explicitly invited participants to share other relevant examples and experiences. The interview was framed as a way to help us understand what works and does not work for students and they were encouraged to suggest improvements for the future. The closing question was: *have the mathematics support seminars changed how you feel about your future as a teacher of mathematics?* This was developed with the aim of encouraging participants to summarise key learnings from the support seminars and discuss implications for their teaching.

While we acknowledge that directly assessing participants' knowledge pre- and post-intervention, would have allowed greater insight into any knowledge gains, onerous data collection prior to the intervention may have had a detrimental impact on participation rates. We do not assume that students' self-assessments of their mathematical knowledge or teaching competence are accurate or would align with more formal assessments of same. Instead, we see value in investigating students' self-perceptions as a means of investigating connections across knowledge, disposition and intended approach to instruction.

Data analysis

Directed content analysis was employed to manage the potential for multiple and disparate findings arising from focus group interviews (Hennink, 2014). This involves the development of codes from pre-existing research questions and also allows themes to emerge from the data during the analysis process (Hsieh & Shannon, 2005). It is appropriate where existing research can be used to inform the initial coding scheme and possible relationships between variables. In this case, research indicates that mathematical content knowledge (e.g., Hiebert et al., 2019; Thanheiser, 2018), previous experiences of learning and teaching mathematics (e.g., Gresham & Burleigh, 2019; Jong & Hodges, 2015) as well as vicarious experiences of teaching (e.g., Bandura, 1997; Yurekli et al., 2020a) will inform preservice teachers' dispositions and their judgements about their own mathematics teaching competence. This literature informed the development of our focus group questions and we identified several predetermined or deductive codes prior to commencing our analysis*: previous experience of learning mathematics; previous experience of teaching mathematics; mathematical content/concepts; self-perceptions of mathematical competence*.

Following transcription of the interviews, each research question was assigned as a heading on a spreadsheet. Each author first examined one transcript each for evidence pertaining to the spreadsheet headings, and the predetermined codes listed above. Relevant segments of transcribed data were input under each spreadsheet headings and labelled with the relevant codes. This initial analysis was carried out in parallel as an exercise in familiarising ourselves with the data. We then progressed to review each other's coding. Disagreements about predetermined codes were few and easily resolved, possibly because this coding drew on manifest meanings rather than latent interpretations of the data (Hsieh & Shannon, 2005). We also used open coding to identify key characteristics of the teaching that participants planned to enact and to label data that appeared to be of importance to participants but which was not captured by the deductive codes. During our individual coding, we highlighted this data and wrote analytic memos, as recommended by Hennink (2014). These memos served as a prompt for individual reflections and joint conversation, and as an audit trail where queries and key decisions were recorded. We worked collaboratively to agree the open codes through discussion. For example, it became clear to both of us that specific elements of the mathematics support seminars were regularly mentioned by participants. In conversation, we reviewed the pertinent data and came to an agreed understanding of inductive codes that were pertinent to answering our research questions. These were: '*alternatives to the standard algorithm', 'collaboration', 'representations', 'sense-making' and 'use of resources'*. We considered whether these codes represented new categories or a subcategory of an existing code (Hsieh & Shannon, 2005, p. 1282). Our analysis indicated that the inductive codes that we identified were subcategories of existing codes. Importantly, this process made it possible for us to identify the cross-cutting nature of these codes. For example, *alternatives to the standard algorithm* was an inductive code that was used to label content that was also coded to *mathematical content*, *previous experience of learning mathematics* and also to segments of transcripts where participants spoke about their future teaching. Codes are shown in italics throughout the findings.

Findings

While findings indicate that the 8 participants who participated in data collection had positive experiences, we do not presume that this was true for the wider cohort, and cannot rule out that only those positively disposed to the mathematics support seminars participated in the focus group interviews. In this section, we address each research question in turn, before presenting a final discussion.

Preservice teachers as learners

This section of our findings focuses on preservice teachers as learners and investigates prior mathematics learning experiences and self-perceptions of mathematical understanding (RQ1) in addition to considering what participants perceived that they learned about the teaching and learning of mathematics from the intervention (RQ2).

RQ1: Prior learning experience and perception of mathematical understanding

In the interviews, we sought to establish a baseline by firstly asking our participants what they felt they learned from the seminars. Analysis of transcripts coded as *Previous experience of learning mathematics,* showed that within this group, prior experiences of learning mathematics were dominated by traditional teacher-centred methods whereby learners were expected to apply computation methods regardless of whether they made sense. Participants referred to learning computation as involving, for example, "one method", "just being given steps to do", "one traditional way". All participants described following steps without understanding. Many also referred to feeling disempowered, or not having experience of figuring things out independently with statements such as "I actually didn't know why you would use it or what it was for" (Róisín) or "I never understood where the zero was coming from" (in multi-digit multiplication) and "I just didn't know why I was doing it" (Sorcha). The codes *self-perceptions of mathematical competence* and *sense-making* also arose in relation to *previous experiences of learning mathematics* but exclusively in a negative manner. Participants referred to teaching approaches that were centred on completing pages of questions without an expectation of, or focus on, understanding, as evidenced in statements such as the following:

> when I was in school it was just a page of sums, and it was just a matter of, who would get them done first into the copy like you weren't even thinking about them (Sorcha)
> so definitely, the making sense of it and understanding why you would do certain things. It's definitely something that we didn't focus on in primary school. (Róisín)

The excerpts provided evidence of lack of experience of concrete materials, and a suggestion from some that gaps in understanding could have been remediated by access to multiple representations (both *use of resources* and *representations* were codes used in the analysis). Participants stated that "I really don't remember using any concrete materials in the classroom for maths" (Eimear) and "there was no focus when I was in primary school on using manipulatives" (Sorcha), or "just having the numbers like the numerals on the board it just was very kind of abstract and I couldn't make the connections" (Meabh).

It is pertinent to note that students' understanding of mathematical concepts was not assessed. Instead, by asking what they felt they had learned, we sought to explore their self-perception, that is whether they felt that their understanding of the content included in the seminars was robust, or lacking in any aspect. As mentioned above, all eight participants mentioned that they followed procedures without understanding why certain steps are carried out. Data coded to *self-perceptions of mathematical content* included comments such as "Oh, I know I do this but I don't know why. Or, for what purpose" (Eimear), "I didn't know what the method was for or like why you would use the method" (Róisín). Participants also regularly referred to their self-perception of poor understanding in mathematics, sometimes related to teachers' assessments of same, including statements such as "I remember struggling with maths in primary school" and "like, my teacher would always say that I struggled with maths" (Ciara); "I wasn't bad at maths but like my teacher would always say that I struggled with maths" (Sorcha). While Ciara's view of her competence seems to align with her teacher's judgement, there is some conflict evident in Sorcha's statement. Unfortunately, the available data does give any insight into whether this is something Sorcha felt at the time, or whether participation in the intervention led her to re-evaluate her previous experiences.

Emerging strongly from the focus groups was participants' concern in relation to teaching mathematics in the senior classes of primary school (3rd–6th class where children are between 8 and 12 years old). Some participants referred to negative emotional states- "kind of panicking, being like, I don't really know the topic to begin with" (Eimear), "the experience kind of scared me a little bit" (Orla) and "I had scared myself that I wouldn't be able to teach senior classes" (Sorcha). Given the acknowledgement of all participants that they did not understand how to multiply or divide multi-digit numbers or add fractions, this is not surprising, but from the perspective of teacher-educators, it is cause for concern.

RQ2: Learning about the teaching and learning of mathematics

Participants identified certain features of the seminars as significant for their learning of mathematics. Data coded as *sense-making* included participant descriptions of seminar activities where they had opportunities to 'figure things out' collaboratively (*Collaboration* was another code used in data analysis). They appeared to value experiencing different methods and recognised that making mathematical connections was important for their understanding. Eimear described how, "you kind of forget things and kind of feel so distant, especially from school". For her, having opportunities to 'figure things out' and '"just getting the chance to actually do a bit of maths yourself" was an important part of (re) learning mathematics. For others, 'figuring things out' was significant because it contrasted with previous ways of learning maths. Sorcha noted that being presented with a word-problem rather than a "page of sums" meant that "you actually had to sit down and think and visualize it." Sorcha did not mention a particular aspect of mathematics with this comment. She appears to be drawing a distinction between procedural and conceptual approaches and is likely referring to the teaching approach, used in all seminars, where students were asked to draw connections between numbers/symbols and other forms of representation (see Sect. "Intervention"). As detailed in Table 1, this included building understanding of whole number and fraction operations by using digital resources or pen and paper to model the underlying mathematical ideas.

All participants seemed to value the collaborative aspect of the mathematics support seminars. Sorcha noted that breakout rooms provided opportunities to ask people questions about "what method they used". In the quote, below Róisin echoes this statement. The collaborative problem-solving discussion described here was a central recurring element and one of the key pedagogical activities of the seminars (Sect. "Intervention").

> I thought it was brilliant that we [...] broke out into breakout rooms, and were able to talk through it with someone else. And because everyone like would have their own way [...]everyone will kind of [...] see it differently. So that was brilliant, you kind of picked up a little trick there from someone else in the group (Róisín)

While Róisín's comment about 'picking up a trick' may appear somewhat reductive, she is referring to alternative ways to complete operations like multiplication or division. Exploring how two different strategies can lead to the same answer is an approach associated with giving explicit attention to mathematical concepts (Yurekli et al., 2020b). What she interprets as a 'trick', we interpret as valuable mathematics knowledge for teaching and note the positive emotion and satisfaction that she associates with it. This is an example of the many data extracts coded to *alternatives to the standard algorithm* which was indicative of students' positive feelings about non-standard methods.

Some participants connected the productive *collaboration* in breakout rooms with the supportive nature of the seminars more generally. This was considered "a safe space" (Orla), which was less "daunting…if you're lacking confidence in maths" (Meabh) when compared with participating in the larger group of 60 students. Working collaboratively on problems appeared to result in positive affect toward mathematics for some students. Bronagh, like others who described 'enjoying' the seminars, noted that "it is actually kind of fun like I would never have classed maths as fun, but I was actually going for and trying to figure it out". We suggest that this affective element of the mathematics support seminars is an important factor in understanding its impact on participants, particularly for students like Róisín who describe how it contrasted with previous experience, "I absolutely loved it because it's not scary it was more of an enjoyable challenge than a scary thing". We acknowledge that relationships between participants may have functioned to funnel the focus group conversation toward a consensus positive view, but highlight again the steps taken by the interviewer to prompt alternative views and to signal openness to critique of the intervention.

The participants identified some features of the teaching approach adopted in the mathematics support seminars as significant to the teaching of mathematics more generally. Participants were asked to consider how tutor practices such as demonstrating, explaining, questions, representations, and connections contributed to their learning. They identified the use of representations and connections to the real-world as important. In relation to *representations*, a code-used during the analysis process, participants tended to agree that "representations are very beneficial. It just made it a lot clearer to understand" (Orla). The idea that representations are a useful tool to support active *sense-making* by students was noted by other students (Eimear, Meabh). Participants also identified the tutor's adoption of a sense-making approach, in which clear teacher explanations followed student exploration and sense-making activities, as important.

> She kind of gives you the questions and then we went through it ourselves and we came back then if we had any problems, she might explain what the maths was, I think it was more figuring it out first before explaining (Ciara)

These sentiments were also echoed by Orla who suggested that "it's best to kind of figure it out for yourself because that way you kind of learn by doing" but tutor explanations of "why we were doing such things" were welcome after students had opportunities to think about the mathematics themselves. In this there is some evidence to suggest that the tutor was likely successful in enacting the teaching strategies of: Facilitating students' connection of ideas to arrive at their own explanations of a general mathematical principle; Demonstrating how different solution strategies to the same problem are similar to or different from each other to illustrate or make a generalization (see Sect. "Intervention"). All seminars did include explicit tutor modelling and clear explanations of general mathematical principles and generalizations, but as suggested by Ciara, explicit instruction by the tutor largely happened after the students engaged in problem-solving themselves.

Preservice teachers as future teachers

Our remaining research questions consider our participants as teachers of mathematics, considering first how they perceive that participation in the mathematics support seminars influenced their competence and confidence in teaching mathematics (RQ 3) and then how they plan to teach mathematics, and whether this has changed because of participation in the mathematics support seminars (RQ 4). Participants were not observed in classrooms. Our analysis focuses instead on their perceptions of confidence and their stated plans for teaching.

RQ3: Perceived confidence, competence and identified causes

The question "Have the mathematics support seminars changed how you feel about your future as a teacher of mathematics?" was explicitly asked. All participants stated that they felt more confident that they would be well-positioned to teach mathematics. For example:

> I feel a lot more confident, and [...] I feel a lot more prepared to teach, and I was very nervous about it but now I wouldn't be. Yeah, I just feel a lot more confident and prepared for it (Meabh)

Participants proffered suggestions of elements that they felt contributed to their increased self-efficacy for mathematics including those presented in Table 2. These can be considered highly-significant as they are what the students themselves identify as causing improvements to their confidence.

The two elements that the most students identified as contributing to increased self-efficacy related to mathematical knowledge. These were the opportunity to address gaps in their understanding and the exposure to *alternatives to the standard algorithm*. As such, there is some evidence that participants perceived themselves to be more confident due to the specialized content knowledge that they were developing.

RQ4: Future teaching of mathematics

The experience of doing mathematics within the seminars appears to have influenced how these preservice teachers plan to teach mathematics. The obligatory modules in both first and second year included explicit content relating to the teacher's role in welcoming and modelling the solving of questions using a range of approaches. However, in these interviews, students commented on the experience of applying multiple strategies/*alternative to the standard algorithm*, rather than just hearing about them, and this experience seems to sharpen their awareness of the role they could play, for example:

> And from the maths sessions, I've kind of realized that there's so many more different ways to do something, because I always just thought there was one simple way, like one traditional way to do it, so I didn't realize you can really kind of break down to a way that suited you and I think from these sessions I've gained a lot of confidence, at like, coming up with new ideas ways to tackle a sum or a problem (Orla)

As evident in the quotes on Table 3, these participants describe their future teaching as involving teaching using *representations*, with attention to students' own strategies and needs, combined with high teacher-expectations. Notwithstanding the fact that these novice teachers may struggle to enact these approaches in the classroom, their statements are significant given the concerns they initially expressed in relation to their teaching of mathematics.

Discussion

This discussion first provides an overview of the key outcomes before providing a discussion of the reciprocal relationships between knowledge, disposition and instructional practice evidenced in this research.

Intervention outcomes

Enhanced MKT and efficacy beliefs

Our analysis shows that participants' previous experiences of learning mathematics emphasized procedures where practicing given algorithms was prioritised (Sect. "RQ1: prior learning experience and mathematics self-concept"). This is relevant as reliance on procedural understanding may contribute to negative disposition (Thanheiser, 2018) and mathematics anxiety can arise from inappropriate use of teaching practices which undermine students' confidence in their own mathematical abilities (Gresham & Burleigh, 2019). In this study, participants described how prior to the intervention they felt concerned about their ability to teach mathematics due to their own limited understanding (RQ1: prior learning experience and mathematics self-concept). Participants reported that the support seminars addressed gaps in their learning (Table 2) and consistently identified alternatives to standard algorithms as significant (RQ2: Learning about the teaching and learning of mathematics). Participants identify improvement in their mathematics teacher efficacy beliefs post-intervention and attribute this to their perceived enhanced mathematical knowledge for teaching (RQ3: Perceived confidence, competence and identified causes). These improvements in the affective domain are positive for the quality of the individual preservice teacher experience and because disposition will be an important factor in sustaining these participants' continued work to develop their mathematical content knowledge (Gresham & Burleigh, 2019). In addition, participant statements attested to their belief that the mathematics support seminars addressed deficiencies in their understanding within the content areas addressed. Their professed increased confidence, however, seems to extend beyond the specific content areas included in the mathematics support seminars as they refer generally to their confidence in mathematics.

Intentions to employ inclusive teaching approaches

Swars et al. (2009) highlight that teachers' levels of mathematics teaching efficacy have been linked with teachers' inclination to employ inclusive teaching strategies that promote mathematical understanding. Significantly, these participants who initially self-identified as having low confidence in mathematics, profess that they intend to teach in ways that are quite different from their experience of schooling (see Table 3). While it is generally not possible to categorise conceptualisations of mathematics and mathematics teaching into strict profiles, these students report favouring non-traditional approaches.

Relationships between knowledge, disposition and instructional practice

This study evidences the reciprocal relationships between teacher knowledge, disposition and instructional practice- experienced and intended (Jacobson, 2017). This research underscores the complexity of the 'instruction' component of Jacobson's model (Fig. 1) and the need for continued investigation of the types of initial teacher education experiences that will result in the enactment of high-quality teaching (Jong & Hodges, 2015).

It appears that the mathematics support seminars created opportunities for students to develop positive dispositions to mathematics- to see mathematics as a coherent, sensible subject and to see themselves as capable of doing mathematics (Kilpatrick et al., 2001). We contend that the approach taken to teaching during the mathematics support seminars, contributed to their mathematics self-efficacy in general, and not just within the content areas covered, though further evidence is required to support this valuable finding.

We suggest that the content of the mathematics support seminars—number operations, decimals and fractions, was instrumental. In foregrounding a sense-making approach that showed alternatives to standard algorithms and emphasized connections between number concepts and procedures (Stein et al., 2017), the preservice teachers had opportunities to appreciate mathematics as sensible and coherent, in a way that was not apparent to them in the rules-based, rote-learning approaches that were emphasised in their previous learning. Our data indicates that improved number knowledge appeared to boost participants' confidence in mathematics more generally. This may seem to contradict the accepted understanding that efficacy beliefs are context-specific (Bandura, 1997). However, it has been suggested that the main focus of the enacted mathematics curriculum in primary schools in Ireland is Number with evidence from the teacher questionnaires in the Trends in Mathematics and Science Study (TIMSS) indicate that more than half of classroom time is spent

Table 2 Elements of the Mathematics Support Seminars that students explicitly stated were helpful in improving their mathematics self-efficacy beliefs

Element	Participants	Examples
Alternatives to the standard algorithm	Ciara, Eimear	New strategies and being like shown that there's a few different ways that you can approach like multiplication and division because […] it was kind of like a relief to know that there was a different, few different, methods […] I was like okay well I understand this method, I have a little bit more understanding of the topic rather than kind of panicking being like I don't really understand the topic to begin with Just being able to see the different methods and the different approaches you can take and teach children different ways. And then, also, ehm, like letting the children kind of come up with their own strategies
Awareness of resources	Eabha	I don't feel as daunted about changing methods or teaching maths because […]we were pointed in directions of so many resources or even things like videos and stuff that we could even watch […] there's more like there to help, you know, teach maths
Addressed gaps in understanding	Róisín, Sorcha, Meabh	To get like my head around this definitely, especially the chunking, I struggled with getting my head around that at first. […] And you actually understand why you're doing it, but it definitely did take time in the first two weeks maybe in the first two sessions to actually step back and be like, forget everything you've already learned and start again. And, but it definitely makes more sense … like when I was doing maths, it was always just the traditional way. […] when we were doing long multiplication, like I always used to just bring down the zero because it was the process. I never understood where the zero was coming from. But then after like the main thing I figured out, that I actually could see where the zero was coming from. But like that it was just different ways of seeing how kids would learn how to do them and like drawn out the picture is when you're multiplying fractions. I found that really, really helped I definitely think I learned lots of new methods of doing multiplication, division and fractions that I didn't know you could do before. And for myself anyway, there's lots of new methods that were a lot easier for me to make sense of as well
Opportunity to do mathematics	Meabh	And I thought it was really useful that in the support groups we went into breakout rooms and kind of were given one or two things to figure out […] I need to actually do it to make sure I understand it
Representations	Meabh	Yeah, I think the visuals that we were kind of shown in the support group and in the lectures was really helpful because I know when I was in school, doing long multiplication and division. Just, just having the numbers like the numerals on the board it just was very kind of abstract and I couldn't make the connections, so seeing like the egg boxes or the diagrams or the app with the cubes, Brainingcamp and complex things like that, seeing it visually made it make sense

Pseudonyms are used for participants throughout. Brainingcamp are digital mathematics manipulatives that were used by the support seminar tutor and by students

Table 3 Characteristics of mathematics teaching that participants plan to enact

Characteristics	Participants	Example
Use of Representations	Meadbh	Yeah, I think I can actually know, pinpoint where I started to lose confidence in my maths ability and it was like, fourth, fifth class-ish. Because we stopped using the manipulatives that we did it for like weight and time, like those areas and length. But for multiplication and division in senior, we didn't use any physical or visual like it was just numbers so that's, I think that was where I started then to not feel confident in math, so I definitely think that I will be doing that when I'm teaching
Children's own strategies	Ciara Róisín Sorcha	I definitely feel more comfortable in being able to support children in any of the ways that they figure out a sum and not shutting them down and doing I feel like we were in school. So I definitely feel more comfortable with it, yeah And then also, ehm, like letting the children kind of come up with their own strategies I know we did that last year but it was interesting to see, like all the different ways I suppose you can do long multiplication or long division versus just kind of the one standard method that we learned I was just so afraid of going into a senior class and then then having a different approach to what I have. But now, like it's so interesting to see the different approaches, they're taking
Attending to children's needs	Orla	Some of the children were really struggling with the traditional method of like long multiplication and from the maths sessions, I've kind of realized that there's so many more different ways to do something, because I was always just taught one simple way, like one traditional way to do it. I realize you can really kind of break it down to a way to do it I can't remember the name of the method that was used, but it just broke it down much more, I just found it much easier, because I had been previously subbing in a fifth class and we were doing long multiplication and a lot of them were struggling, and I think the method that we were taught would have been very beneficial for them
High Expectations	Brónagh	I also learned, like, how important it is, like, me for example, like, I think I'm bad at maths because, like my teacher, I remember her like telling them I struggled at maths and stuff, […] that just like shows like if I'm teaching and I'm saying to children like they're bad at maths […] I'm like, that stayed with me so I just think that's an important aspect to remember as well, like just to encourage all children and like your expectations of them and stuff like that, that I just realized from it

Pseudonyms are used for participants throughout. 'Subbing' refers to substitute teaching, where another teacher covers for a classroom teacher when he/she is unavailable teaching Number (Close, 2013). It seems likely that much of participants' relevant, formative (negative) experiences of mathematics involved the Number domain, and this may have influenced their dispositions to mathematics more broadly. Given that they are preservice teachers with limited other experience to draw from, it seems likely that their mathematics and mathematics teaching efficacy beliefs may be grounded in their perception of their competence in Number.

This is significant because teacher educators must make strategic choices about the focus and nature of the learning experiences that are offered to preservice teachers (Hill, 2010). In our case, while the content of the mathematics support seminars had already been addressed in the standard obligatory module, it appears that these preservice teachers benefited from the extra opportunities to explore the same mathematical ideas in the support seminars. Furthermore, our analysis indicates that the teaching approach adopted, in which tutor explanations followed collaborative 'figuring things out' and sharing of diverse methods signalled an expectation and acceptance of diversity in student thinking. It seems likely that this approach developed participants' understanding of, and disposition towards, non-traditional teaching approaches alongside their content knowledge (e.g., Gresham & Burleigh, 2019). We contend that the mathematics support seminars provided vicarious experiences of a type of mathematics teaching that students had few, if any, prior opportunities to observe and that this experience played a role in developing more positive mathematics teacher efficacy beliefs. While more research in this area is warranted, it is clear that such experiences should be a core part of initial teacher education in mathematics.

Limitations

It is difficult to separate the impact of the concurrent obligatory mathematics module with the impact of the support seminars. Our data relates only to those who volunteered to participate in interviews, and we cannot rule out that these participants volunteered because they were positively disposed to the approach. While the interviewer made efforts to invite critique and disagreement, there was little evidence of this and we acknowledge that the experience of those who did not volunteer for focus group interviews may not have been so positive. Finally, this research was carried out in a particular cultural context which limits the generalisability of any findings.

Conclusion

This research makes an important contribution by highlighting (i) how the nature of learning experiences in initial teacher education can impact preservice teachers' confidence, perceived competence in, and vision of mathematics teaching; (ii) how perceptions of competence in Number appear to influence perceptions of confidence and competence in mathematics teaching more generally. This small-scale research highlights areas of initial teacher education that warrant further theoretical and empirical investigation.

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