# Mathematical knowledge for teaching (MKT):
## Cross-national theory 'borrowing' vs practical insights?

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# What is MKT?

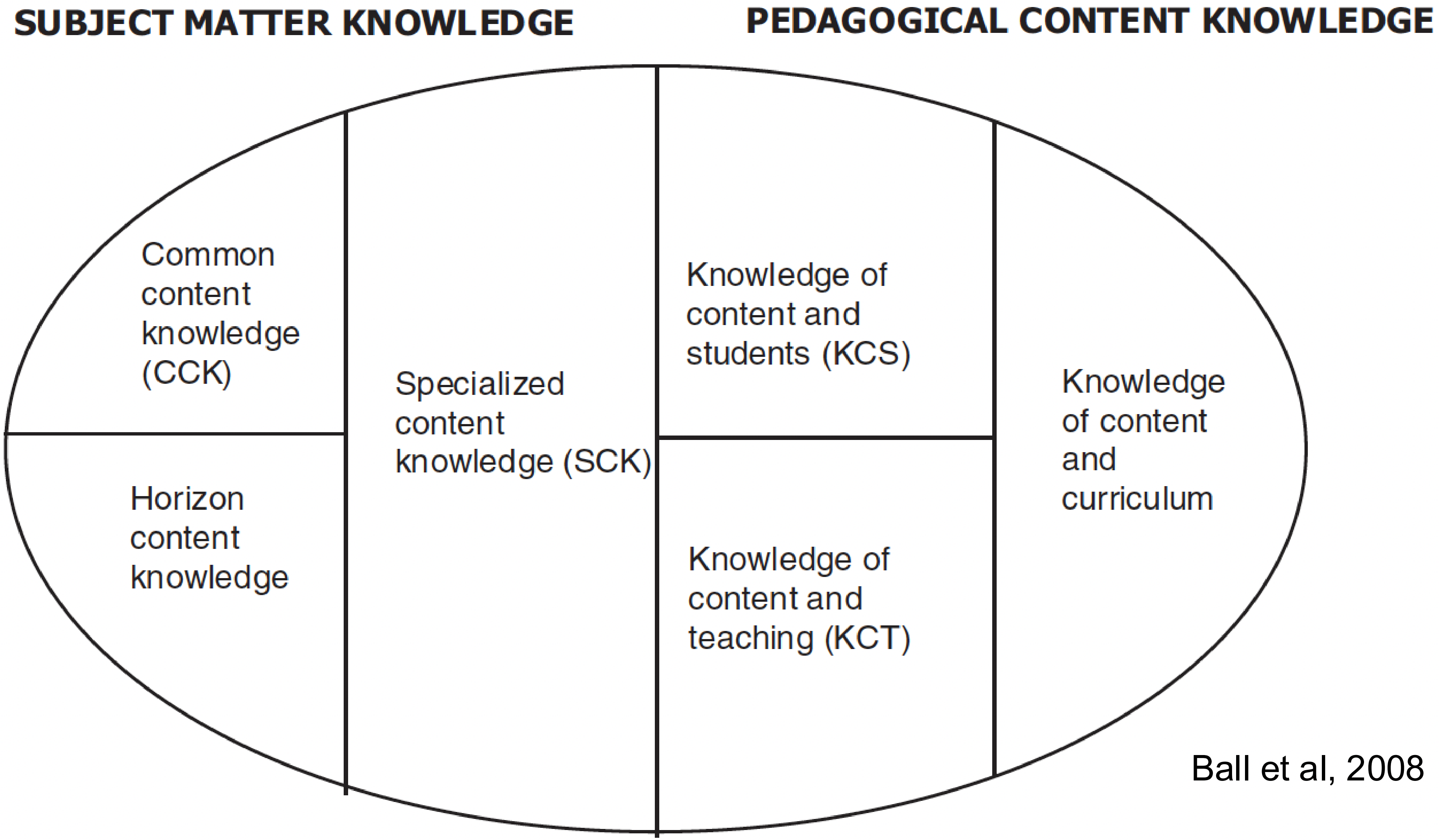
- Building on Shulman’s writing on PCK, Deborah Ball saw as a problem the separation of content and pedagogy in teaching

  *‘leaves teachers on their own with the challenge of integrating subject matter knowledge and pedagogy in the contexts of their work’* (Ball, 2000: 241)

- Argued that solving this problem would require:

  *‘identifying the content knowledge that matters for teaching’*

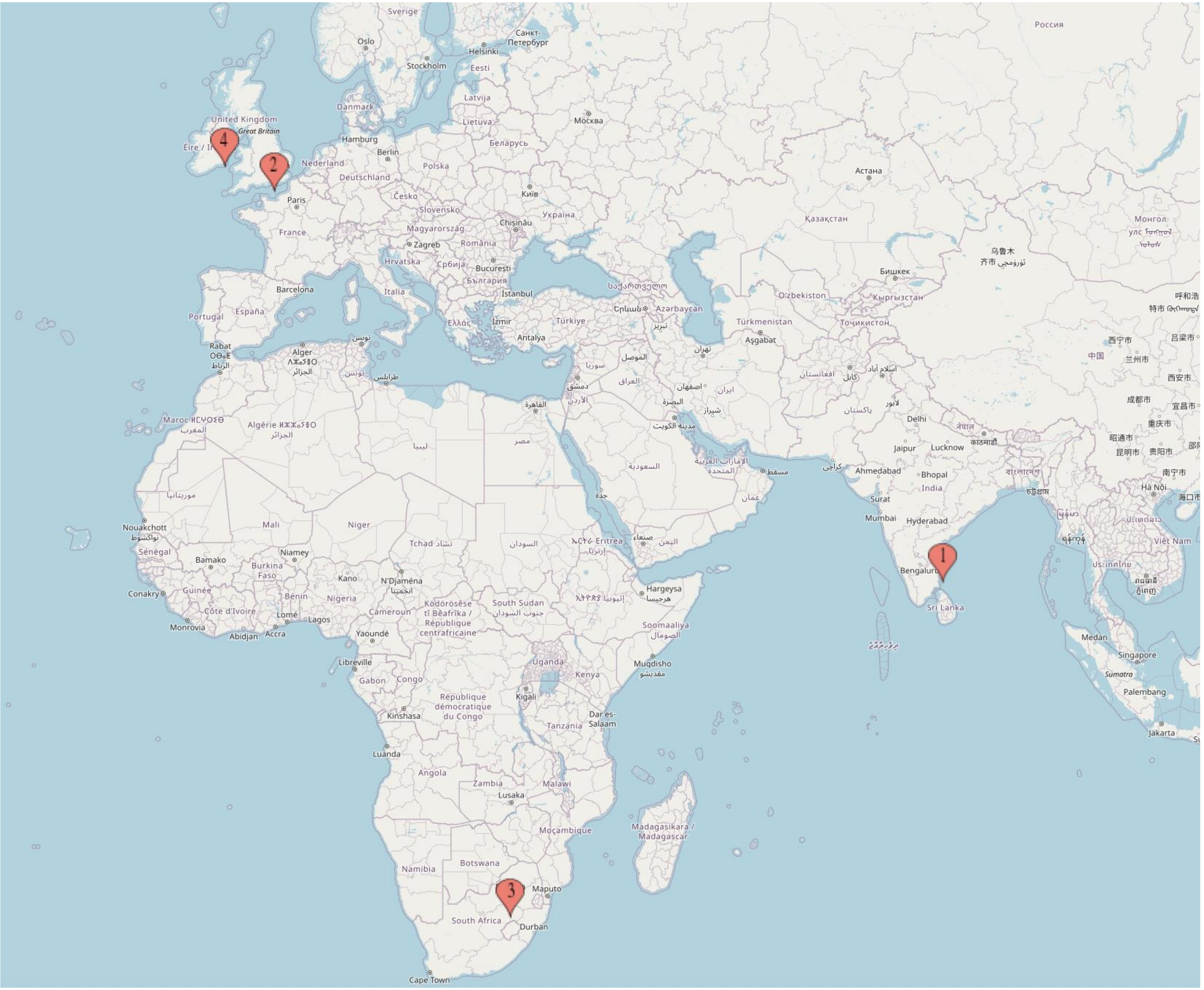
# MKT: The mathematics-related knowledge that matters for good mathematics teaching




Ball et al, 2008

# MKT

# Why am I interested in this story?

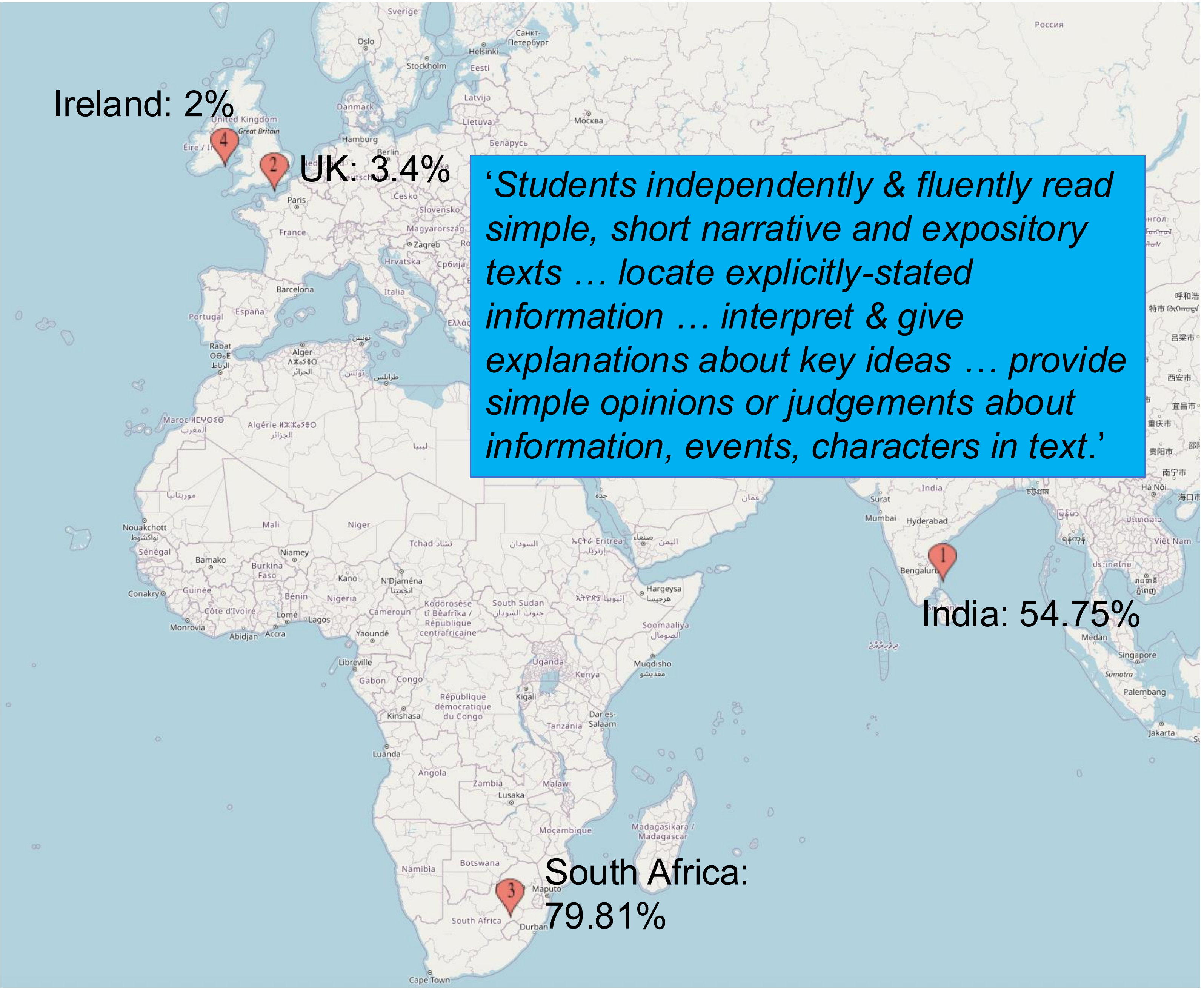


# Learning poverty

**World Bank and UNESCO Institute for Statistics**

**Learning deprivation**, share of children at end primary who read at below the minimum proficiency level:

**Schooling deprivation**, share of primary-aged children out of school. All assumed to be below minimum proficiency level in reading.

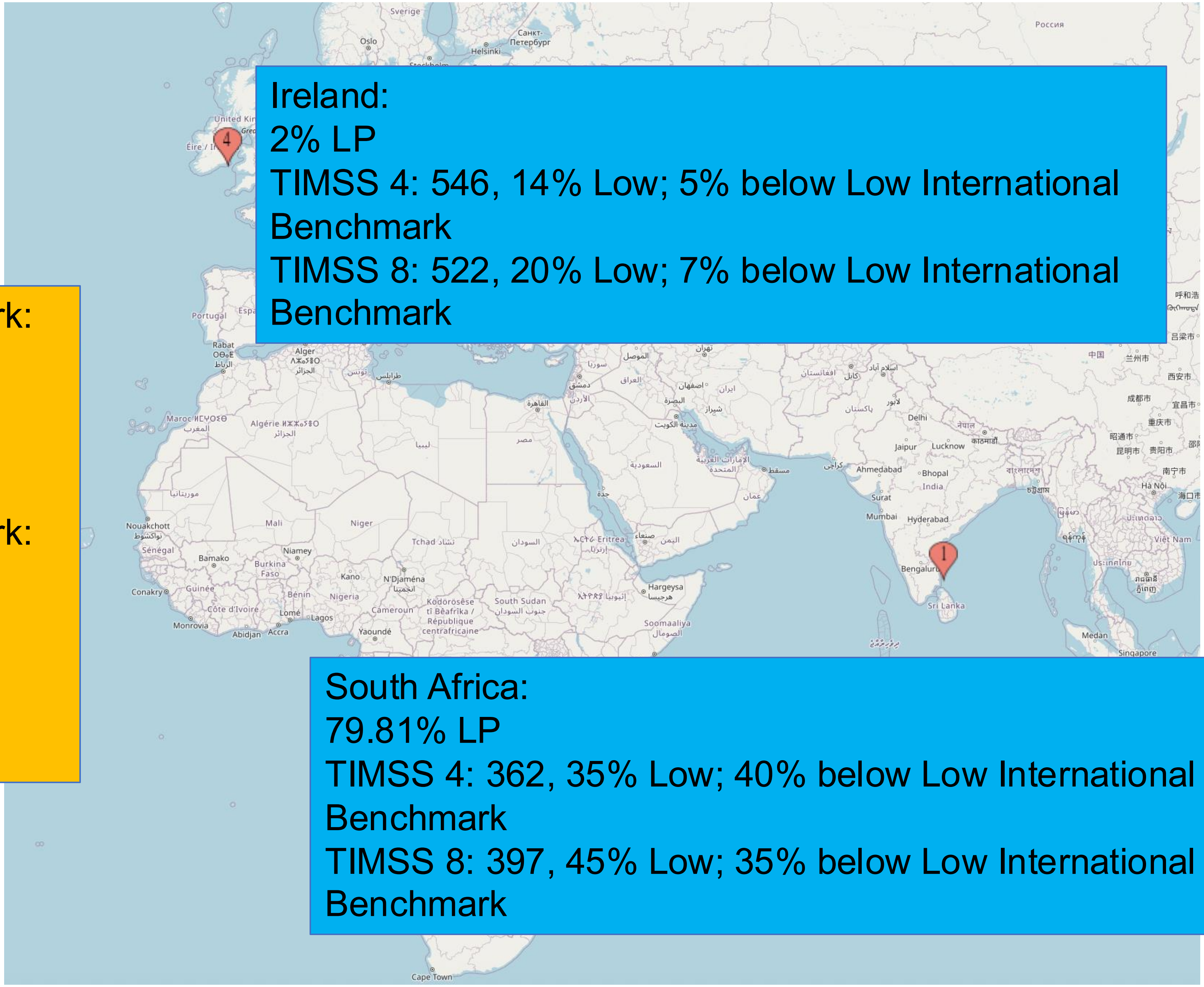



*'Students independently & fluently read simple, short narrative and expository texts … locate explicitly-stated information … interpret & give explanations about key ideas … provide simple opinions or judgements about information, events, characters in text.'*

# Mathematics
## TIMSS 2023

TIMSS 4 Low Int Benchmark: Learners demonstrate basic mathematical understanding

TIMSS 8 Low Int Benchmark: Learners have knowledge of integers, basic shapes, and visual representations




Ireland:
2% LP
TIMSS 4: 546, 14% Low; 5% below Low International Benchmark
TIMSS 8: 522, 20% Low; 7% below Low International Benchmark

South Africa:
79.81% LP
TIMSS 4: 362, 35% Low; 40% below Low International Benchmark
TIMSS 8: 397, 45% Low; 35% below Low International Benchmark

# Majority marginalization from core grade-level outcomes

- UNESCO (2017) estimates - **56%** of the world's children, up to early adolescent age - are not achieving **minimum proficiency** across the relevant grade-levels in mathematics
- Unequally distributed: Less than 20% of children/young adolescents in North America, Europe, parts of Australasia.
- Around 80% in sub-Saharan Africa, parts of central, Southern Asia

↓

Is this issue represented in mathematics education research?

Venkat & Adler, forthcoming

What **theories and frameworks** do we need to understand and address the disparity?

| Ireland | South Africa |
|---|---|
| 30% of schools serve ‘designated’ disadvantaged populations | 87% of public schools are ‘no-fee’ schools serving disadvantaged populations |
| Primary ITE recruits from the top 15-20% of high school graduates | ITE recruits have lower school exit attainment profiles than other UG programmes (Pampallis, 2022) |
| 299.3 mins on average Maths teaching/week in G6 (Kiniry et al, 2025) | Teacher absenteeism, learner mobility, disruptions to schooling common |
| High levels of literacy. Policy rhetoric: expectations of child agency and sense-making | Low levels of literacy. Return to tightly prescribed curriculum ‘coverage’ and pacing. Coverage monitored at district level |

| | |
|---|---|
| Maths attainment in urban [DEIS] schools [20% of pupils] significantly lower than in other schools in TIMSS and national assessments (McHugh et al, 2024) | Significant differences in learning outcomes between schools serving the bottom (> 80%) and top (<20%) socio-economic tiers of the population |

# Differences in context, culture and conditions

Question of interest?
To what extent does research in primary mathematics teacher knowledge/teaching reflect differences in context, culture and conditions?

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Ireland and South Africa:
Contrastive settings, useful for reviewing and reflect on earlier research, raising questions and sharing emerging directions

# Antecedents

- South African work: Settings of ‘majority marginalization’ from core grade-level outcomes in mathematics. Is the issue represented in MER?
- Irish work: Global ‘policy borrowing’ in MER (Quirke et al, under review)
- Strictures of international publication in MER (Niss, 2018)

*‘emergence of an ‘ideal-typical’ research paper that represents a far too narrow and rigid understanding of mathematics education research by not reflecting the multi-faceted reality of the field … detrimental to a field which is far from having found a universal, agreed-upon theoretical underpinning’*

## Primary mathematics teaching and underpinning MKT: long-standing international concerns

- Procedural orientations ‘rules without reasons’, emphasis on producing correct answers
- Poor knowledge of concepts and connections

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## Well-known and widely cited frameworks

- Ball et al (MKT); Hill et al (MQI)
- Schoenfeld (Teaching for Robust Understanding)
- Baumert et al (CoACTIV)
- Rowland et al (Knowledge Quartet)

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Models of good mathematical learning:

- Realistic Mathematics Education & modelling-based approaches
- Strands of Mathematical Proficiency
- Tasks across levels of cognitive demand

# Ireland: Studies of teachers' mathematical knowledge

Hourigan & O'Donoghue's (2007) ITE study

- Test drew on Ball's MKT categories and Rowland's SKIMA
- General strengths on recall and procedural knowledge items, but some basic errors/slips & inability to recall facts/definitions (p.336)

Hourigan & Leavy, 2023

*'classroom practices reflect a narrow approach limited to problem solving as an 'add on', only applied after mathematical procedures had been learned'*

# Borrowed theories: Ireland

- Ball et al's MKT, Rowland et al's SKIMA project and KQ

  procedural strengths, problem-solving and reasoning item weaknesses

- Japanese Lesson Study ideas on problem-solving

  improving teaching through Lesson Study focused problem-solving in classrooms

Delaney (2012), on using MKT measures in Ireland:
'difficult to know which aspects of the structural assumption are country-specific and which are more fundamental to the theory

# Borrowed theories: South Africa

Low performance in primary ITE; limited difference in outcomes between 1st and 4th years: *'need for student teachers to revisit primary school mathematics in a way that provides a deep understanding of key mathematical concepts'* (Bowie et al, 2019)

Used MCK (Ball et al) and cognitive demand levels (Stein et al), linked with SA curriculum to develop assessment and analyse outcomes

79% of grade 6 mathematics teachers showed content knowledge levels below the grade 6/7 band. Problems at the level of 'Common Content Knowledge' (Venkat & Spaull, 2015)

Regional SACMEQ assessments overlaid with Ball et al's MCK categories

- More need for basic content knowledge flagged in South African assessment-oriented studies
- In Ireland, agreement on some need for basic facts and definitions focus, but also, need to extend beyond procedural working into problem-solving

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So – common use of MKT theory across highly disparate settings

SA: Tending not to use MKT instruments and measures
Focus on evaluating teachers' knowledge (Mosvold, 2022)

What about studies of teaching?

# Do studies of the quality of teaching take us into greater understanding?

## Ireland – few studies of mathematics teaching

Use of survey data – e.g. from TIMSS

- largely teacher-led instruction (Eivers, Delaney, & Close, 2014)
- Textbook-based working in 91.5% of 6th classes on most/all days (Shiel et al, 2016)

Survey findings overlaid with MER theories in research reviews guiding curricular reform:

> ‘continued emphasis on lower-order thinking skills and mathematical procedures’ (Dooley et al, 2014: 19)

# New primary curriculum in Ireland

- Recent moves into less detailed curricular prescription. Greater emphasis on teacher professionalism and need to respond to individual pupil needs: general pedagogy
- References a range of global travelling theories re mathematics: need for problem-solving and reasoning, lesson study, strands of mathematical proficiency, cognitive demand and challenge, modelling and curricular integration

# South Africa: Earlier surveys of teaching studies

Comparative study of primary maths teaching across SA and Botswana

- *'teach .. content in a rote-routine way, requiring students to remember number facts and algorithms rather than explaining how these methods work*
- *more concerned about "showing all the steps" than .. having the students think what it means to solve an equation and give an explanation for their answer.*
- *only teaching topics that require the lower levels of cognitive demand and lower grade topics* (Sapire & Sorto, 2012: 441)

MKT

Mathematical Proficiency

Cognitive demand levels

Mathematical Proficiency

Ally & Christiansen (2013):

*'opportunities to develop MP .. are limited both in range and in quality. …concepts and procedures were stated, not explored .. the 'glue' of mathematical proficiency – adaptive reasoning – was given little room to develop'* (p.116)

# South Africa: Earlier survey-oriented studies

Sapire & Sorto's (2012) comparative study of primary maths teaching across SA and Botswana

- *'teach .. content in ...*
  *... work*
  *... to be more concerned about "showing ... steps" than about having the students think what it means to solve an equation and give an explanation for their answer.(p440)*
- *teachers are only teaching the topics that require the lower levels of cognitive demand (p441)*

Rote .. Procedural .. Limited problem-solving and reasoning – commonly stated across both systems

MKT

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- '*teach .. content i… stud…*
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Is this inevitable when similar combinations of global travelling theories are being drawn on?

MKT

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MKT

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And is this drawing on well-known theories connected to the pressures for publication by template?

*…are limited both in range and in …concepts and procedures were stated, not explored .. the 'glue' of mathematical proficiency – adaptive reasoning – was given little room to develop'* (p.116)

# Back to Niss & template recruitment of theory

*If one of the main purposes of conducting mathematics education research is to pave the way for creating better theories that will eventually expand and consolidate our understand of the teaching and learning of mathematics in all its manifestations and under all its boundary conditions, which I believe it is, it is next to insane to force mathematics education research to be locked up in extant theoretical frameworks.' (Niss, 2018: 46)*

## Need for theories and frameworks that help us to study MKT in ways that offer useful insights into the ground

Trajectories of research have led to clearer foci on the problems where theory-building is needed

ITE – KAM Project: Ireland

↓

**Reasoning and problem-solving**
**key problem to understand**

TEDS-M and dispositions survey
Working sheets
Task-based interviews

Wits Maths Connect-Primary Project: South Africa and other intervention studies

↓

**Coherence in teachers' working with mathematics**
**key problem to understand**

Trajectory of studies, classroom and assessment based, with positive learning outcomes leading to scaled up interventions

# Ireland: ITE-KAM study

- Used TEDS-M released items, conceptually oriented, mostly focused on primary middle grades and 1-2 grades above; includes mathematical and pedagogical items. Dispositions and beliefs survey included.
- Baseline B Ed cohort sample (n=411 Primary PSTs)
- 57.6% mean at start of course

More in-depth data came from working sheets and task-based interviews

# Working sheets

**Q3**


$P$ and $Q$ represent two fractions on the number line above.

$$P \times Q = N$$

Which of the following shows the location of $N$ on the number line?

*Check one box*

A.


☐

B.


☐

C.


☐

D.


☐

B Ed facility: 53.5% (n=411)

A: 106. B: 61 C: 23. **D: 220**

Sizeable proportions appear to hold the 'multiplication makes larger' misconception

Rough work sheets


$P \times Q = N$

$P > 1, \; Q > 1$

**Inappropriate example**

**Repeated fraction example**

$0.3 \times 0.3$

$0.3 \times 0.3 = 0.9$

$P = \frac{2}{3} = \frac{4}{6}$

$Q = 4/6$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{4}{6} \times \frac{4}{6} = \frac{16}{36} = \frac{8}{18} = \frac{4}{9}$

$\frac{1}{3} \times \frac{2}{3}$

$0.4 \times 0.6$

$\frac{4}{10} \times \frac{6}{10} = \frac{40}{60} = \frac{4}{6} = \frac{20}{30} = 2/3$

Let $P = 1/3$ and $Q = 3/5$

$\frac{1}{3} \times \frac{3}{5} = \frac{3}{15} < \frac{1}{5}$

→ less than both original values.

**Distinct fraction example**


$P \times Q = N$

$P > 1$ , $Q > 1$

**Inappropriate example**

**Repeated fraction example**

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NA

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COR

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**Distinct fraction example**

let $P = 1/3$ and $Q = 3/5$

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| No working | Inapprop example | Repeated fraction example | | | Distinct fraction example | | |
|---|---|---|---|---|---|---|---|
| | | NA | Inc | Corr | NA | Inc | Corr |
| 254 | 9 | 3 | 3 | 11 | 18 | 44 | 57 |

While most showed no working, the majority that showed working were able to select an appropriate distinct fraction example – so making, at least initial sense of the question.

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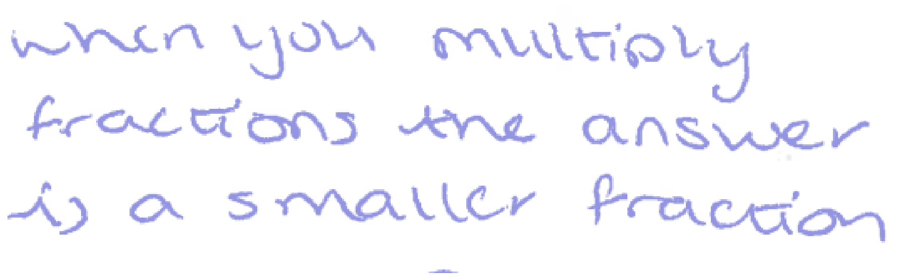
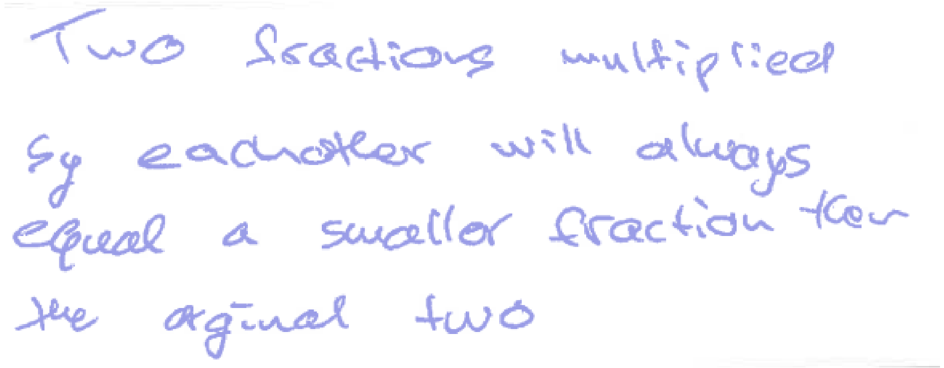



Statements are true for the fractions in the image, but not generally true for all fractions - but language suggests generality.

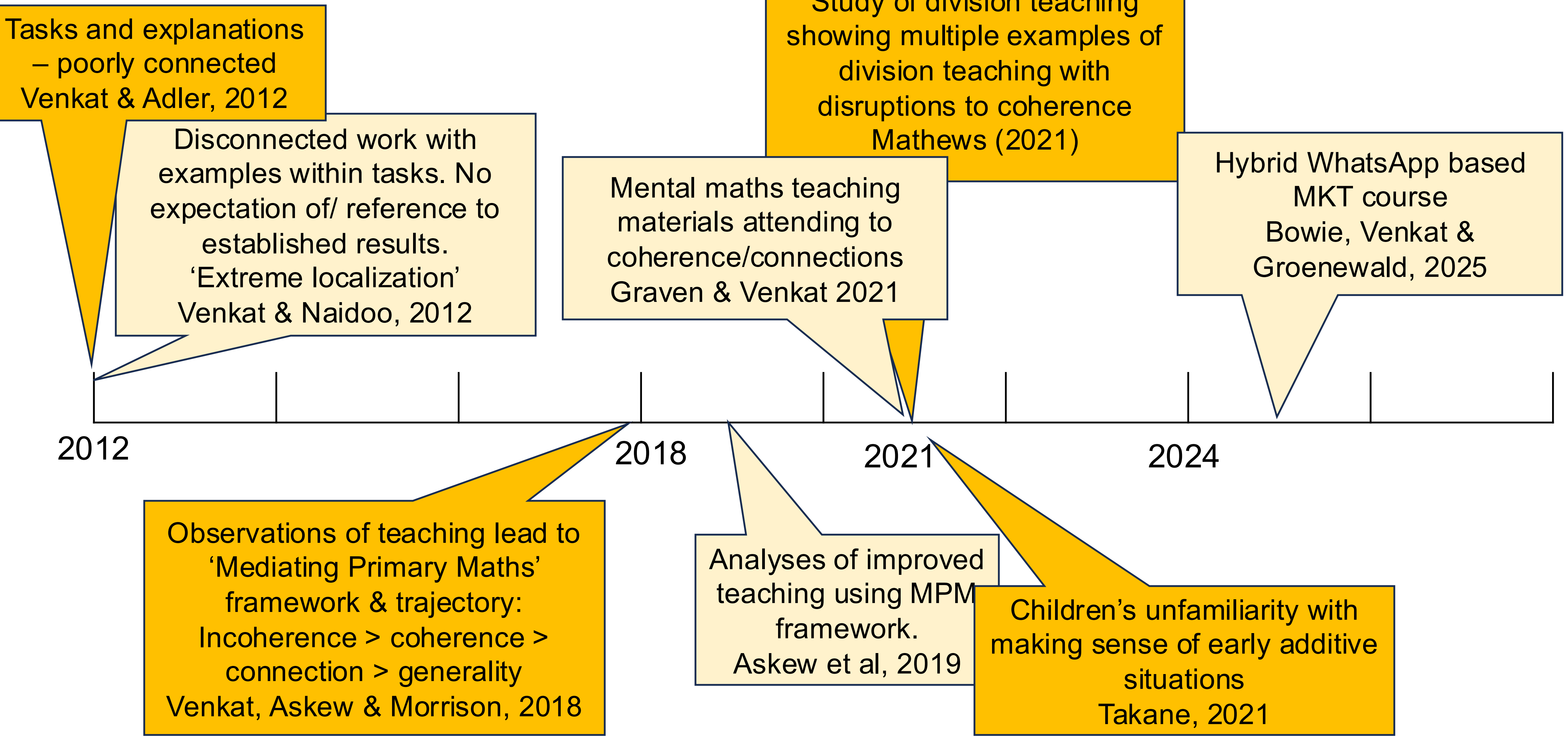
Working from data to theory, not theory to data

Probing teachers' reasoning underlying answers, using working sheets
- Drawing on writing on exemplifying, representing and reasoning

Interviews showed similar willingness to exemplify, represent, and – awareness of when PST did not have an explanation for their steps.

So confirming, but also extending earlier findings.

# South African MKT trajectory towards **coherence** as analytically and practically important



# South African MKT trajectory towards **coherence** as analytically and practically important

Tasks and explanations – poorly connected
Venkat & Adler, 2012

Disconnected work with examples within tasks. No expectation of/ reference to established results. ‘Extreme localization’
Venkat & Naidoo, 2012

Theory, framework and intervention building, studying and testing

Mathews (2021)

Mental maths teaching materials attending to coherence/connections
Graven & Venkat 2021

Hybrid WhatsApp based MKT course
Bowie, Venkat & Groenewald, 2025

2012

2018

2021

2024

Observations of teaching lead to ‘Mediating Primary Maths’ framework & trajectory: Incoherence > coherence > connection > generality
Venkat, Askew & Morrison, 2018

Analyses of improved teaching using MPM framework.
Askew et al, 2019

Children’s unfamiliarity with making sense of early additive situations
Takane, 2021

# South Africa: Current work
# Hybrid WhatsApp based MKT course

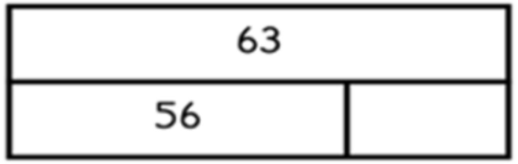
Focused on early grades number concepts – maths and pedagogy, for teachers working in after-school clubs with G4/5 learners on G2/3 concepts

- Written pre- and post-assessment, including comments on different explanations for _ - 23 = 14
- Interim upload of ‘talking hands’ videos
- Interviews with a teacher sample

- Exploring baselines of teachers’ use of PPW and NL models

Tick ALL the number sentences that match the situation represented by the PPW (PPW) diagram:

| |
|---|
| A 63 - ___ = 56 |
| B 56 + ___ = 63 |
| C 56 - 7 = ___ |
| D 7 + 56 = 63 |
| E 63 = 56 + ___ |




| % of teachers who chose: | A, B and E only | A, B, D and E only | | C | D |
|---|---|---|---|---|---|
| Pre-test | 5% | 33% | | 19% | 79% |
| Post test | 2% | 40% | | 7% | 63% |

High proportions including D in their selections indicate teachers who know the missing number for themselves, but have limited understanding of how to set up ***models of*** problem situations

# Comments on 4 explanations for _ - 23 = 14

- The favoured explanation for this task involved telling children to ‘just add’: 64/80 teachers said they liked this explanation
- 62/80 liked the number line explanation that used a model of the problem with reversing to calculate missing number
- 39/80 liked an explanation that involved starting with 37 on a NL and jumping back 23 to get 14.

Interview data, a teacher placed the given numbers in correct positions in a PPW diagram.

When asked to explain why the numbers were put in these positions:

*‘So that they can find the missing number’*

# Making theory practical

- Knowledge of key theories is important.
- Reference to well-known theories is likely necessary in writing for publication
- But over-leaning on particular theories tends to homogenise the stories we can tell in ways that may not be practically useful.
- Broadening the theoretical base is necessary for theory to be practical
- Broadening the theoretical base, in ways that centre on the learning and teaching of maths, is likely the only way to address issues related to majority marginalization

# Thank you!

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