Critical Early Numeracy Outcomes

Supporting Foundational Numeracy Attainment

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1. Attaining Critical Early Numeracy Outcomes

Almost all countries across the globe have early numeracy curricula that detail what they expect children to learn. However, in some parts of the world, most children do not attain the outcomes set out in their curricula.

In this report, we do two things to contribute to addressing this lag in attainment:

1. We outline a set of critical early numeracy outcomes for attainment linked to addition/subtraction and multiplication/division. Children who fail to attain these outcomes struggle to make any progress in mathematics. These outcomes are therefore critical to ongoing access.

2. We illustrate the breadth and depth required in assessing (and therefore, in teaching for) this attainment to ensure strong foundations in early numeracy that provide access to content at the next grade level.

Breadth is achieved through outcomes linked to both Fluencies and Problem-solving. Depth is ensured through outcomes based on solving simple and more complex problems. Key features of the variety of problems that children need to encounter, become familiar with and solve are discussed through illustrative items linked to the critical early numeracy outcomes.

Achieving the Grade 3 critical outcomes depends on achieving critical outcomes in the earlier grades. The critical outcomes for Grades 1 and 2 have been 'backward designed' from the Grade 3 outcomes. They thus offer a roadmap to attainment of the critical early numeracy outcomes in Grades 1-3, rather than representing all the topics set out in national curricula.

The critical early numeracy outcomes (CENOs) identified here are linked to core content in two clusters of early grade mathematics: addition/subtraction and multiplication/division. Achieving these outcomes rests on solving simple and more complex problems (Problem-solving Levels 1 and 2) in these two clusters. This problem-solving connects with an expanding bank of fluencies that support increasingly efficient problem-solving. Mastery across Fluencies and Problem-solving at both Level 1 and Level 2 in one grade is important for access to mathematics in the next grade.

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The number range for Fluencies and Problem-solving increases across Grades 1-3, from 1-10 working in Grade 1, 1-20 working in Grade 2 and 1-100 working in Grade 3. Mastery in each grade means being able to solve a wide range of problems within each number range: problems set in different contexts, problems focused on connecting representations, and problems focused on reasoning as well as calculating. In this report, we illustrate the breadth and depth of Fluencies and Problem-solving skills that are involved in mastery. Attaining the CENOs will require materials and assessments that incorporate this range of problems in teaching programmes.

Using the critical early numeracy outcomes

Stakeholders developing interventions can consider how this specification of critical early numeracy outcomes fits with their country or region's expected curriculum and their own teaching and learning materials. This analysis will indicate if their materials and assessments need adapting to better incorporate the critical outcomes. If additional attention is needed, lesson routines for teaching linked to the CENOs will need to be developed.

Teaching and learning materials are implemented by teachers in classrooms. Stakeholders need to consider their professional development content and approaches. The goal is to ensure that classroom teaching addresses the critical outcomes we want children to achieve. This may require adjusting current lesson planning, and teaching and assessment practices. This process might begin with some baseline assessment of children's competences across the CENOs. Outcomes can indicate immediate and medium-term priorities for attention in teaching. These can be worked into a scope and sequence that guides the development of an intervention lesson model for a trajectory of Fluency and Problem-solving outcomes.

Some CENOs are currently beyond the reach of many children. Our collective work is to gradually move towards their attainment, through combinations of materials and professional development for teachers.

Key ideas underpinning the critical early numeracy outcomes

Three features linked to the critical outcomes in early grade mathematics are:

- Working with representations
- Understanding and using place value
- Building efficiency and fluency with calculation

We describe each of these interrelated features and how they underpin the CENOs in Teaching Briefs 5, 6 and 7 of this report. The key points are: a) representations are central to all mathematical working; b) in early number learning, children need to connect between actions on concrete objects, informal diagrams, words and symbolic representations, and move, over time, to comfortable working with symbolic representations; c) some number representations are particularly useful for addition/subtraction and multiplication/division learning. These are incorporated in our illustrative tasks (part-whole and number line diagrams for addition/subtraction with counters, number lines, 100 squares and column algorithms useful for calculations; equal groups and array diagrams for multiplication/division).

It is important that calculating moves on from counting in ones to more efficient strategies. Place value is central to efficient calculation. Place value understanding goes beyond knowing that numbers can be broken down into hundreds, tens and units. It includes knowing how to jump to, or through, the next ten or the ten before a number, and how to count on, or back, in tens from any number. Ten frames, number lines and 100 squares can be used to build these skills to the level of fluency.

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Fluencies allow children to work efficiently with a range of addition and subtraction problems by the end of Grade 3.

Key representations combined with place value understanding support efficient calculation, attainment of the critical outcomes and access to mathematics beyond Grade 3. Programme designers' check of national curricula and their own schemes of work should include building children's competence with key representations, place value understanding and efficient calculation by the end of Grade 3. Place value is presented as a separate topic in many curricula. Programme designers need to connect place value understandings with increasingly efficient ways of working with addition/subtraction and multiplication/division (detailed in Section 5).

Pathways towards attaining the CENOs for each grade will vary across countries and should be responsive to children's baseline competences and local curriculum specifications. Attainment of the CENOs by the end of Grade 3 is possible for most children but remains an aspiration for too many. The CENOs, illustrative items and teaching briefs aim to make this attainment a reality for many more children.

Background

The CENOs are the outcome of the work of the Numeracy Research and Development (NRD) Expert Panel and follow initial consultations with academics and NGOs with extensive experience of early grade numeracy interventions in contexts where most children currently fail to achieve core early numeracy outcomes. The panel drew from the assessment outcomes developed within the Global Proficiency Framework (GPF). This framework is intentionally more limited than state/national curricula, as its focus is on supporting assessment of minimum proficiencies, but outcomes point to contexts where even these limited outcomes remain ambitious.

In developing CENOs for foundational numeracy, our backward design includes outcomes in the two clusters for Grades 1 and 2 to ensure that the Fluency and Problem-solving skills needed for strong foundations get sufficient breadth and depth of attention in children's prior learning experiences. The CENOs are designed to support staged attainment across the early primary grades. Programme and assessment development teams can use the CENOs and illustrative tasks to inform their design of teaching programmes and formative/summative assessment items that monitor progress towards CENO attainment. Feedback evidence on early grade learning gains on the ground will allow for gradual refinement and expansion of the CENOs. The goal, over time, is for these extensions to move closer towards the breadth of outcomes represented in national curricula.

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2. Introducing the Critical Early Numeracy Outcomes

The addition and subtraction, and multiplication and division clusters include Fluency outcomes and Problem-solving outcomes to be attained in each grade. These two outcomes are explained below. A more detailed discussion of the two outcomes is in Section 4 after our outline of the CENOs and presentation of illustrative tasks linked to them.

Fluencies	Specified skills that children should acquire at the level of near automaticity in each grade. This usually means being able to state an answer within 4 seconds without needing to count or work out on paper.
Problem	Setting up, describing and using models of situations, connecting between representations, and developing increasingly sophisticated and efficient strategies for calculating.

Fluencies and Problem-solving are interconnected and co-developed. Fluencies develop initially through working on problems and require ongoing practice for retention. The Problem-solving outcomes include Level 1 and Level 2 specifications. Progression from Level 1 to Level 2 can involve moves to more complex problems, expanded language and more abstract representations, and more efficient calculation strategies.

It is NOT the expectation that children master all the Level 1 critical outcomes before moving on to the Level 2 critical outcomes. This is important for programme developers to note: Problem-solving Level 2 should not be deferred until children can accurately solve all Problem-solving Level 1 tasks. It is often the case that working on Level 2 critical outcomes reinforces understanding of the Level 1 critical outcomes.

Initial working with practical situations and visual representations support sense-making in children's later work with purely numerical problems. It is therefore important that children start their work in Grade 1, and – where useful – Grade 2, with concrete materials or sketches. A key goal is to link these concrete approaches to numerical representations that gradually become models that children can work with flexibly and efficiently.

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ADDITION & SUBTRACTION GRADE 1 (A&S1)

Fluency

Children can:

- Subitize numbers
- Read, write, count, compare and order numbers up to 20
- Add/subtract 1 to/from numbers up to 20
- Rapidly recall addition and subtraction facts in 1-5 range and bonds of 10
- State doubles of 1-5

Problem Solving

Children can work with addition and subtraction in the 1-10 range

Level 1

Children can:

- Solve problems when the result is unknown (3 + 4 = _)
- Make/draw/work with representations of addition/subtraction (that include number tracks and ten frames)
- Solve addition/subtraction problems by unit counting (e.g. solve 4 + 3 by counting out 4 fingers on one hand, 3 fingers on the other hand, and then counting all fingers to get 7)
- Interpret when to add and subtract from simple language statements (e.g. add 4 and 3 or 7 take away 3)

Level 2

Children can build on Level 1 learning to:

- Solve problems when the change/start is unknown (3 + _ = 7)
- Make/draw/work with representations of addition/subtraction (that include number lines and bar diagrams)
- Solve addition/subtraction problems using known and derived facts (e.g. solve 4 + 3 as 2 more than 5 using 4 + 1 = 5 as a known fact)
- Interpret when to add and subtract from more formal or more complex language statements (e.g. How many more is 7 than 4? or subtract 3 from 8)

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ADDITION & SUBTRACTION GRADE 2 (A&S2)

Fluency

Children can:

- Read, write, count, compare and order numbers up to 100
- Add/subtract 1, 2, 10 to/from numbers in the 1-100 range
- Recall addition and subtraction facts in 1-10 range
- Compose/decompose numbers to 100 using place value
- State doubles of 1-5 and halves of even numbers to 10

Problem Solving

Children can work with addition and subtraction in the 1-20 range

Level 1

Children can:

- Solve problems when the result is unknown (13 + 4 = _; 12 – 5 = _)
- Draw diagrams/write number sentences/ work with representations of addition/ subtraction (that include part-whole diagrams and number lines)
- Solve addition/subtraction problems using known facts and unit counting
- Interpret when to add/subtract from simple language statements (e.g. add 14 and 3 or 17 take away 3)

Level 2

Children can build on Level 1 learning to:

- Solve problems when the change/start is unknown (13 + _ = 17. _ – 5 = 7)
- Draw diagrams write number sentences/ work with representations of addition/ subtraction (that include part-whole diagrams and number lines)
- Solve addition/subtraction problems using known and derived facts
- Interpret when to add/subtract from more complex language statements (e.g. how many more is 17 than 14? or what is the difference between 13 and 8)

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ADDITION & SUBTRACTION GRADE 3 (A&S3)

Fluency

Children can:

- Read, write, count, compare and order numbers up to 1000
- Add/subtract 1-5, 10, and multiples of 10 to/from numbers in the 1-100 range
- Recall addition and subtraction facts in 1-20 range
- Compose/decompose numbers to 1000 using place value
- State the multiple of 10 before/after any number and the nearest 10 for any number in the 1-100 range
- State doubles of 1-10 and multiples of 10
- State halves of even numbers to 20 and multiples of 10

Problem Solving

Children can work with addition and subtraction in the 1-100 range

Level 1

Children can:

- Solve problems when the result is unknown (24 + 15 = _ or 83 - 27 = _)
- Draw diagrams/write number sentences/ work with representations of addition/ subtraction (that include images, part-whole diagrams and number lines)
- Solve addition/subtraction problems using known facts and place value patterns
- Interpret when to add and subtract from simple language statements (e.g. What is the sum of 65 and 17? Or 78 minus 23)

Level 2

Children can build on Level 1 learning to:

- Solve problems when the change/start is unknown (23 + _ = 35 or _ - 16 = 65)
- Draw diagrams/write number sentences/ work with representations of addition/ subtraction (that include part-whole diagrams and number lines)
- Solve addition/subtraction problems using known and derived facts, and use number properties and place value patterns flexibly and efficiently
- Interpret when to add and subtract from more complex language statements (e.g. How much bigger is 47 than 29? Or 45 is 18 less than ?)

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MULTIPLICATION & DIVISION: GRADE 1 (M&D1)

Fluency

Children can:

- Skip count forwards in 2s to 20, starting at any even number in the 1-20 range

Problem Solving

Children can work with multiplication and division using manipulatives and pictures with products not exceeding 12

Level 1

Children can:

- Make equal groups for multiplication stories
- Describe multiplication situations using the language of numbers of groups and size of each group (e.g. 3 bags with 4 apples in each bag)
- Solve multiplication problems, including by unit counting

Level 2

Children can build on Level 1 learning to:

- Make equal groups for multiplication and division stories
- Describe multiplication and division situations using the language of numbers of groups and size of each group
- Solve multiplication and division problems, including by unit counting

MULTIPLICATION & DIVISION: GRADE 2 (M&D2)

Fluency

Children can:

- Skip count forward in 2s, 5s and 10s to 100, starting from any multiple of each of these numbers

Problem Solving

Children can work with multiplication and division presented in pictures and number sentences

These situations involve multiplying/dividing by 2, 5, 10 within 20, 50, and 100 respectively and other factor pairs with products up to 20

Level 1

Children can:

- Connect pictures and number sentences in multiplication and division tasks and word problems (including support for arrays and number lines to model multiplication and division tasks)
- Describe multiplication and division situations (including arrays and number lines) using the language of equal groups (e.g. 3 rows of 4 chairs each, or 4 jumps of 2)
- Solve multiplication and division problems using various strategies, including drawings and unit counting

Level 2

Children can build on Level 1 learning to:

- Connect pictures and number sentences in multiplication and division tasks and word problems (including arrays and number lines) and use them to solve problems
- Describe multiplication and division situations (including extending use of arrays and number lines and using more complex language (e.g. doubling and halving)
- Solve multiplication and division problems using efficient strategies, including the use of skip counting by 2, 5, 10

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MULTIPLICATION & DIVISION: GRADE 3 (M&D3)

Fluency

Children can:

- Recall multiplication and division facts for 2, 5, and 10 up to 20, 50 and 100 respectively

Problem Solving

Children can work with multiplication/division situations presented in pictures and number sentences that use formal language

These situations involve multiplying/dividing involving 1x1 to 10x10

Level 1

Children can:

- Connect number sentences, diagrams and stories of multiplication/division situations including word problems, array and number line images
- Work out products or quotients in number sentences using various strategies such as drawing, repeated addition, known facts, doubling and halving, commutativity

Level 2

Children can build on Level 1 learning to:

- Connect number sentences, diagrams and stories of multiplication/division situations including word problems, array and number line images. Situations may include varied placement of the unknown (e.g. Some friends share 18 candies. If each of them get 3 candies, how many friends were there?)
- Work out missing numbers in number sentences, using various strategies for efficient calculation, including known and derived multiplication facts, doubling and halving, commutativity. (E.g. _ x 3 = 12)

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3. Breadth and Depth: Illustrative Tasks

We have noted that attaining the critical early numeracy outcomes involves mastery of a range of concepts and skills. Too often, reports from countries where early numeracy attainment is currently low point to children being able to correctly complete simpler tasks using rudimentary one-by-one counting approaches, but unable even to attempt more complex tasks.

To address this problem, we focus on two features:

1. Illustrating the breadth of tasks linked to fluencies and problem-solving that children need to be able to solve to attain the CENOs in each grade.
2. Illustrating and commenting on the depth that makes Level 2 problem-solving more complex that Level 1 problem-solving

It is not feasible to comprehensively or exhaustively present a collection of tasks linked to the CENOs. Instead, we present a range of illustrative tasks with commentaries on the breadth and depth of learning that underpins attainment of these outcomes.

In the tables that follow, the critical outcomes for Fluency and Problem-solving linked to the addition/subtraction and multiplication/division clusters in each grade are presented with a range of illustrative tasks. A set of tasks are linked to the Fluency outcomes. The Problem-solving Level 1 and Level 2 tasks, where appropriate, are presented as matched pairs, with a commentary on how either the task, representation or calculation approach can differ across the two levels.

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How to read the illustrative task tables

The tables show the critical outcomes for the Grade 1-3 addition/subtraction and multiplication/division clusters in each grade.

Each table is organized by the three sets of outcomes:

- Fluencies
- Problem-solving Level 1
- Problem-solving Level 2

Each outcome type has a set of skills or concepts associated with it that are detailed under each heading. The tasks presented under each heading illustrate some of the range of questions that can be devised to teach for, or assess attainment of, these concepts and skills. Problem-solving Level 1 tasks represent a simpler and more introductory level of mathematical working than Problem-solving Level 2 tasks.

Illustrative tasks: Addition & subtraction

Across the addition and subtraction clusters for all three grades:

- Level 1 problems include ONLY problems where the result is unknown (e.g. 6 + 3 = [ ]);
- Level 2 problems extend this to include problems with either the change or start is unknown (e.g. 6 + [ ] = 9, [ ] – 2 = 7).
- Level 1 tasks can be associated with more abstract Level 2 work with representations or with more efficient Level 2 calculation strategies and vice versa for Level 2 tasks

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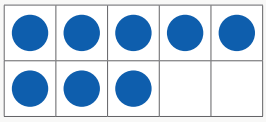
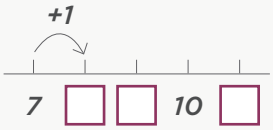
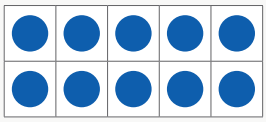
ADDITION & SUBTRACTION GRADE 1 (A&S1)

Fluency

Children can:

- Subitize numbers
- Read, write, count, compare and order numbers up to 20
- Add/subtract 1 to/from numbers up to 20
- Rapidly recall addition and subtraction facts in 1-5 range and bonds of 10
- State doubles of 1-5

How many dots?  *Count forward in ones, starting at 13.* *Count backwards in ones starting at 14.* *Fill in the missing numbers:* *9, 10, _, _, _, _, _, _, 17, 18* *Circle the larger number in each pair:* *18 12 11 17*	*10 + 1 = _* *15 - 1 = _* *6 + 1 = _* *18 - 1 = _* *One less than 6 is _* *One more than 19 is _* *One jump back from 11 is _* 	*1 + 3 = _* *5 - 2 = _* *3 - 2 = _* *2 + 3 = _* *6 + 4 = _* *8 + 2 = _* *Two more than 1 is _* *Three less than 4 is _* *Two jumps forward from 3 is _*
4 and 4 is _ *3 + 3 = _* *Double _ = _ in this picture* 	*More than, less than or equal to?* *7 is _____ 10* *9 is _____ 4 and 4* *1 and 4 _____ 4 and 1*	*8 + [ ] = 10* *[ ] + 5 = 10* *5 - [ ] = 3* *[ ] - 3 = 1*
Double 4 = _ *Double _ = 6*	*Show me three fingers. Show me 5 fingers using two hands.* *I'm going to quickly show you some fingers. How many can you see?*	*Write 8 in words.* *Write 12 in words.* *Write these numbers in numerals: Five, Sixteen, Eleven*

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Problem Solving

A&S1: Children can work with addition and subtraction in the 1-10 range

Level 1	Distinguishing Level 1 and Level 2 tasks	Level 2
Children can: • Solve problems when the result is unknown (3 + 4 = _) • Make/draw/work with representations of addition/ subtraction (that include number tracks and ten frames) • Solve addition/subtraction problems by unit counting (e.g. solve 4 + 3 by counting out 4 fingers on one hand, 3 fingers on the other hand, and then counting all fingers to get 7) • Interpret when to add and subtract from simple language statements (e.g. add 4 and 3 or 7 take away 3)		**Children can build on Level 1 learning to:** • Solve problems when the change/ start is unknown (3 + _ = 7) • Make/draw/work with representations of addition/ subtraction (that include number lines and bar diagrams) • Solve addition/subtraction problems using known and derived facts (e.g. solve 4 + 3 as 2 more than 5 using 4 + 1 = 5 as a known fact) • Interpret when to add and subtract from more formal or more complex language statements (e.g. How many more is 7 than 4? or subtract 3 from 8)
• Yasmin builds a tower with 3 red bricks and 4 yellow bricks. How many bricks tall is her tower? • ***Draw a picture and work out the answer***	**At Level 1, children can directly build Yasmin's tower by putting out 3 objects, then another 4, and count the total.** **At Level 2, the change is unknown so children have to put out 4 objects, add further objects counting on until 9, and then go back and count the number they added.**	• Yasmin builds a tower with 9 bricks. Four of the bricks are red. The rest are yellow. • How many bricks are yellow? • ***Draw a picture and work out the answer***
• Rakhi eats 3 of her 5 cookies. How many cookies are left for her sister? • ***Draw a picture and work out the answer***		• Rakhi eats some of her 7 cookies. She has 4 cookies left. How many cookies did Rakhi eat? • ***Draw a picture and work out the answer***
• Listen as I drop counters into this cup (drop 1, 2, 3). • How many counters in the cup? • Look I am putting two more counters in the cup. • How many counters in the cup now?	**For Level 1, children have to hold the number of counters in the cup in their heads and count on from that number to arrive at the total.**	• There are 6 counters in this cup (show and check). • Watch I'm taking out some counters? • I've got two that I've taken out. • How many counters in the cup now?

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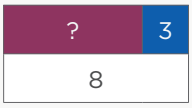
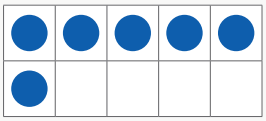
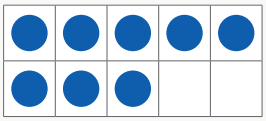
	At Level 2 they know the total and the number removed – some may count back from the total, but some may recall the answer without counting because this number bond has become a Fluency.	
• Split 5 counters between 2 plates. Record what you have done. • Split the five counters in a different way.		• Tom has 2 biscuits out for his friends. Put some more biscuits out on the other plate so he has 5 biscuits. 
• 7 + 2 = _ • 8 – 3 = _  • Make these jumps on a number track? Where do you land?	**For Level 1, children work with number tracks where the separate divisions can be counted.** **At Level 2 children work with strips that represent single digit numbers – this is a more abstract representation.**	• How much longer is the 8-strip than the 3-strip? 
• This tens frame shows 5 + 1.  • Move counters to show 4 + 2. • What other sums can you make with 6 counters?		• How many more to make 10? 
• Add 4 and 3. • Take away 2 from 6. • 2 less than 7? • 4 more than 5? • 2 steps back from 10? • Which is quicker: 5 more than 1 or 1 more than 5?	**For Level 1, the language used is direct: add, take away, less than, more than, etc.** **At Level 2 more interpretation of language is needed: here the language of 'take away from' is used but 2 has to be added to 5.**	• Subtract 2 from 6. • I can see a number on this card. [Teacher shows back of a card.] If I take 2 away from the number on the card I get 5. • What number is on the card? • 7 is _ more than 4.

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- Tell a story for 8 - 5.

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At Level 1, children produce stories that connect with result-unknown numerical problems.

At Level 2, children produce stories that connect with change/start unknown numerical problems.

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- Tell a story for 7 – _ = 5.

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- 8 – 1 = 7
- 8 – 2 = _
- 8 – 3 = _
- 8 – 4 = _
- 8 – 5 = _
- 4 + 5 = _
- 3 + 5 = _
- 3 + 4 = _
- 10 - 9 = _
- 1 + 7 = _

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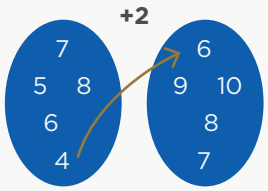

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The problems above, given in practical contexts, all build towards children working with purely numerical problems like the ones here.

Result unknown at Level 1.

Change/start unknown at Level 2.

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- 7 - _ = 1
- 6 + _ = 9
- 2 + _ = 9
- _ + 6 = 8
- _ - 8 = 0

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ADDITION & SUBTRACTION GRADE 2 (A&S1)

Fluency

Children can:

- Read, write, count, compare and order numbers up to 100
- Add/subtract 1, 2, 10 to/from numbers in the 1-100 range
- Recall addition and subtraction facts in 1-10 range
- Compose/decompose numbers to 100 using place value.
- State doubles of 1-5 and halves of even numbers to 10

Write 78 in words.
Write 87 in words.

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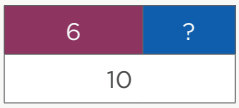
34 + 1 = _
56 – 1 = _
64 – 2 = _
73 – 10 = _
4 and 4 is _
Half of 6 = _

Count backwards in ones, starting at 33



3 + 5 = _
8 – 5 = _
7 – 4 = _
10 + 8 = _
50 + 8 = _
38 = _ + 8
20 + 8 = _
70 + 8 = _
68 = 60 + _

Use the digits 1, 4, 5 to make three different 2-digit numbers. Circle the largest of your numbers.

Circle the numbers that will come up if I count back in 10s from 96:

46, 75, 66, 34, 90, 56, 16

Write these numbers in order, starting with the smallest:

76, 67, 62, 26, 27, 72

91, 89, 19, 81, 80, 90

Problem Solving

A&S2: Children can work with addition and subtraction in the 1-20 range

Level 1	Distinguishing Level 1 and Level 2 tasks	Level 2
Children can: • Solve problems when the result is unknown (13 + 4 = _ ; 12 – 5 = _) • Draw diagrams/write number sentences/work with representations of addition/ subtraction (that include part-whole diagrams and number lines) • Solve addition/subtraction problems using known facts and unit counting • Interpret when to add/ subtract from simple language statements (e.g. add 14 and 3 or 17 take away 3)		**Children can build on Level 1 learning to:** • Solve problems when the change/start is unknown (13 + _ = 17 ; _ – 5 = 7) • Draw diagrams write number sentences/work with representations of addition/ subtraction (that include part-whole diagrams and number lines) • Solve addition/subtraction problems using known and derived facts • Interpret when to add/subtract from more complex language statements (e.g. how many more is 17 than 14? or what is the difference between 13 and 8)

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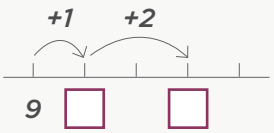
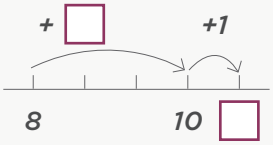
• Asiya read 11 books and Yasmin read 6 books. • How many books did they read altogether? • Mike had 18 marbles but lost 7 of them. • How many marbles did he have left? • It's going to be 11 degrees colder at night than in the day. Today was 16 degrees. What's the temperature tonight? • ***Draw part-whole diagrams for each of these problems. Use them to create a number sentence or number line to solve the problem.***	**At Level 1, children can directly add Asiya's 11 and Yasmin's 6 books.** **At Level 2, children have to interpret ;more' as finding a difference, and not make the error of thinking 'more' always means add.**	• Asiya read 11 books and Yasmin read 6 books. • How many more books did Asiya read than Yasmin? • Mike had 18 marbles but lost some. He has 7 left. • How many marbles did he lose? • The temperature drops by 11 degrees to 16. What was the temperature before? • ***Draw part-whole diagrams for each of these problems. Use them to create a number sentence or number line to solve the problem.***
• Fill in the missing numbers on the number line.  	**At Level 1, children can directly fill in the landing points on the number line, adding 1 and then 2.** **At Level 2 children have to work out the missing jumps by looking at the values on the line.**	• Fill in the missing numbers on the number line. 
• Work practically with bundles of sticks and operation cards. 	**Working with sticks in bundles of ten precedes symbolic problems. Some children will still need to ' 'unbundle' 10s and count in ones. They are not yet fluent with the idea of 10 sticks making up 1 bundle.** **At Level 1 some children will count on in ones to get 14, and then compose 14 into a bundle of 10 and 4 sticks.** **At Level 2 some children will explain that 4 more sticks added to the 6 visible ones will make a bundle of 10, and 4 loose sticks after that will make up the 14. This means that 8 sticks in all are added.**	• Work practically with bundles of sticks and operation cards. 

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<table>
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- 5 + 3 = 8
- 2 + 5 = 7
- 8 – 6 = 2
- 15 + 3 = _
- 12 + _ = 17
- 18 – 6 = _

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In both levels children reason through answers rather than calculate.

At Level 1, given 5 + 3 = 8 they reason that 15 + 3 must be 18.

At Level 2 they reason that adding 9 is the same as adding 10 then subtracting 1 or that subtracting 9 is the same as subtracting 10 then adding 1. Representing these on number lines helps show the logic involved.

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- 5 + 10 = 15
- 17 – 10 = 7
- 5 + 9 = _
- 17 – 9 = _
- 8 + 3 = 8 + _ + 1

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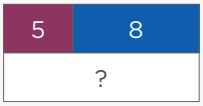
- Write an add sentence that goes with this diagram.



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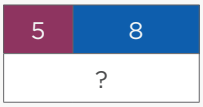
- Write an add sentence and a subtract sentence to go with this diagram.



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- 12 + 7 = _
- 11 – 5 = _
- 3 + 17 = _
- 18 – 9 = _

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The problems above, given in practical contexts, all build towards children working with purely numerical problems like the ones here.

At Level 1 the result is unknown.

At Level 2 the change/ start is unknown.

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- 2 + _ = 19
- _ - 15 = 2
- _ + 17 = 20
- 18 - _ = 3

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ADDITION & SUBTRACTION GRADE 3 (A&S1)

Fluency

Children can:

- Read, write, count, compare and order numbers up to 1000
- Add/subtract 1-5, 10, and multiples of 10 to/from numbers in the 1-100 range
- Recall addition and subtraction facts in 1-20 range
- Compose/decompose numbers to 1000 using place value
- State the multiple of 10 before/after any number and the nearest 10 for any number in the 1-100 range
- State doubles of 1-10 and multiples of 10
- State halves of even numbers to 20 and multiples of 10

Fill in the missing numbers:

668, 669, _, _, _

_, _, _, 802, 803, _

164 = 100 + _ + 4 = 1 hundred + _ tens + _ units

Circle the larger number in each pair

89 101 546 465

Write in the correct symbol <, >, =

100 + 89 _ 100 + 8 + 9

289 _ 20 + 80 + 9

Write Five hundred and fifteen in numerals

Write these numbers in order, starting with the smallest:

276, 672, 627, 726, 267

What is double:

4, 9, 20, 50

What is half of:

12, 18, 40, 7063 + 5 = _

63 – 4 = _

78 + 4 = _

52 – 5 = _

67 + 10 = _

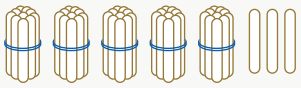
83– 10 = _

56 – 30 = _

Write the multiple of 10 (tens number) that comes after: 65, 27, 98

Write the multiple of 10 (tens number) that comes before: 56, 72, 89

How many sticks do you have after each step [take away a bundle of ten each time]. Circle the totals in a 100 square.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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Problem Solving

A&S3: Children can work with addition and subtraction in the 1-100 range

Level 1	Distinguishing Level 1 and Level 2 tasks	Level 2
Children can: • Solve problems when the result is unknown (24 + 15 = _ or 83 -27 = _) • Draw diagrams/write number sentences/work with representations of addition/ subtraction (that include images, part-whole diagrams and number lines) • Interpret when to add and subtract from simple language statements (e.g. What is the sum of 65 and 17? Or 78 minus 23) • Solve addition/subtraction problems using known facts and place value patterns		**Children can:** • Solve problems when the change/start is unknown (23 + _ = 35 or _ - 16 = 65) • Draw diagrams/write number sentences/work with representations of addition/ subtraction (that include part-whole diagrams and number lines) • Interpret when to add and subtract from more complex language statements (e.g. How much bigger is 47 than 29? Or 45 is 18 less than _ ?). • Solve addition/subtraction problems using known and derived facts, and use number properties and place value patterns flexibly and efficiently
• There are 36 apples and 15 peaches in the bowl. How many fruits are there altogether?	**At Level 1, children write the number sentence that models the problem and calculate the answer.** **At Level 2, children write the number sentence that models the problem, re-order to create friendly numbers (3+17) and (12+8) that allow for efficient calculation.**	• There are 3 bananas, 12 pears, 7 oranges, 8 apples in the box. How many fruits are there altogether?

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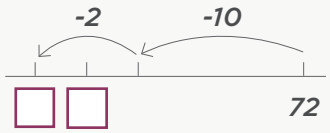
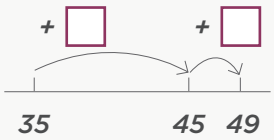
• Fill in the missing numbers on the number line.  • Complete this sentence: 72 – 10 – 2 = _ • And this sentence: 72 – 10 – 2 = 72 - _	**At Level 1, children can use number lines to solve result-unknown problems. They can represent their working using number sentences and connect equivalent number sentences. Some children will need reminding to use adding/subtracting tens patterns rather than counting in ones.** **At Level 2, children can use number lines to solve start- and change-unknown problems. They can represent their working using number sentences and connect equivalent number sentences. They can recognize and mark in jumps of ten on the number line.**	• Fill in the missing numbers on the number line.  • Complete this sentence: 35 + _ = 49 • And this sentence: 35 + _ + _ = 35 + _
• Tom is 23 cm taller than Anne. Anne is 71 cm tall. How tall is Tom? • Jawad has R83. He spends R15 on stickers. How much money does he have left?	**At Level 1, the problems allow children to directly add or subtract the given numbers to get the answer.** **At Level 2, the problems require children to sort out how the given values are related: Do we have a whole and a part? Or do we have two parts. Part-whole diagrams are useful for this.**	• Tom is 23cm taller than Anne. Tom is 91cm tall. How tall is Anne? • Jawad has R83. He buys a pack of stickers and has R19 left. How much did he spend on stickers?
• Draw a part-whole diagram and a number line sketch that shows: 53 – 15 = _ . • Show how to calculate the answer using jumps on a number line.	**At Level 1, children are able to set up part-whole and number line models of problems. They can solve the problem using jumps on a number line, and may also be able to solve using written or mental column algorithms or place value decomposition.**	• Draw a part-whole diagram and a number line sketch that shows: _ - 23 = 14. • Use jumps on the number line to calculate the answer.

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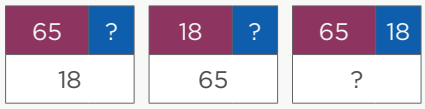
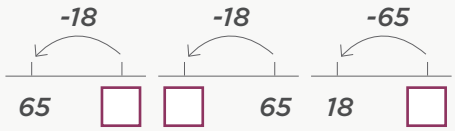
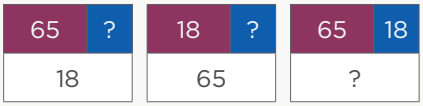
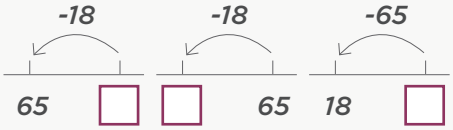
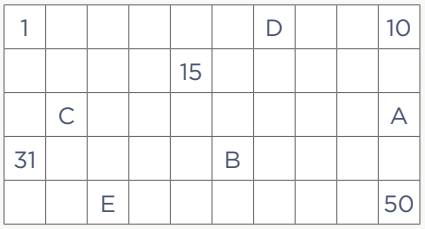
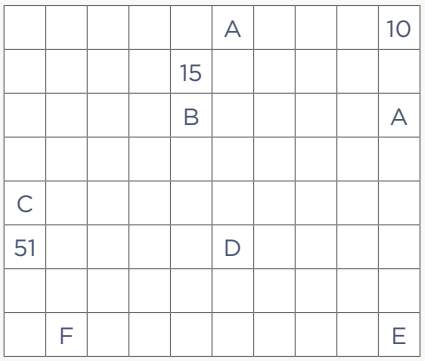
	At Level 2, children are able to set up part-whole and number line models of problems. They can use the part-whole model or number line model to decide how to solve the problem. They can solve the problem using jumps on a number line, and may also be able to solve using written or mental column algorithms or place value decomposition.	
• Which part-whole diagram matches 65 – 18 = _ ?  • Which number line matches 65 – 18 = _ ?  	**At Level 1, children can connect result-unknown problems with part-whole and number line models.** **At Level 2, children can connect change and start unknown problems with part-whole and number line models.**	• Which part-whole diagram matches 65 - _ = 18 ?  • Which number line matches _ - 65 = 18 ? 
• Practice adding and subtracting 10.  • 15 + 10 = _ • 50 - 10 = _ • 31 + 10 = _ • A(30) - 10 = _ • C - 10 = _ • B + 10 = _ • D + 10 = _ • E - 10 = _	**This Level 1 problem is a step forward from a simple Fluency problem involving counting on in 10s from numbers circled on a 100 square. Here, children practice adding and subtracting 10 from numbers in a nearly blank 100 square. This means they have to work out what numbers represent A-E, and then add/subtract 10 from these numbers.** **At Level 2, children can figure out efficient tens and ones jumps to get from a given number to positions A-F. For example, A is 15 – 10 + 1 = 6.**	• Write the numbers for A to G without counting the squares.  • A = • B = • C = • D = • E = • F =

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- 22 + 7 = _
- 31 – 13 = _
- 8 + 39 = _
- 54 – 39 = _

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The problems above, given in practical contexts, support children in working sensibly with purely numerical problems. At Level 1, children can solve a range of result unknown calculations.

At Level 2, children can solve a range of change/start unknown calculations.

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- 22 + _ = 19
- _ - 13 = 18
- _ + 39 = 48
- 54 - _ = 15

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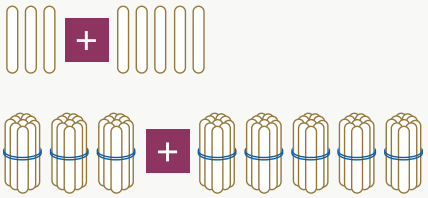

3 + 5 = 8

3 tens + 5 tens = _ tens

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At Level 1, images support children to see connections between addition of units and addition of tens or hundreds.

At Level 2, children can work with purely numerical representations and use place value language and connections to solve problems.

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- 3 + 5 = 8
- 9 – 2 = 7
- 30 + 50 = _
- 90 – 20 = _
- 70 – 40 = _
- 63
+ 24

3 and 4 is 7. 6 tens and 2 tens is 8 tens. Eighty-seven

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Illustrative tasks: Multiplication & division

Across the multiplication and division clusters in Grades 1-3, different aspects separate Level 1 and Level 2 Problem-solving.

Grade 1

- Level 1 problems include ONLY multiplication; Level 2 problems include both multiplication and division.

Grade 2

- Level 1 problems in multiplication and division provide support to effectively use and talk about representation such as equal groups, arrays and number lines; Level 2 problems extend the use of these same representations to solve problems using more efficient strategies.

Grade 3

- Level 1 multiplication and division problems include straightforward calculations using any strategy (e.g., 4 x 5 = _), and use of key representations such as arrays and number lines; Level 2 problems include missing number problems with increasingly efficient strategies (e.g., 6 x [ ] = 30) , and extended use of representations including non-traditional features.

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MULTIPLICATION & DIVISION: GRADE 1 (M&D1)

Fluency

Children can:

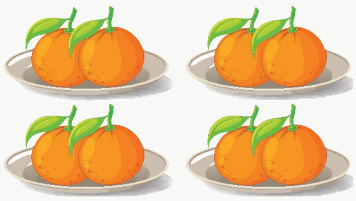
- Skip count forwards in 2s to 20, starting at any even number in the 1-20 range

Skip count by 2s starting from 2 ***Skip count by 2s starting from 14***	***Circle the numbers that you say when you count in 2s***

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Problem Solving

M&D1: Children can work with multiplication and division using manipulatives and pictures with products not exceeding 12

Level 1	Distinguishing Level 1 and Level 2 tasks	Level 2
Children can: • Make equal groups for multiplication stories • Describe multiplication situations using the language of numbers of groups and size of each group (e.g. 3 bags with 4 apples in each bag) • Solve multiplication problems, including by unit counting		**Children can build on Level 1 learning to:** • Make equal groups for multiplication and division stories • Describe multiplication and division situations using the language of numbers of groups and size of each group • Solve multiplication and division problems, including by unit counting
• Describe what you see in this picture using the numbers 2, 4 and 8. 		
• Put two apples into each bag. • How many bags of apples are there? • How many apples are in each bag? • How many apples are there altogether?	**In Level 1, children put the apples/bottle tops into equal groups and are supported to use equal groups language as an introduction to early multiplication ideas. Children may use unit counting to solve the problem.**	• Each bag can hold three apples. There are 6 apples. • How many bags did you fill using 6 apples? • How many apples are there in total? • How many bags? • How many apples in each bag?

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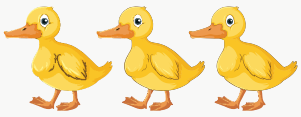
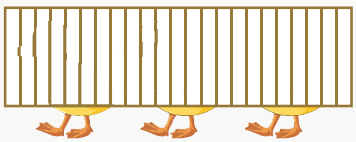
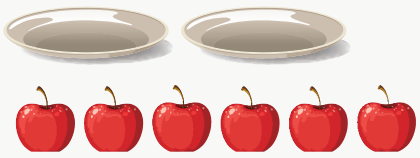
	Level 2 problems introduce children to early division ideas and accompanying language. Children either find the number of equal groups into which a group of objects can be divided (first problem) or the size of each group (the latter two problems).	• Here are 6 apples.  • Share the six apples out equally among the three children. • How many apples are there in total? • How many children? • How many apples does each child get?
• Here are 3 plates. Put 4 bottle tops on each plate. (with concrete materials provided) • How many groups of bottle tops are there? • How many bottle tops are in each group? • I have ___ groups of ___ bottle tops.		• Share 12 bottle tops (concrete materials provided) equally onto 3 plates. How many bottle tops on each plate? • How many bottle tops do we have in all? • How many plates? • There are ___ bottle tops on ___ plate.
• How many legs does each duck have? • How many ducks are there?  • Finish this story: • Each duck has ___ legs. • 3 ducks have _____ .	**At Level 1, children work on multiplication scenarios using ducks and their legs. Naturally occurring groupings of things (such as legs on a horse, or fingers on a hand) are useful ways to illustrate concepts of grouping.** **At Level 2, children work on a division scenario where only legs can be seen, and they have to figure out how many groups of two legs make up the 6 legs.**	• Some ducks are standing behind a fence. • How many legs do you see?  • Finish this story: • There are ___ legs. • 6 legs belong to ___ ducks.
• Act out these stories and fill in the gaps:  • I have 2 plates. I put 3 apples onto each _____. I have ___ apples altogether.		• Act out these stories and fill in the gaps:  • I have 6 apples. I shared them out equally onto ___ plates. Each plate ends up with ___ apples on it.

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MULTIPLICATION & DIVISION: GRADE 2 (M&D2)

Fluency

Children can:

- Skip count forward in 2s, 5s and 10s to 100, starting from any multiple of each of these numbers

Skip count by 2s from 10 up to 30

Skip count by 10s from 30 up to 100

Skip count by 5s starting at 5 up to 50

Skip count by 5s starting at 15 up to 50

Circle all the numbers that come up when we count in 5s.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Problem Solving

M&D2: Children can work with multiplication and division presented in pictures and number sentences.

These situations involve multiplying/dividing by 2, 5, 10 within 20, 50, and 100 respectively and other factor pairs with products up to 20

Level 1	Distinguishing Level 1 and Level 2 tasks	Level 2
Children can: • Connect pictures and number sentences in multiplication and division tasks and word problems (including support for arrays and number lines to model multiplication and division tasks). • Describe multiplication and division situations (including arrays and number lines) using the language of equal groups (e.g. 3 rows of 4 chairs each, or 4 jumps of 2) • Solve multiplication and division problems using various strategies, including drawings and unit counting.		**Children can build on Level 1 learning to:** • Connect pictures and number sentences in multiplication and division tasks and word problems (including arrays and number lines) and use them to solve problems. • Describe multiplication and division situations (including extending use of arrays and number lines and using more complex language (e.g. doubling and halving) • Solve multiplication and division problems using efficient strategies, including the use of skip counting by 2, 5, 10

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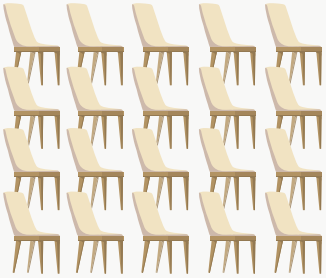
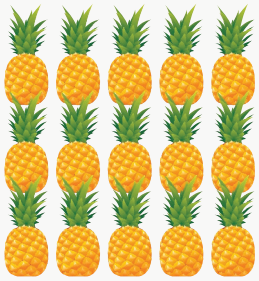
• Each basket has 5 apples. • How many baskets are there? ___ • How many apples altogether? ___  • 5 x 4 = _ • There are 20 apples. • How many equal groups are there? ___ • How many apples are in each group? ___  • 20 ÷ 4 = _	**At Level 1, children solve simple multiplication and division problems that support them to see equal groups of objects in pictures.** **At Level 2, children solve simple multiplication and division problems that encourage them to see equal groups in arrays and find answers by skip counting.**	• There are 4 rows of 5 chairs each.  • Count the chairs by rows: 5, ___, ___, ___ • How many chairs altogether? • 4 x 5 = _ • You have 20 chairs. Arrange the chairs in 4 equal rows. • Draw a picture. • How many chairs in each row?
• There are 5 baskets with 4 apples each. How many apples are there? You can use drawings to help you solve the problem. • Divide 20 apples into 5 equal groups. How many apples in each group? You can use drawings to help you solve the problem.	**Level 1 working either includes supporting diagrams or points children towards creating these diagrams to aid working.** **Level 2 word problems can go beyond imagining arrays and introduce children to more complex language like that of doubling and halving.**	• Sarah has 3 apples. If she doubles the amount of apples the next day, how many apples will she have? • Sarah has 6 apples. If she eats half of the apples, how many apples will she have left?
• How many rows of pineapples? • How many pineapples in each row? • There are ___ rows of ___ pineapples each. • There are ___ pineapples altogether. 	**At Level 1, children learn how to use arrays and number lines by being supported to use an appropriate language to describe them. Notice that the number line is fully marked, so children only need to interpret the picture.**	• Here is a strip of 5 star stickers. • How many strips would I need to buy to have 15 stars?  • How many stars are shown? • If I add one more row of stars, how many stars will there be altogether? 

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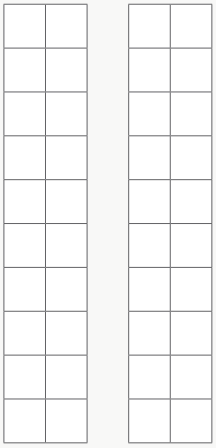
- Use the arrays to solve:



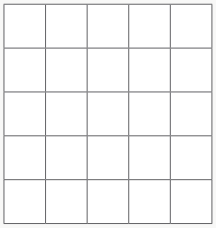
- 10 x 2 = __ + __ + __ + __ + __+ __ + __ + __ + __ + __ = __

- 2 x 10 = ____ + ____ = ____
- 2 x 10 = ____

- How many jumps are shown on the number line? ____
- How long is each jump? ____
- What number do we reach? ____

At Level 2, children are asked to extend their knowledge of arrays and number lines to solve tasks. Notice that the number line is not fully marked, requiring children to draw in the arrows and moving into using skip counts.

- Colour in to show 4 rows of 5 each.



- Start at 0, and mark the number line to show 3 jumps of 5 each.
- What numbers do you touch? What number do you end up at?

- 2 x 4 = 8
- 5 x 5 = 25
- 3 x 10 = _
- 2 x 5 = _
- 5 x 4 = _
- 4 x 10 = _
- 2 x 6 = _
- 5 x 6 = _
- 7 x 10 = _

At Level 1 children solve simple multiplication calculation problems with 2, 5, and 10 using drawings and unit counting.

At Level 2, children solve simple multiplication and division calculation problems using more complex strategies like skip counting and relationships between multiplication and division. These problems continue to support children in working sensibly with numerical problems.

- 3 x 4 = _
- 10 ÷ 2 = _
- 50 ÷ 5 = _
- 4 x 3 = _
- 15 ÷ 3 = _
- 90 ÷ 10 = _
- 12 ÷ 3 = _
- 12 ÷ 2 = _
- 20 ÷ 2 = _
- 12 ÷ 4 = _

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MULTIPLICATION & DIVISION: GRADE 3 (M&D3)

Fluency

Children can:

- Recall multiplication and division facts for 2, 5, and 10 up to 20, 50 and 100 respectively

9 × 5 = _ *4 × 10 = _* *6 × 2 = _* *8 × 5 = _* *7 × 10 = _* *1 × 10 = _* *6 × 1 = _* *1 × 9 = _* *_ × 5 = 30* *_ × 2 = 18* *6 × _ = 12* *_ × 10 = 80* *_ × 10 = 100*	*40 ÷ 5 = _* *90 ÷ 10 = _* *24 ÷ 2 = _* *12 ÷ 2 = _* *25 ÷ 5 = _* *_ × 10 = 100* *Which of these numbers are in the 2 AND the 5 times table?* *14, 22, 25, 30, 34, 35, 43, 54, 55, 70, 91, 100*

Problem Solving

M&D3: Children can work with multiplication/division situations presented in pictures and number sentences that use formal language

These situations involve multiplying/dividing involving 1x1 to 10x10

Level 1	Distinguishing Level 1 and Level 2 tasks	Level 2
Children can: • Connect number sentences, diagrams and stories of multiplication/division situations including word problems, array and number line images • Work out products or quotients in number sentences using various strategies such as drawing, repeated addition, known facts, doubling and halving, commutativity.		**Children can build on Level 1 learning to:** • Connect number sentences, diagrams and stories of multiplication/division situations including word problems, array and number line images. Situations may include varied placement of the unknown (e.g. Some friends share 18 candies. If each of them get 3 candies, how many friends were there?) • Work out missing numbers in number sentences, using various strategies for efficient calculation, including known and derived multiplication facts, doubling and halving, commutativity. (E.g. _ x 3 = 12)

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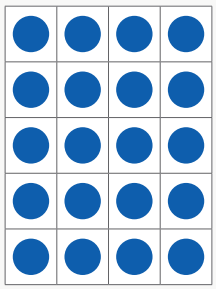
- 5 groups of 4 is _.
- 6 groups of 4 is _.



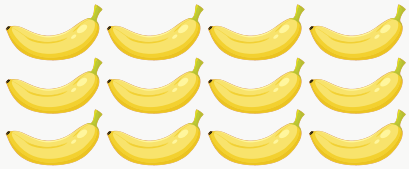
At Level 1, children solve multiplication problems which are related to each other when they are explicitly supported to visualize the relationship.

At Level 2, children solve similar related problems without any support. They also may be asked more advanced questions based on relating two or more Fluency products and patterns in them.

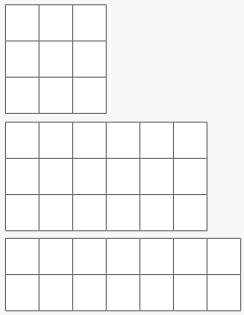
- 6 groups of 7 is 42.
- 7 groups of 7 is _.
- 7 groups of _ is 56.

- Rakhi shared 15 pencils with some children. She had no pencils left. Did she share with 2, 5, or 10 children?

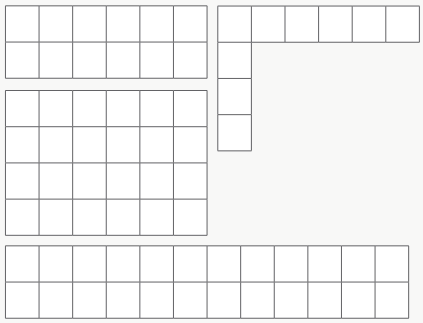
- Write a multiplication and division story about the bananas below.



- Create a number sentence for each array and solve it.



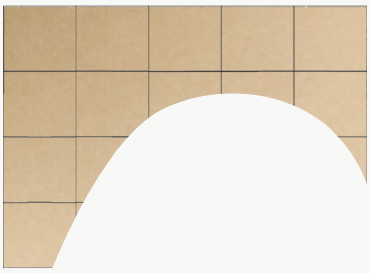
- This shows 2 x 6, which of the arrays shows 4 x 6.



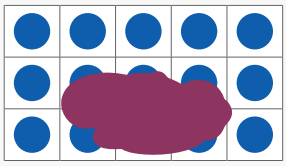
At Level 1, children match or produce a numerical statement/ story for a given array.

At Level 2, children work with arrays that are partially hidden, supporting children to move away from counting individual objects. They also produce number sentences for the images.

- Here is a torn piece of paper. How many small squares do you think it has? Write a number sentence.



- How many dots are there?
- Write the number sentence.



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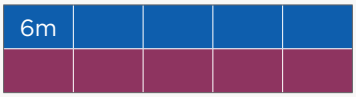
• How many jumps of 2 does it take to get to 18? • _ x _ = _  • How long is the orange rod?  	**At Level 1, children are provided with a number line marked with the starting and end numbers and equidistant marks or bars to solve problems that require repeated iteration.** **At Level 2, children solve a problem independently using the number line, where only 0 and the first numbers are marked. They will need to interpret the numerical problem on the number line and solve it by skip counting. More generally, children can work with bars to solve more complex problems and write appropriate multiplication/division sentences. The solution, once again, supports moves to skip counting or recall of fluency facts.**	• Solve 2 x 9 using the number line below.  Write the number sentence. Solve it. • How many 6m pieces of wood can be cut from a 30m plank? 6m
• Joseph has 6 books, and each book has 5 pages. How many pages are there in all? • There are 5 friends at a party. They have 45 pieces of candy. They want to share them equally. How many pieces of candy will each friend get?	**At Level 1, children solve multiplication and division word problems where the situations are simple and calculations straightforward. They can even recall the multiplication facts to solve them.** **At Level 2, children solve more complex problems which require finding the number of equal groups.**	• Joseph has some books. Each of these books has 6 pages. There are 30 pages total. How many books does Joseph have? • There are some friends at a party. They share 45 pieces of candy equally. Each friend gets 9 pieces. How many friends are at the party
	Level 2 includes problems that require modelling a different situation, like scaling up a recipe.	• Marco is making soup for 3 people. The recipe for 1 person uses 10 grams of salt. How many grams of salt will he need to make soup for 3 people?

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• Draw an array or number line to help you solve the problems, if needed. • 12 ÷ 2 = _ • 12 ÷ 6 = _ • 18 ÷ 3 = _ • 18 ÷ 6 = _ • 2 x 8 = _ • 3 x 8 = _ • 24 ÷ 8 = _ • 2 x 4 = _ • 3 x 4 = _ • 4 x 4 = _	**At Level 1, children solve multiplication and division calculation problems using a strategy of their choice, involving small numbers (with a factor or quotient less than 5).** **At Level 2, children solve missing number or missing operation problems. These problems continue to support children in working sensibly with purely numerical problems.**	• _ x 2 = 100 • _ x 5 - 55 • _ x 2 = 50 • _ x _ = 20 • _ x _ = 30 • _ x _ = 15 • _ ÷ 10 = 8 • 40 ÷ _ = 8 • 70 ÷ _ = 7 • Fill the blank spaces with +, - , × or ÷ • 5 _ 5 = 25 • 18 _ 10 = 8 • 5 _ 5 = 10 • 60 _ 6 = 10 • 2 _ 7 = 14 • 45 _ 9 = 5 • 8 _ 10 = 80

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4. Fluency and Problem-solving in the CENOs

The addition/subtraction and multiplication/division clusters each include Fluency outcomes and Problem-solving outcomes to be attained in each grade. Fluency skills, which most children should acquire at near automaticity level, involve rapid recall. These skills are developed within problem-solving that incorporates key representations, rather than learned as rote, disconnected facts. This is because flexible problem-solving requires knowledge of connections that cannot be acquired in rote approaches.

Problem-solving includes attention to setting up, describing and using models of situations, connecting between representations, and developing increasingly sophisticated and efficient strategies for calculating. Using the patterns and relations in the base ten number system facilitate efficient working and are a critical part of foundational learning and supported by growing confidence with key representations.

Fluencies and Problem-solving are interconnected and co-developed. It is through working on problems that fluencies are developed: for example, adding 2 and 5 might initially be a Problem-solving Level 1 task for a Grade 1 child, until they can recall that the sum of these numbers is 7. At this point knowing the sum of 2 and 5 makes it a Fluency. Over time, a growing bank of Fluencies allows problem-solving to become more sophisticated and efficient. Fluencies need to be revisited and practised so that children retain rapid recall of these facts. Without some regular practice, children tend to fall back into counting-based approaches. Working on more complex problems expands the bank of fluencies that can be drawn on within problem-solving.

The Problem-solving outcomes include Level 1 and Level 2 specifications. These two levels are included to point to key routes in which work within a grade needs to progress over time. At the first level, simpler problem forms are linked to each criterion, while at the second level, more complex problem forms are introduced. Progression from Level 1 to Level 2 happens along several dimensions – summarised in the table alongside:

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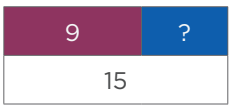
TABLE 1: Aspects that differentiate Level 1 and Level 2 problem-solving

Level 1	Level 2
• Simpler problem forms • Introductory language • More concrete representations • Calculation using counting-based strategies	• More complex problem forms • Expanded language • Increasingly abstract representations • Calculation using efficient number relation-based strategies

In Level 1 simple problems and key additive and multiplicative representations such as number lines and arrays are introduced. These problems can be used to acquire initial fluencies, and to extend the bank of fluencies through tasks that require extensions from known facts. Level 2 includes more complex problems. It is important to note that there isn't a clear demarcation between Level 1 problem-solving tasks and Level 2 problem-solving tasks. Children sometimes solve simpler tasks using more abstract representations and/or more efficient calculation strategies, and this should be encouraged. Conversely, children who continue to work with inefficient counting-based approaches on more complex tasks or tasks in higher number ranges will need focused attention to improve their fluency skills. The Grade 2 Level 2 Problem-solving task below points to the differences between Level 1 and Level 2 ways of working.

Problem:
Ben ate 15 grapes. Nic ate 9 grapes. How many more grapes did Ben eat than Nic?

• Counting-based working • Kemi sketches 15 grapes. She crosses out 9 grapes. She counts the remaining grapes and writes in 6 as the answer	• Efficient calculation-based working • Niki sketches a part-whole diagram:  • She writes: 15 – 9 = _ • She says aloud to herself: '9. 10' and opens 1 finger. She then says ' 10. 15' and opens her other hand showing 5 fingers. She writes in 6 as the answer and says 'Nine plus six equals fifteen'.

Progression

- Kemi's sketch of 15 grapes, then crossing out 9 involves one-by-one counting in each stage, and then again, to count out the remaining 6. This is what children do as they begin to learn to move beyond 10 with their counting and calculating, but it is a time-consuming approach.
- Niki's part-whole sketch shows understanding of 15 as the whole and 9 as one part in this problem. She writes 15 – 9 = _ as the calculation to carry out, and is then able to 'bridge through ten' to quickly produce 6 as the answer. Her written work and talk show that she sees the relationship between 15 – 9 = _ and 9 + _ = 15.
- Niki's work with representations, her calculation strategies and her talk all point to Level 2 ways of working on this problem in contrast to Kemi's work with more concrete Level 1 visual representations and counting-in-ones approach to calculate the answer.

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Problem-solving Level 2 should not be deferred until children can accurately solve all Problem-solving Level 1 tasks. The illustrative tasks in Section 3 contrast Level 1 and Level 2 Problem-solving. The gap between the two levels can be small, allowing children to progress to Level 2 quite quickly, possibly even within one lesson. Additionally, working on Level 2 critical outcomes reinforces understanding of Level 1 critical outcomes.

Work on the first problem below which involves a result-unknown calculation [3 + 4 = _] might be immediately followed up by the second problem which involves a change-unknown calculation [3 + _ = 7]. Talking with the children about what is the same and what is different across the two problems builds understanding of part-whole relationships.

Yasmin builds a tower with 3 red bricks. She adds 4 yellow bricks. • How many bricks tall is her tower?	**Yasmin builds a tower with 3 red bricks. She adds some yellow bricks. Now her tower is 7 bricks tall.** • How many yellow bricks did she add?

Initial working with practical situations and visual representations supports sense-making in children's later work with purely numerical problems. It is therefore important that children start their work in Grade 1, and – where useful – Grade 2, with concrete materials or sketches. A key goal is to link these concrete approaches to numerical representations that gradually become models that children can work with flexibly and efficiently.

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5. Teaching Brief: Working with Representations

Working with representations is central to all mathematical working. The critical outcomes detailed in this report are linked with the addition/subtraction and multiplication/division clusters in the early grades. Mastery of these clusters is supported by learning to work with key representations.

Here we outline the trajectory of children's work with representations of number and of addition/subtraction and multiplication/division in the early grades in two parts:

- Learning number names, number symbols and the quantities that these represent, and counting using these names/symbols
- Using representations to:
  - Give children ways of making sense of number sentences and how the quantities within them are connected, and
  - Solve addition/subtraction and multiplication/division problems

Number names, number symbols and counting

Working on the early arithmetic operations depends on familiarity with some early number representations and skills.

These include:

- Counting to work out how many there are in a set and knowing that the final number represents the total quantity
- Knowing the order of number names
- Subitizing (i.e. immediately recognizing small quantities, or quantities in familiar arrangements without counting – e.g. dice dot numbers or numbers on fingers)
- Recognizing and identifying number symbols (can say '4' when shown the numeral/pick out '4' from a set of 1-10 cards)

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Accurate counting to quantify involves matching the number word sequence with the items being counted. Young children, after counting a set of objects, will often respond to the question 'How many?' by counting the set out again. Knowing that the last number in their count represents the total quantity is an important idea that needs establishing through many experiences of counting. In the course of this counting, children become more familiar with the order of the number word sequence.

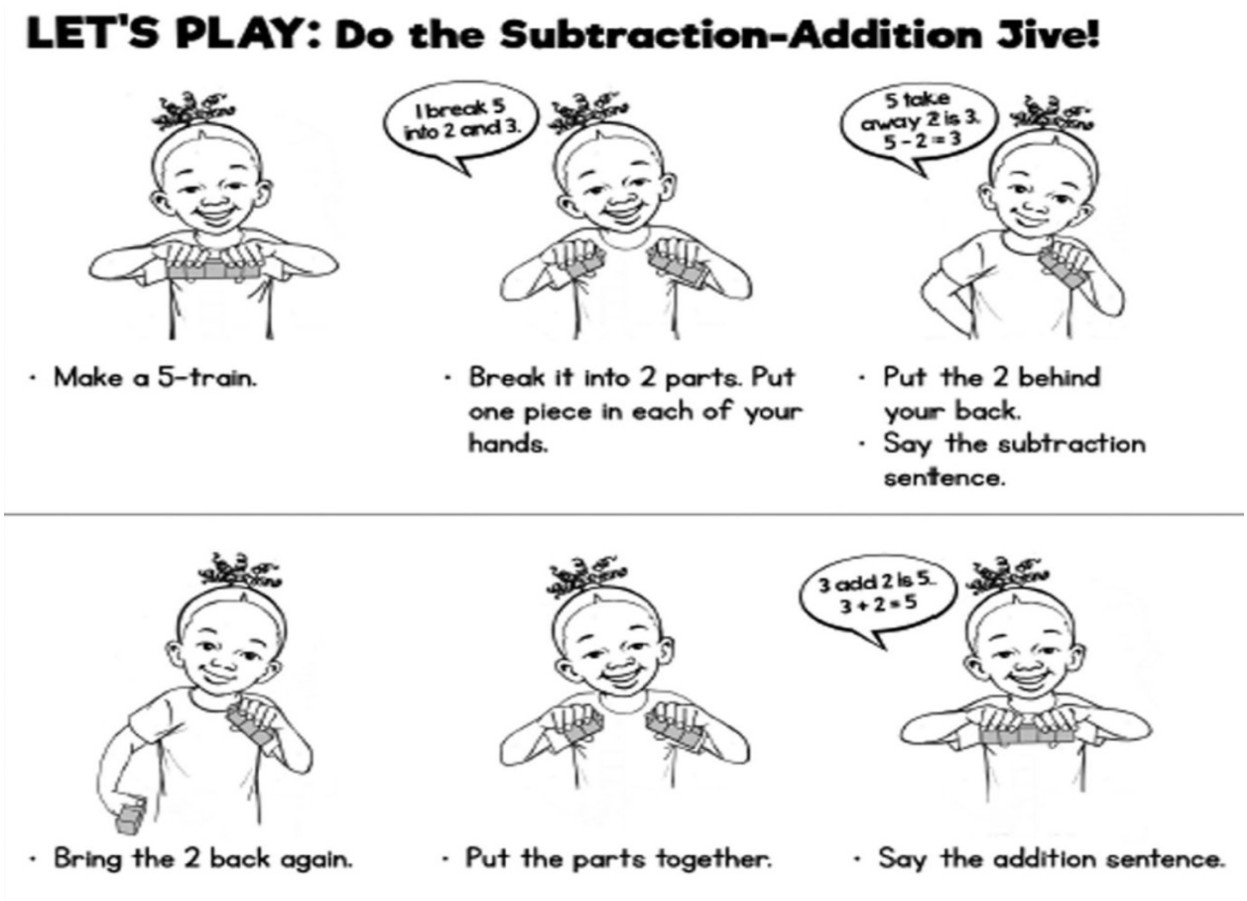
Number symbols and their meanings are the gateway to early number learning. A key part of early grade 1 instruction involves ensuring that children are familiar with the 1-10 number symbols in terms of recognition and identification. Children make meaning of number symbols when they understand the quantities that these symbols represent. This means being able to match a number symbol with number names and with concrete or pictorial representations, e.g.:

Show me three fingers?	**Which number card matches this number of dots?** 	**Draw a tower that is 3 squares tall.**

Familiarity with number symbols, number names and with counting is critical for achieving the Fluency and Problem-solving critical outcomes.

Using representations

A. Representations help children to make sense of number situations. Materials from the Magic Classroom Collective project in South Africa (Porteus, 2023) show an activity that helps children to make sense of number sentences.



Train activity using multilink cubes, from Porteus (2023)

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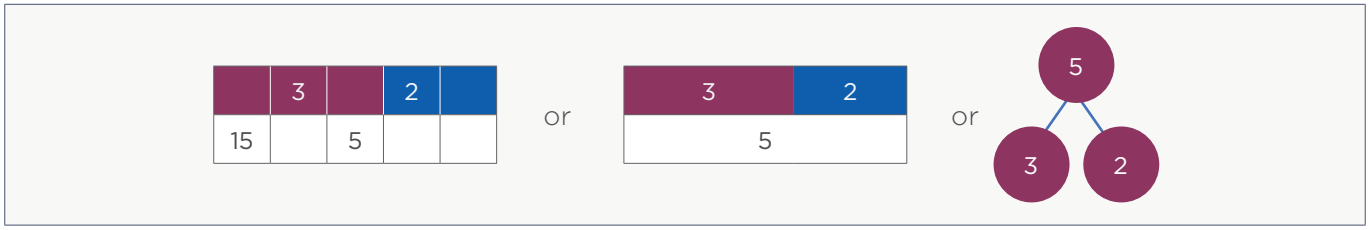
In this activity, children make connections between the actions of removing part of a train, using words to tell a story that represents their actions with the result of those actions, and writing a number sentence linked to the actions and story. Connections are also made to the 'reversing' action and the number sentence that goes with this. This simple activity, suitable for use in Grade 1, helps children to make sense of number sentences like 5 – 2 = 3 and to see connections between this number sentence and number sentences like 3 + 2 = 5. Repeating the activity with 'Put the 3 behind your back' also makes connections with 5 – 3 = 2 and 2 + 3 = 5.

As children become familiar with the initial activity, a quick sketch of a part-part-whole diagram representation can be linked to the actions in this activity, and later, the more abstract triad representation:



Gradually, the diagram and then number sentences – which are quicker to draw/write than making trains with cubes all the time – overtake the concrete activity. It is important for children to connect adding with 'joining' or 'combining' and subtracting with 'taking away'. This provides the base for understanding later working with addition/subtraction number sentences. Here, concrete actions with cubes are linked to the part-part-whole and number line diagrams. Careful sequencing of representations is important, alongside language that helps children to make sense of the connections. Children will not be able to use representations 'automatically' without this kind of support.

It is common-place now to talk about the importance of children encountering mathematics through concrete, pictorial and abstract (CPA) forms. A common misconception is that they are stages children have to work through and then grow out of. It is more helpful to think of CPA as comprising mini cycles of learning to go through. For example, a Grade 1 child may need to work initially with concrete materials to become familiar with numbers to 10, but once confident can go on to work with single digits in more pictorial and abstract forms. That same child might return to initial concrete working with numbers up to 20, but not for single digit problems.

B. Representations in solving addition/subtraction and multiplication/division problems help children in three ways:

- Making sense: representations first help children to make sense of how quantities in a problem are related to each other
- Reasoning: where a missing value needs to be calculated, the initial representation then helps children to reason about what value is missing: part or whole.
- Calculating: deciding which operation to carry out (rather than guessing the operation)

These three steps are illustrated below for a Grade 3 word problem. The reasoning that supports children with solving the problem is also explained.

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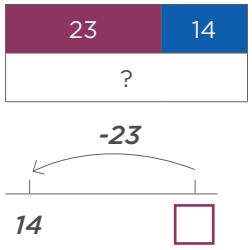
Problem:
Nic has a bag of marbles. He gives 23 marbles to Asiya. Nic has 14 marbles left in his bag. How many marbles did he have at the start?

STEP 1: Making sense

- A part-part-whole or number line sketch can be used to make sense of how the numbers in this problem are related.
- In this case, the missing number must be larger than the two given numbers because Nic 'loses' a part of what he started with and is left with another part.

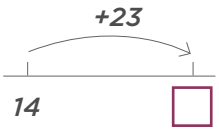


STEP 2: Reasoning

- Looking at the part-part-whole diagram, I see I can put 23 and 14 together to make the missing number. This problem is easier to solve than _ - 23 = 14.
- On the number line, I see that I can start at 14 and jump forward 23 to get to the missing number

STEP 3: Calculating

- Some children will represent their calculation using a number sentence:
- 23 + 14 = _ or
- 14 + 23 = _
- Others will use a number line to represent their calculation:



- Children can calculate 14 + 23 using Fluency with adding 10 or multiples of 10:
- 14 + 10 + 10 + 3 or 14 + 20 + 3

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6. Teaching Brief: Place Value

Place value is a central part of number learning at early primary level. Place value in the decimal number system offers a succinct way of writing numbers and offers the basis for efficient calculating by using tens (and later, hundreds, thousands, etc) as 'benchmark' numbers and as 'friendly' numbers for easy and efficient calculation.

A core idea of place value in the decimal number system is grouping units into tens. This idea should be introduced in Grade 1 and expanded in Grades 2 and 3. In this section, we briefly describe key steps in the building of place value understandings that allow for efficient calculation in the critical outcome clusters.

Representations, as already pointed out, are central to all mathematical concepts including place value. Ten frames (pictured below) are important to introduce in the Grade 1 1-10 number range because they:

- Allow children to see quantities in relation to 5 and 10
- Support development of number relationships in the 1-10 range

Grade 1

Counting out items can be done in many contexts, but doing this in ten frames allows children to see spaces filling up in relation to 5 and 10. Teaching using ten frames should encourage children's talk about the number of counters and the number of empty squares, showing the same situation using their fingers and writing early number sentences.

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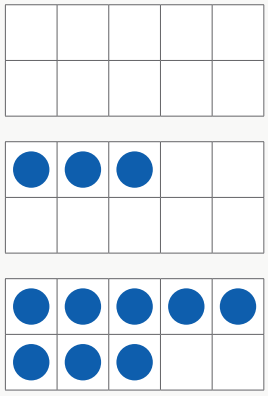
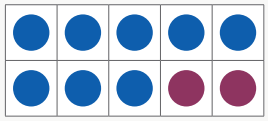


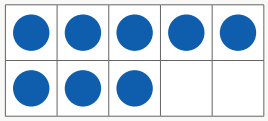
- Begin with discussion of the empty ten frame: that there are 5 squares in the top row and 5 squares in the bottom row, and that this makes 10 squares altogether. This ten frame connects with two hands with five fingers on each, and with the number names and number symbols to 10.
- Numbers in a ten frame. Subitizing (e.g. seeing 3 blue counters here) can be extended in ten frames to '5 plus' combinations, for example, knowing immediately that there are 8 blue counters in the image below because there is one full row (5) and 3 blue in the bottom row.
- Questions can also focus on bonds of 5 using the top row (e.g. 3 counters and 2 empty squares. 3 and 2 together make 5) and bonds of 10 (10 spaces filled with 8 blue counters and 2 red counters).

Working with counters in two colours links with seeing quantities made up of two parts in the 1-10 range and writing number sentences about these.



- What number sentences can I see in this picture?
- 5 + 3 = 8
- 8 + 2 = 10
- 10 – 2 = 8
- 10 – 8 = 2

Through becoming familiar with bonds of 10, children will be able to answer questions like this G1 Fluency problem with a ten frame:



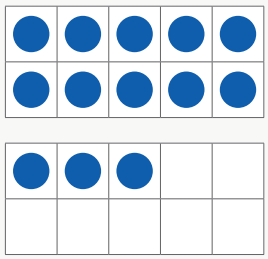
- How many more to make 10?

The aim, over time, is for children to solve problems like 4 + ___ = 10 or 8 - ___ = 3 as pure number calculations mentally, with a mental image of the tens frame supporting their thinking.

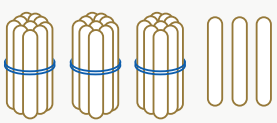
Grade 2

Two ten frames can be used to represent numbers in the 1-20 range and to see these numbers relation to 10 and 20 as benchmark numbers. For example, 13 = 10 + 3, 18 = 10 + 8 and 18 = 20 – 2. The similarity of 8 and 18 (8 is 2 less than 10; 18 is 2 lesson than 20) and the difference of 10 between them can also be seen in the ten frames.

We introduce stick bundles as a second key place value representation in Grade 2.



- How many dots in all?



- How many sticks?

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Ten frames can be used to check if children understand 2-digit numbers as composed of tens and ones (30 + 2 = 3 tens + 2 ones) rather than just as the digits 3 and 2. Tasks like the one below are useful for assessing children's understanding of the place value structure that underpins numbers. These understandings can then be followed up with the use of Tens and Units symbolic representations, with these expanding in Grade 3 to the inclusion of Hundreds, Tens and Units.

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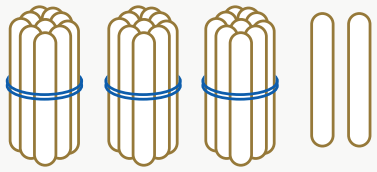
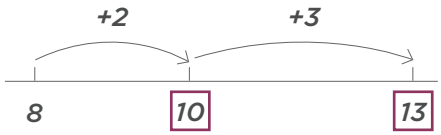
<td>  </td>
<td>  </td>
<td>• Which picture shows 32?</td>
</tr>
<tr>
<td colspan="3">T U
3 2</td>
</tr>
</table>

Ten-frame and bonds of 10 support children in developing strategies for efficient calculations in the 1-20 range. Efficient calculation of 8 + 5 can involve the following steps, with options also for representing these steps on a number line.

<table>
<tr>
<td colspan="3">8 + 5</td>
</tr>
<tr>
<td>• 8 + 2 = 10
• This is a Fluency developed in Grade 1.</td>
<td>• With 2 added, split 5 into 2 and 3 (also a Grade 1 Fluency.)</td>
<td>• Calculate 10 + 3 = 13.
• This is a Fluency to develop in Grade 2.</td>
</tr>
<tr>
<td colspan="3"> 
</td>
</tr>
</table>

The empty number line can consolidate the experience of working with Tens-Frames or matchstick bundles. Asking children to add numbers, like 8 + 7 or 9 + 1 + 2, first using concrete materials and then on the number line leads to the development of a strategy of bridging through 10 (and later, bridging through multiples of 10, e.g. 38 + 7).

Repeatedly taking away tens or adding tens and keeping track of the changing quantity is a Fluency at Grade 2, and important for efficient calculating that moves on from counting in ones. Tasks like the one below using base ten blocks help to develop this fluency, and recording results on a 100 square and then a number sequence emphasises the number pattern that makes adding and subtraction 10 easy.

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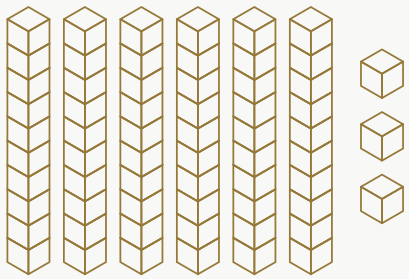
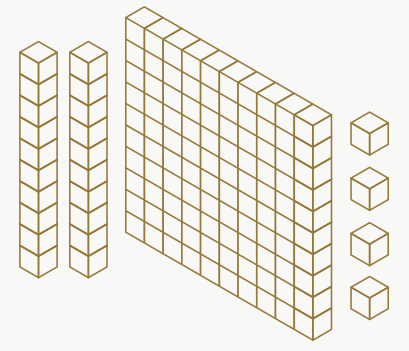


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- 63, 53, 43, 33, ...

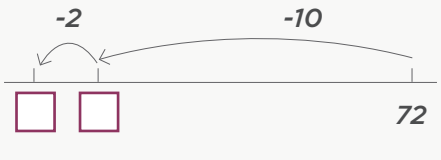
Grade 3

As the number range expands in Grade 3 to counting and recognising numbers up to a 1000 and calculating in the 1-100 range, base ten blocks or images based on these blocks are useful for showing how 10 units are composed into a ten, and 10 tens are composed into a hundred. Tasks in the context of base ten blocks can, once again, be used to check children's understanding of place value structure. For example, a common area of difficulty for children relates to learning that base-10 blocks are not positional in nature (that is, the order of placing the objects does not change the number they represent, unlike numerals where changing the position of the digit changes the value of the number). The following problem checks children's understanding of the quantity each different block type represents and how they together represent a number in the symbolic form in the decimal number system:



- What number is shown here: 214 or 124?

Adding and subtracting using base ten blocks can be mirrored on a number line. For example, for 72 – 12 using base ten blocks, I can take away one ten-block and two unit-blocks and am left with 60. On a number line, these steps can be represented like this.



- Jump back 10 from 72, land at 62. Jump back 2 from 62, land at 60.

The same idea can be extended on a number line to 72 – 15 = _ , with the inclusion of a bridging through ten step, as shown below. This calculation approach uses a sequence of fluencies developed earlier: subtracting 10, choosing the relevant bond of 5 to bridge through 10 (2 and 3 here), knowing that 62 – 2 = 60 and 60 – 3 = 57.

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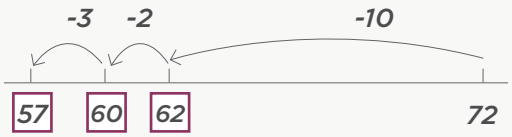

The same calculation can be carried out using the column algorithm. For column algorithms, it is useful for children to be aware of how 'exchange' works with base ten blocks, for example, knowing that 72 = 70 + 2 and 60 + 12. When we 'borrow' from the tens in calculations like: 72 – 15, we are essentially decomposing 70 into 60 + 10 and then 1 ten into 10 ones. Thus turning 72 into 6 tens and 12 units allows us then to calculate like this:

$$\begin{array}{r} {}^{6}\not{7}\,{}^{1}2 \\ -\ 1\ \ 5 \\ \hline 5\ \ 7 \end{array}$$

Once children have a sense of the size of the quantities that underpin hundreds, tens and units, they can move to working on artifacts like the 100 square, and moving between jumps on this chart and symbolic number working with tasks like this:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Which number sentence matches the movement shown by the arrows in this diagram:
- 47 + 20 + 3 = 64
- 47 + 2 + 3 = 64
- 47 + 2 – 3 = 64
- 47 + 20 – 3 = 64

Place value is useful within multiplication as well, but much of this use comes into play beyond Grade 3 level of the critical early numeracy outcomes. Important precursor ideas for this use relate to Grade 3 fluencies such as skip counting by 10. This Fluency can be used to figure out 'neighbouring' products to 10 x a number. The following task is an example. For example, if children know that 10×8 can be represented by the array shown and equal to 80, they can use this to consider what the array for 9 × 8 should look like, and how the product of 10x8 should be adapted to find the answer to 9 x 8.

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7. Teaching Brief: Efficient and Fluent Calculation Strategies

In Grades 1 and 2, the aim is to build a bank of known addition and subtraction facts that children can use as a base to derive further results from. In Grade 1, the focus is on flexible and efficient calculation in the 1-10 range. In Grade 2, this extends to the 1-20 range.

From 1 + 1 to 9 + 9 there are 81 addition facts (and their related subtraction facts) that children need to become fluent in to confidently work with numbers to 100 in Grade 3.

Rather than treating these as 81 separate facts to commit to memory, it is more effective to see these as connected results. Connections allow initial learning to support further learning.

Below, we present a trajectory of strategies that build from initial effectiveness into efficiency, and ultimately, fluency

Grade 1

Count all

- 6 and 2 – put out 6 counters, put out 2 more, count all 8
- Notice that this involves three sequences of counting from 1: counting out 6, counting out 2 and then counting out 8.

As children become more familiar with how addition and subtraction work, they should be encouraged to move to more efficient ways of counting and then calculating:

Count on in ones

- 6 + 2 = _ . Put 6 in your head, open a finger as you count 7, and another finger as you count 8.

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Count back in ones

- 9 – 2 = _ . Put 9 in your head, open a finger as you count 8 and another finger as you count 7.
- The three separate counts reduce here to one count on or count back sequence, so this is more efficient than **Count all**.

A next step is for counting to be fluent enough to work without fingers or counters.

Quickly add 1 or 2

- As counting becomes more fluent, then adding 1 only means saying the next number in the counting sequence: 6 add 1: 6, 7. Adding two only means a count of two, which children should be able to keep track of without needing fingers or counters.

Quickly subtract 1 or 2

- If children can count back fluently, then subtracting 1 only means saying the previous number in the counting sequence: 8 – 1: 8, 7. Subtracting two only means a count back of two, which children should be able to keep track of without needing fingers or counters.

After this, a range of **flexible strategies** can come into play that make use of properties of addition and subtraction.

For addition: Put the larger number first

- 2 + 6 is easier done as 6 + 2

Growing familiarity with these facts lead to some results being established as fluently recalled facts. These form the initial bank of **fluencies in Grade 1**.

Learn pairs that add to 10

- Use ten frames to learn 9 +1, 8 +2, 7 + 3, 6 + 4, 5 + 5 and their reverses

Learn doubles to 5.

- Double 4 is 8 – show on a tens frame

Learn splits of 6, 7, 8 or 9 into '5 and a bit'

- Use ten frames to learn 6 = 5 +1, 7 = 5 +2, 8 = 5 + 3, 9 = 5 +

Grade 2

In Grade 2 the fluencies established in Grade 1 offer foundations that children can use as they move into the 1-20 range. This move will likely involve some reversions into count on, or count back, approaches until the new results also become established as recalled facts. This process builds the bank of results that are known at the level of fluencies.

Add 10 and a single digit

- 10 + 4 = 14 show why this works using two tens frames

Add two single digits each greater than 5 by bridging through 10

- Split one number to make the other add to ten and add the rest:
- 8 + 6 = 8 + 2 + 4 = 10 + 4
- This can be shown with jumps through 10 on a number line

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Subtract a single digit from 11 to 19 by bridging back through 10

- 14 – 6 = 14 – 4 – 2. Model on a number line

Connect bonds of single digit numbers to the 11-20 range

- 4 + 3 is 7, so 14 + 3 is 17

Add 9 by adding 10 and subtracting 1

- 7 + 9 = 7 + 10 – 1
- (7 + 9 might also be calculated by bridging through 10 in this form or as 9 + 7)

Grade 3

Learn doubles of 5, 6, 7, 8, 9

- Double 7 is double 5 plus double 2, 10 + 14. Show on two tens frames

Connect bonds of single digit numbers to 1-100 range

- 4 + 3 is 7, so 34 + 3 is 37 and 84 + 3 = 87

Use bonds of single digits to add multiples of 10

- 4 + 3 is 7, so 40 + 30 is 70. Show with base 10 blocks.

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