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Theme
Mathematics for a Connected World:
Integration, Challenges and Possibilities

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Foreword

It is with great pleasure that we present the proceedings from the Tenth Conference on Research in Mathematics Education in Ireland (MEI 10), which took place in Dublin City University during Maths Week in October 2025. Established in 2005, the MEI conference serves to promote and facilitate discussion on mathematics teaching and learning across the continuum from early childhood education through third level education and beyond. At this milestone of the 10th MEI conference, we acknowledge the significant contribution of the original founding members of MEI, for establishing, maintaining and growing this conference - an important contribution to the growth and quality of mathematics education research in Ireland. We are delighted that MEI has become recognised as a significant biennial event in the CASTeL calendar.

The conference theme, *Mathematics for a Connected World: Integration, Challenges, and Possibilities*, was selected as there is scope for mathematics education to play an important role in responding to contemporary ecological and societal issues. MEI 10 offered opportunities to respond to evolving educational and policy landscapes and to reflect on progress, on ‘sticky problems’ and possible ways forward in a changing, and complex, world. The conference provided a space to consider both disciplinary and interdisciplinary teaching and learning, in addition to the role of mathematics across subjects and real-world contexts.

The conference involved three keynote presentations, three symposia, one workshop, and 40 paper and poster presentations. The keynote presentations were delivered by Dr Thérèse Dooley, Professor Geoff Wake and Dr Mairéad Hourigan. While Professor Wake calls for attention to architecture of learning for rehumanising mathematics, Dr Hourigan argues for a repositioning of mathematics in integrated STEM learning. In addition, a presentation by Emeritus Associate Professor Dr Thérèse Dooley and MEI founding member reflected on the growing mathematics education community in Ireland and recurring and evolving themes evident in MEI papers across the last two decades.

In total, these proceedings contain 52 papers from over 100 national and international authors. Papers underwent a peer-review process and were edited and refined by authors, before being accepted for publication in this electronic conference proceedings. The papers document research at all levels of education - early childhood education, primary and secondary schooling, third-level education, in addition to teacher education. These papers are testament to a vibrant, committed research community.

We acknowledge the significant work completed by all of the Organising Committee and Scientific Committee, and others within DCU, who assisted with the preparation for and facilitation of MEI 10. We are grateful to the invited speakers who contributed so generously to the conference and to all of the authors who responded to the call, submitted and reviewed papers, and participated in the conference.

We look forward to meeting with you all again for MEI 11 in 2027!

Siún Nic Mhuirí, Lorraine Harbison and Miriam Ryan

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Designing architectures for learning: rehumanising mathematics

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This article explores important issues in the design of architectures for learning (Wenger, 1998). I draw on our research team's evolutionary design of a comprehensive approach to learning for teachers and students that was found to improve learning for all students and with increased gains for those from economically disadvantaged backgrounds. Our work is informed by sociocultural theories of learning that put human engagement in mathematics at the centre and consider mathematical thinking at the boundaries between different social practices.

My concern here is how we might reconsider mathematics as having meaning and utility for learners and problematise what it means to teach in such a curriculum (re)design. In doing so I exemplify an approach that has emerged from our work over the last 15 years that has paid attention to making mathematical activity visible in ways that allows students and teachers to engage in dialogic discussion of their own and each other's learning and understanding of mathematics.

Keywords: Socio-cultural theories of learning, design research, dialogic learning

Introduction

This article draws on a substantial body of work, primarily design-based research, over the last ten to fifteen years, that has focused on a significant problem particular to high-stakes education in England. In essence this problem is that of students who, at the end of compulsory schooling at age sixteen, have failed to reach a threshold of competence in mathematics in the long-established GCSE examinations. After eleven years of schooling their mathematical competence, confidence, and relationship with the subject are extremely poor (Boli, 2020; Boli & Golding, 2024; Crisp et al., 2023). If these students remain in state education beyond the age of sixteen, they are obliged to follow courses that lead to a resit of the examination that to them, has just demonstrated their failure. Students on these courses, in general, are highly demotivated and often go through what seem endless cycles of study, (resit) examination, and failure. I will use our research in this area to provide an answer to the bigger, and more general, question: how do we design for learning in a way that re-humanises mathematics for our students?

Learning and architectures for learning

Initially, let me explain the two phrases I have used in the title of this article, 'architectures for learning' and 'rehumanising mathematics'. These have been fundamental in informing our work, particularly as they prompt us to question what we mean by the often-unquestioned terms 'learning' and indeed 'mathematics' or at least as practised in school. The mathematics education research literature, and the wider 'grey' policy literature around mathematics in school, curriculum, pedagogy and learning proceeds as though these are fixed, stable and uncontested. This, lack of critical questioning, in our view is problematic, and indeed – as I hope to illustrate – lies at the heart of the problems that our work seeks to address.

First, I turn to the second clause of my title: that is, 'rehumanising mathematics'. My use of this term has been informed by the work of the designer Thomas Heatherwick and his treatise *Humanise. A maker's guide to building our world*. (Heatherwick, 2023). In this work he puts forward the proposal that we should adopt a more human-centred approach to design,

in particular in the case of the built world, to ensure that architecture combats the soulless, depressing, and unsustainable nature of much modern architecture (as he characterises it). Here I put forward, a parallel view that in our design for learning mathematics we need to (re-) connect mathematics with learners to allow them to truly engage with mathematics in ways that are meaningful to them. Heatherwick, in attempts to reach out to the public and make his argument (see, <https://heatherwick.com/studio/news/humanise-campaign-launched-to-stop-the-spread-of-boring-soulless-buildings/>), talks of ‘boring’ buildings and suggests that good design needs to ensure that “a building should be able to hold your attention for the time it takes to pass by it.’ In order to achieve this he argues that a building must be interesting from three distances: (1) City distance of over 40 metres, (2) Street distance of around 20 metres, and (3) Door distance of about two metres. To parallel this, I suggest that mathematics as a subject domain in our schools needs to ensure that it captures the interest of students (and teachers as well as others) at three levels of separation. These are:

(1) as a school subject. Mathematics needs to be seen to have clear *use value* beyond the *exchange value* of the subject (Williams, 2011). It needs to be visible as a potentially empowering subject that makes connections with our lives, (both present and imagined futures) and as a domain of study with use, inherent interest, and indeed beauty. Current curriculum statements often make claims in such directions, with carefully crafted words, that do not often reflect the reality of classroom implementation. Such words need to lead to a reality such that, from a distance, for students, teachers and the public more widely, mathematics as a domain of study should signal its dynamic potential in the many different aspects of our lives that it can permeate.

(2) as a coherent course of study. Mathematics, for too many students, is seen as a collection of disconnected rules and procedures (Skemp, 1976) that need to be mastered¹ As our research determined, the students, whose (lack of) mathematics learning we were addressing, saw no underlying structure in the mathematics that they were being asked to revisit. (A simple example of this I often refer to, is that the majority of them had little idea of the different structures of odd and even numbers). In our design of mathematics, we need to signal that there *is* some overarching structure that is more than the simple clustering of topics with implicit structures that we, as teachers, understand but remain mysterious to our students. Rather, we need to ensure that students understand how their current learning connects to where they have come from mathematically, where they are going to, and how it is structured (much in the way that in the gaming world, adventure games ensure that players have a sense at all times of what they are doing and why, as well as understanding how that will lead them to ultimate success).

(3) in classroom activity. The argument I develop here is to rethink mathematical classroom activity as explorative: so that mathematics classrooms are more like workshops in which we engage in doing mathematics and being mathematical. This contrasts with what turns out to be a common experience for many students around the world (Hiebert et al. 2003; Mullis et al. 2016). Typically, mathematics lessons involve a period of teacher direct instruction that includes demonstration of how to apply newly introduced mathematical principles followed by a period during which students often work individually, practising application of the new rules and procedures. In this article I argue for a social theory of learning, more of which below, so that mathematics learning is considered a collective enterprise in which learners engage collaboratively in mathematical activity.

¹ I use the word ‘mastered’ here deliberately as this has political significance in England.

Here I note the parallels that may be drawn with Burkhardt and Pead's (2020) categorisation of design at three levels:

Strategic design that at a system level designs structures that can facilitate implementation of learning programmes.

Tactical design that designs approaches and the mechanisms that will support direct interaction with learners

Technical design that focuses on *the materials* that help teachers work directly with students and their learning.

In other words, the architectures, the structures, and the close to practice details, that not only facilitate learning, but provide interest and give meaning to what it means to engage with mathematics.

This brings me to the phrase "architectures for learning", which I appropriate from the work of Etienne Wenger, in particular from his seminal text, *Communities of Practice. Learning, Meaning, and Identity* (1998). Important in this regard is his theory of learning that addresses questions of what it means to learn considering, as he does, that this is at the heart of any *community of practice*.

Etienne Wenger's conceptualisation of "architectures for learning" extends his social theory of learning into the realm of intentional design, particularly for organizations and education. He argues that learning cannot be directly designed, but it can be designed **for**, meaning that it can be facilitated or frustrated by the social infrastructures that we design and put in place. This perspective shifts the focus from merely designing curricula to designing environments that foster inherent human learning through social participation. Wenger positions a conceptual architecture for learning as a tool for designers that outlines general questions, choices, trade-offs, and the basic components or facilities that need to be in place to support learning. He identifies four fundamental dualities that constitute the basic dimensions of designing for learning, each representing an inherent tension that designers must address:

Participation and Reification: These are two complementary avenues for influencing the future of a practice and consequently the trajectories of participants. Our designs must balance the creation of artefacts (reification) e.g., curriculum structures, tools such as lesson plans, tasks and so on), with opportunities for active involvement and meaning making (participation). Excessive focus on reification without participation can lead to "meaninglessness" or "brittle" understanding.

The Designed and the Emergent: Design creates planned structures and intentions, but learning in practice is inherently emergent and cannot be fully controlled. The challenge is to include and leverage this emergent quality, balancing prescription with the inventiveness of practice.²

The Local and the Global: No practice is entirely global; engagement is always local. Design for learning must connect these localities and their diverse "knowledgeabilities" to broader contexts without assuming one practice subsumes another. This leads to the paradox that "No community can fully design the learning of another," yet "No community can fully design its own learning" (Wenger, 1998, p.234).²

² These two dualities, the designed and the emergent, and the local and the global, provided significant tension in our research that sought to measure the effectiveness of our planned intervention. We somehow, had to ensure some compliance with common ways of teaching and learning, whilst also allowing for variation at a local level so that learners local participatory needs were catered for.

Identification and Negotiability: Design inherently addresses issues of power in ways that shape communities and gives rise to meaning making of both individuals and collectives. It provides a vision of identity, influencing what people identify with, and their ability to shape shared meanings (negotiability). As designers we must, therefore, consider how to promote and distribute ownership of meaning, as this profoundly impacts learning. Teachers as end-users of our designed products need to be particularly sensitive to the emerging collective meaning making and developing identities of learners.

Wenger further suggests that good learning environments should be built around three key ways in which collectives as a community of practice connect: engagement, imagination, and alignment. Engagement requires that as designers we provide opportunities and tools that help people take part, work together, and build on past experiences. Imagination requires that we help learners think beyond their immediate tasks so that they can see the bigger picture, reflect on what they do, and explore new ideas or possibilities. Alignment means that we provide an environment that ensures organisation in ways that everyone's efforts are coordinated and so that they can work toward shared goals, with clear roles and responsibilities. Together, these three elements help create strong, ongoing learning communities.

Wenger emphasises that designing for learning is about facilitating an organic, social process with "learning as a living experience of negotiating meaning," (ibid, p. 229) not merely on reified forms. The emergent communities of practice are based on shared activity, experiences and negotiation. Our designs must balance the dualities explicated here, avoiding excessive focus on one pole (e.g., too much reification without participation, or too much local engagement without broader alignment). Fundamentally, Wenger suggests we need to move beyond a narrow view of "knowledge acquisition" and consider the lifelong, identity-transforming nature of learning across diverse contexts. In essence, Wenger's notion of architectures for learning implicitly advocates for a shift from a mechanistic, transmission-focused approach to a more organic, socially embedded, and meaning-centred one. As designers of programmes for learning this implies that we should seek to create environments where the inherent human drive to learn, make meaning, and form identity can flourish through active participation, reflective imagination, and purposeful alignment within and across communities. Wenger suggests, in a way that I recognise in Heatherwick's work, that designing for learning needs to cultivate a vibrant, adaptable ecosystem in which new relationships (identities) with knowledge are forged in ways that are both predictable and emergent. In other words, we need to design for learning in a way that rehumanises mathematics.

Having offered some brief insights into the thinking and ideological positions of Heatherwick and Wenger, I now turn to how Heatherwick's critique of modern architecture (and of architects themselves) in my analysis might parallel the current state of mathematics education in our schools. This comparison is especially relevant when considering the ongoing challenge we face with students who leave school having struggled or failed in mathematics: a group I refer to as the *mathematically disadvantaged*. Unfortunately, these students are also, disproportionately among the *economically disadvantaged*. In our own research (to be discussed later), approximately half of the students retaking their final mathematics examination came from families identified as economically disadvantaged³ (Wake et al, 2023). I argue that for these students, we must *rehumanise mathematics*, to approach it in ways that align with Wenger's conception of learning as a socially situated

³ In general approximately 30% of teenagers come from economically disadvantaged families, whereas of resit GCSE maths students this rises to almost 50%.

process of meaning-making. Through engagement in, and contribution to, communities of practice, learners not only acquire knowledge but also shape their identities and develop trajectories of *becoming* within the sociocultural practices that sustain those communities.

The design

In this section I aim to illustrate how mathematics learning was designed *for*, in line with principles of the social theory of learning outlined above.

The latest iteration of our designed programme became known as [Mastering Maths](#) and in addition to focusing on learning of mathematics as a socio-cultural practice was further informed by ideas of dialogic learning (Alexander, 2008; Mercer and Littleton, 2007). Theoretical ideas and research into dialogic learning views education as a collaborative process in which knowledge is co-constructed through sustained, reflective dialogue among learners and teachers, fostering critical thinking and mutual understanding. It emphasises egalitarian meaning making through talk in ways that democratise the classroom and prioritise and deepen conceptual understanding. Design of lessons and tasks in the Mastering Maths programme were informed by approaches that tried to make real such learning communities and ways of learning in classrooms.

It is important to highlight that although here I will focus on the design of classroom materials, that is at the closest to practice level of design (at the technical level in Burkhardt's categorisation), the Mastering Maths programme, is notable for its complete and comprehensive approach providing structures for teacher learning in communities of practice based on a modified form of lesson study (Wake and Joubert, 2023a, 2023b).

As well as adhering to the principle of developing classrooms as socially inclusive communities of practice engaging with mathematics, the whole programme was based on a clearly explicated set of five Key Principles (KP) that informed every aspect of the work providing a golden thread running through all parts of the programme. These principles were developed together with the (General Further Education (GFE)) college sector, *for* the sector, addressing the priorities of those who work in GFE colleges and teach the GCSE Maths resit students. These principles inform issues relating to how we focus on the underlying structures of the mathematics at issue as well as how we wish to see social behaviours in classrooms (with implications for appropriate pedagogies).

KP 1: Develop an understanding of mathematical structure

KP 2: Value and build on students' prior learning

KP 3: Prioritise curriculum coherence and connections

KP 4: Develop both understanding and fluency in mathematics

KP 5: Develop a collaborative culture in which everyone believes everyone can succeed

Among the designed (reified) artefacts that support the classroom collective (and those that support teacher lesson study communities) I draw attention to:

[Twelve carefully designed lessons \(lesson plans\) and lesson study support](#) providing 12 detailed lessons plans that include some commentary on design intentions.

[Lesson videos](#): edited video clips for each lesson to provide a vision of classroom practice and insight to key ideas and intentions of implementation of teaching.

[Brief overview of lesson design features](#), a two-page summary of the lesson plan, which can be used by teachers in the classroom

[Key principles and research questions](#) for each lesson (to guide lesson studies)

[Lesson materials](#) (lesson plans, powerpoint slide-decks, student materials and so on)

[Self-study materials](#) that can be used to introduce the design of each lesson to teachers

These designed resources are what allow for, and support, the programme's architecture for learning to be implemented and emerge in the reality of the classroom. Of course, teacher behaviours in the classroom, are essential in mediating the designed for community of practice, but given the restrictions of space here I will only refer to the design of the teacher development programme if, and when, necessary

To illustrate how all of the design principles referred to throughout this paper thus far might manifest in classroom activity considered as a community of practice I take just one lesson that focuses on the learning of fractions and ratios. This tackles the problem that students often get confused with the different ways in which part-part and part-whole relationships can be represented. Students, working in pairs or threes, are asked to start the activity by matching the fractions to the ratio statements on a printed worksheet (Figure 1). These describe how a baguette may be shared in different ways between two people, Amy and Bikram. As you might imagine a significant proportion of students match the ratio 1:2 to the fraction $\frac{1}{2}$, the ratio 2:3 to $\frac{2}{3}$ and so on. After students have worked for a while and many expose this type of misconception the diagram cards (Figure 2) are introduced by the teacher who points out that there needs to be consistency in each row of the template. This results in students, by necessity, having to engage in rethinking their initial response.

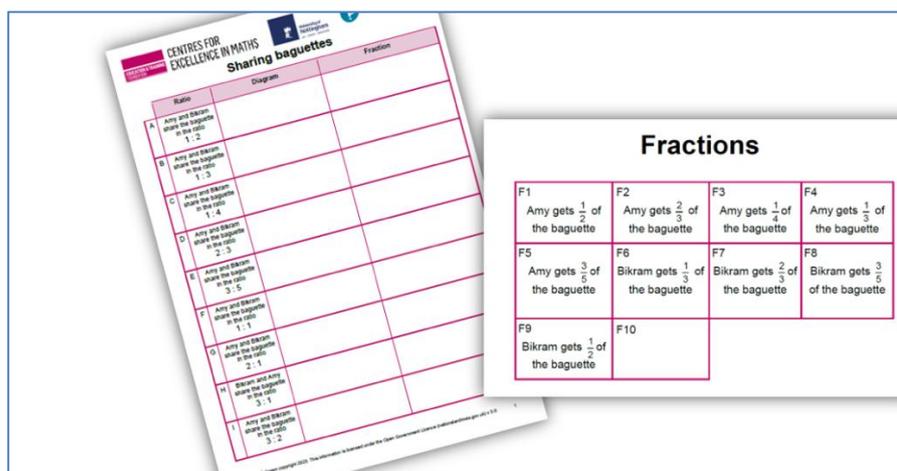


Figure 1. Initial stages of an activity addressing the structure of ratio and fractions

The diagrams (Figure 2) provide insight into mathematical structure (Key Principle 1) and their use encourages students to see links between mathematical concepts, rather than viewing ideas of fractions and ratio as entirely separate content. This is important in supporting a coherent and connected curriculum (Key Principle 3) and is essential in the FE sector, where there is limited curriculum time. The lesson structure follows a brief input by the teacher who introduces the context and sets the students working for the next 45 minutes or so, working with the fraction cards, the diagrams and finally the word descriptions (Figure 2). Note, that we explicitly point out to teachers our designed intention that they make far less direct contribution to the lesson than they usually do. There follows a whole-class teacher-led *discussion* of the mathematics which in turn is followed by a period of students working on a practice examination question(s). Significantly the lesson, as do all Mastering Maths lessons, has substantial periods where students collaboratively explore the mathematics. Rules are introduced about turn-taking and encouraging students to build on each other's thinking.

These designed architectures for learning aim to address Wenger’s dualities outlined above as well as taking a particular stance on classroom learning communities that prioritise student engagement above all: we seek to engage students with mathematical structures that a range of carefully selected models/representations can provide.

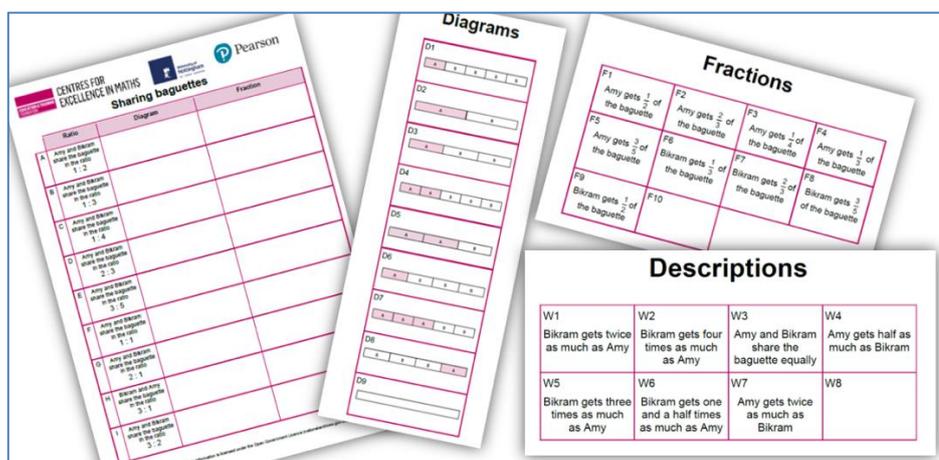


Figure 2. The extended task that introduces connecting representations and word descriptions

Testing the Mastering Maths design for learning

Over the period 2018 – 2023, the UK Government funded the [Centres for Excellence in Maths programme](#) (CfEM). As part of the Delivery Group, the University of Nottingham mathematics education research team provided research expertise across the programme that initially explored four different pedagogic approaches to teaching GCSE resit Maths: a Mastery approach, contextualisation of maths, use of technology and, motivating and engaging students. Within what was seen as a pilot trial, the team replicated the approach used in their earlier work, [Maths for Life](#). We developed a set of five lessons for the first three themes and another of the CfEM partners developed a set of short activities designed to motivate and engage students.

The pilot trials ended abruptly as the first Covid lockdown in the UK was put in place and the research was put on hold for some months. The learning from the three pilot trials informed the design of the CfEM large-scale randomised controlled efficacy trial, for which design work began in January 2021, beginning with five *teaching for Mastery* lessons that ultimately formed the basis for Mastering Maths.

The CfEM efficacy trial began in October 2021, with 147 teachers of GCSE mathematics in Further Education colleges taking part. It was a three-armed trial involving control, partial intervention and full intervention groups. The teachers in the partial intervention attended two half-days of professional development and were given seven mastery lessons and required to teach five of them to their GCSE classes. The teachers in the full intervention group in addition to those in the partial intervention also participated in five lesson study cluster group meetings, led by a Lead Teacher. The students taught by teachers in both intervention arms and control arm had their GCSE scores collected in Summer 2022 and analysed.

The three-armed randomised controlled trial detected improved outcomes equivalent to one month of additional learning for students taught by teachers who had taken part in the most intensive intervention programme (the intervention programme that involved lesson study). This increased to two months of additional learning for the most deprived students (as

measured by the most commonly used measure of deprivation in the UK). These results contrast with those for the partial intervention arm of the trial. For students taught by the partial intervention teachers there was no discernible gain in learning. This leads to the conclusion that it is the collaborative nature of teacher learning that is supported by lesson study that makes the difference to both teachers, and ultimately, their students. This appears to validate the theoretical underpinnings of the entire design of the programme, that is that of a socio-cultural and collaborative approach to learning for both teachers and students. A full account of the research and a report of the study, along with all outcomes is given [here](#) (Wake et. al, 2024)

A remaining question is “why does the Mastering Maths intervention make most difference for students from the most disadvantaged backgrounds? The design team hypothesises that this lies in an overall design that prioritises giving students time to talk with others (both students and teachers) about their emerging understanding of mathematics. The team suggests that previously these students have potentially had little opportunity to talk with each other about any of their mathematics and have been disengaged in ways that have been detrimental to their learning. This outcome is one that validates the careful design approach that ensured close attention was paid to supporting dialogic learning with teachers facilitating much peer-to-peer discussion of mathematical ideas and concepts.

Reflecting on designing architectures for learning

In an article of this scale and scope it is only possible to signal meaningful issues at a general and necessarily high level (that is, remote from the technical aspects of design). The technical aspects of design are necessarily detailed and are in alignment with the contextual rules, both explicit and implicit, that allow the design to become operational in our classrooms. This is much like architects as designers: they are required to comply with many explicit rules, building regulations in relation to health and safety, accessibility, as well as implicit rules, client (and public) expectations and so on. The genius of expert design is that it meets all such requirements whilst still ensuring that it aligns with the designer’s intentions: in our case that it ensures as far as is possible that mathematics learning becomes a socially collaborative experience that facilitates the voice of learners and their deep engagement with, and struggle to understand mathematical concepts.

First, as a designer for learning it is important to have a clear vision of what you expect learning to look like in the classroom: this necessitates having a theory of learning that can guide your design. Not only do you need to be able articulate what you mean by learning, it is important that this can be achieved by students in classrooms. Equally, as in the work of any designer / architect, we need to be as fully cognisant as we can be of the context for which we are designing. In my experience, contrary to what might be thought, design constraints are essential as we strive for good design. It is careful exploration and understanding of the context for which we are designing that allows this. This takes time and opportunities to try things out in (relatively rapid) cycles of research trials and feedback that allow us to ensure our designs will work in the day-to-day realities of courses of study and classrooms. Perhaps the overriding constraint in educational design is the extent to which innovative designs can deviate from current and expected practice. There are inherently strong conservative forces that mitigate against radical design, particularly at a classroom level. Mastering Maths tapped into a climate of dissatisfaction with regards to the outcomes of GCSE resit students, and a consequent willingness of teachers to take some risks (results could hardly get much worse), and our research trial legitimised teachers to try the new proposed approaches. In this way we were, perhaps more able to be perhaps more radical than would otherwise have been the case.

In pursuit of a sociocultural approach to learning this depends, as Wenger pointed out, on consideration of engagement, imagination and alignment. This requires us to consider how our classroom communities will engage with mathematics (by community, here, we need to consider carefully roles and behaviours of teachers *and* learners). As we pointed out briefly, students' behaviours when working on mathematics tasks were raised in classroom conversation in terms of expectations of who should contribute and when, and indeed that all contributions were to be valued and built upon. In addition to designing for mathematical development in ways that provide insight into mathematical structures and concepts, we also need to consider how our reified artefacts can mediate not only engagement with mathematical ideas to support learning trajectories, but also how we design for collaborative engagement and experiences.

Further to this, it is important, to consider what we control in our design and what and how we design for variation. Fundamentally, we seek both uniformity and diversity. Uniformity is to some degree, essential in seeking to develop a learning collective. Especially when working towards external assessment, we are guided by tight curriculum specification: we need to design in ways that will carry us to the final goal; however, we need to be sensitive to the diversity of our learners. In Mastering Maths, our teacher learning process of cycles of lesson study allowed teachers opportunities to engage with the 'designed' and also work within a 'designed envelope of permissible variation'. Teacher engagement in understanding how to strike a balance between the 'global and emergent' was facilitated in our study by the cycles of lesson study that were part of the programme and it was potentially these that led to the most effective implementation and the resulting greatest impact.

Finally, I would like to comment that our research has heartened me in that it was found to make a positive difference to learners' scores, especially for learners from the most disadvantaged backgrounds. It has been fortunate that we have been able to sustain a stream of funding over many years to work towards such careful and comprehensive design that has enabled us to do this. Our Mastering Maths study suggests that we *can* (re-)humanise mathematics for students, I would encourage others to take up the challenge.

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From last place to first: Repositioning the M in integrated STEM.

Using contemporary research as a guide to navigate curriculum change

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Agreement exists that all learners need access to quality STEM education in an ever-changing world. There is a call for more integrated approaches to STEM learning that reflect the nature of society's 'wicked problems'. Acknowledging Irish STEM education policy's ambitious vision for integrated STEM education for over a decade, its capacity to affect change was limited in the absence of compatible curriculum guidance. However, as we move closer to the embedding of a formal STEM curriculum at primary level that champions attention to STEM disciplines alongside integrated STEM, it is timely to pause and consider the role of mathematics within integrated STEM. This paper discusses the body of research focused on the challenges and possibilities in relation to meaningful mathematics integration. The literature supports a retrospective review of various cases, units of STEM inquiry, developed alongside pre-service and practicing Irish primary teachers, examining the role assumed by mathematics, the integrity of mathematics included, alongside potential for enhancement. To conclude, key learnings from contemporary research inform discussions regarding critical features of quality STEM teacher education at this crucial juncture.

Keywords: Mathematics education, curriculum change, integrated STEM education, pre-service primary teachers, practicing primary teachers

Introduction: The evolution of integrated STEM education

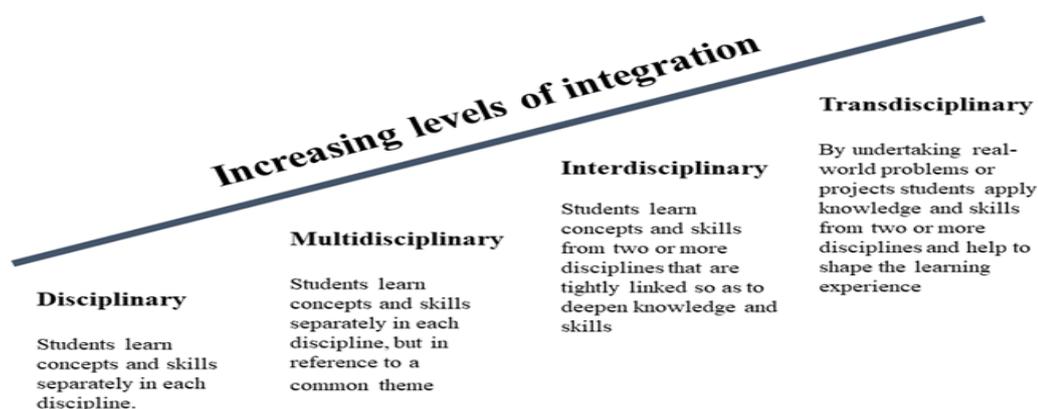
STEM [Science, Technology, Engineering and Mathematics] education has been in the international limelight for decades. While the initial rationale was solely economic, firmly focused on increasing STEM careers to service the STEM pipeline, over time, the need to develop a STEM literate society, where all citizens can critically engage with ever increasing amounts of information and data has become the impetus for quality STEM education for all (Falloon et al., 2020; Honey et al., 2014; Maass et al., 2019; Murphy et al., 2024). This is reinforced by the reality of "a rapidly changing world" where "many students will likely have future jobs not yet in existence- or even imagined" (Hilton & Hilton, 2024, p. 259). While traditionally experiences of STEM in school curricula were of discrete or 'siloed' STEM disciplines, there have been increasing calls for a more integrated approach to STEM education for some time given that an interdisciplinary approach closely aligns with the use of STEM disciplines to solve real-world issues (Breiner et al., 2012; English, 2016; Murphy et al., 2024; STEM Task Force, 2014; Tytler et al., 2019). There is agreement that integrated STEM education should engage learners in authentic and motivating contexts, providing opportunities to develop an understanding of the nature of the STEM disciplines, the relationship between them, and the interdisciplinarity of real-world problems (Just & Siller 2022; Hilton & Hilton, 2024). Equally, the acknowledged potential to develop much sought-after twenty-first century skills required for the future workforce has resulted in its wide promotion (Bybee, 2013; English, 2016; Honey et al., 2014; Maass et al., 2019; Ross et al., 2025).

However, alongside international trends of integrated STEM within school curricula, is the assertion that complete integration of STEM is ‘a step too far’ (Hourigan et al., 2022, p. 704). There is some agreement that rather than have an entirely integrated STEM curriculum, there is merit in learners connecting their knowledge and understandings learned in individual STEM disciplines to their integrated STEM experiences (Hourigan et al., 2022; Bybee, 2013; English, 2016; McComas & Burgin, 2020; Murphy et al., 2024; Tytler, 2020).

As integrated STEM education is a relatively new field, there are varying definitions and conceptions of what constitutes integrated STEM education (Breiner et al., 2012; Fitzpatrick et al., in-press; Hourigan et al., 2022; Sanders, 2009). However, rather than expect that all disciplines would be included, there is general agreement that an integrated STEM task should integrate at least two of the STEM disciplines (Sanders, 2009; Hilton & Hilton, 2024; Murphy et al., 2024; Roehrig et al., 2021). The degree of STEM integration can also vary, as illustrated by Vasquez et al.’s (2013) levels integration (figure 1). A multidisciplinary approach involves STEM disciplines working separately to address a common theme (e.g., Theme: Sound. Science: Exploring vibrations and how sounds are made; Mathematics: statistics investigation ‘What musical instruments do pupils in our school play?’). However, the compartmentalisation of subjects does not support learners to experience the connections. An interdisciplinary approach involves more tightly linked integration of the knowledge and skills from STEM disciplines allowing content from one STEM discipline to support learning in another, thus providing a more integrated experience e.g., ‘How does noise vary in schools across the day?’ Finally, a transdisciplinary approach focuses on solving an authentic, real-world problem relevant to the learners’ lives or project where they use relevant knowledge and skills from across the STEM disciplines, thus providing a highly integrated learning experience e.g., exploration of spaces within the school to identify potential for an enhanced learning environment (see Morrin & Liston, 2024).

Figure 1.

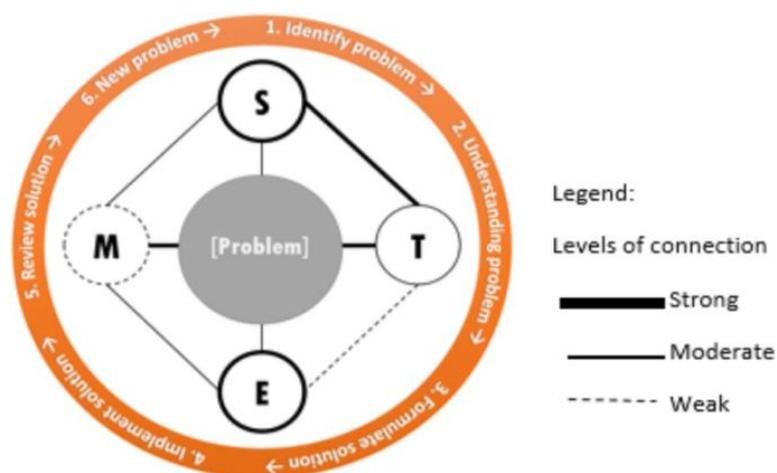
A continuum of STEM approaches to curriculum integration (adapted from Vasquez et al., 2013)



While several frameworks exist with many common elements between them (an emphasis on real-world problems, development of key competencies etc.), Tan et al.'s instructional framework for STEM integration is particularly useful when examining the role and influence of specific disciplines within an integrated STEM task. Tan et al.'s (2019) S-T-E-M quartet compares the STEM disciplines to the instruments in a string quartet. Using this analogy, teachers can consider how the STEM disciplines work together, which takes the lead in terms of deep disciplinary learning as well as the strength of the relationships between these during integrated STEM problem solving. The visual for the framework uses the thickness of markings to communicate the depth of learning within disciplines (circle around each discipline) and strength of inter-disciplinary connections promoted (line between disciplines). This reflects the dynamic nature of integrated STEM education, with emphases varying depending on the nature of the problem presented.

Figure 2.

The S-T-E-M Quartet Instructional Framework (Tan et al. 2019)



Primary STEM education in Ireland

Reflecting international trends, for the last decade, Irish STEM education policy has advocated for an integrated approach to STEM education, with the STEM Education Policy Statement (2017-2026) (DES, 2017, p. 12) setting an ambitious vision that 'Ireland will be internationally recognised as providing the highest quality STEM education experience for learners...' However, the significant lag in curricular changes to enact this vision has meant a mismatch has existed between policy and classroom enactment of integrated STEM education at primary level. For example, the Irish primary curriculum (DES, 1999a) made no explicit mention of integrated STEM education. In this curriculum, while mathematics was a discrete subject, science was positioned within the curricular area Social, Environmental and Scientific Education (SESE) along with History and Geography; and technology and engineering were not official curricular areas. However, this primary curriculum did promote integration and the respective STEM disciplines incorporated skills compatible with integrated STEM education e.g., science: inquiry-based approach, design and make (DES, 1999b); mathematics: applying and problem solving, communicating and expressing, and integrating,

connecting and reasoning (DES, 1999c). While acknowledging increased access to professional development and pockets of integrated STEM practice, the absence of formal guidance from a designated curriculum meant that a traditional, disciplinary approach to STEM teaching prevailed at primary level (Fitzpatrick et al., 2023b.; Flanagan et al., 2024b; Hamilton et al., 2021; Hourigan et al., 2022; O’Dwyer et al., 2023). This is reflected in the STEM Implementation Plan to 2026 (DE, 2023) reporting that during the 2022 consultation process, teachers communicated a desire for access to examples of what STEM education can look like. However, in recent times, there has been a significant closing of the void between policy and curriculum within the Irish context. Firstly, STEM education has been introduced as one of five broad curriculum areas within the new Irish Primary Curriculum Framework (NCCA, 2023b). Another landmark was the publication of the draft Science, Technology and Engineering Education Specification (NCCA, 2024), which is to sit alongside the new Primary Mathematics Curriculum (NCCA, 2023b). This curriculum marks the first formal curriculum inclusion of Technology and Engineering as strands within Irish primary education, with support for these disciplines reflected in the espoused pedagogical practices of *scientific inquiry*, *design thinking* and *computational thinking*. The framework identifies mathematics as fundamental, providing ‘...the foundation for science, technology, and engineering and is the study of the relationships, connections and patterns around us’ (NCCA, 2023a, p. 18). The structures of the STE specification (NCCA, 2024) reflect valuing both disciplinary and interdisciplinary STEM understandings, with dedicated learning outcomes for each of the disciplines alongside the introduction of a new interdisciplinary area ‘The nature of STEM.’ It also promotes dedicated time and space for the application of disciplinary knowledge and connection of learning across the STEM disciplines within integrated STEM. This draft STE specification emulates a social justice perspective, seeking to build on children’s innate curiosity, providing them with the agency to engage with problems of interest to them, promoting active, child-centred investigation of their local environments and the wider world and consideration and communication of viable and creative solutions, thus empowering them to become informed and active citizens (NCCA, 2024). Irish primary children have clearly communicated a strong desire for these kinds of experiences during curriculum consultation (Leavy et al., 2023; NCCA, 2023c).

Acknowledging the complexity of integrated STEM practice

However, despite the welcome advancement of STEM policy and curriculum in Ireland, challenges remain. Studies concur that both practicing and pre-service teachers struggle to design quality integrated STEM problems, particularly problems that retain individual disciplinary integrity and promote deep learning (English, 2016; Fitzpatrick et al., 2024a; Fitzallen, 2015; Honey et al, 2014; Roehrig et al., 2021). Reinholz et al. (2018) acknowledge that even cross-disciplinary teams find designing ‘good’ integrated STEM tasks challenging. Issues include teachers’ lack of exposure to integrated STEM education as either learners or teachers (Fitzpatrick et al., 2024b; Hourigan et al, 2022; O’Dwyer et al., 2023), leading to a lack of understanding of what integrated STEM education ‘looks like’ (Falloon et al., 2020, p. 371). Other barriers include incompatible teacher beliefs, limited knowledge and efficacy for teaching integrated STEM and inadequate integrated STEM curriculum guidance and support

(Anderson & Makar, 2024; Fitzpatrick et al., 2024b; Flanagan et al., 2024b; Ní Ríordáin et al., 2016; O'Dwyer et al., 2023).

There is growing concern around STEM discipline balance within integrated STEM tasks. While initially, there was a suggestion that engineering was being overlooked within integrated STEM practice, over time, there is increasing evidence to suggest that integrated STEM tasks tend to focus on science and/or engineering (Anderson & Makar, 2024). Alongside this, is concern that mathematics remains the 'silent M' in STEM (Shaughnessy, 2013). This is ironic given ongoing reference to mathematics being 'foundational to all STEM subjects' (Anderson, 2024, p. 23). Amid increased attention on STEM disciplinary balance and integrity within the literature, studies consistently lament the limited role that mathematics tends to assume in integrated STEM education.

A spotlight on the role of mathematics in integrated STEM education

The realities of STEM

While the concept of mathematics integration with other disciplines is not new (Dooley et al., 2014), teachers report that mathematics is the most difficult STEM discipline to integrate in STEM tasks (Goos et al., 2023; Goos, 2024). Careful examination of the literature identifies various trends. Firstly, mathematics has been under-represented in integrated STEM tasks, with many lacking any focus on mathematics. On occasions where mathematics was integrated in STEM experiences, it was generally 'relegated...to a subservient role in service of the other STEM subjects' (Anderson & Makar, 2024, p. 3). Mathematics frequently assumed the role of 'tool' (English, 2016; Fitzallen, 2015; Goos et al., 2023; Maass et al., 2019), where 'students used mathematics they already knew to support learning or problem solving in the other STEM disciplines' (Goos, 2024, p. 338). Stohlman's (2020) analysis of integrated STEM tasks over a 10-year period revealed that mathematics integration generally involved the application of prior mathematics (mostly statistics and measurement) knowledge rather than develop new mathematical knowledge. Goos (2024) confirms that while mathematics as a 'tool' is a valid role, it is important that it does not become its only role. Kristensen et al. (2024) propose a framework to support an analysis of the role assumed by mathematics within integrated STEM experiences. It contrasts the role of mathematics acting as a 'tool', where familiar mathematics is used to support learners' understandings of other STEM disciplines with mathematics as a 'primary aim' or 'goal' where the integrated STEM focuses on developing new mathematical competencies, where one or more of the other STEM disciplines provide the context.

It is possible that mathematics may assume both roles i.e., mathematics as a 'primary goal' and a 'tool' within an integrated STEM task (Kristensen et al., 2024). Goos (2024) adds a third category, identifying the potential of mathematics to be an integrative bridge between STEM disciplines, where mathematical modelling serves as a bridge between the STEM disciplines as well as between the mathematics and the real world.

Another source for concern is that the mathematics requirement in integrated STEM tasks tends to be below grade level expectations, in some cases only requiring basic counting or measuring skills (Falloon et al., 2020; Fitzpatrick et al., 2024a; Ross et al., 2025). Just and Siller (2022, p. 16) captured the role assumed by mathematics in integrated STEM as 'minor

matter and a means to an end’, generally used for computation. They argued that learners, as future citizens in an increasingly complex world, would need opportunities to develop their ability to think mathematically. These issues may be associated with Ross et al.’s (2025) observation that mathematics is often the final STEM discipline to be considered when designing integrated STEM experiences. Illustrating this, Fitzpatrick and Leavy’s (2025, p. 23) study demonstrated that pre-service primary teachers’ over-focus on the new STEM discipline, engineering, resulted in mathematics being ‘pushed to the peripheries.’ However, the challenges of applying disciplinary mathematics knowledge and skills within integrated STEM settings were exemplified in their subsequent inability to recognise the lack of appropriate grade-level mathematics learning alongside considerable difficulties in identifying methods of promoting more meaningful mathematics integration despite having completed all compulsory mathematics education courses.

While in theory, any STEM subject could be a starting point for integrated STEM, instances where mathematics was foregrounded in integrated STEM experiences were rare (Anderson, 2024; Falloon et al., 2020; Goos, 2024). Hence, mathematics was seldom the ‘primary goal’ of the integrated STEM experience (Kristensen et al., 2024). Goos (2024, p. 338) raises concern that mathematics tends to be ‘incidental, invisible, or even devalued’ in integrated STEM tasks, given that education seeks to prepare ‘...young people for a world that is inherently interdisciplinary and in which mathematics plays a central role in helping us to solve complex social, ethical and scientific problems.’

Illuminating the M in steM

Given these concerns, there is a growing focus on exploring how best to increase the visibility and level of engagement with mathematics in integrated STEM curriculum. Consensus exists that learners must experience ‘the power of mathematics to model, reason, generalise and predict’ (Anderson, 2024, p. 23). There are increasing calls for integrated STEM education to use authentic integrated STEM tasks or problems, grounded in real world contexts (Treacy, 2021). For example, the International Commission for the Futures of Education (2020) recommend increased integration of curricula focused on ‘the problems facing the planet’ (Hilton & Hilton, 2024, p. 264) as part of the solution to tackle misinformation and combat tendencies to ignore science. Hence, it is desirable that learners receive opportunities to acquire mathematical understandings while ‘grasping its importance for solving real-world problems and shaping society’ (Larsen et al., 2024, p. 86). Mathematics can play a key role in supporting learners given that ‘In today’s digital world, the volume and availability of information is increasing and much of it involves claims based on mathematics. This has implications for mathematics education and STEM education more broadly’ (Hilton & Hilton, 2024, p. 258). This is particularly relevant to the Irish context, given Fitzmaurice et al.’s (2021) finding that many secondary students believe that mathematics was neither necessary nor useful for their education or careers. Gravemeijer et al. (2017) highlight that mathematics education, rather than prioritise mathematics competencies that can be completed by digital technologies, should prioritise the development of twenty first century skills to support learners to problem solve, engage in critical thinking, and reason mathematically.

To this end, research has sought to reveal various means of foregrounding mathematics within integrated STEM education, providing opportunities for meaningful mathematics learning through engagement with integrated experiences. One possible approach is through the promotion of responsible citizenship (Choy and Cooper, 2024; Larsen et al, 2024; Li, 2025; Maass et al., 2019). To achieve this, Anderson (2024b) recommends that learners should be given opportunities to investigate local issues relevant to their lives. Larsen et al.'s (2024) study provides examples including the reduction of plastic waste in the local environment, where learners investigated if the issue constituted a problem and formulated solutions and, in some cases, acted on these. It is suggested that over time, students could engage with more complex global issues e.g., understanding refugees and migration patterns (Steffenson, 2024). Mathematics can play a key role in addressing these open-ended problems. For example, mathematical modelling is a valuable tool that can be used to support learners to actively engage in meaning making for a range of possible purposes including describing a situation, making predictions, designing a solution, planning an event or making an informed decision. Such experiences provide insights into the key role of mathematics, facilitating learners to engage with information and data, critically assess claims to inform conclusions and actions, thus supporting 'effective socially conscious decision-making' (Maass et al, 2019, p. 876).

There is also some focus in the literature on the potential to problem pose or to modify existing mathematics tasks to promote the meaningful integration of mathematics and uphold its integrity within integrated STEM tasks. These proposals recommend that mathematics should be the starting point of planning. For example, Choy and Cooper (2024) encourage teachers to 'notice' the conceivable STEM affordances in everyday mathematics tasks, highlighting that the possibility to 'strengthen the 'M' in STEM abounds in everyday mathematics classrooms' (p. 58). Fitzpatrick et al. (2024a) also propose the potential to *STEMify* mathematics tasks, where teachers interrogate previously developed mathematics tasks further to reveal their capacity for meaningful integrated STEM learning. Stohlmann (2018) proposes 'integrated steM' where mathematics is integrated with at least one of the other STEM disciplines, where the focus remains on mathematics while being supported and enhanced by the other STEM disciplines. Additionally, Larsen et al. (2024, p. 91) identifies the potential of Kristensen's (2024) framework as a reflective tool during planning, where the mathematical goals for integrated STEM tasks are explicitly defined from the outset and the framework serves to promote an intentional focus on and even the prioritisation of mathematics 'as the primary objective' when designing integrated STEM. Fitzpatrick et al. (2024a) also recommend intentional selection of the mathematics disciplinary learning outcomes to promote the integration of grade-appropriate mathematics.

In situations where mathematics has been ignored or its coverage is tokenistic in integrated STEM experiences, Just and Siller's (2022, p. 15) study identified opportunities for rich mathematics learning if tasks were 'slightly transformed.' Similarly, Fitzpatrick et al. (2024a) demonstrate how scaffolded retrospective reflection and discussion supported pre-service teachers to *Mathify* STEM tasks, to include meaningful grade-level appropriate mathematics integration. Building on this, Ross et al.'s (2025) study revealed primary teachers' capacity to exploit incidental mathematics learning opportunities or teachable moments, thus deepening mathematics learning during integrated STEM experience beyond what was

originally planned for. There is much potential for these strategies to be applied to strengthen the integrity of the mathematical focus within integrated STEM experiences.

While integrating mathematics with other disciplines is not a novel approach, the research corpus suggests that there remains much to be learned about how to design ‘good’ interdisciplinary tasks, particularly those featuring mathematics (Goos et al., 2023). Deep engagement with the literature not only provides guidance in future design of interdisciplinary mathematics but also provides criteria to reconsider and critique the role of mathematics in previously designed integrated tasks. I now offer some examples from my own experiences in collaboration with STEM colleagues.

Using the ‘M in STEM’ literature as a lens to explore integrated STEM experiences

Given that integrated STEM education is a nascent area within the Irish context, the lack of curriculum guidance meant that early efforts to engage in integrated STEM were done in a vacuum. However, the development of a new Department of STEM education in MIC in autumn 2017, with all members having a primary background in one of the constituent S-T-E-M education disciplines, marked the start of a STEM journey, where STEM teacher educators, alongside pre-service and practicing primary teachers, explored what integrated STEM could look in Irish primary classrooms. The following case studies of integrated STEM education showcase some of our integrated STEM work. Each case illustrates a different unit of STEM inquiry.

Case 1: *I’m a Celebrity, Get me out of here: Up for a challenge?* [Spring, 2018, University-based Collaborative Lesson Study, mathematics teacher educators, pre-service primary teachers, 5th class pupils] (Hourigan & Leavy, 2020a, 2020b; 2021a, 2021b)

The popular reality TV show called ‘I’m a Celebrity, get me out of here’ served as a theme that connected five consecutive lessons within this unit of inquiry. Each lesson was launched using a different ‘bushtucker trial’ or challenge connected to different features of the reality TV show e.g. jungle camp and targeting various mathematical understandings. The context and driving questions sought to promote interest and motivation to engage in inquiry. Challenges focused on: (1) An investigation exploring which food box had a better chance of being catapulted across the flooding river into camp (Hourigan & Leavy, 2020b); (2) the design of a water tank for a challenge considering its perimeter, area and volume (Hourigan & Leavy, 2020a); (3) determining the area of various [irregular shaped] zones (trial areas and camps), rescaling and redesigning in response to producers’ criteria (Hourigan & Leavy, 2021a); (4) exploration of the relationship (statistical association) between jump height and jump length (Hourigan & Leavy, 2021c).

Case 2: *How Slow is your parachute?* [Autumn 2019, School-University Partnership, STEM teacher educators (science, technology and mathematics teacher educators, primary teachers, pre-service primary teachers, 3rd class pupils] (Hamilton et al., 2021; Leavy et al., 2021).

This integrated ‘Designing a Parachute’ STEM inquiry incorporated geometry concepts within scientific inquiry and engineering practices. Initial exploration of the first parachutes promoted a scientific investigation to determine ‘How does the angle of the parachute frame

affect the speed of descent?’ Subsequently, further analysis of modern parachutes stimulated an engineering design process exploring ‘How does the size of the parachute canopy affect the speed of descent?’ (Leavy et al., 2021).

Case 3: *Busy Helping Bees* [University-based Collaborative Lesson Study, mathematics teacher educators, primary teachers, pre-service primary teachers, Spring, 2020, Senior Infant pupils] (Hourigan & Leavy, 2021c; 2023a).

This unit of STEM inquiry, on designing a series of five STEM-based lessons focused on the theme of bees. The everyday context of the bee population in a park being impacted by a storm sought to motivate engagement with various tasks that included: (1) designing and building a strong hive for the bees (Hourigan & Leavy, 2021c); (2) using knowledge of shadows to determine appropriate locations for planting flowers in the park (Hourigan & Leavy, 2023a); (3) using spatial awareness and coding to guide the bees between flowers; (4) measuring the shortest distance to be travelled by bees when collecting nectar and pollen from flowers.

Case 4: *Learn About Bees from the Bees* [University-based Collaborative Lesson Study, mathematics teacher educators, pre-service primary teachers, Spring, 2023, 6th class pupils] (Leavy et al., 2024; Fitzpatrick et al., under review).

The real-world context of the honeybee based in hives the school’s local environs was used as a stimulus for a 5-day integrated STEM unit of inquiry. Following two preparatory lessons (Lesson 1: All about Bees (Science lesson), Lesson 2: Statistical concepts: Mean and Median), the remaining 3 lessons were integrated STEM lessons, using locally generated data of the conditions in local bee hives (data collected using conveyance technology from hive sensors) as a context. The math action technology (Common Online Data Analysis Platform (CODAP)) supported engagement with the large data sets to explore various driving questions e.g., ‘The beekeeper thinks one hive is in trouble. Focusing on temperature, which hive (if any) should she open?’ ‘Is there a relationship [statistical association] between the living conditions in the hive? [temperature and noise]’ (Fitzpatrick et al., under review; Leavy et al., 2024).

Each of these cases could be considered integrated STEM units of inquiry. However, each differs in its level of integration (with evidence of both multidisciplinary and interdisciplinary integration (Vasquez et al., 2013)) and the disciplinary emphasis within (with mathematics and science assuming lead and supportive roles throughout (Tan et al., 2019)). There is much potential for the relevant literature to inform a retrospective review of the position of mathematics within each of these units of STEM inquiry, revealing the role assumed by mathematics, the mathematical knowledge and skills needed/developed, and further opportunities for mathematics integration. Engaging in a research-informed exploration of the selected cases activates an awareness of the promise for rich mathematics tasks to promote STEM literacy through their focus on mathematical modelling alongside opportunities to develop twenty-first century skills (e.g., Hourigan & Leavy, 2020a). However, it reveals further openings to *STEMify* various mathematics investigations (e.g., Hourigan & Leavy, 2021a; 2021c) alongside potential to *Mathify* integrated STEM lessons and tasks within the respective

units of inquiry (e.g., Hourigan & Leavy, 2021b; Leavy et al., 2021). This review reinforces that when mathematics is presumed as an underlying discipline, and not explicitly planned for (e.g., Hourigan & Leavy, 2023a), opportunities for meaningful mathematical experiences are missed. It also facilitates an examination of the task features of mathematics-centred integrated STEM tasks (e.g., Hourigan & Leavy, 2020b; Leavy et al., 2024). In particular, Case 4 demonstrates the capacity of a sustainability context in driving meaningful statistical investigations using real-world data. It also highlights the role that technology can play in supporting statistical reasoning, shifting the focus from calculation to analysis and statistical meaning making, prioritising conceptual understanding, critical thinking and communication. This unit of inquiry showcases the usefulness, relevance and worth of statistics in everyday life in informing decision making, providing reciprocal opportunities for learners to develop statistical literacy alongside science understandings. This societally relevant unit of inquiry showcases the capacity of a local sustainability context to drive meaningful integrated STEM learning and to promote responsible citizenship (Fitzpatrick et al., under review; Leavy et al., 2024).

Engagement in a retrospective review process showcases further potential to enhance integrated STEM tasks by engaging in iterative cycles of implementation, reflection and problem posing. It also informs more mindful engagement with the balance and quality of disciplinary integration when working with both pre-service and practicing primary teachers going forward, ensuring that STEM integration is not tokenistic and promotes an awareness of the uses of mathematics (and the other STEM disciplines) to solve real world problems while developing desirable disciplinary competencies.

Working towards quality integrated STEM education, with an eye firmly on mathematics

While no universal definition of integrated STEM exists, there is broad agreement that quality integrated STEM education should use real world contexts to promote problem solving, while considering the balance of STEM disciplines integrated and protecting the integrity of these. As we move closer to the alignment of Irish education policy and primary STEM curriculum, it is apt to consider how best Irish pre-service and practicing primary teachers can be supported during the upcoming embedding of the proposed STE specification and integrated STEM education.

It is essential that longitudinal professional learning opportunities provide needs-led support for practicing primary teachers on their individual STEM journeys (Murphy et al., 2024; Smith & Browne, 2024). Firstly, teachers require opportunities to develop their STEM disciplinary knowledge for teaching alongside compatible beliefs, prior to delving into integrated STEM practice (O'Dwyer et al., 2023; Smith & Browne, 2024). While professional learning is presently underway to support primary teachers to embed the Primary Mathematics Curriculum (NCCA, 2023b), similar provision is needed for all STEM disciplines, given that strong disciplinary knowledge is essential to support teachers to identify and exploit STEM integration opportunities (Fitzpatrick et al., 2024b; Flanagan et al., 2024a, b; Honey et al., 2014; O'Dwyer et al., 2023). Professional learning must include carefully selected integrated STEM tasks that model disciplinary balance and maintain disciplinary integrity and provide teachers with opportunities to critique these. It is particularly important that teachers receive ample

exposure to authentic real-world integrated STEM problems where mathematics is foregrounded, given that research has confirmed that it cannot be assumed that mathematics will be meaningfully integrated (Goos et al., 2024). Various approaches to configuring the teaching of disciplinary concepts around an integrated STEM project also requires attention (Murphy et al., 2024). While providing access to quality supports and resources is important, it is essential to recognise practicing primary teachers' relevant experiences and understandings (Hourigan et al., 2022). Hence, these professional learning experiences must empower and support primary teachers to work collaboratively in their school contexts (Dooley et al., 2014; Flanagan et al., 2024b; Larsen et al., 2024; Smith & Browne, 2024), to notice the potential within their current classroom practices for STEM inquiry and to use these as a springboard to problem pose meaningful integrated STEM tasks of interest to the children in their classes. The practices of *STEMifying mathematics* and *Mathifying STEM* hold much promise to acknowledge current valuable practice and to further enhance these during task design. Alongside promoting these practices among primary teachers, there are rich opportunities for primary teachers to use their work in other curricular areas outside STEM, e.g., geography, global education, as a starting point for authentic integrated STEM experiences, to consider how to further *Mathify* these to promote authentic problem solving in pursuit of informed and active citizenship (Fitzpatrick et al., 2024a). Equally, facilitating the implementation of designed integrated STEM activities is crucial given evidence that it is often only after the chance to implement reform practices in one's own school environment that teachers are persuaded of their merits (Flanagan et al., 2024b; Hourigan & Leavy, 2023b; Hourigan et al., 2022; O'Dwyer et al., 2023; Smith & Browne, 2024). During integrated STEM practice, experienced primary teachers will have the capacity to identify and exploit additional opportunities for mathematics learning, beyond those planned for, engaging in 'contingency' or 'thinking on their feet' (Rowland, 2014; Ross et al., 2025). This practice, alongside cycles of implementation and reflective practice will promote ongoing development of integrated STEM practice. Throughout, practicing primary teachers' expertise and creativity must be recognised and their agency exploited, thus facilitating them to truly take ownership of their integrated STEM practice (Hamilton et al., 2021; O'Dwyer et al., 2023), incrementally assuming the role of curriculum creators rather than mere curriculum implementers.

Equally, all future primary teachers require access to high quality STEM education during initial teacher education to prepare them to engage in proposed integrated STEM practices (Fitzpatrick, 2024; Fitzpatrick & Leavy, 2025; Kurup et al., 2019; Murphy et al., 2024; NCCA, 2023a; 2024). Relative to established primary teachers, pre-service primary teachers will require substantially more support in developing their integrated STEM practices (Fitzpatrick & Leavy, 2025). Following the development of appropriate knowledge and beliefs for teaching the respective STEM disciplines, given the siloed nature of their pre-tertiary STEM education, initial teacher education must provide pre-service primary teachers with ongoing opportunities to experience quality integrated STEM education as learners (DE, 2023; Fitzpatrick et al., 2024a; Fitzpatrick & Leavy, 2025). These experiences will model integrated STEM tasks, providing valuable insights regarding what integrated STEM could look like and facilitating pre-service teachers to move into the role of teacher over time, through discussions that draw on their disciplinary understandings, attend to disciplinary balance and the

maintenance of disciplinary integrity. In particular, pre-service teachers require many experiences of authentic real-world integrated STEM problems where mathematics is foregrounded, to facilitate them to see the potential for mathematics to assume a leading role (e.g., Fitzpatrick et al., under review; Leavy et al., 2024). Subsequently, given research that demonstrates the power of trialling and witnessing the impact of recommended reform approaches in real classrooms (DE, 2023; Hourigan & Leavy, 2019; Fitzpatrick, 2024; Fitzpatrick & Leavy, 2025), pre-service primary teachers require scaffolding across the collaborative design and implementation of integrated STEM tasks. Post-implementation guided discussion, and critical reflection must be prioritised, providing pre-service teachers with opportunities to analyse the task in terms of the curriculum-appropriate mathematics learning leveraged thus informing further capacity for problem posing within the context of integrated STEM (e.g., *Mathifying STEM*) (Fitzpatrick et al., 2024a; Fitzpatrick & Leavy, 2025; Hourigan & Leavy, 2020a; 2021). Substantial collaboration will be required across STEM education faculty to ensure a cohesive experience.

In all of this, while acknowledging that not all integrated STEM experiences need to integrate mathematics, given the lessons learned from the literature, intentional actions must be taken to ensure mathematics does not ‘fall silent’ within integrated STEM experiences, given the necessity to prepare future citizens for a world where mathematical literacy is fundamental (Fitzpatrick & Leavy, 2025; Goos et al., 2024). Fitzpatrick et al. (2024a) provide practical guidance to support both pre-service and in-service primary teachers in capitalising on opportunities to promote mathematical understandings from integrated STEM education alongside maximising the capacity of mathematics to support STEM understandings. If implemented as intended, where mathematics is identified as the ‘foundation’ of the other STEM disciplines (NCCA, 2023a), there is much promise for a strong reciprocal relationship between mathematics education and integrated STEM education. In this scenario, engagement with integrated STEM tasks with inbuilt opportunities to use and/or develop appropriate mathematical understandings will naturally incorporate many of the pedagogical practices advocated within the Primary Mathematics Curriculum (e.g., productive disposition, cognitively challenging tasks, maths talk) where children can develop a range of the mathematical proficiencies (e.g., productive disposition, conceptual understanding, adaptive reasoning, strategic competence) (NCCA, 2023b). Equally, genuine implementation of the Primary Mathematics Curriculum centred around child-centred pedagogies and the development of higher order thinking skills will simultaneously promote STEM literacy.

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Mathematics for interculturality: Collaborative online international learning in linear algebra for finance, security and defence strategies

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Activities involving virtual and blended international collaboration, encompassing Collaborative Online International Learning (COIL) and Blended Intensive Programmes (BIP), furnish higher education institutions with more inclusive and environmentally sustainable alternatives to conventional physical mobility schemes. This paper presents the design and implementation of a COIL project in linear algebra for economics, finance, security, and defence, co-created and co-delivered by University College Dublin and University of Turin in November 2024. The 10-hour online extracurricular activity involved 32 first-year students from both partner universities collaborating in mixed virtual teams on case studies and problem-solving tasks in real-world scenarios. After presenting the theoretical framework on which the COIL activity has been built, this paper outlines its development and implementation and concludes with potential future pathways to enhance internationalization within higher education and Mathematics Education research on this topic.

Keywords: Collaborative online international learning, higher education, intercultural education, INVITE learning design framework, mathematics education

Introduction

The internationalization of higher education institutions (HEIs), as defined by Knight (2008), is a widespread, deliberate effort by universities to incorporate international, intercultural, and global perspectives into the core aims, operations, and educational delivery of universities. This trend focuses on providing students with opportunities to interact with a variety of cultures and viewpoints (Vahed & Rodriguez, 2021). The goal is to improve the quality of education and research for everyone and to positively impact society (de Wit & Hunter, 2015). The internationalization of HEIs may benefit students and faculty by fostering the development of soft-skill competencies like team collaboration, communication and presentation skills, problem solving, critical thinking, and responsibility. Moreover, it contributes to the flourishing of a more interconnected, multicultural, and prosperous community of people ready to address global challenges and to contribute to social, cultural, political, and economic progress. To this aim, many HEIs prioritize and leverage on student mobility programs, such as study abroad or international internships. However, only a small percentage of students – between 10% and 13% of undergraduate students in Europe and the US – participate in these opportunities due to financial constraints, language barriers, or personal reasons (European Commission, 2020; Institute for International Education, 2023). Thus, while studying abroad can effectively develop intercultural competence, mobility programs are not inclusive approaches, as most students do not have access to them. The limited participation in traditional physical mobility has pushed HEIs to seek more inclusive internationalization strategies, which have been growing across US and Europe in the last few years. These new mobility programs represent academic modalities where teachers and

students from at least two HEIs participate in shared learning activities in Blended or Virtual Learning Environments. Among those, Blended Intensive Programme (BIP) combines online interactions with face-to-face components and is promoted by the EU Erasmus+ Programme (European Commission, 2023), while Collaborative Online International Learning (COIL) (Hackett et al., 2024) relies entirely on online environments for collaborative projects undertaken by students at their home institutions. Research studies validate the strong international emphasis of BIP and COIL projects and highlight their role in creating valuable experimental spaces to develop intercultural competences (Hackett et al., 2023; Barana et al., 2025a). Increasing concerns about the environmental impact of student and staff mobility have led HEIs to explore sustainable internationalization methods like COIL (Shields, 2019). Furthermore, proposing extra-curricular activities through COIL can add value to learning challenging subjects such as Mathematics (Hilario Perez & Verdejo Gimeno, 2022), as one can discover different approaches resulting from alternative teaching styles. There are some examples of COIL projects in Mathematics and its application, such as “An introduction to mathematical modeling”⁴, involving University of Denver and Universidad del Desarrollo, “Intercultural math & data reporting”⁵, involving University of Potsdam and Pontificia Universidade Católica do Rio Grande do Sul, and “Applying fractal geometry in the construction of sustainable facades”, involving Florida International University and Universidad Cardenal Herrera CEU (Hilario Perez & Verdejo Gimeno, 2022).

In this paper, we contribute to fostering these new forms of mobility and present the design and the implementation of a COIL project in linear algebra for economics, finance, security and defence strategies, co-designed and co-delivered in November 2024 by University College Dublin (UCD) and the University of Turin (UNITO). This initiative has involved a 10-hour online extracurricular project where 32 first-year students from UNITO and UCD worked in mixed teams in a digital learning environment on tasks focusing on the application of linear algebra to case studies and problem-solving activities in the domains of economy, finance, and strategic science. The paper is organized as follows: in the second section, we present the adopted definition of COIL and the theoretical framework on which the activity is built. In the Results section we detail the COIL project design, development and implementation. The final section concludes with potential future directions to foster internationalization in HEIs and Mathematics Education research on this topic.

Theoretical Framework and Methodology

COIL originated at the State University of New York (SUNY) in 2006, leading to the establishment of the SUNY COIL Center to promote its use within the SUNY system (Rubin, 2017; Rubin & Guth, 2022). Initially more prevalent in the US, COIL’s adoption gradually expanded in Europe, partly due to the growth of SUNY’s COIL Global University Partner Network. Rubin and Guth (2022) have provided a list of characteristics of COIL based on best practices. In a nutshell, COIL involves online collaboration between two or more educators from different HEIs in different countries to co-design and co-deliver a course or a set of

⁴ <https://tinyurl.com/4mbs9fzs>

⁵ <https://tinyurl.com/bdf9m667>

online collaborative tasks with shared learning outcomes, typically over a period ranging from four weeks to a full semester. Multicultural teams of students, formed from both institutions, work together online on these assignments, fostering both subject knowledge and intercultural competence within various disciplines. Hackett et al. (2024) proposed the following formal definition of COIL, which we adopted in this research study:

An inclusive, environmentally friendly teaching and learning method used to internationalise the curriculum, in which educators from different educational institutions in different countries connect to co-design and co-facilitate collaborative online learning assignments that are embedded within the curriculum, with the goal of facilitating the development of students' collaborative skills, intercultural competence, and curriculum content learning through collaboration.

Within the INVITE Erasmus+ project, the consortium developed a Learning Design Framework (LDF) for the creation and implementation of virtual or blended international collaboration activities and implemented a training module for HE teachers and staff to create effective BIP and COIL activities in line with this framework (Barana et al., 2025b). It is grounded in the principles of the Analysis, Design, Development, Implementation, and Evaluation (ADDIE) instructional model, and includes aspects to be considered in designing international collaboration activities. The framework consists of six phases, which mirror the ADDIE structure, with the addition of the preparation phase:

1. Preparation: initial conceptualization of the project and identification of partners.
2. Analysis: assessment of the learning needs of international participants, definition of the learning outcomes, and articulation of the intercultural added value.
3. Design: selection of active learning methodologies and choice of digital environments.
4. Development: creation or selection of culturally sensitive and relevant learning materials, and integration of digital technologies to support active learning processes.
5. Implementation: provision of mentorship, facilitation of intercultural collaboration, and ongoing monitoring of students' progress.
6. Evaluation: designing formative and summative assessment strategies, collecting data on learning achievements and participant feedback, and adjusting accordingly.

Results: “Linear Algebra for Economics, Finance, Security and Defence Strategies”

Phase 1 – Preparation

The UCD and UNITO teachers involved in the co-design and co-delivery of the COIL activity attended the INVITE training module in Summer 2024. While attending it and working on the assessment components collaboratively, they found a common ground in their respective institutions for developing an international module and started preparing an initial conceptualization of the COIL accordingly to the INVITE LDF.

Phases 2 & 3: Analysis & Design

The COIL project “linear algebra for economics, finance, security and defence strategies” was aimed at first-year students from the BSc in Economy & Finance and from the BSc in Financial and Actuarial studies attending the MATH10120 linear algebra module at UCD and first-year students from the BSc Strategic and Security Sciences at UNITO

attending the CPS0977 Mathematics module, thus learners with different cultures and skills. Students were expected to have foundational knowledge of linear algebra from their respective modules, as well as basic English comprehension and communication skills. In the analysis phase, we detected the main instructional challenges. The first one was to bridge diverse backgrounds in Mathematics, due to the different national school education systems. The second challenge was linguistic, arising from the mix of English (Irish) mother tongue students, who generally have less experience with oral communication in their education, and non-English (Italian) mother tongue students, who are often exposed to oral presentations. Starting from these challenges, we set the learning objectives (LO) of the activity, including enhancing problem-solving and critical thinking, fostering intercultural communication, applying mathematical methods to real-world economics and finance, as well as security and defence scenarios. The design phase was carried out to consider learners' diverse needs, laying the groundwork for effective knowledge acquisition and intercultural collaboration. Among the choices in this phase, teachers selected tailored collaborative activities to be solved by students working in a mixed team of 3/4 students, at least one per institution, and decided to use a digital learning environment for the delivery of the project. Considering the LOs, the activities and the learning materials were immersed in a scenario: students were divided into teams of secret agents that could pass precious information between Ireland and Italy to design cutting-edge policies and strategies. The teachers designed three tasks to be solved in teams in a timeframe of one month. Task 0 was an initial ice-breaking activity, requiring teams of "secret agents" to collaboratively define and present their team's name, logo, secret headquarters, member roles, vision, and mission. The aim was to facilitate initial interaction and socialization before the introduction of mathematical tasks. Task 1 was about the use of cryptography in the field of security and defence. Task 2 presented a macroeconomics policy problem where teams must explore a one-sector macroeconomic model, analyse real-world global parameters, interpret the resulting values, and justify government policy choices. Teams worked independently, self-organizing their meeting times and methods to solve the tasks. Additionally, two scheduled live meetings and two tutoring sessions were provided. The initial meeting introduced teachers, COIL aims, scope, and group tasks, while the final meeting involved team presentations of their results. Two task-specific tutoring sessions were offered for student support.

Phases 4 & 5: Development & Implementation

The contents of the COIL project were based on real-world tasks and open educational resources (OER). The contextualization made students understand the direct application of linear algebra in cryptography, for the security and defence side, and the driving factors of national economies with a special focus on similarities and differences, for the economic and finance side. Several OERs were used, such as tutorials, open-access papers and books, adapted to make them suitable for the target students and to mix resources from different backgrounds. Live meetings were held online through a web-conference system. A learning management system was adopted for sharing materials, grouping students, providing questionnaires, collecting submissions, gaining feedback, and in general helping in promoting the educational strategies designed. Students could freely choose their preferred tools for collaborative work outside of scheduled meetings. Teachers suggested advanced computing

environments (e.g., Maple, Mathematica), programming languages (e.g., Python), and spreadsheets (e.g., Excel) to enhance productivity and computational task resolution. Students were also encouraged to critically utilize AI tools (e.g., ChatGPT, Copilot, Gemini) for problem-solving and presentation. The activity was delivered on pilot basis in November 2024, once the students completed the linear algebra prerequisites in each institution. The teachers decided to give access to the project to a maximum of 20 students per institution, selected on a voluntary basis in view of an inclusive teaching (first come, first served), and to deliver the course in 10 extracurricular hours, during the Autumn term. 32 students subscribed and the teachers created mixed groups, considering also gender balance. After the initial meeting, the students had 1 week to complete task 0, 2 weeks to complete each task, and 1 week to assemble a presentation, to be discussed at the final meeting.

Evaluation

Students were assessed through individual (initial and final surveys, reflective diaries after each task and a final diary) and group components (three task solution presentations and a final presentation on their teamwork and solution development). Individual assessments addressed collaboration, linguistic, cultural, relational, mathematical, and digital competence. The instructors evaluated each assessed component and provided individual and group feedback. Since COIL activities are still not institutionalized, this project was not credit-bearing. However, to acknowledge the work done and foster engagement, students received a certificate of participation that they can add to their curriculum as an extracurricular activity.

Discussion and Conclusions

In this paper, we presented the development and the implementation of a COIL project co-designed and co-delivered according to the INVITE LDF in November 2024 by UCD and UNITO. Students attending it generally expressed great appreciation for such experience, particularly due to the international collaboration and the use of a real-world scenario for a deeper understanding of theoretical linear algebra concepts. Teachers acknowledged intense commitment from the students' side and an improvement of their linguistic, interpersonal, intercultural and mathematical competencies. COIL and BIPs offer promising alternatives to traditional student mobility for competency development and intercultural learning, yet their effectiveness requires further research through in-depth data analysis. To bridge this research gap and foster internationalization in HEIs, we are analysing the reflective diaries and the individual surveys that students filled out during the activities, and we will correlate our results with available literature on COIL and BIPs.

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Steps towards a mastery learning approach in mathematics service teaching

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Mastery Learning is based on a belief that all students can achieve the same level of content mastery when provided with sufficient time and appropriate resources. We report on an initiative to adopt elements of a mastery learning approach in a first year undergraduate Calculus module for students on Chemistry and Biology programmes, facilitated by digital technology. These included administering a pre-teaching diagnostic assessment, offering regular formative assessments and providing opportunities to retake summative assessments. We discuss students' responses to these additional opportunities to succeed in terms of uptake, improved outcomes and the perceived usefulness of the approach.

Keywords: Mastery learning, calculus, service teaching

Introduction

Since the 1960s many educators have developed instructional programmes around the notion of 'mastery learning'. Bloom stated that

Most students, perhaps over 90 percent, can master what teachers have to teach them, and it is the task of instruction to find the means which will enable students to master the subject under consideration (Bloom, 1968, abstract).

He believed that the cornerstone to the achievement of mastery is the provision of whatever time is needed by each student for learning and that 'instructional alternatives' (in the form of different resources) should be made available to enable each student to learn to a high level. Two particular approaches to mastery have been adopted widely: Learning for Mastery (LFM) (Bloom, 1968) and Personalised Systems of Instruction (PSI) (Keller, 1968). While each includes the division of content into short units with formative assessments to be completed on each unit, they also differ in important aspects. LFM lessons are presented by teachers and thus students move through them at a uniform pace; whereas PSI lessons are typically presented using written materials through which students move at their own pace, revising the content and retaking assessments until they demonstrate the desired level of mastery. As Guskey (2010) explains, allowing students to retake assessments, which is essentially offering them a second chance to succeed, serves as a powerful motivational tool. In addition, the inclusion of regular formative assessments, a core element of mastery learning models, allows students' progress to be monitored systematically and regular feedback to be given, realising assessment *for* learning. Another common element of mastery learning programmes is the administration of a diagnostic (pre-)assessment pre-teaching to determine whether students have the prerequisite knowledge and skills to succeed on the programme (Guskey, 2010). Students whose preassessment scores suggest deficiencies in their foundational knowledge are then given the opportunity to remediate them before continuing with the programme.

In a meta-analysis of findings from 108 studies, Kulik et al. (1990) reported that mastery learning programmes have positive effects on the achievement of students (at all levels) on examinations as well as on their attitudes to both course content and instruction. Moreover, the weaker students in a class appear to benefit more from such an approach.

Groen et al. (2015) also report the positive effect of mastery learning on retention of content. However, the amount of time spent by students on instructional tasks may increase, and self-paced mastery programmes may reduce the completion rates of those on college level programmes (Kulik et al., 1990).

More recently, digital technologies have been used to design adaptive learning platforms, which aim to adapt instruction and assessment to individuals' prior knowledge, educational experience and interests (Aleven et al., 2016). These have the capacity to incorporate elements of the LFM and PSI approaches outlined above in an efficient manner.

Rationale and Aims

Much of mathematics is hierarchical in structure, with later concepts and skills building on earlier ones. Consequently, learning in mathematics must often progress through a structured sequence in which some items of knowledge or skill are necessary prerequisites to subsequent ones. Furthermore, mathematics underpins a wide range of STEM learning activities. Despite this, the under-preparedness of incoming students for the mathematical demands of their undergraduate programmes has been widely (across Ireland, the UK, the US and Australia) and long since reported – a phenomenon regularly referred to as the ‘Mathematics Problem’ – and in fact, appears to be worsening (Treacy & Faulkner, 2015). Even though students may pass university mathematics modules with a grade of 40%, there is anecdotal evidence of a belief among mathematics educators that such a level of competence does not provide an adequate foundation for the future learning of related mathematical concepts nor ensure the ability to apply mathematical knowledge across STEM disciplines.

We wished to increase the competencies of first-year undergraduate science students on our mathematics modules and better prepare them for subsequent mathematics and STEM modules by encouraging and enabling them to master the content rather than to merely achieve a passing grade. However, our capacity for innovation was somewhat constrained by the large number of students (circa 400) taking the modules in question.

Instructional Approach and Methodology

In 2021, we embarked on the redesign of service Calculus modules for first-year undergraduate students on DCU's science programmes incorporating features of both the LFM and PSI models. We adopted a blended-learning approach, utilising DCU's virtual learning environment (VLE) Loop, which allowed us to be flexible and adaptive in terms of variety of and access to materials, and facilitated the provision of individualised pathways through the module to accommodate a diversity of learners. Content is organised into four main units (Pre-calculus, Limits, Derivatives and Integrals) and delivered using (short) pre-recorded videos interspersed with interactive questions. Each week clear instructions are provided to students with regard to the activities and resources with which they should engage, including weekly formative assessment quizzes (which provide instant feedback to students) and recommended reading from a Calculus textbook. These are supplemented by face-to-face Q&A sessions and face-to-face small group tutorials. A diagnostic pre-assessment is administered before the module begins to ascertain whether students have the necessary foundational knowledge (e.g. from algebra, trigonometry etc.) to enable them to

complete the module successfully. If gaps are detected for a particular student, he/she is directed to additional video and written resources addressing those gaps before retaking the pre-assessment. This extra material (the Foundations Unit) is made available over the first few weeks of the module while the module itself proceeds at a reduced pace during this period to allow students time to engage with this revision material. Course credit was awarded for completion of the pre-assessment tests (Ní Shé et al., 2023).

Summative assessment comprised various components including Loop quizzes on the four units over the duration of the year-long module (accounting for 40% of the module grade), with the option of taking each one a second time after a specified interval (typically one week). These quizzes were timed (25-35 minutes) and were available over a 6–8-hour window. Students were provided with automated feedback on their attempts on the tests and were supported to learn content or develop skills in a number of ways before being tested again (by taking an alternative form of the test). Firstly, all resources provided online remained available to students for the duration of the module to allow students to review and revise the material as appropriate and to study at their own pace. Queries could be raised by students in the weekly Q&A session or tutorial about questions they had difficulty completing. Students were also referred to DCU's Mathematics Learning Centre for additional support from a mathematics tutor. This approach is in the spirit of the remediation process reported in Howard et al. (2019) and has similar aims: to allow students to improve based on feedback received.

The data on which we report here was collected for the academic year 2023/24 for 252 students taking the module MS150 Calculus & its Applications, following ethical approval. This module was mandatory for all first year Biology, Chemistry and Environmental Science students (excluding students on the Genetics and Cell Biology programme) at DCU. In 2023/24, there were six Loop quizzes (LQ1-6) with two quizzes on each of the two final units of content (LQ 3,4; LQ5, 6) as these covered more material. The numbers of students taking tests on Units 1-4 at each opportunity and the score associated with each attempt were recorded in Loop. This data was then collated, summarised and analysed using MS Excel. In addition, students were asked to complete a short online survey on completion of the module. This included a number of Likert scale items to collect information on their experience of the module. The survey was administered using a Google form which also collated and summarised the responses collected.

Our research questions were the following: Did students find the mastery learning approach taken (vis-à-vis formative assessments, Foundations Unit material, retake opportunities for summative assessments) useful? Did students availing of the retake opportunities provided for the quizzes improve their performance?

Results

Table 1 below shows students' engagement with the two opportunities provided to complete each of the Loop quizzes and their levels of achievement. For students who attempted the quiz at both opportunities, Table 2 indicates the percentage whose score increased on taking a particular quiz for the second time and the average increase in score

realised. Table 3 below shows the number of students responding to each of these items and the distribution of responses received.

Table 1

Average % score on each test and number of student attempts

	LQ1	LQ2	LQ3	LQ4	LQ5	LQ6
1 st opportunity	44% (#244)	59% (#237)	61% (#226)	72% (#192)	77% (#205)	72% (#216)
2 nd opportunity	55% (#117)	60% (#103)	59% (#84)	75% (#98)	70% (#66)	73% (#90)
Best of 1 st & 2 nd opportunities	55% (#245)	65% (#250)	67% (#236)	78% (#242)	79% (#236)	79% (#244)

Table 2

Percentage of students whose score increased on second attempt at quiz and average absolute increase in score (out of 100)

	LQ1	LQ2	LQ3	LQ4	LQ5	LQ6
Percentage of students	70	65	68	79	51	66
Average increase	32	30	41	39	38	42

At the end of the module, students were asked to complete an online survey on their experiences of the module. As part of this, Likert scale items asked them to rate their agreement with the following statements:

- a) I found the *Foundations Unit* useful for bringing my basic mathematical skills to the right level.
- b) I found the weekly *formative assessment quizzes* useful.
- c) I found the *resit opportunities* for many components of continuous assessment useful.

Table 3

Distribution of responses to Likert scale items on usefulness of formative assessment quizzes, Foundations Unit and resit opportunities for summative assessment quizzes

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
<i>Foundations Unit</i> (152 responses)	28	60	44	11	9
<i>Formative assessment quizzes</i> (152)	38	66	43	2	3
<i>Resit opportunities</i> (153)	94	38	18	3	6

Discussion

132 of 159 (or 83%) of respondents strongly agreed or agreed that they found it useful to be able to resit the summative Loop quizzes showing the degree to which they appreciated this initiative and the ‘second chance to succeed’ it afforded them (Guskey, 2010). This is further supported by the evidence provided in Table 1 of the number of students availing of these resit opportunities. However, their engagement with the second attempts can be seen to decrease, though not uniformly, over time (LQ1-5) suggesting that a weariness or apathy with the assessment structure may have set in. An increase in the numbers of students attempting LQ6 was observed for both sittings, but this may be explained by the fact that it was the last such quiz and thus provided one of the final opportunities for students to improve their overall module grade. In addition, for each quiz, the majority of students who availed of the opportunity to resit were able to improve on their grade (Table 2) and the average increase in score achieved by these students was evident. These increases indicate that the “correctives were successful in helping students remedy their individual learning difficulties” (Guskey, 2010, p.54) for those students at least. Furthermore, there was no evidence that students had a sense of being taught how to pass a test rather than developing a greater understanding and level of skill through the module, as was reported by Groen et al. (2015) as a negative outcome of their implementation of mastery learning.

Boggs et al. (2004) reported that two major obstacles often inhibit the use of mastery-learning as an instruction method – namely, grading multiple versions of tests at various stages of a course and scheduling time for test re-takes. As suggested by Alevan et al. (2016) technology allowed us to address these barriers and support adaptivity in ways that had previously been difficult for teachers to manage. Using the Loop VLE we were able to create multiple versions of quizzes quite easily by means of its randomised question and wildcards within questions features; Loop also facilitated automated grading and the provision of feedback. Moreover, the online nature of the quizzes relieved the pressure on scheduling.

Students also found other elements of the mastery learning approach useful. 104 of 152 respondents (or 68%) found the formative assessments useful, while 88 (or 58%) found the Foundations Unit useful. The inclusion of these elements in the module aims to create the conditions necessary for students to succeed by (i) ensuring students have the necessary prerequisite skills and (ii) monitoring student progress and providing prescriptive feedback throughout the module. Guskey (2010) asserts that research has consistently linked these elements of a mastery learning approach to highly effective instruction. The improvements recorded in Table 2 above reflect findings of Howard et al. (2019) – and raise corresponding questions, which we will address elsewhere. The overall impact on attainment levels in the module, and retention of knowledge is contentious.

Groen et al. (2015) report that mastery learning has been shown to be useful in addressing and counteracting the ‘Mathematics problem’ for many STEM students at an Australian university. The implementation of a mastery learning approach offers similar benefits at Irish universities at a time when the mathematical under-preparedness of incoming students continues to be problematic.

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Supporting teachers to teach mathematics spatially

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This study explored how a professional development (PD) program supported 18 primary mathematics teachers in integrating spatial reasoning into their classroom practice. Grounded in the well-established link between spatial ability and mathematical achievement, the PD aimed to strengthen teachers' conceptual understanding, instructional strategies, and confidence in using spatially rich tasks. The program integrated lesson study cycles with the Pictorial–Concrete–Abstract (PCA) model to guide instructional planning and implementation. Findings revealed a shift in teachers' practice from procedural instruction toward a greater emphasis on conceptual understanding, classroom dialogue, and mathematical reasoning. Participants also developed a deeper appreciation for spatial reasoning as a fundamental component of mathematics learning. These results highlight the potential of combining spatial activities, PCA strategies, and collaborative lesson study within PD to enhance mathematics instruction.

Keywords: Spatial reasoning, spatial ability, mathematics education, professional development, primary teachers

Introduction

Spatial reasoning is a key predictor of mathematical success, influencing skills across geometry, measurement, problem-solving, and algebra (Mix & Cheng, 2012; Uttal et al., 2013; Verdine et al., 2017; Wai et al., 2009). Recent curriculum reforms, such as Ireland's Primary Mathematics Curriculum (NCCA, 2023), now emphasize spatial reasoning as a core component of math education. Despite this, many teachers lack the training and support needed to integrate spatial reasoning into everyday instruction (Bruce & Hawes, 2015; Bufasi et al., 2024a). Although research highlights spatial ability as central to mathematical understanding (Clements & Sarama, 2011; Hawes et al., 2017), professional development (PD) opportunities focused on spatial reasoning remain limited (Bufasi et al., 2024b). Existing PD programs show promise but often fall short in supporting sustained, practical application in classrooms (Bruce & Hawes, 2015; Lowrie et al., 2017). To address this gap, the present study investigated a PD program designed to support primary teachers in embedding spatial reasoning into mathematics instruction. The program combined two complementary frameworks: the Pictorial–Concrete–Abstract (PCA) model and Lesson Study (LS). The PCA model scaffolds learning from visual to abstract representations (Bruner, 1966; Moyer, 2001), while LS fosters collaborative planning, observation, and reflection (Lewis, 2002).

While few studies have focused specifically on applying professional development (PD) frameworks to spatial pedagogy, emerging research highlights their potential. Lowrie et al. (2017) found that emphasizing spatial language and visualization in early years'

classrooms led to increased student engagement and improved spatial task performance. Similarly, Bruce and Hawes (2015) reported positive shifts in teacher beliefs and practices after introducing spatial reasoning modules, with more frequent use of spatial tasks and representations. In contrast, Bufasi et al. (2024b) observed that although participants deepened their conceptual understanding of spatial reasoning, many struggled to implement these insights in practice. Collectively, these studies underscore the need for PD models that go beyond raising awareness to support the sustained, practical integration of spatial strategies in everyday teaching.

Responding to this need, the present study examines how a PD model shaped teachers' understanding, confidence, and use of spatial ability. Drawing on teacher reflections, interviews, and classroom observations, it addresses two research questions:

RQ1: How does a PD focus on spatial reasoning influence teachers' perspectives on mathematics teaching and their use of spatial tasks?

RQ2: How does the integration of strategies such as the PCA approach affect teachers' instructional practices and students' mathematical communication?

By centering spatial reasoning within professional learning, this study offers a practical model for enhancing mathematics instruction through a spatial lens.

Method

We implemented a LS model as the core structure for the PD program, working with 18 experienced female primary teachers from a Latvian public school ($M = 22.06$ years, $SD = 7.84$). Teachers were evenly distributed across Grades 1 to 3 (six per grade). Over six months, participants engaged in a seven-session PD program that combined theoretical input with practical, classroom-focused strategies. Each session introduced key spatial reasoning concepts—such as mental rotation, perspective-taking, and symmetry—alongside the PCA model. To support practical application, sessions included modeled lessons demonstrating how these concepts could be embedded in instruction. For example, in Workshop 3 ("Symmetry through Tangrams"), teachers manipulated shapes (Concrete), sketched mirror images (Pictorial), and articulated symmetry rules (Abstract). Following each session, teachers worked in triads to complete LS cycles, collaboratively planning, teaching, and reflecting on three co-designed lessons that integrated spatial reasoning and PCA strategies. Each lesson targeted a distinct spatial competency and followed a scaffolded progression from hands-on experience to abstract reasoning. To evaluate changes in teachers' understanding and perceptions, we administered a 12-item pre/post questionnaire combining Likert-scale and open-ended items, focusing on: (a) conceptual understanding of spatial reasoning, (b) perceived relevance to mathematics teaching, and (c) confidence in using spatial tasks. Table 1 summarizes the lesson content, spatial focus, and PCA integration across grades.

Table 1

Summary of research lessons by grade, spatial focus, and PCA integration

Lesson Title	Grade	Spatial Focus	PCA Integration
<i>Building 3D Shapes</i>	Grade 1	Spatial visualization, composition	Concrete: block-building tasks; Pictorial: drawing 3D shapes; Abstract: comparing shape properties
<i>Mapping the Classroom</i>	Grade 2	Perspective-taking, spatial orientation	Concrete: physical movement; Pictorial: sketching bird's eye maps; Abstract: directional language and map reading
<i>Transforming Patterns</i>	Grade 3	Symmetry, rotation, translation	Concrete: using pattern tiles; Pictorial: drawing transformations; Abstract: describing transformation rules algebraically

Results

Data were collected from three sources: (a) written teacher reflections during and after PD sessions, (b) transcripts from end-of-program interviews, and (c) classroom observation notes from two Lesson Study (LS) cycles. Using Braun and Clarke's (2006) six-phase thematic analysis, we identified themes corresponding to each research question.

Findings for RQ1: *How does a PD focus on spatial reasoning influence teachers' perspectives on mathematics teaching and their use of spatial tasks?*

Two themes emerged reflecting changes in teachers' beliefs and understanding:

1. Shift from Procedural to Conceptual Instruction

Initially, most teachers emphasized procedures, repetition, and accuracy. Through the PD and LS activities, they redefined their roles as facilitators of reasoning and exploration. Lesson plans and observations showed increased use of open-ended questions, student dialogue, and spatially rich tasks. One teacher noted, "*I now see my role more as facilitating thinking rather than giving answers.*"

2. Broader Conceptions of Spatial Reasoning

Only 22% of teachers initially recognized spatial reasoning as part of their practice. By the end, most described it using terms like mental rotation, transformation, and spatial language. They incorporated strategies such as manipulatives, drawing, and spatial prompts (e.g., "*What does this look like from another angle?*"). However, barriers remained, including time constraints, limited resources, and entrenched views of mathematics as abstract and symbolic.

Findings for RQ2: *How does the incorporation of strategies such as the PCA approach impact teachers' instructional practices and student mathematical communication?*

Two additional themes reflected instructional and student-level shifts:

1. Scaffolded Learning and Student Engagement

Teachers reported that the PCA model improved lesson structure and accessibility. Beginning with hands-on or visual activities helped students build mental models, boosting confidence with abstract concepts. In the “3D Shapes” lesson, for example, students progressed from building with cubes to sketching perspectives and discussing shape properties using formal language.

2. Improved Mathematical Communication and Metacognition

Students became more articulate in expressing and evaluating their thinking, especially during LS reflections. Teachers observed greater use of reasoning language and peer discussion. As one noted, “*My students started asking ‘Why does this work?’ and challenging each other’s ideas.*” PCA-based instruction, combined with collaborative lesson design, supported these outcomes and deepened teachers’ pedagogical insight.

Discussion

This study examined how a PD program supported 18 primary mathematics teachers in embedding spatial reasoning into their instructional practice. Findings highlight the potential of spatially focused PD to meaningfully shift teacher beliefs, classroom practices, and student engagement.

In response to RQ1, the analysis revealed a clear move from procedural to conceptual teaching, with teachers increasingly positioning themselves as facilitators of reasoning and exploration. This pedagogical shift reflects prior research showing that engagement with spatial tasks enhances teachers’ appreciation of spatial reasoning as integral to mathematical thinking (Mix & Cheng, 2012; Lowrie et al., 2017). Teachers broadened their understanding of spatial reasoning—from viewing it as simple visual support to recognizing it as a cognitive process involving mental rotation, transformation, and perspective-taking—aligning with a broader movement toward inquiry-based and student-centred instruction.

For RQ2, the PCA model played a central role in scaffolding complex concepts and supporting student learning. Teachers reported that structured progression from concrete to pictorial to abstract stages helped students build confidence, communicate ideas clearly, and persist with challenging problems. These findings echo research linking spatial instruction with enhanced mathematical reasoning and problem-solving skills (Casey et al., 2011; Clements & Sarama, 2011). Moreover, the Lesson Study (LS) framework amplified these effects by offering collaborative, iterative spaces for teachers to refine instruction, respond to student needs in real time, and integrate new strategies into their teaching.

Taken together, the results suggest that PD models combining conceptual input, hands-on experience, and peer collaboration—anchored in classroom practice—are particularly effective in promoting sustained instructional change. Teachers not only developed a stronger conceptual grasp of spatial reasoning but also enacted practical strategies that improved student communication and metacognitive awareness.

Despite these promising outcomes, the study has limitations. Conducted in a single school with a homogeneous group of experienced teachers, the findings may not generalize across diverse educational contexts. Future research should explore how spatially focused PD performs across varied school settings, teacher profiles, and cultural contexts. Longitudinal studies would also help assess the lasting impact on student learning outcomes.

Nevertheless, this study demonstrates that integrating spatial reasoning into PD—through structured frameworks like PCA and LS—can meaningfully reshape mathematics instruction and deepen students’ engagement and understanding.

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Mathematics, coding, teacher education: Lessons learned

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With the increasing integration of coding (and computational thinking) in curricula around the world, there is a pressing need to better understand how to prepare in-service and pre-service teachers to integrate it meaningfully in the classroom. In this paper, we build on recent research to reflect on the project-based approach taken at our institution since 2017 whereby future mathematics teachers (a) learn to use programming as an “object-to-think-with” for mathematical investigations, (b) reflect on their learning experiences guided with selected readings, and (c) conclude with a collaborative lesson implementation with local in-service teachers.

Keywords: Coding, mathematics teacher education, mathematical investigations

Developing Teachers’ Abilities and Understanding of the Relevance of Coding for Math

Future secondary mathematics teacher Jonathan, reporting on his collaborative lesson planning and classroom implementation with a peer and two Grade 6–7 teachers, noted:

After completing this project ... I am beginning to be convinced that coding belongs in the mathematics curriculum. ... I am beginning to feel as if coding in math is something that is actually effective, and not something that just sounds like a nice idea in the literature. I am excited to do more things like this in the future to see what it looks like to integrate coding with math in other subjects and if it is as seamless and effective as we found our activity to be. (Jonathan’s report, April 2025; our emphasis)

We ask: how can we prepare future teachers to perceive the relevance of coding for mathematics and to feel confident and competent to integrate it into their future mathematics classrooms? This question is pressing given the current context that we briefly elaborate next.

Coding and Computational Thinking in Compulsory Curricula

Coding for (math) learning has a long legacy through Papert’s (1980) work. However, it is only in the last two decades that several countries and jurisdictions have integrated “computational thinking” (CT) or coding as part of their mandatory school curricula, recognizing it as an essential competency of the 21st century (Broley et al., 2023) e.g. as advocated by Wing (2006). The curricular integration of CT and coding has taken various forms—as a separate curriculum (e.g. in England), as a transversal skill across all subjects (e.g. in Sweden), or as part of the mathematics curriculum (e.g. in Australia and in our jurisdiction, Ontario, Canada: Ontario Ministry of Education [OME], 2020). This has created the need in the latter jurisdictions for math teachers to develop pedagogical competencies to teach and integrate coding and CT.

Recent literature reviews highlight, however, that our understanding of math teacher training in CT and coding is still emerging (Broley et al., 2023). For example, Vinnervik (2022) found “a sense of inadequate preparation among the teachers, with concerns being ... their knowledge [and] confidence in [coding] and a lack of teaching approaches that go

beyond fun one-time events” (p. 36), while Maugesten et al. (2023) found that future math teachers focused more on coding for motivation than for students’ math competence. This provides context for our question. In practice, as these conversations and curricular changes were emerging in the mid-2010s, our math department adapted some of its mathematics content courses for future teachers, which we describe in the next two sections.

Using Coding for Pure and Applied Mathematical Investigations—Brock’s MICA I-III

In the Mathematics Integrated with Computers and Applications (MICA) I-II-III courses offered at our institution (since 2001), mathematics majors and future secondary mathematics teachers learn to use programming in order to investigate mathematics conjectures, theorems, concepts, and real-world applications (Buteau et al., 2015; Muller et al., 2009). They complete four or five projects per MICA course, in a weekly 2-hour lecture and 2-hour lab format, and currently use Python language. For example, in their first project assignment, MICA I students formulate or select a conjecture on prime numbers or hailstone sequences and create a program to explore it. This project has been effective due to its “low floor high ceiling” (Papert, 1980) approach that recognizes most students (until recently) start MICA I with no or little programming background. Another example of a project assignment, sometimes in MICA II, is the creation of a program to explore and simulate a prey–predator population model (Lotka–Volterra system of differential equations).

Each MICA course ends with a “passion project” on which students work usually in pairs and on a topic of their own choosing. For example, MICA II students Kylie and Matthew created a program to investigate whether it is better to walk or run in the rain; MICA II student Adam created a program to investigate the inner symmetries of the generalized Mandelbrot set; and in MICA I, future teacher Jessica created an interactive learning environment (including games) for a fictive school student to learn about negative numbers. Such project aligns with Papert’s vision to let students engage in topics that are relevant to them (personally and/or to the community). In fact, in Broley et al. (2022b), we found that, when asked about the MICA project from which they learned the most, more than half the surveyed MICA students identified one of their passion projects, explaining their choice with their rich learning experience (math knowledge, programming skills, and non-cognitive skills).

Overall, the approach in MICA courses is to use programming as an “object-to-think-with” (Papert, 1980), whereby learners refine and expand on their mathematical work through dynamic modification of, and feedback from, their computer program. All MICA projects integrate programming in a way that is essential to the mathematical work at hand (Buteau et al., 2019). As advocated by Papert (1980), students’ learning outcomes through experiences such as in MICA include not only problem-solving skills and new mathematics knowledge but also resilience and other important non-cognitive skills (Buteau et al., 2025). MICA courses’ overall teaching approach also aligns with Papert’s view that teachers should adopt a facilitator role and mainly provide the context for discovery to happen (Buteau et al., 2019). In addition, in Broley et al. (2022a) we found that students considered the most effective teaching aspects for supporting their learning in MICA courses to include individual help in lab sessions; different one-on-one interventions to help them with (math) coding bugs or

misconceptions; and a non-judgmental class atmosphere. Since 2017, the third MICA course was split into two sections, one of which (labelled MICA III*) was designed for future secondary mathematics teachers, which we next focus on.

Brock's MICA III* for Future Teachers: Toward the School Classrooms

To prepare future teachers to integrate coding in their math classrooms, different activities and complementary foci were incorporated in this third MICA course. Coming from MICA I-II, future teachers start this course having developed some skills of coding for math; however, none have experienced coding in their own school math education and most have had no teaching experience in a school classroom. As such, the approach taken in this course includes three components that complement the four MICA math investigation projects: C1) after each project, a guided reflection on their own experience to develop an understanding of the relevance and their learning of math coding; C2) selected readings and activities about mathematics coding in the school (math) classroom to develop initial practical skills for school implementation; and C3) a classroom implementation, in collaboration with local teachers (final project).⁶ We now highlight five aspects of the course that we see as crucial in our approach, and in connection to these three components above.

Develop an Awareness of Papert's "Object-to-Think-With" Concept in Their Own MICA Project Engagement [C1, needed in C3]

MICA III* students complete four mathematics investigation projects, such as using Python to explore the empirical probability of the problem that a random chord in a circle is longer than the side of an inscribed equilateral triangle, and to solve it theoretically. After each project, students are asked to reflect on aspects of the mathematics coding activities and of their learning experiences. For example, after having completed this probability project while coming across the usually upsetting finding that there seems to be multiple answers to this problem (try it! $1/3$? $1/4$? $1/2$?), future teachers are asked to reflect:

Comment on the ways in which programming was useful to your solving of this mathematical problem (indicate ... one or more affordances [of programming for math learning, reading: Gadanidis et al., 2016] that contributed to it). Do you think you could have solved it without the use of programming? Explain.

This supports future teachers' understanding of the added value of using programming in such mathematics context. Furthermore, future teachers are guided to reflect on the learning environment that enabled them to complete their mathematics coding investigation (e.g., they are asked to "identify a 'key' strategy used by the instructor, TA, or a peer helper to support [their] completion of the project and the ways in which it supported [their] work or learning").

Program in a Friendlier Programming Environment for School: Scratch [C2, needed in C3]

The course starts with a 2-hour lab (school) mathematics coding activity in Scratch that aims to introduce future teachers to this coding environment widely used in schools. This is done through a "turtle geometry" approach (Papert, 1980); the lab includes geometric

⁶ For MICA III* course material, including assignment guidelines and readings, see <https://ctuniversitymath.ca/teacher-education-resources/>

constructions (and/or puzzles) of a square, triangle, circle, flower of circles, spirals, and stars. As MICA III* future teachers easily transition to this programming environment, they are prompted to reflect right away on the mathematics coding (e.g., by describing the different coding concepts for math; reading: Brennan & Resnick, 2012), including considering it as another representation of mathematical concepts. Importantly, this lab enables future teachers to envision (for the first time) how coding can be brought in the school classroom, starting in early grades as found in our school mathematics curricula (ME, 2020).

Learn About Mathematics Coding Implementation Approaches, Including Use-Modify-Create Activity Format [C2, needed in C3]

With no experience of coding in their own school math education, future teachers only have one reference for its integration, namely from MICA I-II. Yet the MICA approach is slow and gradual from the ground up, as MICA courses are designed and dedicated to use coding as an “object-to-think-with” for mathematics learning—an approach that, in our view, is not realistic in the school classroom. As such, different math coding implementation approaches are presented during lectures, including the *Use-Modify-Create* coding activity format (reading: Waite & Grover, 2020) with a few illustrations, and which students also experience through a probability coin problem simulation activity, in preparation of their random chord project. This is emphasized because future teachers are expected to integrate this coding activity format in their final project to focus on the math (rather than the coding). It further includes the *4 P’s for Creative Learning*, *Universal Design for Learning*, and *Culturally Relevant Pedagogy* for coding classroom implementation with supportive readings. While the latter are more general implementation approaches, it also includes specific and concrete advice to teachers and principles about integrating and teaching mathematics coding in their school classroom (reading: Gadanidis, 2022, pp. 1–6).

Explore Good School Math Coding Activities in Scratch and Python [C2,3]

It is particularly important to expose MICA III* future teachers to good school mathematics coding activities since they have no reference from their school education; even after MICA I-II, future teacher Nadia (pseudonym) noted at the start of the course: “I do not feel as of right now I am equipped to teach any [math] programming for Grades 6–12 as I ... would not know how to make a lesson out of it/teach it to young students.” Examples of good mathematics coding activities that focus on “bringing the math to life through coding” are presented and discussed in lecture and/or experienced in the labs, particularly before future teachers meet with their local teacher partners for the final project. Selected activities are taken from different resources; for example, the Grades 6–10 activity in Scratch to visually and algebraically explore polygonal numbers (Gadanidis, 2022) and the Grades 7–8 perimeter and area activity in Python (Gadanidis, 2021), both of which following a use-modify-create format (see Fig. 1). As a designer and instructor of MICA III* and instructor of MICA I for over 20 years, I (Buteau) find this aspect of selecting rich, concrete school mathematics coding activities that connect well with the Ontario school mathematics curricula to be the most challenging.

Experience Math Coding with a Teacher, and See it Through Students’ Eyes [C3]

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Developing learning and life skills through mathematics assessment

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Assessment in the core mathematics module for first-year engineering students (155+ students) was redesigned through use of peer review, self-assessment and reflection. These skills are valuable more generally for lifelong learning. As part of the redesign students keep a log book in which they attempt a worksheet each week before the feedback session. They review peers' work in groups in the feedback sessions following a provided solution. After each feedback session, they write some reflective notes in their log book. The log books form 40% of the assessment with the remainder through an exam. The regular structure helped keep students on schedule and the frequent practice meant they were well prepared for the exam. The peer review was beneficial regarding layout of solutions and alternative methods and background reflective processes associated with developing independent and self-regulated learning skills were evident. There is some evidence of positive impact on student confidence.

Keywords: Mathematics, continuous assessment, log book, peer review, reflection

Introduction

The role of mathematics in STEM education can be enhanced by interdisciplinary approaches such as the integration of subject knowledge alongside development of twenty-first century skills (Maass et al., 2019). Such skills include creativity, critical thinking, problem solving, collaboration and self-direction (e.g., Partnership for 21st Century Skills, 2002). This includes working productively with others and monitoring one's own understanding and learning needs; these skills are of particular relevance to this paper, which describes the redesign of a core mathematics module for first-year engineering undergraduates in Queen's University Belfast with the aim of building student confidence through the use of peer review, self-assessment and reflection.

The literature shows that the greater immediacy, frequency and volume of peer feedback compensate for any lack in quality compared to teacher feedback. Dochy et al. (1999) summarise positive effects of self-, peer, and co-assessment and state that their impact on the learning process is the main reason for using them. Falchikov and Goldfinch (2000) conducted a meta-analysis of 48 quantitative studies that included comparisons of marks awarded by peers and teachers. Better correspondence between peer and teacher marks occurred when a global judgement based on well-understood criteria was required rather than marking several individual dimensions. Nicol and Kushwah (2024) argue that students are generating feedback all the time when they compare their current knowledge against some reference information. Moon (2004) contends for the expansion of reflective activities during higher education given their influence on meaningful learning and on the development of learning behaviour. From a simple human perspective, making friends and feeling a part of the degree programme are necessary bases for academic success (Thomas et al., 2017).

This paper describes the module. It provides the rationale for the assessment redesign, this new model being in place since the academic year 2016/17. The methodology for evaluating the effectiveness of the new assessment is presented and the findings are analysed.

Background

Mathematics 1 is a core module (20 CATS points) for first-year aerospace and mechanical engineering students. Teaching takes place over 25 weeks with a two-hour lecture and one-hour feedback session per week. A formal teaching style is generally employed in lectures with effort made to present the material in a logical manner. Numerous worked examples are included to help reinforce the material being taught while typical engineering applications illustrate the usefulness of mathematics to engineers.

The class size has ranged 155 – 195 since 2016. Most have come through the UK A-level system with 52% having at least grade A in A-level Mathematics during the period 2016-2020; this proportion has increased to 75% during 2022-2025. On average, 7% of the A-level students have completed Further Mathematics.

Assessment Redesign

A key motivation for the changes was to provide additional learning opportunities for students of all abilities and backgrounds. Also, the University's education strategy proposed a greater focus on continuous assessment and less reliance on examinations. With these goals in view, the delivery and assessment of Mathematics 1 was redesigned for 2016/17.

The redesign is focused on the feedback session and builds on the principles of time on task, cooperation among students and prompt feedback (Chickering & Gamson, 1987). Academic learning time is related to quality and refers to the time students are actively and successfully engaged in tasks of appropriate difficulty. Students keep a log book in which they attempt a worksheet each week before the feedback session. This ensures engagement with tasks on a weekly basis. They work in groups of four in the feedback sessions where a postgraduate student demonstrates the solutions and students review each other's work and leave feedback alongside. Thus, feedback is provided weekly (frequency) in the feedback session from other students and while they still remember what they were doing (timely). Nicol et al. (2014) place feedback in the context of peer review, defined as an arrangement whereby students evaluate and make judgements about the work of their peers and construct a written feedback commentary. Producing feedback was recognised as just as valuable for learning as receiving feedback. While the latter alerts students to deficiencies in their work, the former engages students in multiple acts of evaluative judgement and applying criteria. This reflective process, involving comparison of own work with others', is a defining feature and key benefit of peer review (Nicol et al., 2014). The format is low risk for the students: if they find the tasks easy, they benefit from the teamwork in class, teaching others and learning to give feedback on their peers' work; if they find the work more challenging, they can receive help both in class and outside class through the School's peer-assisted learning (PAL) mentor scheme. Following experience during the pandemic, a blended approach is now used with half the feedback sessions in-person and half through the virtual learning environment (Canvas), a video-recorded solution replacing the live demonstration in this case. The in-

person groups were selected by the lecturer to ensure a balance of mathematical abilities, based on prior mathematical qualification, and usually stayed the same over the year. In the online environment submissions were assigned randomly for review so students were reviewing different peers' work each week. After each feedback session, students write a few bullet points of reflection in their log book. Written expression facilitates thinking about mathematics more deeply and may promote improvements in procedural and conceptual understanding (Kenney et al., 2014). Students were given a structure for how to review peers' work and were given prompts for what to include in their reflections.

The log books form 40% of the assessment with the remainder through an exam. The log book mark is based on a final reflective piece and the weekly reflections rather than the attempts at the worksheet questions. Attending at least 75% of feedback sessions and attempting at least 75% of worksheets is necessary to pass the module and it is necessary to pass both the log book and exam elements individually. Thus, it is intended to encourage deeper learning based on critiquing of peers' work and aiming for reflection and self-assessment of own work. Students will be practising skills which they need to pass the summative assessment. This approach also provides a framework for productive use of the allocated independent study time (128 hours) in the module.

Evaluation

Impact of the changes can be represented by improvements in the student experience, students being better prepared for second year and being more confident in their academic ability. Data gathered to evidence impact has come through mid-module evaluation, focus groups and post-module questionnaires. While the broad structure of the redesign was established, it was expected to make iterative changes to the module over time. Mid-module evaluation using open-ended questions can provide rich feedback allowing lecturers to make adjustments to benefit the students while still on the module (Sozer et al., 2019).

In 2016/17, two mid-module surveys were conducted in weeks 5 and 18 to obtain students' views on the new aspects of the module and a measure of their confidence to pass. Focus groups to provide further elaboration about the module were held in week 23 of 2016/17 with approximately 35 students from five sub-groups (Further Maths, A-level Maths grade A/B, no A-level Maths, international and resit students) with potentially different needs. A mid-module survey with similar questions to the previous ones was issued to the class in week 12 of 2017/18. Focus groups to investigate any longer-term impact were held in 2020 with nine third- and fourth-year students who had taken the module in 2016/17 and 2017/18. Focus group participants were selected by purposeful sampling. A pre-prepared list of questions was used and the answers were recorded. In 2016/17, the groups were facilitated by PAL mentors (more senior students) and in 2020 facilitation was by an educational developer.

Most respondents recognised the relevance of the skills implemented into the module (Table 1). The proportions were higher in 2017/18 indicating that the lecturer had improved his explanation of the module design to the class. A large proportion of students were confident that they would pass the module (Table 1).

Table 1

Mid-module survey results in 2016/17 and 2017/18 showing percentage of respondents who agree with statement

	16/17 wk 5	16/17 wk 18	17/18 wk 12
I understand why:	n = 132 (74% of class)	n = 106 (59% of class)	n = 99 (59% of class)
reflection skills are important to master	53	48	65
peer assessment skills are important to master	55	49	66
self-assessment skills are important to master	64	57	80
teamwork skills are important to master	80	59	67
Are you confident that you will pass this module?	n = 140 (78% of class)	n = 113 (63% of class)	n = 103 (62% of class)
Very confident	26	23	28
Confident	69	71	63
Not confident at all	5	6	9

The focus group in 2016/17 showed that students considered weaker (no A-level maths, resitters) reported improvement in their mathematical understanding, confidence and organisation. The peer review encouraged students (all sub-groups) to put more effort into clarity and presentation of their work. There was some dislike of the reflective commentary, despite the evidence showing they appreciated its relevance; its format was changed for the following year. Students wished to retain their log books for future reference.

The free-response questions in the mid-module surveys give evidence of a positive student experience. When asked what aspects of the log books and feedback sessions were useful and/or enjoyable, there were 87 responses in 2017/18 and they were collated by thematic analysis (Figure 1). Receiving feedback was the most common response. Reference to peer review may involve both giving and receiving feedback. The second most common response involved viewing other methods, which hints at the self-evaluation implicit within peer review. The theme of personal development includes meeting others in the class and greater confidence. Only six students said they disliked the setup or found no benefit from it.

The following quotations give some depth and authentication of student experience.

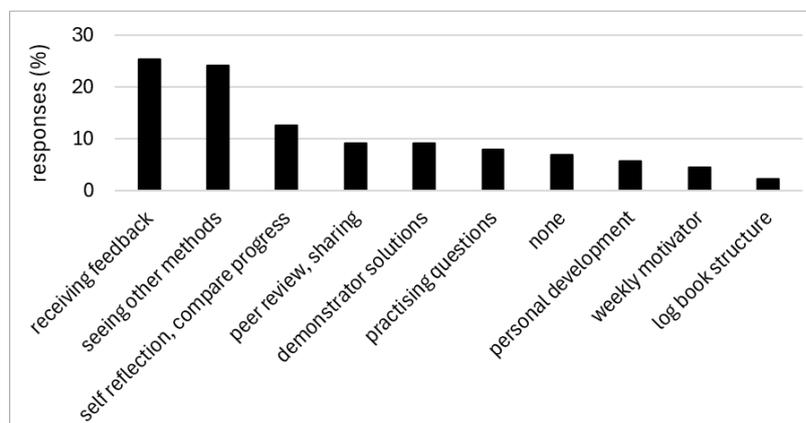
Student A: The fact that there is work that clearly gauges where I am at with the topic and how I need to improve.

Student B: Provides a way to talk to others so you can find out if you are the only one struggling with something or not.

Student C: By evaluating my peers' work I found that their more thorough method and clearer notations helped them reach the correct solution.

Figure 1

Student responses (n = 87, 52% of class) when asked what aspects of the log books and feedback sessions have been useful and/or enjoyable (2017/18)



In the focus group with third- and fourth-year students, they believed that the log books were successful in terms of putting them in the right frame of mind to work regularly and keep up to date. They agreed that they were well prepared for the exam due to the repetitive nature of the worksheets and frequent practice. The work getting progressively more difficult was noted. Working in groups was helpful for first years trying to settle into university. Furthermore, it was comfortable to ask for help in the group and an explanation from someone at the same stage of learning was beneficial. Students were less convinced by the requirement to write a weekly reflection, believing that it became repetitive and could be difficult to identify things to change. They suggested not having a word count and having a few pointers on content. There was some realisation of voluntarily implementing a similar structure of working in later years of the degree although some thought that the maths log book and weekly worksheet involved a relatively large effort. However, the log book was considered a particularly useful reference source given that maths is core to many other modules. There was some impact on confidence. This included the feeling of progressing at the same speed as everyone else and building a foundation from the repeated practice.

Students generally expressed positive attitudes toward the peer review process, aligning with existing literature in context of clear guidance being provided and peer grading not required (Nicol et al., 2014). Many student responses referenced aspects beyond simply receiving feedback – such as exposure to alternative methods and opportunities for reflection – which are processes commonly linked to high academic achievement. This suggests that the redesign holds promise for fostering independent learning (Nicol et al., 2014). A key feature of peer review, the reflective comparison of one’s own work with that of peers – a hallmark of active, self-regulated learning – was notably evident in our findings. Interestingly, students not only compared mathematical methods but also their own progress or struggle relative to others. Our implementation further encouraged students to consolidate their reflections through writing. While reflective writing is well established in many disciplines, it remains relatively uncommon in mathematics education (Kenney et al., 2014).

Conclusions

The assessment in a first-year engineering mathematics module has been successfully redesigned to allow for various abilities by including peer review and reflection on a weekly worksheet. There were benefits regarding mathematical procedure and self-regulated learning skills. Future work may involve further analysis on the longer-term impact of the redesign.

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Examining the levels of cognitive demand in Junior Cycle mathematics examinations

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The Junior Cycle Mathematics curriculum was introduced to schools in September 2018, replacing the previous Junior Certificate programme. Its implementation raised concerns among key educational stakeholders, who feared a potential decline in academic rigour (e.g. McGarr et al., 2022). This study investigates whether the levels of cognitive demand in examination tasks changed following the curriculum reform. A total of 1,257 examination tasks from Higher and Ordinary Level papers were analysed using an adapted version of the Stein and Smith (1998) Mathematical Task Analysis Guide, as presented in O'Connor et al. (2019). Findings reveal little significant difference in the levels of cognitive demand at both Higher and Ordinary Level between the Junior Certificate and Junior Cycle examinations. Low-level cognitive demand tasks dominated the examinations at both Higher and Ordinary Level. The implications of these findings are discussed in relation to the reform's objectives, teaching practice, and student achievement.

Keywords: Junior Cycle reform, cognitive demand, examination tasks

Introduction

The Junior Cycle curriculum reform aimed to balance students' acquisition of knowledge with the development of independent thinking and problem-solving skills (NCCA, 2018). Across subjects, the reform was met with criticism from educational stakeholders, who expressed concerns about a perceived decline in academic rigour (McGarr et al., 2022). This study explores these concerns within the context of Junior Cycle Mathematics by analysing the cognitive demand required of students in Junior Certificate and Junior Cycle Mathematics examinations using an adapted version of the Stein and Smith (1998) Task Analysis Guide, as presented in O'Connor et al. (2019). Our aim is to address the following research questions: (1) What levels of cognitive demand are evident in Higher and Ordinary Level Mathematics examinations?, and (2) To what extent have these levels changed following the reform?

Literature Review

Cognitive demand can be described as “the kind and level of thinking required of students in order to successfully engage with and solve the task” (Stein et al., 2009, p. 1). Stein et al. (2009) posit that if the goal of learning is to think, reason and solve mathematical problems, then students need to engage in cognitively challenging tasks. The cognitive demand of tasks is intrinsically linked to the thinking students must do to complete the task. While there are several frameworks for analysing cognitive demand of tasks in mathematics, this study uses the Stein and Smith (1998) Task Analysis Guide, which categorises tasks as being of low-level or high-level cognitive demand. Low-level tasks involve memorisation or applying known procedures *without* connections to underlying concepts. High-level tasks involve applying known procedures *with* connections to underlying concepts or tasks that

require students to ‘do mathematics.’ The adapted version of this framework, as presented in O’Connor et al. (2019) is shown in Figure 1, and includes additional descriptors for these four categories of tasks.

In recent years, several studies have examined the cognitive demand of Irish mathematics examination papers. Burns et al. (2018) reviewed the Leaving Certificate examination papers from 23 subjects from 2005 to 2010 using Bloom’s Taxonomy. For the Leaving Certificate Mathematics examinations, over 90% of questions were classified as “Apply”, a low-level cognitive skill that is based on executing learned procedures, indicating a strong emphasis on procedural performance. O’Connor et al. (2019) used an adapted version of the Stein and Smith (1998) Task Analysis Guide to investigate the levels of cognitive demand in Higher-Level Leaving Certificate examinations before and after the Project Maths reform, and found an increase in the level of cognitive demand tasks after the reform. Goodwin (2024) further built on this work by using the adapted framework to analyse Higher- and Ordinary-Level Leaving Certificate examination papers from 2018-2023.

Two studies focused on the cognitive demand of Junior Certificate mathematics examinations. Cunningham et al. (2017) examined the impact of the Project Maths reform on these examinations using the PISA and TIMSS Mathematics Frameworks. They found that while Project Maths had some influence on increasing the cognitive demand, lower-order processes still dominated. Garry et al. (2020) examined three Junior Certificate Mathematics papers using Lithner’s (2008) Creative Reasoning Framework, finding that most tasks in the examination required imitative, rather than creative, reasoning. Neither study included papers from the Junior Cycle mathematics curriculum.

Methodology

Junior Certificate papers from 2015-2019 and Junior Cycle papers from 2022-2024 were analysed. In addition, the 2021 SEC sample papers were included to provide additional insight into the Junior Cycle examinations. For the Junior Certificate years, each examination comprised two papers (Paper One and Paper Two), which were treated as a single examination for the purposes of this study. All questions from these papers were analysed, with each part of a question (e.g. (a)(i) or (b)(iii)) considered a separate task. In total, 1,257 tasks were examined. Where available, marking schemes were used to determine the proportion of total marks allocated to each task.

Each task was coded using the framework shown in Figure 1. This is an adaptation of the Stein and Smith (1998) Task Analysis Guide, modified by O’Connor et al. (2019) to make it suitable for coding examination tasks. The first author met with Goodwin who had just completed a study using the same framework (Goodwin, 2024) for advice on its use. She then coded all the tasks, with the second author verifying the coding for one year of papers. The marking schemes were also used as support material to assess the level of cognitive demand evident in a task. Certain codes were not used in the task analysis. The code M1 applied to every task, as they all involved recalling some prior knowledge. The code M2 cannot be applied to examination tasks, as they are set to be solvable within the examination time constraints. The code P4 is not relevant to this context, as the examination tasks focus on

assessing knowledge, rather than developing it. P5 was also excluded as examination tasks rarely ask students to explain their answers, and when they did, this was treated as an increase in cognitive demand. When tasks satisfied descriptors from multiple categories, the task was classified by the highest level of cognitive demand present. Once coding was completed, the proportion of tasks and the proportion of marks assigned to each level of cognitive demand was calculated. Welsh's t-test at a 5% significance level was performed to investigate if the levels of cognitive demand were significantly different following the reform.

Figure 1

Adaptation of Task Analysis Guide (Boston & Smith, 2009) from O'Connor et al. (2019)

Low-Level Cognitive Demands	High-Level Cognitive Demands
<p>Memorisation Tasks</p> <p>M1. Involve either producing previously learned facts, rules, formulae, or definitions or committing facts, rules formulae, or definitions to memory. <i>This descriptor was automatically assumed to be present in each task because every task requires the production of previous knowledge.</i></p> <p>M2. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.</p> <p>M3. Are not ambiguous- such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.</p> <p>M4. Have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned or reproduced.</p>	<p>Procedures with Connections Tasks</p> <p>PC1. Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. <i>This descriptor was applied if the task highlighted a link to students between procedures and underlying concepts, or if the task required students to notice a concept based on the repeated use of procedures.</i></p> <p>PC2. Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.</p> <p>PC3. Usually are represented in multiple ways (e.g. visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. <i>This descriptor was applied if the task included any additional representations of the initial question.</i></p> <p>PC4. Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding. <i>In this descriptor, the idea of not following the procedure mindlessly was focused upon. This descriptor was used if a student was required to use familiar procedures but it was not obvious that the procedure was required from the task or prior experience.</i></p>
<p>Procedures without Connections Tasks</p> <p>P1. Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. <i>In this descriptor experience was taken to be the students' previous experience of completing fundamentally similar tasks in class or previous exam papers.</i></p> <p>P2. Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. <i>In this descriptor it was taken that the lack of ambiguity could come from the fact that the student would have completed many similar questions in class.</i></p> <p>P3. Have no connection to the concepts or meaning that underlie the procedure being used. <i>This descriptor was used infrequently as very few tasks were completely unrelated to any concept or meaning.</i></p> <p>P4. Are focused on producing correct answers rather than developing mathematical understanding.</p> <p>P5. Require no explanations or explanations that focus solely on describing the procedure that was used.</p>	<p>Doing Mathematics Tasks</p> <p>DM1. Require complex and non-algorithmic thinking (i.e. there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).</p> <p>DM2. Require students to explore and to understand the nature of mathematical concepts, processes or relationships.</p> <p>DM3. Demand self-monitoring or self-regulation of one's own cognitive processes.</p> <p>DM4. Require students to access relevant knowledge in working through the task. <i>This descriptor was used when a task contained multiple elements or topics and students were required to use relevant knowledge from a variety of areas in mathematics.</i></p> <p>DM5. Require students to analyse the task and actively examine task constraints that may limit possible solution strategies and solutions.</p> <p>DM6. Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process.</p>

Note. Adaptations made by O'Connor et al (2019). are highlighted in bold and italics.

Findings

For the Higher-Level Examination papers, tasks coded as ‘Procedures Without Connections’ dominated the examinations across all years at 56%, with an average of 3% of tasks coded as ‘Memorisation,’ 27% coded as ‘Procedures with Connections’, and 15% coded as ‘Doing Mathematics’ (see Figure 2). No significant difference was found between the distribution of marks and the distribution of tasks for each level of cognitive demand. The only statistically significant difference in the examinations before and after the Junior Cycle reform was in the ‘Memorisation’ tasks, with the proportion of these tasks decreasing.

Figure 2

Higher-Level examinations. Distribution of tasks by cognitive demand.

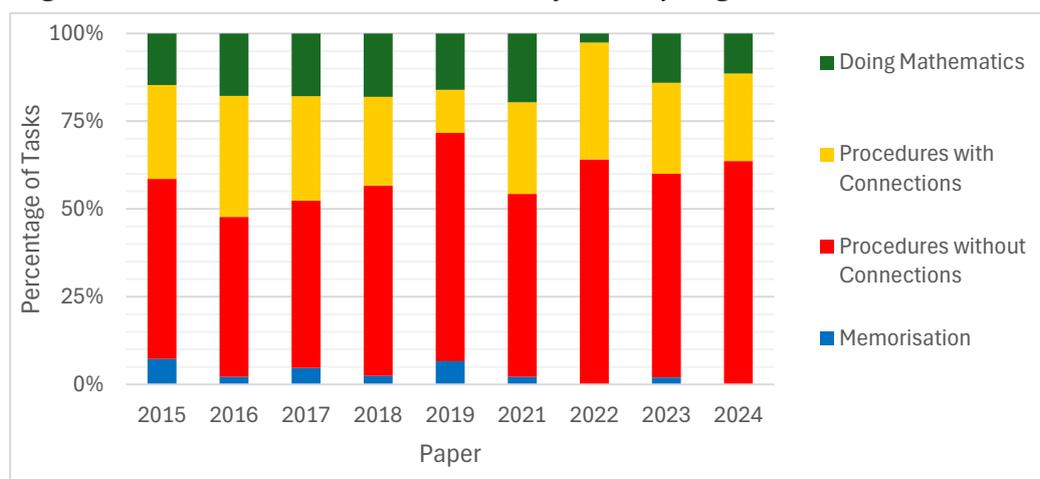
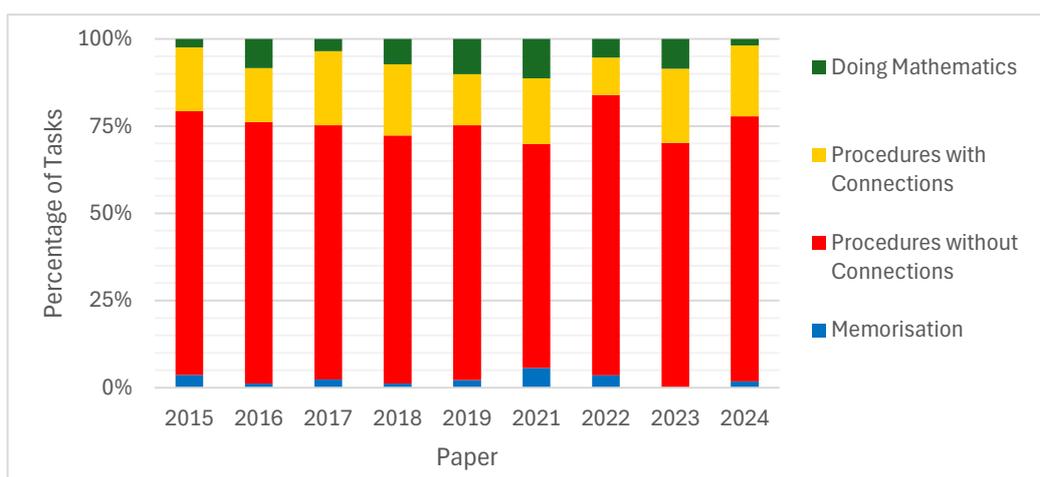


Figure 3

Ordinary-Level Examination papers. Distribution of tasks by cognitive demand.



In the Ordinary-Level Examination papers, tasks coded as ‘Procedures without Connections’ dominated the examinations across all years studied at 73%, with an average of 2% of tasks coded as ‘Memorisation,’ 18% as ‘Procedures with Connections’ and 7% as ‘Doing Mathematics’ (see Figure 3). A significant difference was found between the distribution of tasks and the distribution of marks for questions coded as ‘Procedures with Connections’, with these tasks receiving a disproportionately higher share of marks relative to

their frequency. No statistically significant difference was observed between the distributions of cognitive demand in the Ordinary-Level Junior Certificate and Junior Cycle examinations.

Discussion and Conclusions

The findings of this study suggest there is little significant difference in the level of cognitive demand students can expect in the Junior Certificate and Junior Cycle examinations. The only significant difference identified was a reduction in the number of ‘Memorisation’ tasks in Junior Cycle Higher-Level papers. This is likely due to the fact that students are no longer required to memorise and reproduce the geometric proofs that were on the syllabus for Junior Certificate Mathematics. This aligns with the broader aims of the Junior Cycle reform to move from rote-learning information in favour of conceptual understanding (NCCA, 2018).

The new Junior Cycle Mathematics specification introduced a new ‘Unifying Strand,’ designed to integrate key competencies across all content strands (NCCA, 2018). The learning outcomes associated with this strand range from recalling key concepts and demonstrating procedural fluency (U.1 – U.3) to more complex competencies such as representing and connecting ideas (U.4 – U.6), problem solving (U.7 – U.10), making generalisations and proofs (U.11 – U.12) and communicating mathematical thinking (U.13). It could be argued that these higher-order learning outcomes are not being adequately assessed in the final examinations, suggesting a possible misalignment between the intended goals of the curriculum and the focus of current assessment practices, which future work might address.

Across all examinations studied, many questions and marks were assigned to low-level cognitive demand tasks, particularly ‘Procedures without Connections.’ Our findings agree with those of Cunningham et al. (2017) and Garry et al. (2020) who also found that lower-level cognitive tasks dominated the Junior Certificate Mathematics examination papers they analysed. Introducing more high-level cognitive demand tasks in examinations could motivate teachers to incorporate deeper, more challenging learning into their lessons. However, it is important to recognise that procedural tasks make up a significant part of the curriculum and must therefore be assessed in the examination. It is also important to note that many of the tasks were considered ‘Procedures without Connections’ due to students’ prior experience with them in class. These may have required a high level of cognitive demand from the student during the initial knowledge acquisition phase, with the level of cognitive demand decreasing as students became more familiar with them.

Finally, this study highlights the cognitive demand required to achieve various grades in Junior Cycle Mathematics. In all examinations students could attain a passing grade through completion of low-level tasks alone. At Ordinary Level, students could achieve a C grade (Junior Certificate) or a Merit (Junior Cycle) without any engagement in higher-order thinking. While this may raise concerns about the challenge level of assessments, it also appropriately recognises and rewards students who have mastered key procedures. Procedural fluency supports moderate success, but top grades demand deeper, higher-order engagement.

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Engagement and equity in Irish post-primary school mathematics competitions: An analysis of participation in two competitions

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Mathematics competitions for school students can promote student engagement and enhance problem-solving skills. Several such competitions for students in post-primary schools in Ireland are organised by the Irish Mathematics Teachers' Association. Questions arise about equity: is school participation equitably distributed with regard to socioeconomic status (SES) and gender across different categories of school? The paper examines participation in two of the competitions over the years 2022-2025, with a focus on 2025. Increased engagement is evidenced by growth in numbers participating. However, fee-paying schools and boys' schools are overrepresented, while girls' and mixed schools and those catering for low-SES students are underrepresented. Recommendations are made to promote greater equity.

Keywords: Mathematics competitions, engagement in mathematics, equity

Introduction

Mathematics competitions are a feature of the mathematics education landscape. They can promote engagement with mathematics, develop problem-solving skills, provide appropriate challenges for talented students, and – depending on the format – encourage teamwork and cooperation (de Losada & Taylor, 2022; Soifer, 2017; Thrasher, 2008). The Irish Mathematics Teachers' Association (IMTA), which supports teachers in Irish post-primary schools (catering for students aged approximately 12 to 18), has organised such competitions since 1991. Typically, the competitions are for teams of students from schools in which their teacher is an IMTA member and organises their (voluntary) participation. The continued involvement of many schools over the years suggests that Irish teachers value the activity and at least some students find the competitions engaging and / or helpful. However, questions arise about equity. Is school participation equitably distributed across different categories of post-primary school, and is there gender balance among the student contestants?

The IMTA aims to promote engagement (that is, active participation in and valuing of mathematical activities; see Attard, 2012) for all post-primary students. However, in view of national concerns about performance and gender imbalance among the highest-achieving students – discussed below – the scope of this short paper is limited to two competitions that cater for students taking the most challenging (“Higher level”) Mathematics courses. *Team Maths* is geared to students in their final year of schooling, in which they sit for the Leaving Certificate examination, while the *Pi Quiz* is for those in the final year of the post-primary junior cycle. The aim of the paper is to trace participation in the two competitions over the years 2022-2025, examining 2025 in particular – profiling schools by socio-economic status (SES) and gender – to assess inclusivity, with a view to trying to mitigate imbalances.

Background and Context

Recent research on Irish mathematics education provides both encouragement and warnings. Encouraging aspects include the fact that Ireland’s mean scores in international tests of achievement are significantly above the averages for participating countries (Donohue et al., 2023; McHugh et al., 2024). Also, uptake of the Higher level Mathematics courses has increased greatly from a low base (National Council for Curriculum and Assessment, 2005; Society of Actuaries in Ireland, 2023). However, the increase for the Leaving Certificate coincided with, and is related to, the “bonus points initiative” that gives students achieving a satisfactory grade in Higher level Mathematics an advantage in applying for places on many courses at third level (O’Meara et al., 2020): thus, reflecting extrinsic motivation rather than genuine engagement (Attard, 2012). This warning sign is accompanied by others. In the international tests, while Irish means are high, the standard deviations are small; a comparatively low proportion of the students score at the highest levels. Moreover, among these high achievers, students from low-SES backgrounds are underrepresented (Donohue et al., 2023; McHugh et al., 2024; for context, see Weir & Kavanagh, 2018). A further concern is highlighted in a report by the Society of Actuaries in Ireland (SAI) (2023). It documents that, for the highest achievers both in international tests and in the State examinations, boys figure disproportionately and over the last 12 years this gender gap has generally increased.

The issues are complex and nuanced; discussion of the many factors involved is outside the scope of this paper, which focuses on contributions being made by the IMTA. IMTA membership is available to schools (hence, to their mathematics teachers) and also to individual teachers. A good indicator of the position of the Association nationally is given by school membership; Table 1 shows the number of post-primary schools, categorised by SES and gender, and the number of IMTA members in each category in Spring 2025. Schools designated as DEIS (“Delivering Equality of Opportunity in Schools”) – “deis” means “opportunity” in Irish – are those in the national scheme giving extra support to schools serving low-SES populations, whereas fee-paying schools typically have a high-SES intake.

Table 1

Number of Post-Primary Schools per Category and Number (%) that are IMTA Members

Category	All	SES				Gender	
		DEIS	Fees	Neither	Mixed	Boys	Girls
Overall ^a	724	232	52	440	518	92	114
Members of IMTA	418 (57.7%)	102 (44.0%)	42 (80.8%)	274 (62.3%)	274 (52.9%)	67 (72.8%)	77 (67.5%)

Note: ^aNumbers are taken from the Department of Education’s provisional October returns for 2024-2025 (<https://tinyurl.com/yc5m5cx6>); additionally, two “unrecognised” schools that are IMTA members are included.

Context for the investigation is set by outlining the history of the competitions. Team Maths was started in 1991. The format is that of a table quiz, with teams of four students contesting over eight six-minute rounds. The content is curriculum-related (providing revision for the Leaving Certificate, a selling point for students), though the style is not necessarily

familiar (thus, developing students’ problem-solving skills). The questions are intended to be graduated from easier to harder, aiming to allow all students to experience success but to separate the most able from the main body. Especially for the final two rounds, for which the time limit is challenging, team members typically need to cooperate. Initially, branches of the IMTA ran the competition at the same time but there was no national final. However, since 2005, branch winners have progressed to a national final. The Pi Quiz – geared to the junior cycle – has taken a similar path more recently, starting in 2009 and with the first national final being held in 2023. Further details of the structure and style of the competitions are omitted here; they appear on the IMTA website (<https://imta.ie>; see “Competitions”).

From 2022-23, with a view to facilitating greater engagement, the Council of the IMTA (of which the authors are members) made a series of changes to the organisation of both competitions. For the regional rounds, instead of having just one centre per branch, more centres were set up; the maximum number of participating teams per school was increased from one – or, in special circumstances, two – to four; the competitions were held during rather than outside school hours; and the entry cost was lowered. Rules for progressing to the finals were adjusted; instead of one team per branch (two for large branches), qualifiers were the winning team from each centre plus other high achievers, with the capacity of venues for the finals allowing for around 40 teams. It is appropriate to ask if aims have been met with regard to engagement and whether participation is equitable in terms of SES and gender.

Methodology

As the idea of writing this paper emerged only after the 2025 finals, quantitative data were restricted to those in the public domain (suitably anonymised) to obviate ethical issues. The data were compiled from IMTA publications and records and its website (<https://imta.ie>), and from national statistics. The numbers of schools and of teams entering each competition over the years 2022-2025 were tabulated; participation being voluntary, these figures serve as proxies for school and student engagement. For 2025, each participating school was classified by SES and by gender composition; schools winning Team Maths finals over the years were classified likewise. To investigate equity, distributions across the SES and gender categories were compared with the national distributions, using chi-square tests. Qualitative information was obtained from teachers’ feedback after the finals, with their permission for its use.

Findings

Table 2

Numbers of Schools (and Teams) Participating in the Regional Rounds, 2022-2025

Competition	2022	2023	2024	2025
Team Maths	81 (134)	97 (193)	140 (358)	190 (490)
Pi Quiz	n/a ^a (n/a ^a)	231 (590)	246 (751)	275 (875)

Note: ^aFigures not available prior to the introduction of the final round in 2023.

The numbers of participating schools and teams for 2022-2025 are shown in Table 2. For 2025, the classification in terms of school SES and gender for the regional rounds is

presented in Table 3. Table 4 displays the corresponding analysis for the final rounds (in which the numbers participating were limited by the size of the venues, as mentioned above).

Table 3

Number of Schools Entering the 2025 Regional Rounds, by Competition and School Category

Category	All	SES				Gender	
		DEIS	Fees	Neither	Mixed	Boys	Girls
<i>Team Maths</i>							
No. of entrants	190	22	31	137	113	41	36
% of entrants	100	11.6	16.3	72.1	59.5	21.6	18.9
% of category ^a	26.2	9.5	59.6	31.1	21.8	44.6	31.6
<i>Pi Quiz</i>							
No. of entrants	275	53	30	192	174	50	51
% of entrants	100	19.3	10.9	69.8	63.3	18.2	18.5
% of category ^a	38.0	22.8	57.7	43.6	33.6	54.3	44.7

Note: ^aSee Table 1 for category numbers.

Table 4

Number of Schools Progressing to the 2025 Finals, by Competition and School Category

Category	All	SES				Gender	
		DEIS	Fees	Neither	Mixed	Boys	Girls
<i>Team Maths</i>							
No. of entrants	37	2	17	18	21	12	4
% of entrants	100	5.4	45.9	48.6	56.8	32.4	10.8
% of category ^a	5.1	0.9	32.7	4.1	4.1	13.0	3.5
<i>Pi Quiz</i>							
No. of entrants	41	6	11	24	20	18	3
% of entrants	100	14.6	26.8	58.5	48.8	43.9	7.3
% of category ^a	5.7	2.6	21.2	5.5	3.9	19.6	2.6

Note: ^aSee Table 1 for category numbers.

Chi-square tests of goodness of fit confirm that all differences from the national distributions for SES and gender are highly significant ($p < .01$ or less). For SES, fee-paying schools are overrepresented and DEIS schools most underrepresented; for gender, boys'

schools are over-represented. Similar patterns appear from analysis of all Team Maths winners (19 finals, two years lost to COVID). Of these teams, 14 were from fee-paying schools and also 14 from boys' schools. No team from a DEIS or girls' schools has won, though there were girls on some winning teams from mixed schools. In 2025, the Team Maths winning team was from a mixed school, but all winners in both competitions were boys.

Comments from teachers after the 2025 finals provide points of interest. They reported that students enjoyed participation in a mathematical event (even if not scoring highly) as well as recognising its value in preparing for State examinations. Also, there was enough interest in some schools for teams to undertake training or practice with previous sets of questions.

Discussion

While this study is limited in scope and by its post-hoc nature, it offers initial findings worth exploring further, especially via qualitative research with students as well as teachers. Comparing features of Team Maths and the Pi Quiz with the advantages of competitions – engagement, problem solving, challenges for talented students, teamwork and cooperation – as highlighted in the literature cited above, the two IMTA competitions appear appropriate for at least some students. As students take part voluntarily, participation is deemed to indicate genuine engagement with mathematics, involving intrinsic motivation as well as useful revision. However, there are equity issues; not all students are afforded the opportunity to participate, while for some the competitive element and time pressure may not be suitable.

Participation in the regional rounds, as shown in Table 2, has increased dramatically alongside – and at least partly due to – the recent changes in organisation. However, an upper limit on school numbers is imposed by the requirement of IMTA membership; this is reflected in the figures, especially for categories in which the Association is underrepresented (Table 1). Notably for the finals (Table 4), serving high-scoring teams, the imbalance compared with the national distributions for SES and gender reflects the broader problematic issues identified above: overrepresentation of boys and those of high SES among the most successful students. Many fee-paying schools have a long history of involvement (via their teachers) in the IMTA; hopefully this will continue, but involvement from others could be encouraged and facilitated. For *suitability*, competitive activities do not appeal to everybody; research, while nuanced, points to girls being less well suited than boys (SAI, 2023; Thrasher, 2008). Girls' engagement might be enhanced by other approaches, like those already provided by Maths Week (<https://www.mathsweek.ie>) and Maths Eyes (<https://haveyougotmathseyes.com>).

Conclusion and Recommendations

The IMTA provides engagement with mathematics for a large, and latterly increasing, cohort of school students through its competitions Team Maths and the Pi Quiz, which serve those taking Higher-level mathematics courses. In a small way, this may even help to address the shortage of very high-achieving students in Ireland. However, equity issues appear; fee-paying and boys' schools are overrepresented by comparison with numbers nationally, while DEIS, mixed and girls' schools are underrepresented, especially in the finals.

Recommendations for the IMTA include increasing outreach to and support for DEIS, mixed and girls' schools; providing optional training materials for underrepresented or new

schools; enhancing rewards for participation; monitoring trends and publishing inclusive participation goals annually; and undertaking other activities to promote engagement equitably. For the wider mathematics education community, there is a need for more research on the roles of mathematics competitions and other extracurricular activities in Ireland.

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An investigation into mathematics study resources used by undergraduate students

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During the COVID-19 pandemic, many lecturing staff produced online materials and supports for their students, which they have continued to use despite in-person teaching. This paper reports on a study into which mathematical resources were used by undergraduate students when studying mathematics. An anonymous survey was administered to 691 mathematics and service mathematics students from a range of disciplines and year-groups in Dublin City University. Students were asked about the resources or tools they used when studying mathematics. The findings were that generative artificial intelligence was used by 91% of respondents; mathematics support services by 40%; and other resources by 35%. In this short paper, we focus on these “other resources”, breaking down where students are turning for further support of their learning. Of these, YouTube was by far the most popular, with six times more respondents using YouTube than the next most common resource.

Keywords: Mathematics education, higher education, generative AI, mathematics support

Introduction

In higher education, it is expected that students will spend time outside of formal lectures, tutorials or workshops studying the material in their programme. Lecturers often spend considerable time and effort developing lecture notes, alongside tutorial or homework sheets, videos or quizzes, to guide students in their study efforts. During the COVID-19 pandemic, many lecturing staff developed a range of online material and supports for their students, which they have continued to make available to students despite the return to in-person teaching. In addition, most higher education institutes in Ireland now also have mathematics support on offer to students, which they can use free of charge for additional help with their mathematics study. Particularly in the past year, there have also been huge advancements in generative artificial intelligence (AI). Therefore, it is timely to explore the types of resources that students of mathematics use when studying independently. However, a full exploration of student usage of either generative AI or mathematics support is beyond the scope of this short paper. As a result, the research question we wish to address here is:

RQ: Other than mathematics support and generative AI, what resources do undergraduate students report using to study mathematics?

Background

Researchers have been looking into the study resources selected by students for mathematics for quite some time, although results have differed in recent years, particularly due to the development of more online material as a result of the university closures during the pandemic (Ní Fhloinn & Fitzmaurice, 2021). Ní Shé et al. (2017) studied the resources favoured by first-year service-mathematics undergraduate students across four higher-education institutions in Ireland. A total of 460 students completed their anonymous survey.

Students were asked to list resources they found helpful under the following headings: books, handouts, videos, websites, or other. Handouts were considered helpful by the highest percentage of students, followed by websites and then videos. Similarly, Anastasakis et al. (2017) investigated the resource usage by second-year engineering students in the UK by surveying 201 students. They found that the most popular resources were those provided directly by the university (whether in the form of notes, handouts or a website), but there was evidence of usage of resources such as Wolfram Alpha or videos as well. Göller (2021) undertook a small-scale study in a German university, interviewing 18 mathematics students up to four times during their first year of study, investigating the resources they used for study, the purpose of these resources, and how they contributed to the students' learning successes. They found that students initially tried to solve exercise sheets on their own, and if they experienced difficulties, they used more and more resources to support themselves. The most frequent resources used were working with peers, searching for solutions on the internet or in books, or working through the solutions of others. In Tossavainen et al. (2020)'s study in Finland, a group of 98 students across two campuses were surveyed regarding their study habits in mathematics and the resources they used. They found some differences based on student age, with older students more likely to liaise with their lecturers, while younger students relied more on their peers. They also concluded that higher grades were not associated with the use of any particular resource and were more likely to be based on students' quality of study. Kempen and Liebendörfer (2021) explored the resources relied on by 89 students taking a Linear Algebra module during online teaching due to the pandemic. They found that students rated externally produced videos as highly as they did those produced by the teaching team. Consultation with peers was ranked most highly of all in terms of usefulness by these students, even though the teaching was remote and not in-person at the time.

Methodology

To conduct the anonymous survey, a survey instrument was designed to investigate undergraduate students' usage of mathematics support, generative AI, and other mathematical resources. One of the authors attended mathematics lectures and explained the purpose of the study to students and offered them the opportunity to undertake the survey at that point. The survey was conducted online using Google Forms, although students were offered the option of completing a printed survey if they preferred. Nobody chose the printed option. In addition, students were emailed after the class with a link to the survey, in order to allow those who had not attended the lecture to complete the survey in their own time if they so wished. Overall, the response rate was 40% of all possible students in the cohorts surveyed, although anecdotally, somewhere between 70-80% of students present in lectures completed the survey.

The opening questions of the survey related to profiling questions, such as the respondents' gender, year of study and degree programme. Because the latter was very fine-grained, we have collated the degree programmes into a series of disciplines. The results of this, and the other profiling questions, can be seen in Table 1.

Table 1

Profile details of the sample of survey respondents (n=691)

Gender	Number of Respondents
Male	382
Female	286
Non-binary / Trans / Queer	6
Blank	17
Year of Study	
1	383
2	250
3	20
4	34
MSc (taught)	4
Discipline of Study	
Science	232
Engineering	133
Actuarial/Financial Mathematics	115
Preservice Mathematics Teachers	84
Computer Science	70
Accounting and Finance	30
Economics / Psychology & Maths	8

After these profiling questions, students were asked a number of questions about their engagement with mathematics support services and their usage of generative AI, both for maths and non-maths purposes. The questions were a mixture of open and closed, and the results were analysed using SPSS for quantitative data and General Inductive Analysis (GIA) for qualitative data. The GIA was conducted using Excel and responses were coded under more than one theme as appropriate. The results of these questions are largely outside the scope of this paper. Instead, we will focus on the responses given by students when asked about resources other than mathematics support or generative AI that they used for mathematics study.

Results

Table 2 below shows the responses given by respondents to the question “*Do you use any other tools or services for learning maths that haven’t been mentioned above? If yes, please list them.*”. There were 243 responses to this question, and these were coded using GIA as mentioned above. However, given the nature of the responses to this particular question, the coding ended up largely being a repetition of the resource name given by the

respondent, as many did not elaborate beyond listing the resource. Although not included in this question, the number of respondents who said in earlier questions that they used generative AI and the Maths Learning Centre are given in the table as context for the other resources mentioned.

Table 2

Resources used by undergraduate students studying a mathematics module in Dublin City University (n = 691)

Resource Type	Number of Respondents
Generative AI	629
Maths Learning Centre	267
YouTube	122
Websites	22
Khan Academy	20
Books	19
Online calculators	19
Videos	13
Wolfram Alpha	13
Friends/Family	11
Lecture Notes	11

As can be seen in the table, 91% of respondents had used GenAI for mathematics, and 39% had used mathematics support, while 35% used other resources. Out of those 62 students who said they never used GenAI for maths, 37% said they used mathematics support, and 43.5% said that they used other resources such as those listed above. For those 629 students who used GenAI for maths, at least on occasion, 40% used mathematics support and 34% used other resources such as those listed above.

Although the question about alternative resources was open-ended, many respondents simply stated the resource that they used with no further comment. YouTube was by far the most popular resource after generative AI and mathematics support. A number of students noted that they had several preferred channels that they used for this, such as the student who stated, *“When I need help understanding a specific concept or process, I try to find a YouTube video on how to solve it, preferably made by people whose videos I’ve seen before and explanations I understand”*. Another student pointed out why they preferred to use YouTube, observing *“Youtube is an amazing tool and does the same job as genAI in my opinion except the explanations are graphical as well as verbal”*. Students showed considerable trust in videos from websites such as Khan Academy, with one stating *“Videos online are always a safe way to learn new concepts, i.e khan academy”*. In terms of books, one student stated, *“I read the James Stewart’s Calculus book, i like to study from it because there are more exercises to practice from and better explanation with real world examples”*.

Although online calculators could be used to simply obtain the answer to a calculation without having attempted it, one student pointed out that *“I have used calculators online to check my answers such as gauss Jordan calculators, differentiation and integration calculators”*. Students reported using Wolfram Alpha similarly, saying *“Wolfram Alpha to check if you calculated an integral correctly”*. The guiding influence of family and friends was clear in a few comments, such as the student who said, *“My Dad and Brothers have all studied maths so I get help from them a lot of the time”*.

Discussion and Conclusions

In contrast with earlier literature (Ní Shé et al., 2017; Anastasakis et al., 2017), where students seemed to rely most heavily on notes or handouts provided by the lecturer, this was not the finding in our study. The usage of generative AI while studying maths occurs heavily across all year groups and disciplines in our sample, without exception. Those utilising mathematics support services comes out at about 39%, which is slightly higher than in previous national studies, most notably O’Sullivan et al. (2014). However, considering the “other resources” mentioned by students, the reliance on YouTube videos is what stands out in this study, almost six times more popular than the next most common resource. In fact, those who mentioned “Khan Academy” or “Videos” could also have been included under this heading, although we chose not to do so to provide more finely-grained insights into the specifics of what students were using. It is possible that students did not think to mention notes or handouts, as they were not prompted specifically to do so, and that having previously completed some questions on generative AI, they were already thinking in terms of digital resources – but it may also be the case that there has been a sizeable shift in the types of resources being favoured by students studying mathematics in university. Either way, it is clear that only very small numbers are now using traditional resources such as textbooks for study purposes.

The number of students who cited peers or family support was also very low in our sample, but it should be noted that an additional 15 students who answered questions about mathematics support services earlier in the survey mentioned that they did so with friends, so they would not have included this separately in response to this question. Therefore, this number is artificially low and should not necessarily be interpreted as a lower level of peer-study than other studies (Tossavainen et al., 2020; Kempen & Liebendörfer, 2021) have reported. Indeed, if we include these 15 students with the 11 from Table 2 under “Friends/Family”, we see that this becomes the next most popular option after YouTube videos, although still a long way behind. Without having been asked specifically if they still studied with peers, it is difficult to gauge how much of a shift has occurred in this regard, but it is an area worthy of further research, as study groups and peer support have long been regarded as important supports in students’ mathematical learning.

In terms of future work emerging from the results of this study, it would be of interest to repeat this question in a year’s time to see if there has been a further shift in resource usage due to the propagation of generative AI. It would also be worth investigating if the increase in the amount of notes and handouts available online rather in print and handwritten in class has had any impact upon student engagement with these. Finally, it would be valuable to ascertain

if any of the student usage of YouTube was based on lecturer recommendations, or if they were relying solely upon peers or their own searching to determine which channels were most valuable to their study of mathematics.

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The knowledge and understanding needed to teach mathematics effectively

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Since 2011 the authors have researched and utilised models of teacher knowledge and mathematical understanding to prepare preservice teachers for the effective teaching of mathematics, to identify strengths and weaknesses in their knowledge and understanding and provide a comprehensive framework that underpins and facilitates autonomous lifelong learning. While the literature offers many models of teacher knowledge and understanding, there is no integrated framework that combines both to support the growth of mathematics teachers as learners and educators. We address this gap by introducing a composite model entitled the Understanding and Knowledge for Effective Mathematics Teaching [UKE-MAT] model, which exploits the commonalities and distinctions of knowledge and understanding to ensure comprehensive preparation for teaching mathematics. We demonstrate the applicability of the model in the design and evaluation of an intervention to enhance preservice teachers' knowledge and understanding of the utility value of mathematics.

Keywords: Teacher knowledge, mathematical understanding, effective teaching

Background to UKEMAT

Knowledge and understanding are related characteristics, yet distinct in their own right (Hannon, 2021). Where knowledge is generally seen as the facts, formulas, procedures, and terminology of a subject, understanding is about making connections, seeing patterns, and being able to *apply* knowledge in various contexts (Hannon, 2021; Crooks & Alibali, 2014). Both are required to be an effective teacher. For mathematics teachers in particular, a lack of knowledge or understanding can lead to shortfalls in their communication of mathematical content to their students (O'Meara, 2011).

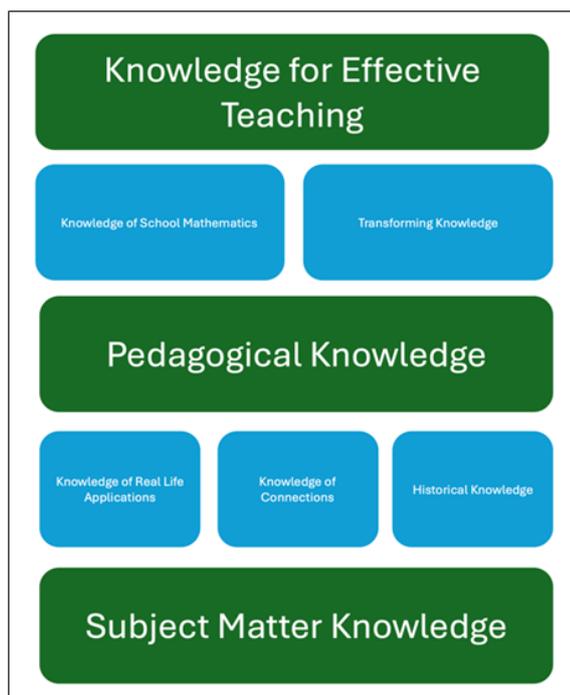
Many models of mathematical teacher knowledge can be found in the literature. Starting with Shulman in 1986 (though not mathematics specific), many researchers (Ball et al., 2008; Rowland et al., 2005) have delineated the spectrum of knowledge a teacher requires to teach effectively, including but not limited to: subject matter knowledge (SMK), pedagogical content knowledge (PCK), knowledge of curriculum, and knowledge of students. In 2011 O'Meara presented the Ladder of Knowledge [LOK] (see Figure 1) as a model that embodied the range of knowledge domains that would underpin the effective teaching of mathematics curricula in the 21st century.

Due to significant changes in the secondary mathematics curriculum in Ireland in 2010, O'Meara built on preceding models of teacher knowledge to create an apposite model of teacher knowledge to address local needs. Many of the models, like Ball et al. (2008) and Rowland et al. (2007) were created specifically for primary school teachers hence a gap existed for one that was tailored for secondary school teachers of mathematics where increased emphasis on ICT and applications was endorsed (O'Meara, 2011). The LOK is informed by the categories and constructs of Ball et al.'s (2008) Mathematics Knowledge for Teaching (MKT) model. The LOK essentially builds on MKT by organising the knowledge

types developmentally, offering a progression or "ladder" through which teachers move as they develop expertise. For example, while SMK features in both models, the nature of LOK indicates that it is an entry point, and no other domain of knowledge can be developed until proficiency in this domain is attained. MKT underpins the ladder by providing the domains of knowledge, while the ladder interprets these as stages in teacher knowledge acquisition. Essentially where MKT is a typology, the LOK is more of a trajectory.

Figure 1

The Ladder of Knowledge



The key features of the LOK are that (1) it outlines SMK as an entry point ; (2) it outlines the knowledge domains that allow teachers to progress from SMK to PCK and onto the knowledge for effective teaching; and (c) it acknowledges the need for teachers to develop a multitude of knowledge domains including knowledge of applications; knowledge of mathematical connections and knowledge of students' cognition and interests in order to be able to teach mathematics effectively. As a result of these features, the LOK is the model of teacher knowledge that partly underpins the UKE-MAT model.

What constitutes mathematical understanding is another long-standing question in mathematics education. Usiskin (2015) classified mathematical understanding into five interlinked dimensions: Skill Algorithm – proficiency at completing a mathematical algorithm to attain the correct answer; Property-Proof – understanding why mathematical processes within an algorithm gives the correct answer; Use Application – understanding when and where it would be appropriate to use the algorithm; Representation Metaphor – being able to support a mathematical explanation with some appropriate visual image/graph/metaphor; History Culture – understanding how a mathematical concept developed over a period of time. This multidimensional framework of mathematical understanding complements the LOK and was chosen as the critical accompanying framework to complete the UKE-MAT

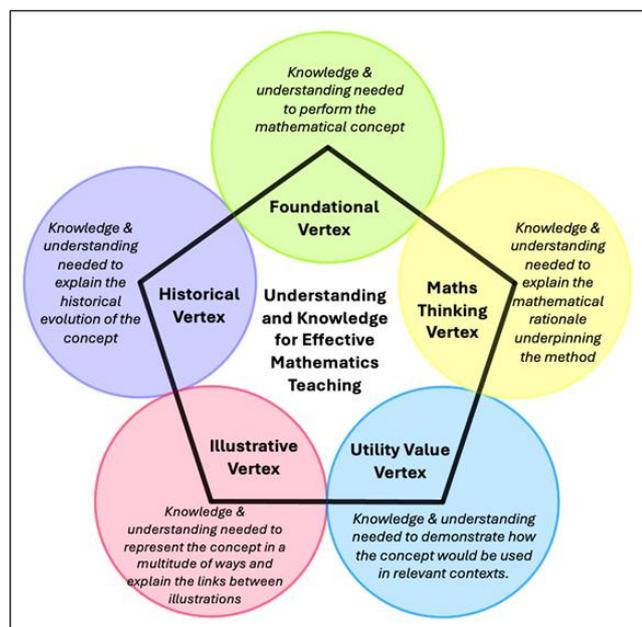
model. This is a very important endeavour as currently no model that amalgamates dimensions of knowledge and facets of understanding for effective mathematics teaching exists. This research aims to fill this gap in the literature.

The UKE-MAT Model

At the centre of the pentagonal UKE-MAT model (Figure 2), we can see the understanding and knowledge for effective mathematics teaching. Each vertex then draws on one or more dimensions of the Ladder of Knowledge and integrates them with a corresponding facet of understanding from Usiskin’s model. For example, the *foundational vertex* in the UKE-MAT model draws on a (1) teachers’ SMK, (2) their knowledge of school mathematics on the taught curriculum, and (3) the skill algorithm dimension of understanding. SMK equips teachers with the requisite knowledge to recognise how a mathematical problem can be solved using different approaches, in addition to a clear appreciation of the relationship between these approaches. Knowledge of school mathematics enables teachers to identify when it is appropriate to extend a previously learned algorithm, introduce a new one, and make meaningful connections to earlier content. This form of knowledge underpins what Usiskin (2012) describes as the skill-algorithm dimension of understanding, which involves the application, comparison, and at times the creation of new algorithms. Each vertex is formulated in a similar manner and collectively all five vertices comprise the absolute knowledge and understanding a teacher needs to teach mathematics.

Figure 2

UKEMAT Understanding and Knowledge for Effective Mathematics Teaching



Proof of Concept – Evaluating and Enhancing Students’ Utility Value of Mathematics

UKE-MAT has been used by the authors in a variety of different ways and here we will report on how it informed the design of an intervention and related pre- and post-tests to evaluate the utility value of fractions of a cohort of 23 Pre-service teachers (PSTs). The tests were constructed to measure PSTs’ understanding of the four primary mathematical

operations with a focus on rational numbers. In the pre-test, PSTs were presented with four different fraction operations ($3\frac{1}{3} + 4\frac{1}{5}$; $\frac{5}{6} - \frac{2}{3}$; $\frac{3}{8} \times \frac{1}{5}$; $\frac{5}{6} \div \frac{2}{3}$) and asked to: (a) Evaluate the problem and show their workings (foundational vertex); (b) Explain the rationale behind the process they just carried out and the reasoning behind each step (maths thinking vertex); (c) Give a meaningful real-life example to represent the mathematical process they just carried out (utility value vertex); (d) Give a graphical/pictorial representation or a metaphor of both the process and the answer (visual representation vertex).

The PSTs completed the pre-test during the second week of the semester and then participated in a mathematics pedagogy intervention for the succeeding three weeks. The intervention was an integral part of a mathematics pedagogy module for PSTs in their third year of a four-year undergraduate programme and was also theoretically underpinned by the UKE-MAT model. The purpose of the intervention was to allow the PSTs to revisit a range of different concepts relating to rational numbers from an advanced perspective (e.g. why do we need to find a common denominator when adding fractions?). Through this intervention we aspired to facilitate the simultaneous development of all aspects of PSTs' knowledge and understanding of fractions. The intervention schedule is detailed in Table 1.

Table 1

Intervention design underpinned by UKE-MAT

Week	Content	Link to UKE-MAT Vertices
1	(a) The importance of teaching for understanding	(a) N/A
	(b) Identifying the knowledge and understanding needed to teach for understanding: The UKE-MAT Model.	(b) All vertices
	(c) Understanding fractions and effective methods for teaching equivalent fractions	(c) All vertices
2	(a) Procedures behind operations of adding, subtracting, multiplying and dividing with respect to fractions	(a) Foundational vertex
	(b) The origins of mathematical operations (addition and subtraction), what they mean, historical origins, and understanding why the processes work	(b) Maths thinking and historical vertex
3	(a) Evolution of mathematical operations (multiplication and division), their meaning and why the procedures work	(a) Maths thinking and historical vertex
	(b) Converting procedural problems in fraction operations to realistic contextualised problems	(b) Utility value vertex
	(c) Visual representations of fractions and fraction operations	(c) Illustrative vertex

The authors conducted an analysis of students' utility value (part (c) of the assessment) of rational numbers prior to, and on completion of, this intervention. Their responses to each of the questions in the pre and post tests were categorised ranging from Type 1 – Type 6. Type 1 indicated a meaningful, realistic example of the problem posed that was not contrived, which reflected a nuanced and comprehensive understanding of where the concept might be used in real life. At the other end of the scale, a Type 6 response indicated a

misconception. Hence their total score in both the pre- and post-test ranged between 4 and 24, with a lower score indicating a more sophisticated level of comprehension of the utility value of mathematics. Analysis of students' pre-test scripts indicted students possessed a limited understanding of the utility value of mathematics, failing to articulate how they could be used in everyday life. However, statistical analysis of the pre- and post-test scores indicted that the UKE-MAT supported intervention had a positive influence on this dimension of knowledge and understanding, despite the short timeframe. The PSTs were significantly more empowered to teach fractions after the programme (O'Meara & Fitzmaurice, 2022).

Conclusion

UKE-MAT is a model that adds to the body of literature on mathematics teacher education by identifying and marrying the domains of knowledge and understanding required by teachers for the effective teaching of mathematics. In this paper, the authors give a brief overview of just one way in which the model has been used in their own preservice teacher education programme, but UKE-MAT offers a valuable foundation for further exploration and innovation in mathematics teacher education, both within preservice programmes and beyond.

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Reimagining STEM education in the early years of primary school through Universal Design for Learning and Lesson Study

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This paper explores how Universal Design for Learning can inform inclusive STEM education in the early years. Drawing on a Lesson Study project in two Irish primary schools with children aged 4-7, the study examines how teachers planned, implemented, and reflected on STEM activities using Universal Design for Learning principles: multiple means of engagement, representation, and action and expression. Findings show that Lesson Study enabled teachers to observe and respond to learner variability, challenging deficit-based assumptions. Through visual supports, structured choices, and multimodal expression, children with additional needs accessed and engaged in STEM meaningfully. Teachers reported greater awareness of pupil diversity and developed more adaptive practices. The research demonstrates that combining Lesson Study with Universal Design for Learning supports inclusive, strengths-based teaching and positions inclusion as a core design principle in early STEM education. Implications are discussed in relation to curriculum reform and teacher professional learning.

Keywords: Universal design for learning, STEM education, Lesson Study, early childhood, primary level, inclusion

Introduction

As early years education becomes increasingly shaped by calls for integration and inclusion, the question of how to embed meaningful STEM (Science, Technology, Engineering, and Mathematics) experiences for all learners is gaining traction. STEM in the early years is widely recognised for fostering inquiry, curiosity, and positive dispositions towards learning (McClure et al., 2017). However, the field remains marked by inequities in both access and representation. Children with additional needs are often excluded from early STEM initiatives and research (Yang et al., 2024). This absence signals a troubling misalignment between the rhetoric of inclusion and the realities of classroom practice.

Drawing on the theme of the MEI 10 conference, which calls for reflection on integration, connections, and the societal role of mathematics education, this paper proposes Universal Design for Learning (UDL) as a framework for reimagining early STEM through a lens of equity. UDL shifts attention away from a deficit view of children and instead focuses on redesigning learning environments to support the diverse ways children think, communicate, and engage (Meyer et al., 2014). This paper makes a case for a strengths-based, theory-informed approach to early STEM education that treats inclusion as a central organising principle rather than an add-on.

Background and Study Context

The Irish education system is in a period of curricular reform, with the Primary Curriculum Framework (Department of Education, 2023) placing increased emphasis on inclusive, integrated learning. It promotes equity of opportunity and participation for all

children, explicitly recognising the need to respond to diverse identities, abilities, and experiences. However, while the framework outlines an ambitious vision, there remains a gap between policy rhetoric and practical implementation, particularly within STEM education, which in the early years of primary school remains in its infancy in Ireland. Teachers report limited professional development and a lack of confidence in adapting STEM content for children with additional needs (D’Agostino & Horton, 2023).

The research presented in this paper investigates how teachers apply UDL when planning inclusive STEM activities. The focus is not only on what teachers do, but on what barriers they identify and how they respond to learner diversity. The UDL framework is structured around three principles: (a) multiple means of engagement, (b) multiple means of representation, and (c) multiple means of action and expression (CAST, 2024). The study seeks to examine how pedagogical approaches informed by UDL can contribute to participation within STEM education.

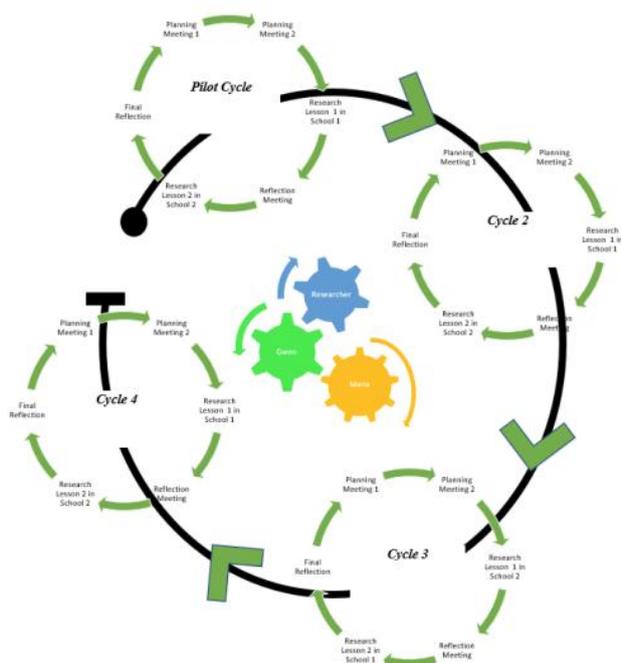
Methodology

Lesson Study (LS) is a collaborative form of professional development that prioritises student learning over teacher performance (Dudley, 2013). Rooted in Japanese education, LS involves teachers engaging in a cycle of planning, teaching, observing, and reflecting on a ‘research lesson’ to better understand and improve classroom practice. Unlike traditional professional development models, LS foregrounds pupil engagement, participation, and learning as central to teachers’ professional growth. Common features of LS include active teacher inquiry, long-term engagement, and collective reflection. Studies show LS supports teacher responsiveness to diverse learning needs (Dudley, 2013), and fosters more effective classroom differentiation and inclusive teaching, including for pupils with additional needs (Schipper et al., 2017). These attributes position LS as a robust and research-aligned methodology for improving instruction and analysing practice in authentic educational contexts.

This research was conducted in two rural Irish primary schools, focusing on the author’s classroom with Junior and Senior Infant classes (ages 4-7). Pseudonyms were used to protect participants’ anonymity, with the teachers referred to as Gwen and Maria. Despite having 15-20 years of experience with younger children, all participants were new to LS. The study centred on integrated STEM education, also a novel approach for all participants. Research lessons were developed and subsequently retaught with another cohort of Junior and Senior Infants. The eight-month research design comprised a pilot study (Cycle 1) and three full LS cycles (Cycles 2–4). Each LS cycle lasted approximately six weeks (Figure 1). During each cycle, the participants engaged in two planning meetings prior to the first teaching of the research lesson in one of the participating schools. Subsequently, a reflection meeting revealed observations and learning, which in turn informed lesson changes. The reteach of the research lesson in the other school was followed by a final reflection meeting. Data were collected through multiple methods, including interviews, collaborative meetings, observations and field notes.

Figure 1

LS Cycles



Semi-structured interviews were conducted with Gwen and Maria, at the beginning and end of the study, and with the school principal at the end. These interviews aimed to explore the impact of LS, experiences of STEM education, and children's engagement. All interviews were audio-recorded and transcribed. Collaborative planning and reflection meetings were integral to each LS cycle. Planning sessions focused on anticipating children's understanding and designing tasks. Reflection meetings, enabled teachers to evaluate lesson effectiveness, discuss children's responses, and revise future plans. These meetings were also audio-recorded and transcribed. Lesson observations formed a core part of the LS process. Teachers used observation templates focusing on children's engagement, disposition and skill development. Observation data provided insight into children's thinking and interactions. Field notes and a reflective journal supported reflexivity and critical analysis throughout.

Data were analysed using Braun and Clarke's (2006) six-step approach: familiarisation with the data, generating initial codes, searching for themes, reviewing themes, defining and naming themes, and producing the final narrative. Interview and meeting data were transcribed and field notes were expanded. Initial codes were identified and grouped into candidate themes through an iterative, reflective process. Memos were written to capture analytical insights, which supported the development of overarching themes. Themes were then reviewed to ensure they faithfully represented the dataset and formed a coherent analytical narrative.

Findings and Discussion

This research explored how the UDL principles, multiple means of engagement, multiple means of representation, and multiple means of action and expression, were enacted

during a series of LS cycles focused on STEM education in Junior and Senior Infant classrooms. LS provided a rare opportunity for teachers to closely observe children's learning processes and to challenge pre-existing assumptions about their capabilities. Across the cycles, teachers repeatedly reflected on how children's reasoning and participation exceeded their expectations, particularly in terms of prior knowledge and problem-solving. 'We were underestimating how their little minds can work' Gwen noted, signalling a shift from deficit-based to strengths-based thinking.

Multiple Means of Engagement

This shift in mindset directly aligns with UDL's principle of multiple means of engagement, which focuses on stimulating interest and motivation for learning. Inquiry-based, hands-on, and open-ended STEM tasks engaged a wide range of learners, including those typically disengaged during traditional lessons. Teachers observed that children were 'completely absorbed in their creations.' At the same time, LS exposed challenges for some children with additional needs, who found aspects of these lessons overwhelming, echoing concerns in the literature around STEM's inclusivity (Park et al., 2017). LS created a space for collaborative problem-solving, allowing teachers to refine instructional approaches iteratively to better address learner variability.

To support engagement, teachers introduced story-based scenarios such as 'The Gingerbread Man needs a boat' which offered narrative-driven entry points to STEM tasks. Cycles Three and Four saw increased flexibility in grouping arrangements, allowing children to choose between working alone, in pairs, or in groups depending on their comfort and preference. Children were also given choices in how to participate and plan, drawing, talking, role-play, or using digital tools, enabling more personalised routes into learning.

Multiple Means of Representation

The principle of multiple means of representation was enacted through teachers' adaptation of content delivery to meet varied learner needs. In the pilot cycle, one child with additional needs and English as an Additional Language was observed to lose concentration easily and struggled particularly with group tasks. As Gwen reflected during the post-lesson meeting, '[Child A] was very distracted and distracted others'. Upon questioning after a floating and sinking lesson, it became evident that the child had no understanding of the key vocabulary. As a result, the conceptual aims of the lesson were missed. This moment was significant, as teachers recognised that without deliberate focus on language, STEM content could become inaccessible, particularly for children whose first language was not English and for some neurodiverse learners. From that point onward, they embedded explicit vocabulary instruction into lesson planning and into play sessions, ensuring that key terms were taught, reinforced visually, and practised before and during activities.

In later cycles, this translated into more consistent use of visual aids, real objects, and explicit language to introduce and reinforce scientific terms. For example, during the 'Make a Chair for Baby Bear' task in Cycle Four, teachers used photos, 3D models, and hands-on materials to present different chair types. Two children with additional needs were given concrete supports and visual prompts to complete the task independently. These adaptations

reflect Donegan-Ritter's (2017) call to incorporate visual and tactile supports, and to design tasks that are both accessible and appropriately challenging. Across the later cycles, differentiated questioning, controlled task complexity, and the provision of structured materials became more common, illustrating teacher responsiveness as advocated by Schipper et al. (2017).

Multiple Means of Action and Expression

The final UDL principle, multiple means of action and expression, was clearly demonstrated as teachers recognised the diverse ways children articulated understanding. Some children verbalised reasoning with confidence; others expressed ideas through manipulation of materials or visual representation. One child with additional needs led a group and explained why an object floated, indicating both cognitive and social engagement. Teachers came to value these multimodal expressions, recognising that conventional assessment methods often fail to capture the full breadth of children's thinking. Flexibility in how children communicated their ideas was welcomed, and revisions were welcomed as part of the learning process.

LS meetings reinforced the importance of this principle by allowing teachers to jointly reflect on observed pupil behaviours. Teachers began to embed differentiated tasks as standard practice rather than reactive supports. By Cycle Two, the deliberate use of 'case pupils' (Dudley, 2013) helped focus observations on children with a range of learning profiles, including those with additional needs. These focused observations demonstrated how different children accessed the knowledge and skills embedded in STEM lessons.

Conclusion

This study contributes to emerging understandings of how inclusive STEM education can be meaningfully realised in early childhood settings through the integration of LS and UDL. By focusing on learner variability, it moves beyond dominant gender-focused narratives in STEM and highlights the broader, often overlooked, dimensions of inclusion in STEM. The findings demonstrate how structured collaborative inquiry enabled teachers to identify and remove barriers to participation, improve their responsiveness to diverse needs, and reconceptualise inclusion as a core component of high-quality STEM teaching.

For policy and practice, this research underscores the importance of embedding inclusive design frameworks such as UDL within professional learning models like LS. Policy initiatives should support sustained, school-based collaborative learning structures that prioritise children's diverse ways of engaging, accessing, and expressing learning. In doing so, early years STEM education can become not only more equitable, but more effective for all learners.

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A vision for early childhood mathematics in the Republic of Ireland

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This paper presents a vision for Early Childhood Mathematics (ECM) in the Republic of Ireland (ROI). Despite recent policy reform calling for ECM to be included in early childhood education initial education programmes (DE, 2024), little is known about the attitudes and mathematical competencies of those teaching these programmes. PLÉ, the Association of Academics in Early Childhood Education and Care in Higher Education in Ireland, have been investigating the nature of early mathematics provision in ROI with their members, with several recommendations arising from this work. This paper describes the broad vision for ECM agreed upon by PLÉ members during a world café event in 2024, requiring systemic change. The what (recommendations), how (steps to implement change) and who (key stakeholders) were identified, creating a vision for ECM to inform future policy, frame professional development and aid in the development and design of appropriate mathematics undergraduate modules.

Keywords: Early childhood education; initial professional education; mathematics

Introduction

In Ireland, Early Childhood Education (ECE) has become a focal point for educational policymakers, particularly in the areas of STEM (GoI, 2017) and numeracy education (GoI, 2024b). 2019 saw the introduction of the *Professional Award Criteria and Guidelines for Initial Professional Education (Level 7 and Level 8) Degree Programmes for the Early Learning and Care (ELC) Sector in Ireland* (DES, 2019). This document asked providers of initial professional education (IPE) to ensure that pre-service early childhood (EC) educators develop a ‘Knowledge of children’s different strategies of learning (play-based, social learning, early literacy and **numeracy**, language acquisition and multilingualism)’ (p.16). However, this same document states that the guidelines do not ‘seek to create absolute homogeneity across higher education professional awards in ECEC’ (p.15); therefore, undergraduate programme coordinators can choose how to present this content to students, leading to variability in the delivery of this content.

Findings from O’Neill and Gillic’s (2024) research exploring the perspectives of ECE lecturers, who are members of an Early Childhood Mathematics Education (ECME) SIG, found that participants reported possessing high levels of mathematical self-efficacy. It was acknowledged that to support student engagement and learning concerning ECME, they, as lecturers in this area, should possess high levels of mathematical content and pedagogical content knowledge too. Interestingly, despite claiming high levels of self-efficacy in mathematics, some participants reported being anxious about teaching ECME on their courses. Finally, participants recommended that a stand-alone mathematics module be included in all IPE undergraduate programmes and that numeracy be afforded the same importance and visibility as literacy in their programmes.

This paper builds on the findings of O'Neill and Gillic's (2024) report. It provides an account of the perspectives of a broader cohort of ECE lecturers (N =45) towards mathematics and its inclusion in ECME in pre-service ECE programmes. A vision for ECME in Irish EC pre-service programmes, arising from participant recommendations, is presented here.

Literature Review

While traditionally regarded as the preserve of primary and secondary education, the appropriateness of mathematics for early childhood education is now well established (Lundqvist et al., 2023), with teacher educators expected to share emerging understandings around mathematics in the early years with prospective early years educators (Whyte et al., 2018). Contestation exists around the distinction between mathematics and numeracy, and which should be the focus in the early years. However, Dunphy (2018, p.106) explains, 'while numeracy is important, education and curricula at all levels, including preschool, should encompass a broader view of mathematics and learning'.

Developmentally appropriate support for mathematics within the early years is recognised as being more informal than that of primary education (Lundqvist et al., 2023). Drawing on Ginsburg and Ertle (2008), mathematics pedagogical content knowledge (MPCK) within the early years is unique in that learning opportunities for mathematics frequently occur spontaneously during play and interactions with children (Jenben et al., 2022), requiring educators to engage in 'intentional teaching' (Knaus, 2017). Intentional teaching requires educators to make explicit the mathematical ideas that emerge during play, supporting children's understanding of them (Cohrsen et al., 2013, cited in Knaus, 2017). Drawing on the findings of the National Research Council in 2009, Dunphy (2018) explains how, while most children can learn mathematics in their early years, this potential is frequently limited through a lack of intentional or supported opportunities to engage with mathematics within ECE settings. Problematically, from a review of the literature by Linder and Simpson in 2018, both the mathematical content knowledge and mathematical knowledge for teaching (MKT) were found to be somewhat lacking.

Specific mathematics education is essential for early years educators (Whyte et al., 2018), however, considerable variation exists among programmes with mathematics frequently integrated with other subjects (Maxwell et al., 2006, cited in Whyte et al., 2018). Uncertainty exists around both which mathematics content is appropriate for the early years and how mathematics should be taught in this environment (Knaus, 2017). The learning opportunities for mathematics in the early years can support children's later mathematics understanding. Number (encompassing whole numbers, operations, and relations) and geometry (comprising spatial thinking and measurement) have been identified as two key content areas for mathematics education in the early years (Dunphy, 2018).

Methodology

The world café method is frequently used for public consultation or organisational change processes (Löhr et al., 2020) and was selected for its rigour, relevance and speed (Schiele et al., 2022). The process facilitates informal conversation among groups regarding a

particular problem or topic, in this instance a consideration of the recommendations made by the PLÉ Maths SIG.

Invites to participate were issued via PLÉ, the Association of Academics in Early Childhood Education and Care in Higher Education in Ireland. 45 individuals contributed to a world café at a day-long event in late 2024. All participants were female and teaching on early childhood degree programmes in third-level institutes in the Republic of Ireland. The world café session gathered participant perspectives on the six recommendations from earlier research (see O'Neill and Gillic, 2024). Participants were asked to consider, discuss and reflect on these recommendations and identify how they might be achieved. Small groups reviewed draft recommendations and as per Brown and Isaacs (2005) the world café method recorded participants' contributions, supported of cross-pollination and connected the diverse perspectives of contributors.

Findings

The purpose of this study was to determine a collaborative and collective vision for early childhood mathematics in the Republic of Ireland. Central to ideas were the visibility of mathematics on EC graduate programmes and clarity about how EC mathematics is conceptualised. Both are needed to inform future policy, frame professional development and aid in the development and design of appropriate mathematics undergraduate modules. Building on antecedent findings (O'Neill and Gillic, 2024), this project attempted to agree on a vision with a wider cohort of participants. From the consensus achieved, several key findings emerged that confirm and refine recommendations, including:

1) *Mathematics should be given the same level of importance as literacy on EC graduate programmes.* Agreement is needed on when, what, how, and why mathematical content is introduced. A shared understanding of approach and pedagogy must be established for graduate programmes, aligned with Aistear (NCCA, 2024) and the national Qualifications Advisory Board requirements.

2) *Specific knowledge, skills and pedagogy are required to support young children's fundamental math skills.* It was agreed that EC educators require a certain level of mathematics content knowledge, making this an essential part of initial education. A greater awareness of pedagogical strategies, the role of teachable moments, and practices that support mathematical learning through real-world and experiential learning was deemed crucial.

3) *A distinction should be drawn between mathematics and numeracy.* As per Knaus (2017), mathematics should be explicitly defined as a subject with distinct language, skills, and processes, while numeracy should be recognised as a life skill. While the term numeracy is favoured in EC policy documents in the Republic of Ireland (for example, GoI 2017; 2024a; 2024b), educators require mathematics content- and pedagogical- knowledge to notice, identify and support numeracy in EC settings. As such, educators are learning about mathematics to support numeracy.

4) *Separate, discrete mathematics modules within degree programmes are proposed.* The breadth and depth of mathematics that young children are capable of engaging with is unlikely to be covered within partial or blended modules, such as STE(A)M modules

or modules that focus on Literacy *and* Numeracy. However, only a revision of programme credits can determine where and how a standalone mathematics module could be included in graduate programmes. It was agreed that this is a long-term recommendation, requiring further debate and discussion.

5) *A greater focus on the birth to 3-years age range is required.* The group agreed that a dearth of information exists about the birth-3 age range, suggesting any research carried out by PLÉ should be directed here. This may include carrying out empirical research, scientific literature reviews or designing and facilitating professional learning events focused specifically on babies and toddlers.

The findings above relate to *what* needs to be done. The data generated during the world café event also addressed *how* this might be achieved. Consensus emerged under three headings: professional learning (PL), stakeholder partnerships, and clear and consistent messaging about EC mathematics. Provision of PL opportunities for EC HEI lecturers, including workshops, communities of practice, and collaborative research, was suggested, as well as additional resources and support for teaching. In addition, capacity building for early childhood educators (ECEs) and parents was advised. Exploring how recommendations might be achieved, participants identified key partners for collaboration, including: National policy and regulatory bodies, Higher Education and Training Institutions, Early Childhood Education (ECE) Settings and Placement Coordinators, Qualified Professionals and those in Training. Finally, participants in this study identified the need to establish and maintain EC mathematics within national EC discourse. PLÉ and the early childhood mathematics SIG will play a significant role in achieving this goal.

It should be noted that there was some dissent among the group. Debate arose regarding terminology, i.e. whether the term numeracy or mathematics is more appropriate in EC contexts, even at graduate level. The connotations of the term mathematics and implications of its introduction were discussed, for example, minimum mathematics grades as an entry requirement to early childhood education programmes. Further, the need to include parents, early childhood educators in practice and other professionals in the sector, while beyond the purview of PLÉ, was identified as an important element in fore fronting EC Mathematics within the national discourse.

Conclusion

In conclusion, this study highlights the importance of giving mathematics education the same priority as literacy in early childhood education and pre-service training programmes. It emphasises the need for a consistent, developmentally appropriate approach that includes dedicated mathematics modules and special attention to children from birth to three years old. The findings support the development of strong professional learning opportunities, the building of meaningful partnerships among stakeholders, and clear, consistent messaging to help embed Early Childhood Mathematics Education (ECME) into national policies and practices. The recommendations specifically call for a dedicated standalone mathematics module in early childhood degree programmes and focus on meeting the learning needs of infants and toddlers. Although there are ongoing debates about terminology

and how to implement these ideas, the main goal remains for policymakers, educators, families, and practitioners to work together. This collaboration can help make mathematics a natural, accessible, and engaging part of early childhood education in Ireland. Moving forward, continued effort, open communication, and shared commitment are essential to turn these recommendations into real improvements that will benefit children's learning and support the wider early childhood sector.

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Creative mathematical writing: An interdisciplinary approach to geometry and language in an Irish-medium classroom

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This study examines the integration of mathematics and language through creative mathematical writing in an Irish-medium (IM) primary classroom. By embedding geometry in real-world contexts across writing and mathematics disciplines, the research examines how students aged 10–12 used their stories to represent and apply their conceptual understanding of 2D and 3D shapes, spatial reasoning, and measurement. Recognising assessment as an integral part of teaching and learning, this study is an example of a planned interaction along the assessment continuum. A qualitative analysis was employed to formatively assess student-authored stories. Findings revealed strengths in procedural application, alongside misconceptions of 2D and 3D shapes and unit conversions when scaling up to real-life models. Creative writing emerged as a high-insight strategy for uncovering students' concept images and provided a rich window into their understanding. The study underscores the potential of creative writing to address the dearth of specific assessment resources for IM contexts.

Keywords: Creative mathematical writing, assessment, geometry, Irish-medium education, integration

Background

This paper contributes to the theme of *Mathematics for a Connected World* by exploring how creative writing bridges numeracy and literacy to enhance conceptual understanding. Communicating is foregrounded in the recently introduced Irish *Primary Mathematics Curriculum* (Department of Education [DE], 2023) highlighting how language and mathematics learning are deeply intertwined. In this study, we exemplify how we planned an assessment opportunity for students to communicate their mathematical understanding. This comprised creating stories with a focus on the geometry and measure strands, known areas of conceptual difficulty in Ireland (Pitsia & Lysaght, 2021). The context for this research is in an Irish-medium (IM) school in the Gaeltacht, where Irish tends to be the first language of the students.

A particular challenge in IM settings is the lack of authentic resources available in Irish, including those for mathematics assessment (Nic Aindriú & Ó Duibhir, 2023). This makes it difficult for teachers to meet the demands of the revised mathematics curriculum, where assessment plays “an integral” role (DE, 2023, p. 34). A further challenge is the tension between viewing language as a deficit and recognising it as a resource in minority language contexts (Nic Aindriú, 2024; Ní Ríordáin, 2018; Planas & Civil, 2013).

Against this backdrop, the study explores how creative mathematical writing can serve as both a learning and assessment tool within an IM classroom. Students used their own

language as a resource in their creative writing to make sense of 2D and 3D shapes, measurement, and spatial reasoning, and construct their own mathematics problems in story form. This approach aligns with the *Primary Language Curriculum* (Department of Education and Skills [DES], 2019) which emphasises the importance of meaningful communication and recognises language as central to learning.

Conceptual Framework

The study reflects Carr and Lee's (2012) Learning Stories framework, positioning creative writing as a valuable tool to assess understanding in context where student-authored stories offered insights that might not surface through traditional assessments. Situated as a "Planned Interaction" (DE, 2023, p. 35) along the assessment continuum, we utilised a revelatory task (Dooley, 2022), in this case, creative mathematical writing, to gain an insight into the specifics of students' geometrical concept images (Tall & Vinner, 1981). In IM contexts, this approach is also consistent with a language as resource viewpoint (Planas & Civil, 2013), recognising students' own language as a cognitive and cultural asset that allows them to express, construct, and consolidate mathematical meaning through narrative.

Methodology

The study involved one primary school in a Gaeltacht area where Irish is predominantly the language of home. A total of 18 students aged 10–12 participated in two creative writing workshops delivered through Irish by the *Fighting Words* team. Utilising the *Write to Right* pedagogical approach, the entire class initially collaborated to create an opening paragraph for a shared story, devised the setting, plot, and characters. Beyond this, the students worked in groups to continue the story or create their own story if they preferred. The only requirement was that the stories contained a problem or dilemma that needed mathematics to solve.

Data were the set of all four student-authored stories (originally in Irish, then translated). A qualitative content analysis (Schreier, 2014) was conducted to uncover students' concept images as they were revealed in the narratives they constructed. Two of the authors built a coding frame based on the key aspects of geometry that students would have encountered in their mathematics classes (see Table 1). The authors cross-referenced their interpretation of students' thinking by referring to the original Irish-version of the stories. Inter-rater reliability tested agreement and ensured the consistency and validity of the codes.

Table 1

Building the Coding Frame (Based on Schreier, 2014)

Coding frame	<p>Contains three main categories based on key areas of the Shape and Space Strand, and the Measures Strand of the curriculum: spatial awareness and location, shape, and measuring.</p> <p>Main categories divided into five initial subcategories based on the aspects of mathematical proficiency in the <i>Primary Mathematics Curriculum</i> (DE, 2023); adaptive reasoning, strategic competence, conceptual understanding, procedural fluency and productive disposition.</p> <p>After trialling the coding frame, a decision was made to focus on one aspect of mathematical proficiency - ‘conceptual understanding’ as this was the aspect most likely to provide insight on student concept images (Tall & Vinner, 1981).</p>
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At the end of the analysis process, the authors were left with a series of coded story excerpts which allowed for an illustration of the concept images that the students had of this topic.

Findings

The common story starter was based on a racing narrative. Three groups continued with this storyline whereas one group created their own story about planning for a party. Real-world problems encountered were navigating the obstacles in the racecourse and calculating the dimensions of rooms so as to order the correct size marquee.

Shape

Students often confused two-dimensional (flat) shapes with three-dimensional (solid) objects, attributing physical characteristics (e.g., driving cars, having depth) to 2D shapes such as squares and rectangles. For instance, characters like “Ronnie Fox the Square” and “Galaxy Bar the Rectangle” were portrayed engaging in real-world actions, implying a misunderstanding of their spatial properties. However, some groups showed partial understanding by correctly identifying features of 3D shapes (e.g., "cubic blocks" or "circular wheels") or later correcting earlier misconceptions (e.g., describing Galaxy Bar as a cuboid). These contradictions often oscillated mid-narrative, offering opportunities for cognitive conflict and reflection (Vosniadou & Brewer, 1992).

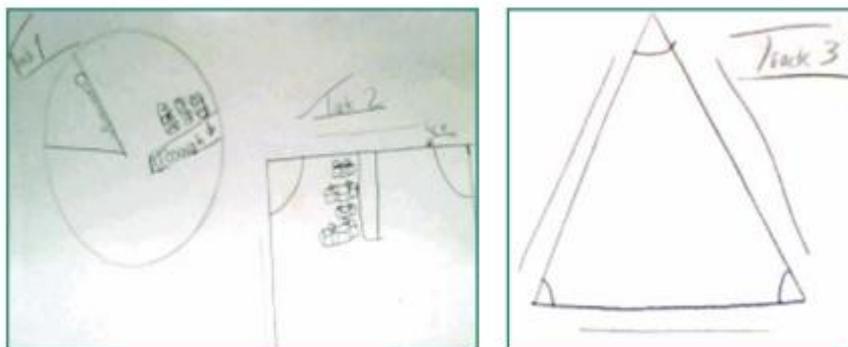
Spatial Awareness and Location

Students incorporated mathematical terminology and concepts such as acute, right, and reflex angles into story contexts (e.g., racecourse turns in Figure 1 below). While students could name and describe angle types e.g. “There was a hexagonal mat, and they had to drive around the six sides of the hexagon. Each angle of the hexagon was 120 degrees, making 720 degrees in total” they occasionally misapplied them in context such as misunderstanding the implications of a 360° turn. “What would they do? Mmmm... around 360 degrees or straight

through the diameter... What would be the fastest way?" It was not clear if students recognised that to travel 360 degrees would bring the driver back to the starting point.

Figure 1

Students' Illustrations of Racecourse Turns



Measuring

Students applied mathematical reasoning to calculate perimeter and area in meaningful contexts such as ordering a marquee for the party: “They would have to measure the entire place themselves before the designer arrived. They went into the living room and took out the plan they had made before. All the measurements were far too small.” They demonstrated correct use of formulas (e.g., perimeter of a square, area of rectangles) and translated measurements into story contexts: “She wanted the cloth to be longer, so she said to herself, “Maybe I’ll tell the designer it’s 15m long and 6m wide. So 90m²!” However, challenges emerged in transitioning from 2D area to 3D capacity. For instance, in estimating the volume of a swimming pool, students did not provide a measurement for depth, with their calculation appearing to confuse m² with m³: “They needed 5000 litres of water as the pool was 2000m², 50m long and 40m wide”. Students incorrectly concluded that the answer was 5,000 litres rather than 5,000 m³. One cubic meter is equivalent to 1000 litres. Consequently, they should have multiplied the volume value by 1000 which would have resulted in 5,000,000l (double that of a typical Olympic-sized swimming pool). Similar difficulties arose in calculating the depth of a pothole as “265 feet deep”. Even allowing for creative licence, the students did not appear to have a concept of how deep that measurement might actually be in the real world (nearly double the height of The Skyline Tour walkway at Croke Park which is 44 meters - approximately 144 feet - high). Another point of note is that the students used feet in this instance rather than metres as in the curriculum.

Discussion

This interdisciplinary approach aligned closely with the aims of the new curriculum and addressed several ongoing “sticky problems” in Irish mathematics education such as underperformance in geometry and measurement. Rather than assessing predetermined knowledge, this approach sought to understand students' reasoning and misconceptions through their own words and representations. Embedding geometric ideas in stories

illuminated conceptual understanding in a non-threatening way and enabled students to use their own mathematical language in contexts that were familiar and of interest to them. Their stories offered insight into not just what students understood, but how they thought, eliciting rich, process-oriented evidence of student thinking to reveal in some cases flawed or incomplete concept images (Tall & Vinner, 1981) particularly in fluctuating between 2D and 3D concepts in both shapes and measures.

Implications

A limitation of this study is the small sample size and the one IM context. However, we believe that creative mathematical writing is a practical, scalable, assessment strategy for teachers and that future research might explore its use across other strands (e.g. data, algebra). To realise this potential, there is a need for effective and sustained teacher professional learning on planned assessment interactions utilising creative mathematical writing as a revelatory task.

Conclusion

This study investigated how creative mathematical writing can reveal students' conceptual understanding and misconceptions in geometry and measures. Their stories offered a powerful lens into student thinking, making misconceptions visible in a low stake engaging format. While students showed strength in applying formulas in more procedural settings within the confines of traditional classroom mathematics, they needed support to connect conceptual understanding with mathematical modeling of tasks in real-world contexts. These insights can inform targeted teaching and formative feedback strategies.

Creative writing provides a student-centred and interdisciplinary pathway for learning. It not only supports curriculum goals but responds to broader educational challenges particularly in the IM setting where, as previously noted, there is a dearth of authentic assessment resources. In a connected world, mathematics must connect—to language, to life, and to learners. Creative mathematical writing is one way to build that bridge.

Acknowledgements

The compilation of short stories generated by the students can be accessed at https://tairseachcogg.ie/Home/ResourceDetails?_HYtIf125x=INuvtCx8OS4Y9aYOWDgzmlvupeJczgO4APCgYA9lX3HVvb91eiNznansojzdha7B#

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Designing for diversity: A self-study on UDL implementation in primary mathematics within a collaborative professional learning community

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This self-study explores how three primary teachers in a Professional Learning Community (PLC) applied Universal Design for Learning (UDL) principles in mathematics education within their classroom practice. Thematic analysis of reflective journals showed that sustained PLC engagement strengthened collegiality, accountability and instructional practice. Participants integrated UDL strategies leading to greater student motivation and autonomy. The study highlights cross-school collaboration and structured professional learning as key to inclusive, responsive teaching.

Keywords: PLC, UDL, primary mathematics, collaboration, inclusive pedagogy

Introduction

Contemporary classrooms encompass a broad range of learner needs, abilities and cultural backgrounds. The Universal Design for Learning (UDL) Guidelines, developed by the Center for Applied Special Technology (CAST, 2024), offer a proactive, evidence-based framework to foresee and eliminate barriers to learning. UDL is based on three principles: (i) multiple means of engagement (why), (ii) representation (what) and (iii) action/expression (how). UDL encourages flexible, inclusive environments that give learners choices in how they access and demonstrate knowledge, aiming to address learner diversity by incorporating such design from the very beginning in all settings (Flood & Banks, 2021; Pusateri, 2022). Curriculum developments in Ireland reflect a policy commitment to embedding UDL in primary education. The Primary Mathematics Curriculum incorporates UDL language, such as “multiple means of expression and representation” (NCCA, 2023, p.29). However, it offers limited practical guidance for implementation. This places responsibility on teachers to interpret and apply the framework, often without sufficient continuous professional development (CPD). Research highlights that while understanding UDL theory is essential, effective application depends on sustained professional learning opportunities, which remain limited in both initial teacher education and CPD (Flood & Banks, 2021; Pusateri, 2022).

Maths4All launched an online Professional Learning Community (PLC) in November 2022 to support teachers in applying UDL in mathematics. PLCs are collaborative groups focused on improving student outcomes, sharing responsibility and engaging in data-informed reflection. They offer safe spaces for sharing experiences and refining practices (DuFour, 2004). Two participants were founding members and became co-facilitators after six months, delivering nine nationwide online sessions through Clare Education Centre over the following year. The third participant joined during one session. The PLC then evolved from a public forum to regular meetings among members, demonstrating a commitment to ongoing CPD and UDL in mathematics. The participants teach in diverse primary settings, including senior mainstream, vertical multigrade and special education in an infant school.

Methodology

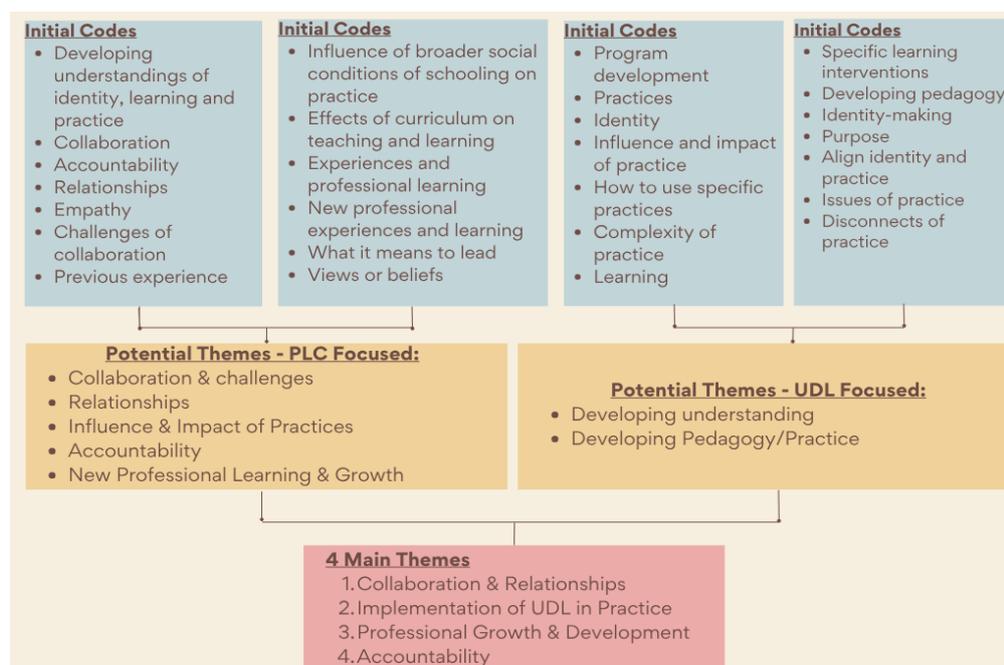
This qualitative study employed a self-study methodology to investigate how three teachers critically reflected on their mathematics teaching within a PLC framework. Pithouse-Morgan et al. (2014) noted that self-study emphasises reflexivity and co-construction of knowledge through shared experiences, advocating a shift from solitary to collaborative inquiry. Each participant kept a reflective journal documenting insights and challenges in implementing UDL in mathematics lessons. Acting as critical friends, participants provided constructive feedback and questioned assumptions, which fostered reflective thinking and professional growth (Kelley et al., 2022). Journal entries were written across thirty online sessions from multiple perspectives, as members and facilitators, while the PLC developed over the two years. Multiple voices enrich understanding and offer diverse data (Pithouse-Morgan et al., 2014). Facilitators’ meeting notes were also used to identify themes.

Data Analysis

Through thematic analysis, guided by Braun and Clarke’s (2016) six-step process, common themes about pedagogical change were identified. Participants first reviewed their journal entries, selecting insightful reflections on UDL and the PLC. These entries were examined collaboratively, resulting in twenty-seven initial codes related to the research question. From these, seven potential themes emerged. These were refined into four main themes with sub-themes. The identified themes were supported by specific journal examples. This process deepened the analysis of professional learning, helping participants recognise patterns and consider how the PLC influenced their understanding and application of UDL.

Figure 1

Initial Codes and Resultant Themes



Findings and Discussion

Thematic analysis of the data identified four main themes: Collaboration and Relationships, Implementation of UDL in Practice, Accountability, Professional Growth and Development. These themes will be explored in more detail.

Collaboration and Relationships

Analysis of participant journal entries revealed a high level of collaboration within the PLC. Participants valued the chance to work with teachers from different school settings and year groups, an opportunity often limited by the isolated nature of school structures. Exposure to perspectives beyond their immediate context was seen as beneficial for professional development, as it allowed for the exchange of diverse strategies and helped break down the professional silos that can hinder growth. Participants in the PLC shared successes, setbacks and classroom experiences in a supportive environment. Both meeting notes and reflective journal entries frequently referenced this practice. Participants exchanged strategies, tools (e.g., virtual manipulatives, visuals, math games) and resources that enhanced their understanding of UDL and practical ways to identify and reduce barriers for students in their classes. Creating a shared digital space (Padlet and Google Drive) facilitated the organisation and access of materials. Participants expressed great appreciation for the inspiration gained from other educators in the PLC. The encouragement from peers helped sustain motivation and growth mindset, especially when adopting new approaches as described by Participant A:

In sharing my experiences with the group, I get to hear about reflections and observations that others have about what I have shared. I also feel that it is a great space to ask questions, both about things that others share and about things that I might be looking for some help with, within my own classroom.

The meetings consistently nurtured a warm, collegial environment, and participants conveyed a shared sense of psychological safety. This aspirational outcome was not guaranteed at the outset. Feedback shows that their involvement in the group has been overwhelmingly positive, as evidenced by the following comment from Participant C:

I always look forward to checking in and catching up with the others. Due to the friendly and supportive vibe among the group, it does not feel like an onerous task to conduct these meetings. I am enthusiastic about trying new things after each meeting.

As group members became more familiar with one another, there was an increased openness in contributions, due to the regular interactions that fostered meaningful professional relationships. These reflective conversations fostered professional growth and encouraged collective problem-solving as outlined here by Participant B:

What challenges did I face? Time to get around to each group to hear their discussions about where to place the prisoners (as part of the game, Release the Prisoners) and what they noticed. What might I do differently next time? Maybe next time I could get them to record some of their ideas about where to place prisoners in their mathematics journals, as suggested in the group.

By collaboratively identifying obstacles and brainstorming solutions, the PLC functions as a professional space for iterative inquiry and adaptation (McLaughlin & Talbert, 2006). Logistically, task division within the group has been equitable and collaborative, fostering strong relationships that enhance the PLC's effectiveness. However, this equal distribution has occasionally complicated decision-making, as members of the PLC may hesitate to take the lead. Research indicates that teachers in a PLC can initially struggle and may feel "side-lined" (Lovett, 2011). Another challenge noted by participants is the time commitment outside of working hours, which, together with limited opportunities for collaborative lesson planning, can hinder effectiveness of PLCs (DuFour, 2004).

Implementation of UDL in Practice

As illustrated in the following quotes from participants C and B, respectively, there were notable shifts in both instructional practices and thinking. The journal entries reveal increased awareness and application of UDL principles, leading to positive outcomes for students in the classroom and provide evidence of the significant impact of participating in a UDL-focused PLC in primary mathematics.

The group read an article titled "What is UDL in the Classroom?" and discussed it during the session. I found this helped us to connect theory, represented by the CAST guidelines and the New Mathematics Curriculum, with our classroom practices.

Possibly the most important reflection of all is that I feel the students in my class have benefited from my participation in the UDL PLC. My classroom is becoming more and more inclusive as I embed UDL principles in my lessons. I am trying out new things that I wouldn't have known about if it were not for the UDL PLC.

A prominent theme emerging from the data was the intentional use of multiple methods of engagement, representation and expression. Participants reported incorporating visual aids, virtual manipulatives and picture books to support mathematical concepts, ensuring accessibility for students with different learning styles. These practices reflect a growing understanding of how adaptable teaching approaches can remove barriers to learning and improve understanding. For example, Participant A stated:

I found that virtual manipulatives were great and allowed me to offer greater choice to students in terms of how they engaged with tasks and how they expressed their learning. I found that students of all ability levels were actively engaged in the lessons where they had a choice of using virtual manipulatives.

A particularly notable sub-theme was the increased provision of choice and autonomy in mathematics activities, with a gradual shift towards offering students options for engaging with content or demonstrating their understanding. Using mathematics journals for students to showcase their learning in various ways enhanced motivation and ownership of learning, especially for those who previously disengaged from traditional mathematics lessons and assessment. The significance of goal setting and activating prior knowledge was another consistently reported practice among participants.

Professional Growth and Development

Sharing prior experiences and practices fostered meaningful personal growth. Discussing lessons about accessibility and engagement required personal reflection both before and during their descriptions. Throughout the sessions, participants took opportunities to share, facilitate discussions and present their classroom practices and research. This approach represented a significant step in professional development as it involved taking leadership within the group and openly sharing pedagogical methods. Participant A reflected:

I shared my experience teaching a lesson on factors using virtual manipulatives with the group. Reflecting on the lessons allowed me to evaluate what went well regarding accessibility and engagement. This process reinforced the importance of considering barriers and making tasks inclusive for all students. I also thought about what I would change if I were to teach the lesson again and what I would like to explore next regarding using virtual manipulatives with the class. Gaining feedback and support from the group gave me the confidence to keep trying new things.

Initially, two of participants relied heavily on prepared slides as a presentation tool. As their confidence and comfort in their roles grew, they gradually moved away from dependence on presentation aids. This transition allowed for more genuine connection, where a dialogue-based approach led to more meaningful discussions and sharing of ideas. Additionally, participants gained validation through discussions and constructive feedback, which enhanced their professional development and motivated their collaborative learning experience.

Accountability

Another prominent theme identified was the role of the PLC in fostering a heightened sense of professional accountability. Regular meetings motivated participants to apply UDL principles in their classroom practice consistently. Participant C noted:

I find that the regular meetings help keep UDL to the forefront of my mind. As well as learning so much during each session, it challenges me to keep learning more about UDL between meetings so that I can engage in the discussions with others who know so much about it. In order to be able to contribute to the group discussions, I also push myself to try out new ideas in the classroom.

Participants reported feeling encouraged by the practices shared by PLC members in different schools, viewing these insights not as benchmarks for comparison but as sources of inspiration for their development. Although there is potential for negative self-comparison, the dominant response was one of motivation and encouragement. Notably, some participants began to incorporate the CAST guidelines for UDL into their discussions, prompting others to engage more deeply with these frameworks, such as printing the guidelines for regular reference. Compared to other CPD formats, participants regarded the PLC as providing a distinctive form of sustained and meaningful accountability, as noted by participant B:

First time hearing about CAST Guidelines 3.0. It is lovely to see play and regulation so important in the guidelines. I am also loving the OIDE-led Muinín workshops, but in the PLC, there is more time to chat and reflect. Especially as the group becomes more familiar, people are sharing more.

Conclusion

This study emphasises the vital role that a sustained and collaborative PLC can play in enhancing teachers' understanding and practice of UDL. Participants built more confidence in using UDL through regular engagement, reflective dialogue about successes and challenges and sharing resources. The findings support existing research highlighting the significance of both theoretical knowledge and practical support in professional development (Flood, 2021; Pusateri, 2022). The study also underscores the relational and emotional aspects of teacher learning, where psychological safety and peer encouragement serve as enablers of pedagogical change. Although time commitment and clear decision-making processes remain challenges, this study suggests that self-directed, context-responsive professional learning can effectively foster inclusive, student-centred practices. These outcomes reinforce the case for broader integration of UDL-informed PLCs within continuing professional development.

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A shared struggle: Teacher perspectives on the challenge of teaching problem solving in senior cycle mathematics

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Developing students' problem-solving skills is widely recognised as a vital component of effective mathematics education, particularly at senior cycle level. This study explores the perceptions of nine post-primary teachers regarding the teaching of problem solving within the Leaving Certificate Higher Level Mathematics (LCHLM) syllabus. While all participants expressed strong support for the value of problem solving, they identified significant barriers to its implementation. Chief among these were insufficient instructional time and the demands of an overcrowded curriculum, which limit opportunities for deeper, student-led engagement. Additionally, teachers reported that many students are unfamiliar with problem-solving approaches and often hesitant to engage with problem-solving tasks, preferring more procedural or guided methods. These challenges highlight a disconnect between curricular intentions and classroom realities. Addressing these barriers is essential if problem solving is to move from being an aspirational goal to a meaningful part of students' mathematical experience.

Keywords: Problem solving, teacher perspectives, Leaving Certificate Higher Level

Introduction

The ability of students to solve problems is one of the most fundamental goals of teaching mathematics, but also one of the most elusive (Stacey, 2005). In recent decades, problem solving has become a central component of mathematics educational reforms worldwide and this is rooted in the belief that problem solving incorporates mathematics as a whole, more so than more 'traditional' approaches that emphasise procedural mastery (Lester & Cai, 2016). Despite increased attention in both research and curriculum development aimed at enhancing problem-solving, students' abilities in this area historically remain underdeveloped and continue to show a clear need for significant growth and refinement (Lesh & Zawojewski, 2007). Additionally, teachers' confidence and beliefs regarding problem solving (Stipek et al., 2001), their lack of training regarding how to embed problem solving into everyday teaching (Johnson et al., 2019), and a lack of classroom time to engage in inquiry-based learning activities (Shiel & Kelleher, 2017), also potentially act as inhibitors towards the successful integration of problem solving into the mathematics classroom. To gain insight into the experience of Leaving Certificate Higher Level Mathematics (LCHLM) teachers when attempting to embed problem solving into their classroom, this paper reports on a selection of teachers' perceptions towards the instruction of problem solving.

Mathematics Curriculum Reform

According to Cavanagh (2006, p. 121), 'the path to reform of mathematics teaching is slow and replete with challenges.' Since the late 1980's, mathematics curricula around the world have been changing to move away from what were deemed outdated curricula with an

overemphasis on skill mastery and procedural fluency towards a more child-centred teaching and learning environment, where conceptual understanding and problem solving were key elements (Cavanagh, 2006). In the United States in 1986, the Commission of Standards for School Mathematics was established in a bid to improve the quality of mathematics in schools by setting out a new vision of school mathematics as an active process and involved changes to mathematical content, teaching methodologies and assessment practices (Toumasis, 1997). The mathematics curricula of many countries around the world were influenced by these changes, including the U.K. (Noyes et al., 2013) and Australia (Anderson, 2009). In 2010 in Ireland, a new nationwide post-primary mathematics curriculum was introduced to modernise a perceived out-dated and teacher-centred curriculum (Lyons et al., 2003), by simultaneously altering the content, the teaching methodologies, and the national assessment strategies. Additionally, guided by existing research (see Conway & Sloane, 2005) and in alignment with many curricula around the world at this time, an increased emphasis was placed on the importance of real-world applications and the development of problem-solving skills within the new curriculum. Currently, at Junior Cycle level, problem solving is highlighted in the rationale of the specification, the aim and objectives of the specification, as an element of the unifying strand that connects each of the content areas within the specification, and as the central focus of the two classroom-based assessments. Equally, at Senior Cycle level, problem solving and being able to apply mathematical understanding is at the core of the specification document and is stated as a clear objective of the document.

The Role of the Teacher During Curriculum Reform

Ernest (1989) noted that teachers' beliefs and perceptions are key factors that shape their actions in the classroom and thus have a substantial bearing on any initiative or reform that aims to alter classroom practice. In fact, Handal (2003, p. 47) went so far as to say that 'these beliefs appear to be cogent enough to either facilitate or slow down educational reform' and thus can have a significant impact on the enactment of any reform. It can take time for teachers to build the confidence needed to adjust their teaching approaches, and in many cases, they are also required to engage with new content themselves which can be an additional challenge (Thompson et al., 2013). This process of professional growth is continuous and takes time, involving both learning and re-evaluating as they respond to evolving expectations and classroom realities. Moreover, pre-service teachers during times of curriculum reform often express apprehension about implementing the reform changes, particularly when such reforms diverge significantly from the pedagogical approaches and learning experiences they encountered during their own secondary education (Johnson et al., 2020).

Methodology

This study reports on an aspect of a larger study that utilised document analysis and teacher interviews to gain insights into teachers' perceptions of teaching LCHLM. For this paper, only the elements of the teacher interviews which specifically related to problem solving will be presented. Flores (2005) stated that teachers' perceptions of a reform influence the degree and manner that they implement it and for this reason teacher interviews were

employed to gain insights into the opinions and challenges that the teachers face when trying to embed problem solving into their classroom.

Teacher Interviews

Semi-structured online interviews were conducted, and recorded, with nine LCHLM teachers to probe their perception and enactment of the curriculum. The nine teachers were recruited via social media and volunteered to participate in the study. The teachers had between 6- and 33-years' experience teaching mathematics and between 5- and 25-years' experience of teaching the LCHLM syllabus. Topics such as their opinions of the new curriculum, perceived associated pressures of teaching LCHLM, the specific content of the syllabus, the time allocated to cover the syllabus, their pedagogical approaches, and assessment strategies that they utilise were all discussed. It should be stated that while these interviews provide valuable insight, the small sample size and specific participant contexts limit the study's generalisability to the broader population of mathematics teachers.

Analysis of Teacher Interviews

To analyse the data a reflexive thematic analysis approach was used, which allows codes and themes to emerge inductively from the data rather than being pre-defined. Soon after each interview, a member of the research team reviewed and corrected the transcript for accuracy, providing an initial opportunity to engage with the data. Once all interviews were transcribed, the researchers independently re-read them multiple times, noting emerging trends and noteworthy passages. To ensure reliability, the researchers independently used NVivo to systematically code the data, identifying significant elements as preliminary codes. These codes were refined collaboratively through discussion. The researchers then grouped related codes into emergent themes. Once the emergent themes were identified, the researchers discussed their findings to decide whether to keep, combine, refine, separate, or discard themes. Finally, any disagreements were resolved through consensus, and each theme was given a clear and concise name.

Findings

Overall, all the teachers interviewed stated that they viewed problem solving as an essential part of the curriculum and a key skill that all students should have to develop. Teacher 4 noted that 'problem solving is actually very important' whereas Teacher 6 stated that problem solving is 'huge, it's very, very valuable.' Teacher 5 spoke about the long-term benefits for the students by highlighting that the LCHLM syllabus is very difficult and that if students could develop their problem-solving skills, then 'if they can master that, I guarantee you that they'll fly through four years of third level.'

While the teachers acknowledged the importance of problem solving and recognised it as a key skill for students to develop, several expressed concerns about students' abilities in this area. Teacher 5 remarked that many students' problem-solving skills are 'very weak.' Similarly, Teacher 1 observed that even in higher level mathematics classes, when it comes to problem solving, 'some of the weaker kids, you have to guide them a bit more.' They elaborated further, highlighting that student engagement with problem solving varies significantly depending on the group's ability level, noting that 'you get different levels of

engagement' based on the class. Echoing this perspective, Teacher 5 commented on how some students are resistant to problem-solving tasks, saying they 'don't like it because it pushes them,' and emphasised that developing these skills 'takes a lot of time' and that students often 'resist' teachers' efforts to involve them in such activities. Teacher 9 added that in their exams, the final 20% is allocated to problem solving questions, but among the 'less good groups,' when it comes to those questions, 'nobody' manages to solve them.

Another challenge highlighted by the teachers was the difficulty of fitting problem solving into class time. Teacher 2 shared their enthusiasm for using problem solving tasks, expressing that they 'wish we had more time to do them in class' and that, although they would love to include more, they 'don't have time to.' Similarly, Teacher 3 reflected on the nature of problem solving itself, suggesting that it 'has to kind of come from yourself' and remarking that they felt that 'you can't teach problem solving.' Nonetheless, they stressed its value in the classroom, acknowledging that 'you do need time to problem solve' and lamenting that, while they would love to spend a double class on problem solving regularly, 'in reality, you just don't have that time.' Teacher 6 echoed these concerns, pointing to time constraints as a barrier to developing students' skills, stating they do not have 'enough freedom of time ... to develop their problem-solving skills' to the extent they would like. Finally, Teacher 9 offered a slightly different perspective on the time debate, noting that while they try to engage Junior Cycle students in problem solving, by the time students reach the Leaving Certificate, 'they don't want you to spend an hour having the crack [trying to figure out a problem], they want to do their subject and move on.'

Conclusion

While the participating teachers in this study expressed strong support for the value of problem solving—viewing it as a critical skill for students' mathematical development—they also highlighted several persistent challenges in embedding it meaningfully within the LCHLM syllabus. A central issue identified was the constraint of classroom time. Teachers noted that effective problem solving requires space for students to think deeply, explore different strategies, and grapple with uncertainty—processes that are inherently time-consuming. In contrast to traditional approaches, where solutions are often demonstrated and replicated, problem solving demands a slower, more exploratory pace that is difficult to maintain within the time pressures of a packed curriculum.

Compounding this challenge is students' relative unfamiliarity with problem solving as a mode of learning. Teachers reported that many students lack the confidence, resilience, or experience to engage productively with problem solving tasks. Some are hesitant to persist through initial confusion or struggle, while others expect more direct guidance and clear-cut answers. This resistance, whether due to prior learning habits or external assessment pressures, further complicates efforts to foster a classroom culture where problem solving is a routine and valued part of mathematics learning.

These findings underscore the reality that, while there is broad professional commitment to promoting problem solving in mathematics education, its successful integration into daily classroom practice remains fraught with difficulty. Time constraints,

curriculum demands, and students' limited prior experience with problem solving all serve as significant barriers. However, the importance of cultivating students' problem-solving abilities—both for their mathematical development and their broader critical thinking skills—makes this an area that cannot be overlooked. If problem solving is to be more than an aspirational goal, it must be supported through thoughtful curriculum design, sustained professional development, and a realistic alignment of classroom expectations with the time and resources available. Addressing these issues is essential to ensuring that problem solving becomes a central, lived experience in the mathematics classroom, rather than a peripheral ideal.

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Interest and confidence in primary mathematics: Exploring student attitudes through clustering

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This study explores Irish primary students' attitudes toward mathematics, focusing on two key dimensions: interest and confidence. Using the TIMSS 2015 attitudinal scales, data were collected from 136 sixth-class students across two primary schools in Ireland. On average, students reported slightly higher confidence than interest, with no significant gender differences. Cluster analysis identified three attitudinal profiles: Low Interest–High Confidence, Low Interest–Low Confidence, and High Interest–High Confidence. These profiles provide a nuanced perspective on students' emotional engagement in mathematics, highlighting the limitations of relying solely on average scores. While gender distribution across clusters varied descriptively, no statistically significant associations were found. Findings support the use of affect-responsive teaching strategies and recommend integrating emotional and motivational assessments into policy and practice.

Keywords: Mathematics attitudes, interest, confidence, attitudinal profiles

Introduction

Affective dimensions in mathematics, i.e., how students feel about and engage with the subject, critically influence their engagement and learning (Renninger & Hidi, 2016). Among the most critical affective factors, interest and confidence play pivotal roles in shaping students' attitudes, behaviors, and persistence in mathematics learning (Hidi & Renninger, 2006; Pinxten et al., 2014).

Interest reflects emotional engagement and a willingness to explore ideas (Hidi & Renninger, 2006), while confidence involves belief in one's mathematical ability, shaped by personal experience, emotional state, and social feedback (Bandura, 1997). Students who find mathematics interesting are more likely to persist through difficulty and enjoy problem-solving (Hidi & Renninger, 2006). Higher confidence levels can mitigate mathematics anxiety, suggesting that interventions focusing on self-belief and success experiences can help break the cycle of disengagement (Dowker et al., 2016). Both interest and confidence are linked to student success (Fredricks et al., 2004; Hidi & Renninger, 2006).

In primary years, when attitudes are still forming, affective experiences can internalize students' self-beliefs, willingness to engage with challenges, and their later performance (Dweck, 2006; Lazarides et al., 2020). Fredricks et al. (2004) emphasize that emotional and cognitive disengagement often go hand in hand, leading to academic withdrawal if not addressed early. International assessments such as the Trends in International Mathematics and Science Study (TIMSS) now integrate affective dimensions, acknowledging the role that emotional and motivational factors play in mathematical learning outcomes (Mullis et al.,

2016). The affective dimension of mathematics learning is echoed in Ireland's new Primary Mathematics Curriculum (PMC), which emphasizes positioning dispositional constructs, such as confidence and interest, as essential to mathematics learning (NCCA, 2023). The PMC also explicitly highlights the importance of fostering a productive attitude toward mathematics for the effective development of mathematical proficiency. While interest and confidence are often studied separately, recent research highlights that their interaction may yield even greater insights than their isolated effects. (Lazarides et al., 2020; Pinxten et al., 2014; Renninger & Hidi, 2016). For example, high confidence alone does not guarantee persistence if interest is lacking, and vice versa. This study builds on such findings by analysing the co-occurrence of interest and confidence among primary students using cluster analysis, a person-centered technique. This approach aligns not only with international assessments like TIMSS but also with the PMC's emphasis on nurturing positive mathematical identities, underscoring the need for empirical insights into how these affective constructs co-exist in classrooms. The study addresses the following research questions:

- (1) What are the levels of interest and confidence reported by primary students (aged 11-12) in mathematics learning?
- (2) What distinct affective profiles emerge when interest and confidence are examined jointly, using cluster analysis?

Methodology

This quantitative, cross-sectional study examined 136 sixth-class students (60 boys, 76 girls), aged 11–12 years (mean age = 11.4), from two urban primary schools in Ireland. Participants reflected a mix of academic and socio-economic backgrounds. Data were collected using a questionnaire adapted from TIMSS 2015, which assesses mathematics interest and confidence of 10-11-year-olds using two scales: 'Students Like Learning Mathematics' and 'Students Confident in Mathematics.' Each scale consists of nine items. One statement was excluded from the Confidence scale to reduce teacher-related bias. Consequently, 17 statements were included: 9 for Students Like Learning Mathematics (LM) and 8 for Students Confident in Mathematics (CM). Students recorded their level of agreement for each statement with four response options: Agree a lot, Agree a little, Disagree a little, and Disagree a lot. The questionnaire was administered in a pen-and-paper format. The study received ethical approval from Dublin City University Research Ethics Committee (DCUREC/2022/022). Ethical procedures ensured participant anonymity and confidentiality.

Data were analyzed using IBM SPSS Statistics Version 29 using descriptive statistics, t-tests, correlations, and K-means cluster analysis. Responses were recorded on a 4-point Likert scale (1 = Strongly Disagree to 4 = Strongly Agree). Composite scores for each scale were calculated by averaging responses, after reverse-coding negatively worded items. This resulted in a mean LM_score and CM_score per student, each ranging from 1 to 4.

Findings

Students' Interest and Confidence in Mathematics

Descriptive Statistics: The mean score for interest in mathematics (LM_score) among the 136 participating students was 2.74 (SD = 0.72), while the mean score for confidence in mathematics (CM_score) was slightly higher at 2.93 (SD = 0.74). This indicates that, on average, students felt more confident than interested in mathematics. These results suggest that while students tend to feel capable in mathematics, their emotional engagement with the subject may be comparatively lower.

Paired Samples Analysis: Confidence vs. Interest: A paired sample t-test revealed a statistically significant difference between confidence and interest scores, $t(135) = -3.244, p = .001$. The mean difference was -0.193, indicating that on average, students rated their confidence significantly higher than their interest in mathematics. The correlation between LM_score and CM_score was moderate ($r = .545, p < .001$), suggesting that while related, the constructs represent distinct aspects of students' attitudes. The effect size for this difference (Cohen's $d = -0.278$) was small to moderate, highlighting a meaningful, though not large, divergence in affective orientation toward mathematics.

Gender-Based Differences: Independent samples t-tests were conducted to examine gender differences in LM_score and CM_score. For interest, boys (M = 2.80, SD = 0.71) scored slightly higher than girls (M = 2.69, SD = 0.73), but the difference was not statistically significant, $t(134) = 0.843, p = .401$. Similarly, for confidence, boys (M = 2.99, SD = 0.70) also scored higher than girls (M = 2.88, SD = 0.77), though again the difference was not statistically significant, $t(134) = 0.939, p = .349$. Both p -values exceeded the 0.05 threshold, with small effect sizes (Cohen's $d = 0.15$ for interest, 0.16 for confidence), suggesting that boys and girls reported similar levels of interest and confidence in mathematics overall.

Affective Profiles of Learners Based on Cluster Analysis

To determine whether distinct groups of students could be identified based on their interest and confidence levels, a cluster analysis was conducted using K-means clustering. Using the elbow method, a 3-cluster solution (K = 3) was selected as the optimal representation of the data for the most meaningful representation. These results revealed three distinct profiles. Table 1 shows the distribution of students across the clusters. Nearly half (44.9%) of the students demonstrated both high interest and high confidence (Cluster 3), while the remaining students were split between those who were confident but not interested (25.7% - Cluster 1) and those who were disengaged on both fronts (29.4% - Cluster 2).

Table 1

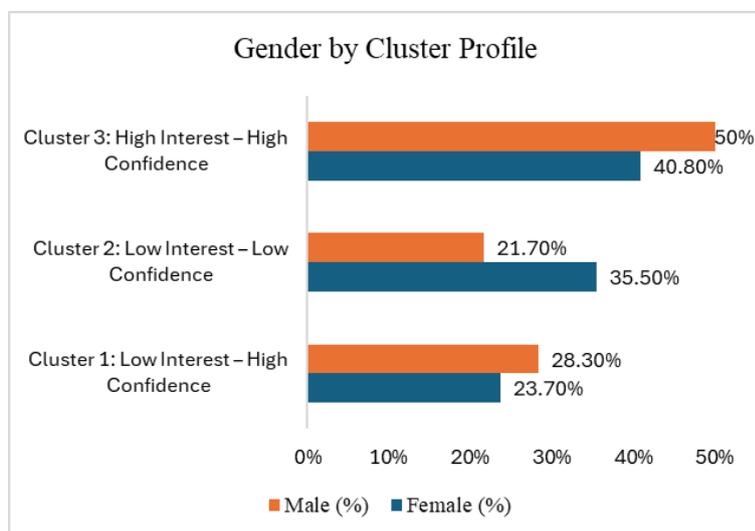
Distribution of students across the clusters

Cluster	Description	Mean Interest score	Mean Confidence score	Cluster composition
Cluster 1: Low Interest, High Confidence	Appear to feel capable but not particularly emotionally invested in the learning process.	2.22	3.07	N = 35 (25.7%)
Cluster 2: Low Interest, Low Confidence	Represent students who are disengaged or struggling.	2.21	2.00	N = 40 (29.4%)
Cluster 3: High Interest, High Confidence	Appear to be positively engaged with mathematics, both emotionally and cognitively.	3.38	3.46	N = 61 (44.9%)

To determine whether gender influenced the distribution of students across the three clusters, a cross-tabulation of cluster membership by gender was conducted, followed by a chi-square test for independence. A visual representation of this distribution is in Figure 1.

Figure 1

Gender Distribution Across Clusters



The cross-tabulation shows that a slightly higher proportion of boys were in Cluster 3 (high interest and high confidence), while a greater percentage of girls were in Cluster 2 (low interest and low confidence). Cluster 1 showed a relatively balanced gender distribution. To test for statistical significance, a chi-square test was conducted, yielding a value of $\chi^2 (2, N = 136) = 3.106, p =$

.212. This result indicates that there was no statistically significant relationship between gender and cluster membership. Therefore, gender was not a significant factor in determining the attitudinal profile to which a student belonged.

In summary, the analysis revealed that, on average, students reported moderate levels of interest and confidence, and mean confidence scores were slightly but significantly higher than their mean interest scores. No significant gender differences appeared in scores or cluster

membership. Descriptively, more girls appeared in Cluster 2 and more boys in Cluster 3, though this pattern was not statistically significant.

Discussion and Conclusion

This study shows that while students generally report confidence in mathematics, emotional engagement may lag behind. While the students in this study generally indicated positive attitudes, a noteworthy difference was observed between interest and confidence, with confidence being rated higher. This aligns with earlier research suggesting that students may feel capable of tackling mathematical tasks but still lack emotional engagement (Fredricks et al., 2004; Pinxten et al., 2014; Renninger & Hidi, 2016). Cluster analysis offered a more nuanced view than group averages, revealing meaningful affective profiles that reflect varying combinations of emotional and cognitive dispositions. Aligning with the work of Lazarides et al. (2020), the findings validate the notion that students' affective experiences are complex and categorizing students by demographics or averaging responses can overlook significant variability within groups.

Students in Cluster 1, characterized by low interest yet high confidence represent those who feel competent in mathematics but are emotionally disengaged. According to Bandura (1997), confidence can be bolstered by repeated success or teacher affirmation, even if intrinsic interest is lacking. This pattern of competence without enjoyment (Pinxten et al., 2014) may stem from a lack of motivation in the subject, potentially undermining their long-term engagement and deeper learning (Fredricks et al., 2004). Classroom practices that fail to stimulate curiosity or provide autonomy might reinforce competence without enjoyment (Pinxten et al., 2014). Addressing this disconnect calls for pedagogies that foster interest through meaningful and open-ended problem-solving experiences. Cluster 2 (low interest, low confidence) represents those who may be particularly susceptible to mathematics anxiety and negative self-concept, aligning with findings from Dowker et al. (2016). Dowker et al. (2016) note that the emotional toll of low confidence in mathematics can manifest early, leading to an increased risk of forming negative emotional ties with mathematics. Cluster 3, the largest group, includes confident, interested students with an ideal profile associated with achievement, resilience, and enjoyment of mathematics, fostering persistence and success (Bandura, 1997; Dweck, 2006).

The analysis showed no significant gender differences in interest or confidence, supporting studies of diminishing gaps in mathematical attitudes among younger students in many countries (Mullis et al., 2020). Nonetheless, the descriptive trend observed in the cluster profiles, particularly a higher proportion of girls in the low-interest, low-confidence group, echoes broader concerns about confidence gaps and stereotype internalization (Lazarides et al., 2020). These patterns warrant attention, even in the absence of significant mean differences.

This study shows that primary students engage with mathematics in emotionally diverse ways, highlighting emotional and motivational variety even within similar demographic groups. The study has important implications for teaching. From a practical standpoint, these findings suggest that evaluating affective factors may help teachers identify

disengaged students, though further research is needed to confirm their impact on learning outcomes. Differentiated instruction should encompass more than just academic performance; it must also consider motivational and emotional engagement. For instance, rich mathematical tasks can enhance students' emotional ties to mathematics (Kaur, 2025).

This study is limited by its cross-sectional design, reliance on self-report, and a convenience sample of two schools, which restricts causal interpretation and generalizability. Future longitudinal research could track how these profiles evolve over time and relate to mathematics performance. Students' attitudes evolve, and tracking profiles over time may reveal developmental shifts not evident in single-time-point designs. Nonetheless, the findings support integrating affective measures, such as confidence and interest, alongside achievement scores in practice and policy, which align with the aims of the Irish Primary Mathematics Curriculum. Such integration could help design more holistic support systems, as suggested by existing research. Professional learning programmes must equip teachers with strategies to enhance both confidence and interest, recognizing that affective development is as critical as cognitive growth. Ultimately, effective teaching must strike a balance between supporting students' mathematical proficiency and understanding their motivation.

However, since performance data were not collected, such suggestions are grounded in existing theory rather than conclusions from this dataset alone. Future research should investigate the connections between affective profiles and achievement, as well as how classroom practices may influence students' movement between profiles. Recognizing the emotional diversity within mathematics classrooms is crucial to cultivating capable, curious, and confident learners.

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Investigating probabilistic justice in elementary school students: A study of probability and fairness understanding through games

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This study aims to explore the concept of probabilistic justice and its impact on elementary school students' understanding of fairness. Using two games—rock-paper-scissors and a specially designed dice game—it investigates how fifth-grade students evaluate fairness in probabilistic contexts and how their perceptions of fairness evolve through engagement with these games. The study focuses on the role of probability in shaping fairness judgments and examines how children interpret outcomes when probabilities are unequal. Given the limited research on fairness in probabilistic scenarios, this study seeks to contribute to the understanding of how children develop probabilistic reasoning and how this reasoning influences their sense of justice. The findings are expected to inform educational practices aimed at teaching fairness and probability from an early age, with implications for the development of critical thinking and moral judgment in children. Data collection has been completed, and analysis will follow using a qualitative observational approach.

Keywords: Probabilistic justice, children's fairness reasoning, probabilistic reasoning, children's moral development

Introduction

Fairness is a foundational concept in both social and educational contexts, deeply influencing children's cognitive and social development. While fairness is often viewed as the equal distribution of outcomes, a growing interest is being placed on probabilistic justice, that is, how individuals evaluate fairness when outcomes are governed by chance or uncertainty rather than fixed rules or equal distributions. This term is used to describe fairness judgments in probabilistic contexts, especially when randomness, unequal odds, or perceived bias are involved.

Children's understanding of fairness is influenced by their cognitive development. According to Piaget (1932), young children tend to associate fairness with equal distribution, but as their cognitive abilities evolve, they begin to understand the subtler aspects of fairness, including the role of chance and probability. This study seeks to investigate how elementary school students conceptualize fairness in the context of probabilistic events. Specifically, this research will focus on probabilistic justice by using simple games such as rock-paper-scissors and a specially designed dice game to examine how students evaluate fairness based on probabilistic outcomes.

This study is particularly timely given that research in moral development has primarily focused on equality-based concepts of justice (Kohlberg, 1969). In contrast, there is a significant gap in research on how children perceive fairness when it involves probabilistic or uncertain events, such as games of chance (Tversky & Kahneman, 1974). By exploring

these concepts, the study aims to provide new insights into how children's understanding of fairness evolves when influenced by uncertainty and chance.

The problem this research seeks to address is the limited understanding of how children conceptualize fairness, especially in the context of probabilistic events. While fairness is often associated with equality and distribution, the role of probability and chance in determining fairness has not been sufficiently explored in children's moral development. This study aims to address this gap by investigating how elementary school students evaluate fairness in games with probabilistic outcomes and how this understanding evolves with age.

The study will explore the following research questions:

1. How do elementary school students evaluate fairness in games of chance (rock-paper-scissors and dice games)?
2. How do students perceive the role of probability in determining fairness?
3. In cases where the probabilities are unequal (e.g., a biased dice), do students recognize the imbalance and perceive the game as unfair?
4. How does students' understanding of fairness evolve as they engage with probabilistic scenarios?

Theoretical Framework

The concept of fairness has been explored extensively in the field of moral development, with foundational work by Piaget (1932) and Kohlberg (1969) suggesting that children's understanding of fairness evolves with cognitive development. These theories primarily focus on equality as a central component of fairness. However, the concept of fairness in probabilistic scenarios, where outcomes are determined by chance rather than equal distribution, has not been thoroughly investigated.

In traditional studies of fairness, equality-based justice is the dominant concept. Piaget's (1932) theory of cognitive development suggests that children initially see fairness as equal distribution, and as they grow, they begin to understand that fairness may also involve recognizing varying circumstances and unequal distributions. Kohlberg's (1969) stages of moral development build upon Piaget's work, positing that children's moral reasoning matures over time, evolving from a focus on direct equality to understanding abstract concepts of fairness, including principles that might allow for unequal distributions in certain contexts. These works provide a strong foundation for understanding how children first engage with fairness in terms of equality.

However, this understanding has largely been confined to the domain of moral development, and there is a gap in the literature when it comes to probabilistic fairness—fairness evaluated in probabilistic or uncertain contexts. The work of Tversky and Kahneman (1974) on probabilistic judgment offers insights into how people assess fairness when outcomes are uncertain or based on probability. Their work laid the groundwork for understanding how individuals make decisions under uncertainty, which is crucial for evaluating fairness in probabilistic scenarios. Tversky and Kahneman introduced the concept of heuristics and biases, which refers to the shortcuts people take when making judgments

about uncertain situations. This concept has implications for understanding how fairness might be perceived in probabilistic contexts, particularly when the odds are unequal or when there are biases in the perceived fairness of outcomes.

Barrouillet and Camos (2001) further explored how children's cognitive development influences their understanding of probability. They found that, as children mature, they become better at evaluating probabilistic outcomes and understanding the inherent uncertainties involved in decision-making processes. This developmental progression is essential for grasping how children come to view fairness when the outcomes are influenced by probabilistic factors.

Recent research has expanded these developmental and cognitive perspectives by highlighting the role of subjective and contextual influences on children's probabilistic reasoning. Nikiforidou (2023) discussed the view that young children cannot reason about probability, emphasizing that their interpretations are often intuitive and embedded in lived experiences. Similarly, Kingston and Twohill (2022) demonstrate that children frequently rely on personal desires, emotional responses, and prior experiences when interpreting probabilistic tasks, rather than formal logic. These findings suggest that fairness judgments in probabilistic contexts are shaped not only by cognitive development, but also by how children emotionally relate to outcomes and perceive chance within social interactions. Further extending this view, Paparistodemou et al. (2008) showed that children's ideas of fairness in random scenarios are dynamically constructed through both visual-spatial reasoning and interactive game design. In their study, children actively negotiated between deterministic ideas of fairness (e.g., symmetry, equal placement) and stochastic ones (e.g., randomness, unpredictability), sometimes modifying randomness to make outcomes seem 'fair.' This suggests that children's reasoning about probabilistic fairness is fluid and responsive to contextual cues, rather than fixed or purely logical. Integrating these perspectives supports a more nuanced understanding of how probabilistic fairness emerges at the intersection of cognitive growth, intuitive reasoning, and subjective interpretation.

This study builds upon these foundational theories, applying them to the understanding of fairness in probabilistic contexts and investigating how this understanding emerges in elementary school students. The goal is to extend the existing theories on fairness to include probabilistic judgments and to understand how children's cognitive development impacts their perceptions of fairness in uncertain, probabilistic contexts.

Methodology

This study employed a qualitative observational approach to investigate how elementary school students reason about fairness in probabilistic contexts. Data were collected from two 5th-grade classrooms in a public elementary school, each consisting of 24 students (ages 10–11). The games were played in pairs, and data collection took place during regular class sessions. The data collection will occur in two phases:

1. **Rock-Paper-Scissors Game:** Students will play rock-paper-scissors in a repeated manner (20 rounds). After each round, students will be asked to assess whether they think the

game was fair and whether the outcomes were based on chance. This will help in evaluating how they perceive fairness in repeated events with equal probabilities.

2. Dice Game: In the dice game, students will use a specially designed biased die with unequal probabilities (e.g., two faces showing a 2 and three faces showing a 3). Students will play 20 rounds of the game, and after each round, they will discuss whether they perceive the game as fair. They will also be asked to justify their answers, particularly when the probabilities are unequal.

Data will be collected using a combination of pre-game and post-game questionnaires, observational notes taken during the games to record students' interactions and responses, and group discussions to explore students' reasoning behind their fairness evaluations.

Fifth-grade students (ages 10–11) were selected because they are developmentally at a transitional stage where abstract thinking and reasoning about fairness and probability typically begin to emerge. According to Nikiforidou (2023), children at this age increasingly attempt to reconcile intuitive beliefs, personal experiences, and formal aspects of randomness, making them particularly suitable for studies that examine fairness in probabilistic contexts.

The data will be analyzed using thematic analysis following the approach proposed by Braun and Clarke (2006). An interpretive-inductive strategy will guide the identification of recurring patterns in how students perceive and explain fairness and chance across both game tasks (rock-paper-scissors and a biased dice game). All data sources – classroom discussions, observation notes, and questionnaires – will be reviewed iteratively and coded based on emergent themes.

The analysis will be structured around four themes: Fairness, probability reasoning, subjective interpretations, and justification strategies. These include concepts such as fairness as equal distribution, contextual fairness judgments, probabilistic reasoning (e.g., recognizing uneven outcomes in a biased die), and emotionally or experientially driven responses (e.g., attributing outcomes to luck or strategy).

In addition to thematic patterns, the analysis will consider the developmental aspects of children's reasoning. Some students may demonstrate early fairness thinking by equating fairness with equal outcomes, while others may provide more context-sensitive explanations or probabilistic justifications. Subjective expressions such as emotional responses or beliefs in personal control (e.g., "I won because I used a strategy") will be interpreted as evidence of intuitive or affect-based reasoning rather than logical probabilistic judgment. This approach allows for a nuanced understanding of how children integrate cognitive, moral, and emotional dimensions when evaluating fairness in chance-based games. Data collection has been completed, and analysis is currently forthcoming.

Expected Contributions

This study aims to contribute to a more nuanced understanding of how elementary school students reason about fairness in probabilistic contexts. By examining students' interpretations of fairness in both equal-probability (rock-paper-scissors) and unequal-probability (biased dice) games, the research will provide insights into how probabilistic

reasoning, intuitive beliefs, and emotional responses shape their fairness judgments. The findings are expected to inform mathematics education and moral development research by highlighting the developmental emergence of ‘probabilistic justice.’ Additionally, this work can guide educators in designing learning environments that encourage children to reflect critically on fairness, chance, and uncertainty.

Conclusion

This study seeks to explore the relationship between probabilistic reasoning and fairness in elementary school students. By investigating how students perceive fairness in games with probabilistic outcomes, the research will offer a deeper understanding of how children conceptualize fairness and how probabilistic thinking contributes to their moral development. The results will also inform educational practices aimed at fostering an understanding of fairness and probability from an early age.

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Primary mathematics curriculum in Ireland: The Shape and Space strand matters more than you might think

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Although Ireland performed above average in mathematics in PISA 2022, students scored lowest in the space and shape subscale, indicating a need for greater spatial awareness support. Given the spatial-mathematical association, we examined how spatial skills relevant to the new primary mathematics curriculum (mental rotation, spatial visualisation, and spatial orientation) predict sixth-graders' place value (PV), arithmetic (Ar), geometry (Ge), and word problems (WP) performance and the role of gender. Data from 100 sixth-graders in Ireland showed that mental rotation significantly predicted PV and Ar; spatial visualisation predicted Ar, Ge, and WP; and spatial orientation predicted performance across all areas. After accounting for spatial skills, females' Ge and WP scores were predicted to be higher than males'; spatial visualisation was a significant predictor for females, while spatial orientation was a significant predictor for males. Our findings underscore the value of emphasising spatial skills to support all students' mathematics performance.

Keywords: spatial ability, primary mathematics curriculum, space and shape, PISA

Introduction

Despite Ireland's above-average performance in overall mathematics in PISA 2022, students scored lowest on the space and shape subscale—consistent with the 2012 cycle when mathematics subscales were last reported (Donohue et al., 2023, pp. 21, 47). This suggests that spatial content is relatively challenging, highlighting an area for improvement in mathematics education in Ireland. Furthermore, spatial ability (e.g., the ability to 'recall, generate, manipulate, and reason about spatial relations; Gilligan-Lee et al., 2022) has been shown to predict students' mathematics performance at all educational stages (Newcombe et al., 2019). However, a great amount of research has focused on which spatial skills are most relevant to mathematics performance in early childhood or secondary education (Newcombe et al., 2019) with studies on the middle years (age 11 to 13 years old) resulting in conflicting findings. In our study, we examined whether three spatial skills—mental rotation (the ability to rotate 2D and 3D objects mentally), spatial visualisation (the ability to perform transformations of spatial objects that involve multiple steps), and spatial orientation (imagining the appearance of objects or scenes from a different viewpoint)—predict students' performance across place value, arithmetic, geometry, and word problem tasks. A trend identified by Donohue et al. (2023) is that male students in Ireland outperformed female students in all cycles of the PISA mathematics assessment (Donohue et al., 2023, p. 21). This persistent gender gap in mathematics at the national level, along with previous, albeit limited, evidence of gender differences in the spatial-mathematical association during the middle years (Harris et al., 2021), prompted us to examine the role of gender.

Interestingly, our examination of the role of mental rotation, spatial visualisation, and spatial orientation on mathematics achievement is timely, as these three spatial skills are embedded within the newly introduced shape and space mathematics curriculum strand in Ireland (NCCA, 2023). For example, mental rotation is essential in sixth-grade mathematical tasks that require students to recognise whether a shape is rotated or reflected. Spatial visualisation is evident in mathematical activities that require complex mental manipulation of spatial information, such as transforming 2D and 3D shapes and interpreting diagrams. Spatial orientation is central to outcomes related to realising positions on the coordinate plane, navigating directions, and visualising how geometric shapes appear from different perspectives. Therefore, our study could shed light on how the spatial content of the new primary mathematics curriculum (NCCA, 2023) can not only improve students' spatial thinking skills but also their overall mathematics (Gilligan-Lee et al., 2022) and STEM (Science, Technology, Engineering, and Mathematics) performance (Zhu et al., 2023).

Literature review

Spatial ability has been associated with mathematics achievement (Newcombe et al., 2019). However, which spatial skills are most crucial for sixth-grade mathematics performance remains unclear. For example, Mix et al. (2016) did not find mental rotation and spatial visualisation to predict overall mathematics performance. Instead, visuospatial working memory was the strongest predictor of overall mathematics scores, with perspective-taking—an aspect of spatial orientation—also contributing significantly, though to a lesser extent. The findings of Mix et al. (2016) suggest that static, memory-based and not dynamic, object-based spatial processes may be more relevant to the mathematical demands at this developmental stage. Yet, Cirino et al. (2016) reported that mental rotation directly predicted sixth-grade general mathematics performance and appeared to have an indirect effect on fraction performance by supporting procedural computation and symbolic comparison. Focusing on geometry and number sense tasks, Harris et al. (2021) found that mental rotation and spatial visualisation predicted the mathematics performance of slightly younger students (fifth grade). The outcomes from the studies by Cirino et al. (2016) and Harris et al. (2021) show that examining specific mathematical domains, in contrast to Mix et al.'s (2016) study, may reveal distinct spatial skill contributions that remain undetected when analysing overall mathematics performance.

Furthermore, there are indications that the spatial-mathematical relationship may vary between male and female students during the middle years of schooling (Harris et al., 2021). While Harris et al. (2021) did not find gender differences in the mathematics scores of fifth-grade students, distinct gender patterns in how spatial skills predicted mathematics performance emerged. For geometry, mental rotation predicted the scores of male students, while spatial orientation predicted the scores of female students. For number sense, males' performance was predicted by mental rotation and spatial visualisation, while for females, only spatial orientation again predicted their performance. These findings suggest that male and female students may use different spatial strategies (object-based vs. egocentric) when solving geometry and number sense tasks (Harris et al., 2021).

Our study extends prior research by examining the role of mental rotation, spatial visualisation, and spatial orientation across a broader range of sixth-grade mathematical domains within the context of Ireland, while also assessing the role of gender. To this end, we investigated the following research question: Do mental rotation, spatial visualisation, and spatial orientation predict sixth-grade students' performance in place value, arithmetic, geometry, and word-problem tasks, and what is the role of gender?

Method

We conducted a cross-sectional study using data on spatial and mathematics scores collected prior to spatial training. One hundred sixth-grade students (74 female, 26 male, 0 other; $M = 11.9$ years, $SD = 0.2$) from five classes in two primary schools in Ireland participated in our study. We obtained written consent from parents or guardians and informed all students that they could withdraw from the study at any time without providing a reason. Teachers led the administration of the assessment materials, for which we provided training prior to the data collection. All students completed both spatial and mathematical assessments in their regular classrooms on the same day. To measure spatial ability, we used the Spatial Reasoning Instrument (SRI; Ramful et al., 2017) comprising 30 tasks, 10 each assessing mental rotation, spatial visualisation, and spatial orientation. To assess mathematics performance, we used a curriculum-aligned battery of 15 tasks covering place value (ordering and locating numbers on number lines), arithmetic (operations with decimals and fractions and solving simple equations), geometry (area, volume, symmetry, and missing shape dimensions), and word problems (real-life applications involving cost, space, and fractions).

Key Findings

To investigate the predictive role of mental rotation (MR), spatial visualisation (SV), and spatial orientation (SO) on students' place value (PV), arithmetic (Ar), geometry (Ge), and word problems (WP) performance and the role of gender, we conducted four hierarchical multiple regression analyses (one for each mathematics area). In each model, the dependent variable was the respective mathematics score. We entered gender (dummy coded; female = 0, male = 1) in Step 1, added the three spatial measures (MR, SV, and SO) in Step 2, and included interaction terms between gender and each spatial measure in Step 3. As shown in Figure 1, there were some gender differences in mathematics performance, with males scoring higher in PV and females scoring higher in Ge and WP. However, in Step 1 of the regression, gender alone did not significantly predict performance in any mathematical area (PV: $R^2 = 0.04$, $p = .052$; Ar: $R^2 = 0.00$, $p = .974$; Ge: $R^2 = 0.01$, $p = .260$; WP: $R^2 = 0.02$, $p = .232$).

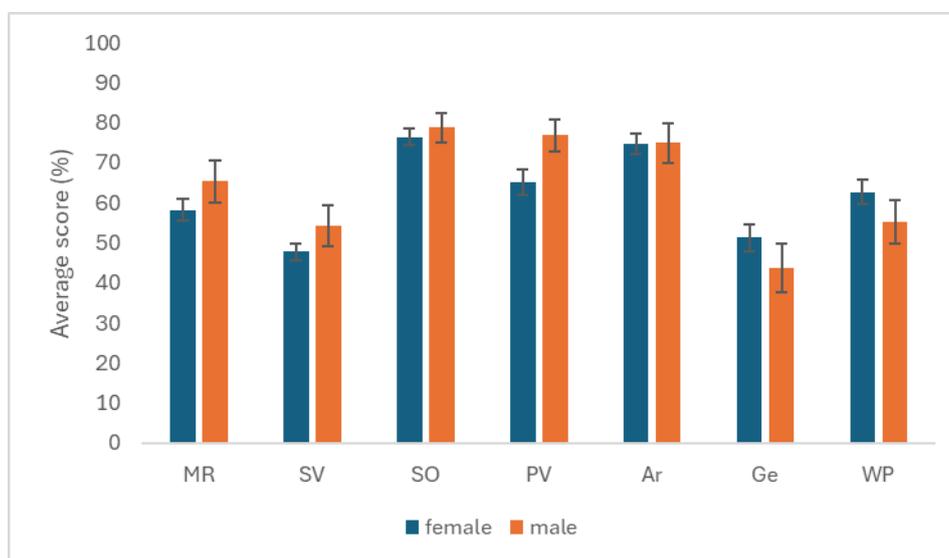
In Step 2, adding spatial skills significantly improved model fit (PV: $\Delta R^2 = 0.24$, $p < .001$, Ar: $\Delta R^2 = 0.37$, $p < .001$, Ge: $\Delta R^2 = 0.32$, $p < .001$, WP: $\Delta R^2 = 0.29$, $p < .001$). Mental rotation significantly predicted place value ($b = 0.15$, $p = .045$) and arithmetic ($b = 0.23$, $p = .004$) scores, but not geometry ($b = 0.18$, $p = .079$) or word problems ($b = 0.11$, $p = .252$). Spatial visualisation significantly predicted arithmetic ($b = 0.20$, $p = .018$), geometry ($b = 0.31$, $p = .007$), and word problems ($b = 0.21$, $p = .049$), but not place value ($b = 0.09$, $p = .231$). Spatial orientation significantly predicted scores in all four mathematical areas (PV: $b = 0.22$, $p = .020$, Ar: $b = 0.20$, $p = .046$, Ge: $b = 0.32$, $p = .023$, and WP: $b = 0.41$, $p = .002$).

Furthermore, gender became a significant predictor of geometry ($b = -1.01, p = .028$) and word-problems ($b = -0.89, p = .036$), with males scoring higher than females, but remained non-significant in place value ($b = 0.48, p = .131$) and arithmetic ($b = -0.31, p = .335$).

In step 3, we added interactions between gender and the three spatial scores (MR, SV, and SO) in the regression models. Gender-spatial interactions were significant only for arithmetic performance ($\Delta R^2 = 0.12, p < .001$; $SV \times \text{Gender}: b = -0.51, p = .013$; $SO \times \text{Gender}: b = 1.01, p < .001$). Follow-up analysis revealed that for females, arithmetic performance was significantly predicted by MR ($b = 0.29, p < .001$) and SV ($b = 0.33, p = .001$) but not by SO ($b = 0.04, p = .744$). For males, SO was the only significant predictor ($b = 1.06, p < .001$) (MR: $b = 0.03, p = .889$, SV: $b = -0.018, p = .230$).

Figure 1

Average scores by gender across spatial and mathematics measures



Note: MR = mental rotation, SV = spatial visualisation, SO = spatial orientation, PV = place value, Ar = arithmetic, Ge = geometry, WP = word problems. Error bars represent the standard error of the mean.

Post hoc power analyses using G*Power indicated sufficient power (.86–.98) to detect the observed effects ($\Delta R^2 = .12-.37$).

Discussion

A revised mathematics curriculum was recently introduced in Irish primary schools, which places greater emphasis on spatial reasoning skills (NCCA, 2023). This shift highlights the need to examine how the spatial abilities of mental rotation, spatial visualisation, and spatial orientation, integral to the new curriculum, predict sixth-grade students' performance across place value, arithmetic, geometry, and word problems while also considering the role of gender. Our findings showed that spatial skills collectively predicted students' performance across all four mathematical domains, accounting for 37% of the variance in arithmetic, 32% in geometry, 29% in word problems, and 24% in place value. Gender also appeared to play a role. This outcome aligns with previous studies (Newcombe et al., 2019) and highlights the importance of spatial ability for mathematics achievement in the Irish education system.

Notably, each spatial skill showed a distinct pattern of association with mathematical performance. Mental rotation predicted performance in place value and arithmetic but not in geometry and word problems, suggesting its relevance to tasks requiring comparison of decimal places, understanding the number line, and procedural fluency (Cirino et al., 2016). This association is supported by theoretical accounts proposing that mental rotation facilitates flexibility in transforming arithmetic problems and reasoning about numerical magnitudes (Newcombe et al., 2019). Our findings also align with Harris et al. (2021), who found mental rotation to predict fifth-graders' number sense performance. However, unlike Harris et al. (2021), we did not find a predictive relationship between mental rotation and geometry performance. Spatial visualisation predicted arithmetic, geometry, and word problems scores, demonstrating a broader influence on mathematics performance compared to mental rotation. This outcome reinforces prior theory highlighting the importance of spatial visualisation for mentally representing and manipulating complex visual representations in mathematical problems (Newcombe et al., 2019). Therefore, contrary to Mix et al.'s (2016) suggestion that object-based spatial skills (mental rotation, spatial visualisation) may be less relevant for sixth-grade mathematics, our results indicate their continued importance at this educational stage. Finally, we found that spatial orientation predicted performance across all mathematical areas. This outcome provides evidence of spatial orientation's broad importance for mathematics achievement, an understudied spatial skill in mathematics education, contrasting previous findings indicating its limited contribution (Mix et al., 2016; Harris et al., 2021).

As for the role of gender, we did not observe gender differences in mathematics scores. However, when controlling for spatial skills, our model predicts that female students' geometry and word problem scores would be higher than those of males. Furthermore, self-identified gender appeared to influence the relationship between spatial skills and mathematics performance only in the case of arithmetical tasks. Female students' arithmetic scores were predicted by spatial visualisation, while males' scores by spatial orientation. This outcome suggests that both genders relied on different spatial skills to solve arithmetic tasks. The opposite pattern of gender moderation observed in our study, compared to Harris et al. (2021), suggests that potential differences in the spatial strategies employed by male and female students may vary depending on contextual factors, such as instructional practices, teacher expectations, or broader sociocultural norms related to gender and mathematics.

Altogether, our findings suggest that spatial skills have a predictive role in broad sixth-grade mathematics performance, underscoring the importance of explicitly supporting spatial awareness in sixth-grade mathematics instruction. While the non-random sampling and uneven gender distribution limit the generalisability of our findings, the outcomes have important implications for practice, showcasing the value of attending to the spatial elements of the mathematics curriculum. Strengthening students' spatial awareness may further support performance across a wide range of mathematical areas. Future research should qualitatively investigate how students think spatially about mathematics and how teachers could support students in leveraging their spatial skills when engaging with the subject. Our quantitative findings suggest that teachers should encourage all students to use spatial strategies to foster equitable spatial skill development and potentially bridge the gender gap as identified in national assessments, such as PISA (Donohue et al., 2023, p. 21). In conclusion, the

predictive role observed for spatial skills in our study suggests that enhancing spatial awareness through the new primary mathematics curriculum may support both spatial reasoning skills and broader mathematics achievement.

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A comparative study of the cultural script of year 4 mathematics in Japan and the UK

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Lesson Study (LS) is a globally recognized collaborative research model that fosters pedagogical improvement. This study introduces *Transcript-Based Lesson Analysis* (TBLA), an emerging LS methodology that utilizes classroom transcripts from observed lessons to uncover the "cultural script" of pedagogy. Building on TBLA research conducted in Japan, Singapore, and Malaysia, this study turns its focus to the UK—a more multicultural and heterogeneous educational context. The research involves observations and transcript analyses of primary mathematics lessons from two schools in England, collected during a year-long sabbatical in collaboration with the University of Nottingham. While data analysis is ongoing, the study aims to reveal underlying cultural codes of mathematics teaching and contribute to the limited body of TBLA research in European contexts. Ultimately, the research seeks to support global pedagogical exchange by helping educators recognize their own cultural scripts and consider adapting effective practices from other countries to enhance student learning.

Keywords: Cultural script, cultural code, TBLA, collaborative research

Lesson Study

Lesson study, first adopted as a professional development methodology to help in the teaching of mathematics and arithmetic in one of Japan's first teacher trainer higher educational institutions has been used as a collaborative research exercise in the country since 1872 (Kusanagi, 2021). It is widely believed that Nara University of education was the first place where lesson study was introduced as a collaborative method of teacher training to help with the pedagogy of mathematics in the Meiji period. Since then, it has been used widely throughout Japan to train pre-service teachers at almost 80% of universities in Japan that offer teacher training courses.

In the UK and perhaps Ireland however, the history of lesson study is much shorter. It is generally believed that lesson study was first introduced by Professor Pete Dudley of Cambridge University in 2001 (Dudley, 2014) in the UK which later spread to Ireland. Professor Dudley used Lesson Study as a professional development tool for the teaching of English. His work had a significant impact on teacher practice and pupil learning in the UK. Professor Dudley has since accrued a large dataset of lessons that work from a wide source of schools through the UK and are available publicly on Camtree (Cambridge Teacher Research Exchange) an online data source of lesson studies on all types of lesson subject areas managed by a small team in Cambridge. However, Professor Geoff Wake of the University of Nottingham took a different approach and is now seen as one of the key people in the adoption of Lesson Study for mathematics at schools across England.

TIMSS and PISA

Several of the big data studies like TIMSS (Trends in International Mathematics and Science Study) and PISA (Programme for International Student Assessment) show England and the UK to rank poorly in maths when compared to other OECD countries (Richardson et al.; 2020). A key issue is that England's performance gap between the highest and lowest achievers in TIMSS 2019 was notably wider than in many other countries (Richardson et al., 2020). Additionally, while other systems have seen improvements, the mathematics performance of England's lowest-achieving students has declined over time (Hodgen, 2022), in contrast to trends elsewhere (OECD, 2013). Hodgen et al. suggest that this low achievement may be due to a long list of factors including socioeconomic disadvantaged students, particularly due to those eligible for free school meals (2022). Despite numerous policy efforts aimed at reducing educational inequality and improving outcomes for disadvantaged students the problem at low mathematical attainment remains. Addressing the issue at of understanding at the foundation level of maths could be a possible solution.

An alternative approach

Perhaps what is needed at this stage is an alternative way to teaching key elements of mathematic understanding, something different to what learners may be used to, something that is known to work well in other countries. The renowned piece on comparative studies “the Teaching Gap” by Stigler and Hiebert (2009) identifies areas of maths pedagogy that work well in three countries and make repeated mention of lesson study used widely in Japan. They introduce the term ‘*cultural script*’ which they recognise as the DNA of teaching, or elements of pedagogy that are seen as optimal ways of instruction past down from generation to generation. If the cultural script can be identified successfully then it could be presented to teachers in other countries and adapted to local learning environments.

Many areas in the UK and to some extent Ireland, continue to struggle with the teaching of mathematics (Hodgen et al, 2022). Wake (2020) has realised that something must change. Lesson study is increasingly being seen as a possible solution to the problem of low attainment in mathematics which Hodgen et al (2022) realise can hinder “individuals as in terms of future employment, earnings and life chances” (Dearden et al., 2002 in Hodgen et al, 2022). It is widely believed that lesson study can help to make a change and improve the pedagogy of foundation level maths with a focus on early to mid-year primary school age groups. Lesson study research is often conducted at key stage 2 age group (7-11) (Austin, 2017; Murphy et al, 2017; Wood, 2017) the age of children where key mathematical concepts are usually introduced.

Cultural Script simply replacing teachers does not guarantee better teaching quality (Stigler & Hiebert, 2009). Improving instruction requires more than just learning new theories, methods, or tools—it depends on effectively connecting theory with practice while incorporating culturally embedded factors that make these approaches successful (Hiebert &

Morris, 2012). However, these cultural influences are often subtle or hard to identify. As Stigler and Hiebert explain:

much of what happens in the classroom is determined by a cultural code that functions, in some way, like the DNA of teaching. This could be why simply changing teachers will not automatically produce a change in results (Stigler & Hiebert, 2009, p.12)

This cultural code is known as the teaching script—the shared beliefs, values, and habitual practices in classrooms among members of the same culture (Sarkar Arani, 2017; Stevenson & Stigler, 1992). Although the teaching script (or cultural script of a lesson) shapes instructional practices, it often operates unconsciously. Stigler and Hiebert (2009) argue that this script cannot be uncovered solely through teacher reflection or researcher observation, as it is so deeply ingrained in a culture that it goes unnoticed. Reflection alone cannot reveal what is taken for granted, and the insights gained are typically limited to specific individuals or local contexts (certain schools or study groups), making them difficult to generalize.

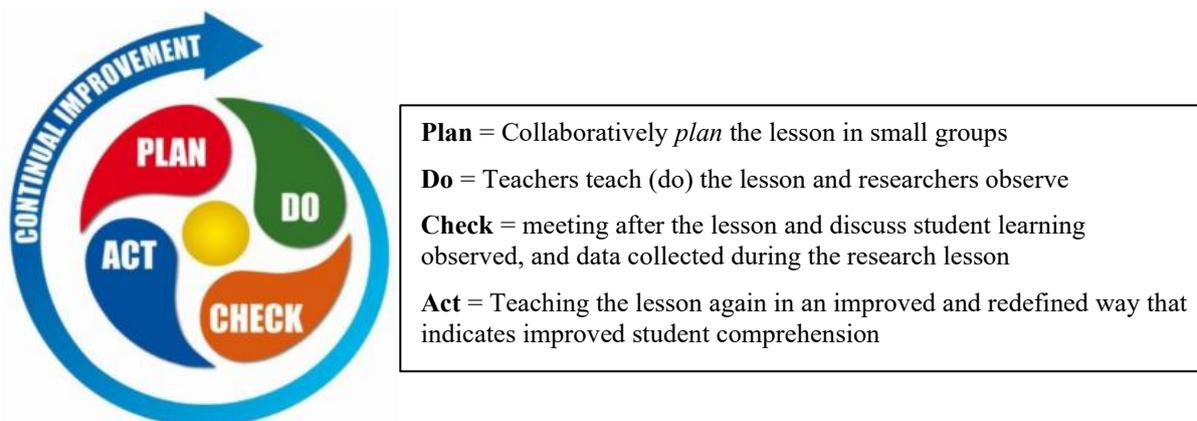
Lesson study is now a highly regarded and global research field that continues to gain interest worldwide. As collaborative research, Lesson Study holds the potential to make change in the classroom and improve on student understanding at the local and national level. However, as Stigler and Hiebert (2009) suggest, if we can determine good pedagogy in one country and adopt it in another the realm of possibility for change becomes much wider.

Transcript Based Lesson Study

This short paper will focus on an area of Lesson Study called transcript-based lesson analysis. Transcript based lesson analysis (TBLA) is a relatively new and emerging area of lesson study that takes a slightly different approach to the traditional format. With lesson study that traditional format involves the well-known PDCA (*Plan, Do, Check, Act*) cycle of Planning the lesson in a small group of researchers and teachers, (figure 1) This first step in the cycle is a collaborative approach to lesson planning as it involves several teachers of varying levels of experience and together, they come up with a lesson plan. The second stage (do) is the instruction of the research lesson which was planned together in step 1 (plan). The research lesson is observed, video and audio recorded and then analysed by the research team in the most pivotal of the four-step cycle (check). Together the team identifies issues in the lesson where learners need help in understanding the main content of the class. Finally, the research lesson is instructed again with advice received from the check-phase and areas that previously needed attention is addressed.

Figure 1.

Four-stage cycle of Lesson Study. Also known as Shewhart & Deming cycle (Johnson, 2016)



Transcript based lesson analysis differs from this by adopting just three steps from the PDCA cycle (figure 1). It is a much simpler process and involves just the DC and in part A sections from the four-stage cycle above. With TBLA, researchers do not interfere in the planning of the lesson, which is left entirely to the teachers. They do though ask for permission to observe the lesson where they set up a series of audio and video recorders that focus on the teacher, the learning and interaction within the class and produce a transcript of the lesson. This written transcript is the main source of data and is distributed to the research team and teachers who then discuss areas within the transcript to identify the cultural script of the concepts raised. It helps if the research team is from a wide range of backgrounds and nationalities each of whom may have different insights of what that cultural script may be. With TBLA this novel approach to the more formal construct of lesson study, there is huge potential to make a change in actual pedagogy and improve teaching. TBLA is research on teaching, not research in teaching (Sarkar Arani et al; 2017; 2019)

TBLA studies from SE Asia

Previous studies on TBLA provide a cross-cultural learning opportunity for all nationalities of researchers involved to learn from each other how to analyse a lesson, deliver evidence-based suggestions for its improvement and look culturally at what goes on in the classroom (Sarkar Arani et al 2025; 2019; 2017; & Sarkar Arani, 2017). An analysis of a lesson through different lenses and researchers from different social-cultural backgrounds can help educators to find their own approach to a global standard or method of improvement and change the cultural script of teaching. In this context, lessons could be described as an instance of pedagogical reasoning that drives customised teaching for personalised learning. This may help to understand how we know what we know and, as educators, if our information is reliable. Some may refer to this as evidence-based, (Newmann and Associates, 1996; Sarkar Arani, 2017).

The main aims are to reveal the cultural scripts about mathematical learning that are held by, but often invisible to, members of a culture; and to address educators' conceptions of teaching mathematics across cultures. The qualitative data analytical approach identifies the idea that focus on a single problem during the lesson enabling opportunities for deeper learning

and explores the value of, or problems associated with epistemological decisions therein. Research like this stands to make an important contribution, both through its novel method, and in its specific ideas about desirable qualities of mathematics lessons and how these might differ from the viewpoint of individual cultures.

In Progress

The main researcher of this study has worked on several similar TBLA based lesson studies in Singapore, (Sarkar Arani & Lander, 2019), Malaysia (Sarkar Arani et al, 2024) and Japan (Sarkar Arani et al, 2017) that focus on the cultural script of pedagogy by teachers in the Far East and Southeast Asia. However, as this is a relatively new area of research, as of now there are few studies that focus on identifying cultural scripts in the UK and or the European context, a much more multi-culturally and heterogenous society than that of the countries where such studies have been conducted so far. The author is a full-time professor in Japan but at the time of writing is currently on sabbatical at the University of Nottingham, UK. He received ethical permission to visit a school in the East Midlands area of England where he observed a maths class at the year four level (ages 8-9) on July 11th 2025. As described in the abstract data collection for this study is ongoing. After data collection is complete, transcripts and post-lesson discussions will be distributed to the international board of researchers and any cultural script of mathematical concepts identified will be introduced.

Ultimately, this area of Lesson study emphasizes the details, progression, and context of lessons, avoiding broad generalizations. Instead, it relies on concrete observations and dialogue from classroom practice to guide reflection. Through this process, teachers collaboratively develop and share professional knowledge, enhancing teaching practices collectively. This paper presentation although still in progress, will present cultural script identified in a mathematics class in England and compare that to findings in Japan and Malaysia.

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Pre-service teachers' insights into epistemological challenges of using empty number lines to solve additive tasks through jump strategies

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This study investigates the effect of the MM-WiL (Mental Mathematics – Work Integrated Learning) intervention on South African pre-service teachers' (PSTs') understanding and utilisation of Empty Number Lines (ENLs) in teaching the Jump Strategy (JS). Based on the Mental Starters Assessment Project (MSAP), the study employed pre/post-test data from 43 PSTs and post-intervention reflections. Analysed using the “obstacles to learning” framework (Bingolbali et al., 2011), the results indicated a modest improvement in ENL-related performance and identified key epistemological challenges, such as directionality and abstraction. While PSTs recognised the value of ENLs in fostering conceptual development, the findings emphasise the need for more structured pedagogical scaffolding within teacher education programmes.

Keywords: Learning Difficulties, mental mathematics, pre-service teacher education, empty number line

Introduction

There is a growing emphasis on developing the mental strategies of South African Foundation Phase PSTs and on effective teaching methods. Mental strategies such as the JS encourage adaptable thinking and enhance learners' conceptual understanding when integrated effectively into instruction (Venkat & Mathews, 2024). To address this need, the MM-WiL project was designed to improve PSTs' mental strategy instruction through the MSAP. Central to this is the ENL, a visual tool that helps learners transition from concrete to abstract thinking (van den Heuvel-Panhuizen, 2008). This study examines the use of ENLs by PSTs to teach JS. It attempts to answer the following question: What learning obstacles have PSTs faced when employing ENLs to solve additive tasks using JS?

Literature

MM-WiL project

The MM-WiL project emerged from earlier national efforts to standardise mathematics education in South Africa, particularly through the PrimTEd Standards and the MSAP resources, which promote six key mental strategies. Our university embedded these strategies, with particular emphasis on JS, into a 6-hour component of a third-year Foundation Phase methodology course, reaching 80 PSTs annually. In 2024, these PSTs completed the same pre- and post-tests designed for Grade 3 learners, which they would then teach in various schools as part of the intervention during teaching practicum, focusing on identifying items they found most challenging. These PSTs also completed a post-intervention reflection as part of this project (Venkat & Mathews, 2024).

Additive reasoning

Additive reasoning is understanding and manipulating relationships between quantities using addition and subtraction. It involves recognising that numbers can be composed and decomposed flexibly, such as through part-part whole relationships. Learners must also grasp the inverse relationship between addition and subtraction to reason effectively about quantities in different contexts (Ching & Nunes, 2017). This reasoning develops from early counting strategies to more sophisticated mental strategies that reflect understanding of numerical structure (Van Dooren et al., 2010).

Jump strategy (JS)

Using the N10 method, the JS supports learners in tackling addition and subtraction by decomposing numbers into tens and ones, making the process more manageable (Blöte et al., 2000). Instead of calculating the total in a single step, learners incrementally “jump” along a number line (NL), maintaining the first number and splitting and adding the parts of the second. For example, to solve $56 + 13$, a learner places 56 on the ENL, then adds 10 to reach 66, followed by 3 to arrive at the final answer of 69. This method supports the transition from concrete counting to more abstract mental calculations by leveraging part-whole understanding, base-ten structures and the development of fluency and equivalence reasoning (Venkat & Mathews, 2024).

Empty number line (ENL)

The ENL is a powerful visual tool that helps learners represent and organise their thinking when solving addition and subtraction problems. Unlike pre-marked NLs, the ENL allows learners to construct their jumps and placements, encouraging flexible strategy use and number sense (van den Heuvel-Panhuizen, 2008). As learners gain confidence, these visual and physical jumps gradually become internalised, allowing them to perform similar operations mentally, what Freudenthal called “using one’s head” rather than just “in the head” (Treffers & De Moor, 1990). This transition reflects a deeper understanding of numerical structure and is a hallmark of developing mental arithmetic fluency. The awareness, or lack thereof, regarding the structure of the ENL influences whether PSTs with experience encounter difficulties in using it and in solving additive tasks.

Theoretical framework

This study used the “obstacles to learning” framework (Bingolbali et al., 2011) to analyse PSTs' reflections on using ENLs to teach the JS. The framework identifies three interrelated sources of difficulty: epistemological, psychological, and pedagogical. Epistemological challenges arise from the abstract nature of number structures, such as understanding jumps in tens. Psychological obstacles include learners' reliance on counting in ones or low confidence in mathematics. Pedagogical barriers stem from instructional choices, where procedural teaching of ENLs can limit conceptual understanding. These interconnected obstacles highlight the importance of scaffolding and reflective teaching practices.

Methodology

This study took place at an urban South African university in the third year of study, involving 43 third-year PSTs who completed both pre- and post-tests along with reflections. These PSTs participated in a mathematics intervention programme as part of their coursework and teaching practicum. The four-year Bachelor of Education (B.Ed.) programme for teachers includes two years dedicated to teaching primary school mathematics content, followed by two years focused on fundamental pedagogical practices and methodology. These students' mathematical background is built from the initial two years of content knowledge. Most students enter the course with mathematical literacy, and some have mathematics from high school. Before data collection, all PSTs completed the necessary ethical consent documentation for the project, which has ethics number H21/11/45. The names of the PSTs were anonymised and referred to as Student 1 (S1) through Student 43 (S43). Data collection involved the PSTs completing a pre-test before attending 6-hour lectures and tutorials on JS. The same PSTs then completed a post-test. The pre- and post-tests were used from the MSAP Grade 3 assessment items, focusing on using NL to solve additive problems. Both tests consist of 30 items. The first 20 questions focus on fluency, requiring immediate answers, while the following five questions involve strategic calculations used to determine the most efficient strategies. Subsequently, the last five questions employed logical reasoning, patterns, and connections between mathematical concepts referred to as strategic thinking. These PSTs implemented the intervention during a six-week teaching practicum in Grade 2 and 3 classrooms across various South African schools. As part of the intervention, PSTs were required to plan and teach at least two mathematics lessons focused on mental strategies, specifically using the JS and ENLs. PSTs were encouraged, though not formally assessed, to reflect on learners' engagement with ENLs as part of their reflection journals and to collect anecdotal evidence on learner responses.

The data used in this study utilised the scores from the pre- and post-test, along with the reflections post-teaching practicum. Data analysis, for the quantitative component, we compared the mean scores on NL-related test items from pre- to post-test. The class size (N=43) allowed for paired comparisons; a simple gain score was also calculated. Qualitative data was analysed using thematic analysis of their comments, based on various questions. The reflections were submitted as part of a structured practicum assignment. PSTs responded to prompts including “What was difficult for you in teaching Jump Strategies?” and “What worked well?” A total of 43 reflections were submitted, and 16 included descriptions of challenges specific to using ENLs. Sometimes, a single PST may provide multiple responses pertinent to a singular obstacle or contribute responses across various obstacles. We examined PSTs' reflections on the ENL use, separating positive comments from difficult comments. Next, we categorised reflections into epistemological, psychological, and pedagogical obstacles, and identified recurring responses within each category. In other words, each time a response is received, we would record a tick and tally the number of ticks to ascertain whether there were predominant or less predominant responses (see Table 1).

Insights

For the purpose of this study, we decided only to include data from the pre- and post-test that dealt with using ENLs as this arose as a major theme from the initial analysis of the reflections. Pre- and post-test results indicate a modest improvement in PSTs ability to solve ENL-based problems after the lectures. On relevant test items (e.g. marking or interpreting ENL jumps), the average score increased from 80.34% (pre-lectures) to 85.42% (post-lectures). This gain of about 5.08% points suggests that the PSTs teachers strengthened their understanding and skills in using ENLs for mental calculations. Whilst the paper focuses on the difficulties, 16 of the 43 PST's noted that using the ENL assisted learners in making sense of the additive tasks. Some of the PSTs noted that learners' use of ENLs helped them “to comprehend abstract ideas” (S17), “break down numbers into their tens and units” (S9) and “it helped them see exactly how the other learners were able to get to the answer mentally” (S18). The rest of this section will discuss insights about difficulties working with ENL from the reflections of the same group of PSTs. Only two categories of difficulties became visible: epistemological and pedagogical obstacles.

Table 1

Description of responses from 16 PSTs who identified difficulties

Obstacles	Direction on ENL	Jumping on the ENL	Moving away from the use ENL	Jumping backwards	No exposure to ENL	Use of inappropriate resources
Epistemological Responses	9	8	8	2	3	
Pedagogical Responses						1

Table 1 indicates the two obstacles evident from PST's responses in column 1. Columns 2 to 6 are the themes that emerged from the responses of the 16 PSTs.

Epistemological Obstacle Analysis

Epistemological obstacles refer to difficulties that arise from the nature of the mathematical concepts or representations themselves (Bingolbali, et al., 2011). 30 out of 31 coded PST responses reflected epistemological obstacles. Three predominant difficulties emerged for the PSTs in utilising the ENL in JS. The primary challenge identified by numerous PSTs was the learners' difficulty in determining the appropriate direction to follow on the ENL when performing addition and subtraction (9 responses). One of the PST noted that “some learners found it difficult to use number lines, going in the wrong direction when adding and subtracting which is essential to understanding jump strategies” (S2). This indicates a conceptual misunderstanding of how subtraction and addition are represented on the ENL, an obstacle arising from the representation of the operation.

The second difficulty several of the PSTs highlighted was learners' inability to jump correctly on the ENL (8 responses). PSTs described this difficulty in different ways. Some PSTs

mentioned that “*some students may find the idea of "jumping" between numbers on a number line confusing*” (S7). The same PST mentioned “*another challenge lies in ensuring that all students understand the increments they are supposed to "jump" by*” (S7). The PST noted that “*these students might struggle with the spatial and sequential aspects of the number line, making it hard for them to keep track of their jumps or understand how the increments relate to the overall problem*”(S7). Another PST mentioned, “*while some learners were able to quickly grasp the concept of making jumps on a number line, others were finding it confusing and difficult to apply*” (S8). Learners' difficulty in executing these jumps on the ENL hinders their ability to create their jumps and placements, which promotes flexible strategy application and number sense (van den Heuvel-Panhuizen, 2008).

The final predominant response articulated by the PST's was that learners struggled to move from the ENL to solving additive tasks mentally (8 responses). For example, one PST noted that “*while the number line is a helpful visual tool, many learners became dependent on it and struggled to transition to mental strategies*” (S3). Here, learners' reliance on concrete or visual representations prevents them from mentally transitioning from these representations to solving tasks mentally. Bingolbali et al. (2011) note that the abstractness of the concepts contributes to epistemological difficulties.

Two less predominant responses to the epistemological difficulties related to developing the concept of using the ENL to solve additive tasks through JS. PST's first less predominant response was that learners were not exposed to the ENL as a representation in solving additive tasks. One PST noted, “*adapting to the different preparedness levels of the learners was one of the biggest challenges. Some learners found it easy to understand how to use number lines and separate numbers into smaller parts, while others found it more difficult*” (S15). The inability to separate the number inhibits learners' ability to use the N10 method when tackling addition and subtraction by decomposing numbers into tens and ones, making the process more manageable (Blöte et al., 2000).

The second, less predominant response is that learners struggled to jump backwards on the ENL when solving subtraction tasks. One PST noted, “*many children didn't realise that they needed to move backwards for subtraction and forward for addition*” (S1). These responses fed into learners' struggles to escape these epistemological obstacles.

Pedagogical obstacle analysis

Pedagogical obstacles identified by the PSTs included using inappropriate teaching approaches (Bingolbali et al., 2011). Across all the responses, one response pointed to this particular obstacle. The reflection indicated “*to illustrate number lines and jumps on the chalkboard was somehow time consuming, and it was not always capturing learners' attention as effectively as modern resources*” (S8). This response highlights the pedagogical challenges associated with the role of representations in addressing additive tasks. The PST raises two issues regarding teaching learners to solve additive tasks with the ENL as a resource. The first issue is time. The PSTs discussed pacing in task completion. In South Africa, teachers in underprivileged schools struggle with pacing, which is significant. (Hoadley, 2003). The PST additionally emphasises that the ENL fails to engage modern learners as a resource capable of captivating their attention.

It is noteworthy that only one PST explicitly acknowledged a pedagogical challenge. This may reflect a lack of awareness among PSTs about their own instructional decisions, a common issue among early-stage teachers. These findings suggest that PSTs may underreport or misrecognise pedagogical barriers without targeted support, highlighting the need for more reflective practice and observation-based feedback during their training.

Conclusion

This study affirms the educational value of ENLs in enhancing PSTs' instructional strategies and learners' additive reasoning. The MM-Wil intervention provided a strong foundation for integrating mental mathematics into classroom practice, though persistent epistemological challenges, such as directionality and abstraction, remain. These issues are not unique to South Africa; international research also highlights the difficulty PSTs face in connecting conceptual understanding with representational tools (Van den Heuvel-Panhuizen, 2008). Strengthening teacher education programmes to address these challenges is essential for realising the full pedagogical potential of ENLs in early mathematics instruction.

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Measuring students' procedural and problem-solving skills on entry to third level education using a pre-approved diagnostic test

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Like many other countries internationally, the second level (12-18 years) mathematics curriculum in Ireland changed from one where rote learning and procedural skills were the focus to a curriculum which placed an emphasis on deep conceptual understanding and problem-solving in real life contexts (Cunningham et al., 2016; Faulkner et al., 2021). Faulkner et al. (2021) developed a diagnostic test to examine and compare beginning undergraduate students' procedural and problem-solving skills after the introduction of these curriculum changes. This study found that incoming undergraduate students performed statistically lower on average when engaging with problem-solving questions when compared with their performance when engaging with procedural questions. The authors acknowledged that more longitudinal quantitative research needed to be carried out to examine this difference in performance. Using this pre-approved diagnostic test, this paper will detail initial quantitative findings from the first two years of a larger PhD project.

Keywords: Procedural skills, problem-solving skills, diagnostic testing

Introduction

Internationally there have been concerns regarding the low level of students' mathematical skills coming from second level, particularly students who are entering third level institutions (Treacy et al., 2016; Carr et al., 2013; Faulkner et al., 2010; NCCA, 2005). Society has become more reliant on, and influenced by, advances in science and technology, both of which largely depend on mathematics, and so it becomes more important that the mathematics education offered to students should reflect this (NCCA, 2005). There have been growing concerns in Ireland regarding mathematics in the Senior Cycle of second level education, particularly, the number of students receiving low grades in the Leaving Certificate Ordinary Level (OL) mathematics examination (NCCA, 2005). As a result, the second level mathematics curriculum in Ireland changed from one where rote learning and procedural skills were the focus to a curriculum where deep conceptual understanding and problem-solving in real life contexts were the focus (Cunningham et al., 2016; Faulkner et al., 2021). Faulkner et al. (2021) examined students' performance after the introduction of these curriculum changes by developing a diagnostic test to examine and compare students' procedural and problem-solving skills on entry to third level education. It was found that students performed statistically lower on average when engaging with problem-solving questions in comparison to their performance when engaging with procedural questions. This research aims to use this pre-approved diagnostic test to further investigate procedural and problem-solving skills of students on entry to third level education in Ireland.

Second Level Mathematics Curriculum in Ireland

Second level education in Ireland comprises of the Junior Cycle (three-year syllabus), Transition Year (optional one-year) and the Senior Cycle (two-year syllabus). The Junior Cycle concludes with Junior Cycle Final Examinations (formerly known as the Junior Certificate) at approximately 15 years of age and the Senior Cycle concludes with Leaving Certificate examinations at approximately 18 years of age. Both the Junior Cycle and Leaving Certificate examinations are prepared by the Irish State Examinations Commission. All students sitting these exams receive the same exam at their chosen level. For Junior Cycle, mathematics can be taken at Higher Level (HL) or OL. For Leaving Certificate, mathematics can be taken at three levels, Foundation Level (FL), OL or HL.

A mathematics curriculum initially entitled “Project Maths” was introduced into second level schools across Ireland between 2008 (when the curriculum was introduced into 24 pilot schools) and 2015 (when all strands of the new curriculum were assessed in both the Junior Cycle and Leaving Certificate examinations). One of the aims within the Leaving Certificate mathematics syllabus is:

to develop mathematical knowledge, skills and understanding needed for continuing education, life and work. By teaching mathematics in contexts that allow learners to see connections within mathematics, between mathematics and its applications to real life, it is envisaged that learners will develop a flexible, disciplined way of thinking and the enthusiasm to search for creative solutions. (NCCA, 2013, p.6)

This aim is supported by objectives which are designed to enhance the development of students’ mathematical proficiency which comprises of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (NCCA, 2013; Cunningham et al., 2016).

Overall, the mathematics curriculum has been described as:

empowering students to develop essential problem-solving skills for higher education and the workplace by engaging teenagers with mathematics set in interesting and real-world contexts (Cunningham et al., 2016, p.80).

After the introduction of these second level curriculum changes, Faulkner et al. (2021) developed a diagnostic test to examine students’ procedural and problem-solving skills on entry to third level education.

The Diagnostic Test

The Faulkner et al. (2021) diagnostic test was divided into two sections, section A which contains the procedural questions and section B which contains the problem-solving questions. The questions from each section were paired to mirror each other in terms of the mathematical concepts they are testing to compare students’ performance from each section. The procedural skills needed to complete a question in section A successfully were also required to successfully complete the paired question in section B, however, the section B question also contained real-world context (Faulkner et al., 2021). The test contains 9 questions in each section of the test with question 2 and 9 containing two parts. This number

was deemed suitable so that students would be able to complete the test during a 50-minute lecture slot.

The questions for section B of the test were taken directly from previous Junior Cycle and Leaving Certificate examinations from both HL and OL examination papers. Taking the questions from previous examination papers (albeit with slightly amended contexts) resulted in the questions for the diagnostic test directly mirroring the mathematics curriculum's interpretation of problem-solving in the context of a written examination (Faulkner et al., 2021).

Marking the Diagnostic Test

For the research outlined here, correct, incorrect and blank answers were recorded when marking the diagnostic test. A correct answer received 1 mark and both incorrect and blank answers received 0 marks. As there were no attempt marks being awarded due to the comparative nature of the diagnostic test and since question marking has an element of subjectivity, it was decided that the tests would be marked twice, once with a rigid marking scheme (RMS) and a second time with a lenient marking scheme (LMS). The idea of this was to investigate if rounding issues or slips (where a student makes a calculation error with workings) would have an impact on the findings, especially since some of the questions in section B required working with decimals and rounding for the final answer which was not the case with the paired question in section A. Therefore, this issue would be negated with the LMS. Additionally, considerations were made to ensure that the marking schemes were consistent across both section A and section B of the test as a student's result from each section would be compared.

Internal Consistency of the Diagnostic Test

Cronbach's alpha is used to measure the internal consistency of a group of items, this is done by measuring the homogeneity of a group of items (BrckaLorenz et al., 2013). "It is an indication of how well the different items complement each other in their measurement of different aspects of the same variable or quality" (Litwin, 2003, p.22). The value for Cronbach's alpha lies between zero and one, with values closer to zero indicating a lower internal consistency and values closer to one having a higher internal consistency. Cronbach's alpha was calculated across all 22 questions on the diagnostic test, using the RMS and LMS, to measure the internal consistency reliability for this research. The value of Cronbach's alpha is 0.840 using the RMS and 0.849 using the LMS which demonstrates good internal consistency in both cases.

Participants for the Diagnostic Test

All students who undertook the diagnostic test are in year 1 of their undergraduate studies at different levels of QQI awards. The majority of the sample are in a QQI level 8 programme (93.6%) with the remaining students being in a QQI level 7 programme (5.5%) or QQI level 6 programme (0.9%). The sample contains students from 25 programmes of study which ranges from ones which have a large mathematical component, for example, engineering and science, to ones which would have a smaller focus on mathematics (accounting and finance and pharmaceuticals) to more general programmes which cover a

wider range of subjects similar to second level schooling (International Foundation Programme). For their Leaving Certificate mathematics level, the majority of the sample were HL (73.0%), a little under one third of the sample were OL (26.7%) and the remaining students were FL (0.3%).

Data Analysis

The Statistical Software Package for Social Sciences (SPSS Version 29) was used to analyse the data from the diagnostic test. Paired t-tests were carried out to compare means in section A and section B of the diagnostic test to examine students' performance for the overall group using both the RMS and the LMS. The researcher wanted to compare the means to investigate if there were statistically significant differences between students' performance on the diagnostic test within smaller groups (e.g. Leaving Certificate level), however as the data within each group was not normally distributed t-tests were not appropriate. Therefore, the Wilcoxon Signed Ranks Test was used to compare the medians of section A and section B of the diagnostic test within each group, using both the RMS and LMS.

Findings

Paired sample t-tests were used to establish if the whole group of students who undertook the diagnostic test in 2022 and 2023 ($n = 660$) performed statistically significantly better in one section of the diagnostic test when compared to the other. The paired t-tests were carried out twice, once for results using the RMS and once using the LMS.

The results using the RMS showed that there was a statistically significant difference in the students' mean performance in section A ($\bar{x} = 69.45$), when compared to section B ($\bar{x} = 43.80$), with the students' mean performance being statistically significantly better in section A which contained the procedural questions (see Table 1 below).

Table 1

Performance of overall group in sections A and B of the diagnostic test using the RMS.

	QQI level	Section A			Section B			p-value
		N	M	SD	N	M	SD	
Entire Group	n/a	660	69.45	23.37	660	43.80	22.17	$p < 0.001$

Table 2

Performance of overall group in sections A and B of the diagnostic test using the LMS.

	QQI level	Section A			Section B			p-value
		N	M	SD	N	M	SD	
Entire Group	n/a	660	71.42	22.86	660	51.80	23.05	$p < 0.001$

Similarly, the results using the LMS showed that there was a statistically significant difference in the students' mean performance in section A ($\bar{x} = 71.42$), when compared to section B ($\bar{x} = 51.80$), with the students' mean performance being statistically significantly better in section A which contained the procedural questions (see Table 2 above).

Results using the Wilcoxon Signed Rank Test showed that there was a statistically significant difference ($p < 0.001$) in students' median performance in section A, as against section B (see Table 3 below) of the diagnostic test for students who sat mathematics at HL in their Leaving Certificate exam, with students' median performance being higher in section A than section B using the RMS and LMS. This was also found to be the case for students who sat mathematics at OL in their Leaving Certificate exam ($p < 0.001$) using the RMS and LMS. In the case of students who sat mathematics at OL, the median performance is below 20% using the RMS and below 30% using the LMS in section B of the diagnostic test.

Table 3

Comparing median performance of students in section A and section B of the diagnostic test by Leaving Certificate level with the RMS and LMS using the Wilcoxon Signed Ranks Test, where the median is given as a percentage of correct questions in each section.

LC Level	Section A		Section B		p-value
	N	Md	N	Md	
Higher RMS	482	81.8	482	54.5	$p < 0.001$
Ordinary RMS	176	45.5	176	18.2	$p < 0.001$
Higher LMS	482	81.8	482	63.6	$p < 0.001$
Ordinary LMS	176	54.5	176	27.3	$p < 0.001$

Further findings indicate that there was a statistically significant difference in students' median performance in section A, as against section B of the diagnostic test within each programme of study/grouped programmes of study with students' median performance being statistically significantly better in section A, the procedural section of the diagnostic test using both the RMS and LMS. Initial findings also suggest that students entering programmes with a higher minimum grade in mathematics entry requirement tended to have a better performance in both sections of the test.

Discussion and Further Research

The students who undertook the diagnostic test for this research are extremely diverse in terms of their discipline areas and their Leaving Certificate levels and grades. However, all students have come from second level education in Ireland and completed their Leaving Certificate in or after the year 2018 and were exposed to the alternative teaching methods and mathematical content which were introduced with the second level mathematics curriculum from 2008. The quantitative findings presented that on average, students performed statistically significantly better in section A of the diagnostic test, the procedural questions when compared with section B, the problem-solving questions using the RMS and LMS. These findings are in line with the findings from the Faulkner et al. (2021) study which used the same diagnostic test albeit the present research examines a much larger data set.

The diagnostic test was distributed to a third cohort of students in September 2024 which will add to the above data set in the coming months. In addition to the quantitative research outlined above, the overall research project aims to carry out a qualitative investigation using the Faulkner et. al (2021) diagnostic test combined with a 'think-aloud' protocol. The 'think-aloud' protocol requires students to verbalise their thoughts while

engaging with a given mathematical task, allowing one to access what is happening in the student's short-term memory (Cowan, 2019; Ericsson, 2003).

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A preliminary analysis of real-world contexts for fractions used in Northern Ireland and Singapore mathematics textbooks

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Fractions are among the most challenging concepts that children encounter in the primary school years, largely due to the many different fraction conceptualisations. To make mathematics more meaningful and accessible for all students, teachers are encouraged to use real-world contexts and models to bridge the gap between real-world understanding and mathematical understanding. Choosing appropriate real-world contexts that are suitable for model building presents challenges for textbook developers. This paper considers the use of context in the teaching of fractions in two countries: Northern Ireland and Singapore. Two textbooks, one from each country, were analysed with regard to the coherence between how fractions are conceptualised and the contexts or the models used to represent them.

Keywords: fractions, teaching and learning, textbooks, real-life contexts

Introduction

Proficiency with fractions is essential not only for mathematical learning but also for applications across a range of scientific disciplines, career paths, and everyday situations. Yet, fractions are widely recognized as one of the most conceptually challenging areas of mathematics for children in the primary years (e.g., Smith, 2002). It is therefore concerning that despite sustained reform efforts over several decades by researchers, policymakers, mathematics educators and textbook developers, children's performance on fraction arithmetic tasks has shown limited improvement (Siegler & Lortie-Forgues, 2017). Connecting school mathematics to the real-world is considered a crucial factor for meaningful learning (Abaño et al., 2024). However, a substantial body of research challenges the assumption that the contextualised tasks that learners experience in the mathematics classroom are similar to the mathematics that people use in the real-world (e.g., Nunes et al., 1993). Textbooks, often the main resource for teachers, play a pivotal role in guiding pedagogical approaches. This paper seeks to contribute to the discussion by examining and comparing how contexts are used in the introduction of fraction concepts in textbooks from two countries with different education systems – Northern Ireland (NI) and Singapore.

Fraction Constructs

According to Smith (2002, p. 3), “no area of elementary mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios and proportionality”. One of the key issues associated with the challenges of teaching and learning fractions is the multiplicity of fraction constructs (Lamon, 2020). There are at least five different fraction constructs: part-whole, measure, operator, quotient and ratio. For example, as a part-whole construct, the fraction $\frac{2}{5}$ is usually interpreted as two out of five equal parts of the unit (whole). As a measure, $\frac{2}{5}$ is perceived as a distance of two $\frac{1}{5}$ units from zero on

the number line, which presents fractions as numbers. Regarding fractions as numbers also relates to the idea that $\frac{2}{5}$ is the answer to $2 \div 5$ (quotient). As an operator construct, $\frac{2}{5}$ of a unit is found by multiplying the unit by two and dividing the result by five or dividing the unit by five and multiplying the result by two. Finally, as a ratio, $\frac{2}{5}$ represents a multiplicative relationship between two quantities in a particular order. It is crucial for students to encounter the different conceptions of fractions as part of their learning experiences. However, fraction instruction has tended to concentrate on the part-whole construct, and as a result children's understanding of fractions is "teetering on a small, shaky foundation" (Lamon, 2020, p. 32).

Representations, Models, and Contexts

With the aim of broadening students' understanding of fractions, Cramer et al. (2002) recommend providing opportunities for students to work with multiple representations of fractions and translate between and within these various representations: concrete, pictorial, verbal, symbolic and realistic. Similarly, van de Walle et al. (2013) promote the use of different models – area models (e.g., fraction discs), linear models (e.g., fraction strips, numbers lines), and set models (e.g., collections of discrete objects) – to expose students to different ways of thinking about fractions. However, it appears that textbooks often favour the use of area models, leading to a limited understanding of fractions (Watanabe, 2002). This singular focus on the part-whole construct of fractions is also compounded, at times, by a lack of clear connections made between the different representations.

Furthermore, the use of context in mathematics curricula deserves careful attention given that the mathematics in real-life is always in context (Meyer et al., 2001). Borasi (1986) defines context as "the situation in which [a] problem is embedded" (p. 129). Context is usually provided by the text of the problem, but it can also be contained in pictures, diagrams, or tables. Contexts fulfil various, often interrelated, roles, with a single context potentially serving multiple roles simultaneously. These include: (1) motivating learners to engage with new mathematical concepts, (2) offering opportunities to use and apply mathematics, (3) introducing new mathematics, (4) guiding potential solutions, and (5) grounding mathematical understanding (Meyer et al., 2001). Although context is not the main factor influencing difficulty of questions, it can shape learners' responses (Greatorex, 2014).

The failure of learners to pay attention to relevant context-based knowledge in their solutions has been widely documented and is attributed to the artificial contexts that are typical of many school textbooks (Verschaffel & DeCorte, 1997). Whilst artificial contexts can be useful, they may also interfere with and distract from the underlying mathematics, with evidence suggesting that many learners are required to suspend belief and make unrealistic assumptions (Greatorex, 2014). It is argued that the use of "contrived contexts, trivializing examples, and irrelevant pictures" may lead children to "reject mathematics as being contrived, trivial and irrelevant" (Du Feu, 2001, p. 4). Additionally, contexts that are not familiar to learners' experiences have the potential to be "alienating, excluding or exacerbating of disadvantage" (Sullivan et al., 2003, p. 119). In this paper, we examine the contexts used in textbooks and consider how they may promote or hinder students' understanding of fractions. The paper is framed by the following overarching research

question: What are the different contexts used in the textbooks when students are introduced to fractions and how are these contexts aligned with the fraction concepts and models?

Methods

In NI, fractions are first introduced in Key Stage 1 (ages 7-9), within the Number strand of the Primary Curriculum. Whilst the expectation is that pupils will recognise and use simple everyday fractions, teachers have considerable flexibility in interpreting the curriculum to meet their pupils' abilities and interests. In Singapore, only the part-whole concept of fractions is introduced to students, primarily via the area model, at Primary 2 (age 8) under the Number strand of the Primary Mathematics Curriculum. In NI, schools have the freedom to choose their own textbooks, unlike Singapore where one textbook series is used in all schools. The aim was to explore how fractions are first introduced in textbooks as this will influence how the learner's knowledge and understanding develops over time. One textbook was chosen from each country: *New Heinemann Maths 3 Textbook (NHM3)* (Scottish Primary Mathematics Group, 2000), one of the more popular textbooks for Year 4 in NI, and *Primary Mathematics Textbook 2B (PMT2B)*, the current textbook for Primary Two in Singapore (Curriculum Planning & Development Division, 2022). These textbooks were chosen as they are representative of how fractions are first introduced in each of the two countries. Analyses occurred at each item level. Each item was coded using the five conceptions of fractions: part-whole, measure, quotient, operator, and ratio (Lamon, 2020). The representations used were also coded using the categorisation: area, linear, and set (van de Walle et al., 2013). We then took note of the contexts used to introduce fractions in the two textbooks and seek to understand how the contexts used may promote or hinder students' understanding of fractions. Coding was done independently by each author and analyses were subsequently shared and justified through discussion together.

Findings

In this section, the contexts used in the chosen textbooks are presented with an analysis of how the contexts align with the corresponding fraction concepts and models used. There are 20 items in NHM3: the main focus is on the part-whole conceptualisation of fractions; both area and set models are used. One item, requiring students to count in halves along a number line, relates to the measure conceptualisation. Only four real-world contexts are used. As seen in Figure 1, real-world contexts are first introduced in the second item with fruit pies represented by area models (circular regions partitioned into equal parts). The use of context promotes understanding of the part-whole concept of fractions through the area model. In further items, the set model is used to represent the part-whole concept. For example, arrays are used to support student appreciation of the relationship between fractions and multiplication and division. A range of real-world examples is used, as shown in Figure 2, before progressing to word problems with no supporting illustrations.

Figure 1

Context of sharing food (NHM3)

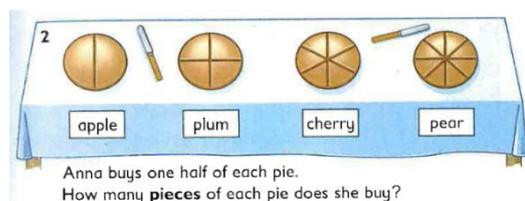


Figure 2

Context of trays of rings and fruits (NHM3)



By contrast, all of the 23 items in PMT2B involve the part-whole conceptualisation and the area model. It is interesting to note that contexts are used primarily to introduce new concepts and skills. For example, as seen in Figure 3, the context of sharing food is used in the chapter opener and the introduction of “half” as the first fraction. The use of contexts then fades away into the use of area models (such as fraction discs and rectangular bars). These contexts align closely with the part-whole concept of fractions and the models used.

Figure 3

Context of sharing food (PMT2B)



The food-sharing context is also used when addition and subtraction of fractions are introduced in the textbook. See Figure 4. It is noteworthy that the contexts used for addition and subtraction of fractions are coherent with the stipulated emphasis on the part-whole concept and area model of fractions.

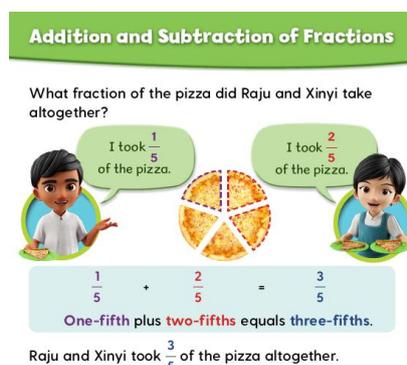
Discussion and Conclusion

In both textbooks, real-world contexts – particularly food-sharing contexts – are used to introduce the part-whole concept of fractions. This has the potential to promote reasoning about the relationship between the number of equal parts and the size of each part because of the coherence between the contexts, concepts, and models. For NHM3, the context also

provides a connection for teachers to discuss fraction equivalences. However, it is possible that the real-world context may obscure these underlying relationships. For example, students may focus solely on the number of parts to answer the question, ‘How many pieces of each pie does she buy?’ (Figure 1).

Figure 4

Context of sharing food for adding fractions (PMT2B)



Whilst the chosen contexts – trays of rings and fruits – in NHM3 are coherent with the array model, it could be argued that they are contrived and may not be relevant to students’ experiences. It is also questionable whether an array of real-world items contributes any more to student understanding of fractions than an array of non-contextual objects such as circles. Such was the case in the first use of the array in NHM 3. Likewise, for PMT2B, the use of pizza sharing for adding and subtracting fractions is questionable because it is more natural to discuss operations of fractions in relation to the measure concept (Lamon, 2020). It is also unrealistic to cut a pizza into 5 equal parts and most people prefer to discuss the quantity in terms of the number of slices, rather than the fraction of the pizza. Hence, this appears to be a contrived context for addition and subtraction.

This preliminary investigation has shown that the kinds of real-world contexts used in textbooks and how they align with fraction concepts and models is very complex. Although tentative, the findings suggest the need to consider how the chosen contexts connect with the fraction concepts and models used. Whilst this study explored the coherence between the contexts, concepts, and models; further research could consider how students may perceive them. What is clear is that there is a need to engage with more research to understand the role of real-world contexts in promoting or hindering conceptual understanding.

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A case study of undergraduate mathematics students' collaboration

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We report on a case study from a research project examining undergraduate mathematics students' collaboration on homework. We examine the collaborative practices of two participants from the larger study who worked together on homework in what we propose is a community of practice. These two participants are chosen due to their aspirations of pursuing a career in research. While preparing future mathematicians has previously been identified as a goal in university mathematics, we examine the issue from the student perspective. Using communities of practice as our theoretical frame, we analyse how the two participants attempted to align their collaborative practices as undergraduates with their imagined futures as research mathematicians.

Keywords: Undergraduate students, communities of practice, collaboration

Introduction

In this paper we report on a case study from a research project in which we examined undergraduate mathematics students' collaboration on homework (Murphy & Meehan, 2024). As part of the larger study, ten recent graduates from a large, research-intensive university in Ireland were interviewed about the collaboration they undertook on homework problems. Reflexive thematic analysis (Braun & Clarke, 2022) was used to identify commonalities in participants' accounts. We found that participants collaborated on homework with *specific* peers who were also their friends and tended to be like them in terms of academic ability and interests. Participants reported working on problems alone initially and collaborating at the point at which they got stuck. All participants felt they benefitted from collaborating. We also identified constraints on participants' collaboration which included doubts regarding the collaborative nature of research mathematics.

Our initial analysis of the interview data was focused on identifying *commonalities* in participants' accounts. However, we also noted idiosyncrasies in participants' collaborative practices and felt that the indigenous way in which participants negotiated their response to constraints resonated with Wenger's (1998) theory of communities of practice (CoP). In this paper, we take the opportunity to explore this aspect of our data further by presenting a case study of two participants, Sullivan and Lynch⁷, who collaborated as undergraduates in a group which we propose can be considered a CoP. We substantiate our claim that this group is a CoP by illustrating the manifestation of three characteristics of a CoP in their practice of collaborating on homework. We chose to focus on these two participants as both spoke of their ambitions to pursue a career in research mathematics. Prior research has identified preparing future mathematicians as a goal of mathematicians (Woods & Weber, 2020). However, in this paper we examine the issue from the student perspective. Specifically, we

⁷ Sullivan and Lynch are gender-neutral pseudonyms.

utilize the theory of CoPs to examine how Sullivan and Lynch attempted to *align* their practice of collaborating as undergraduates with their *imagined* futures as mathematicians.

The Collaborative Nature of Research Mathematics

In popular media, mathematics has traditionally been portrayed as a solitary activity (Barba, 2018). Contrary to this stereotype, there are indications that mathematics is becoming increasingly collaborative. Grossman (2005) reviewed patterns in mathematics research publications and found that the proportion of single-authored publications fell from 91% in the 1940s to 54% in the 1990s. Murphy and Meehan (2024) conducted a review of the authorship of articles published in 2023 in the top twelve “Mathematics” journals as ranked by Scimago JR. Single-authored papers accounted for 18% of this sample, suggesting the continuation of the collaborative trend. Burton’s (2004) interview-based study with 70 mathematicians from the UK and Ireland provides insight into the nature of research collaboration. Burton found that all but four participants reported working either cooperatively across disciplines or collaboratively with other mathematicians. Participants discussed cognitive and affective benefits of collaborating. They also talked about the role of competition within mathematical culture and reported that while the discipline has become increasingly collaborative, competition is pervasive and often co-exists with collaboration.

Theoretical Framework

Stated succinctly: communities of practice (CoPs) are “groups of people who share a concern or a passion for something they do and learn how to do it better by interacting regularly” (Wenger, 2015, p. 1). Wenger (1998) describes three characteristics of a CoP as the *mutual engagement* of participants negotiating a *joint enterprise* over time and developing a *shared repertoire* in the process. *Mutual engagement* involves establishing ways of working. Examples of the *shared repertoire* of a CoP include the discourse used, routines created, and the creation of artifacts. Simplistically, the *joint enterprise* of a CoP can be thought of as its members’ understanding of what the enterprise is about, its aims and ideals. Wenger (1998) cautions that *joint enterprise* goes beyond a “statement of purpose” and is negotiated by the community as they determine “what is important and why it is important, what to do and not to do” (p. 81). Wenger notes that CoPs are not “self-contained entities” (p. 79). Their practice can be influenced by external constraints to which they respond in an indigenous manner.

While *engaging* in practice is one way that an individual can identify as belonging to a CoP, Wenger (1998) presents two other modes of identification: *imagination* and *alignment*. Identifying with a CoP through *imagination* is described as “creating images of the world and seeing connections through time and space by extrapolating from our own experience” (p. 173). *Imagination* can enable us to see our engagement in practice in a different light and may be affected by stereotypes we hold. Identifying with a CoP through *alignment* involves “coordinating our energy and activities in order to fit within broader structures” (p. 174). In Farnsworth et al. (2016) Wenger discusses the example of a girl who *imagined* herself as a marine biologist. In terms of *alignment*, she had to consider what practices she needed to engage in so that she could ultimately *engage* in her dream job.

Using the CoP theoretical framework, we articulate our research questions as follows:

- **RQ1.** How are the three characteristics of a CoP manifested in the practices of a group of undergraduate mathematics students collaborating on homework?
- **RQ2.** How do members of this CoP *align* their practice of collaborating on homework with their *imagined* future as research mathematicians?

Methodology

At time of interview, Sullivan and Lynch were recent graduates of a mathematics degree from a large, research-intensive Irish university. During the interviews, it became apparent that Sullivan and Lynch had collaborated on homework within the same group of peers. This is likely a result of the snowball sampling technique used during participant recruitment, with participants asked to recommend peers who might also consider participating. At the time of interview, both Sullivan and Lynch were pursuing graduate study in the mathematical sciences with the goal of conducting research and obtaining a PhD.

The first author conducted one-on-one, semi-structured interviews with participants which lasted an average of 30 minutes and were recorded and transcribed. Participants were asked to describe their experiences of collaborating with peers – who they collaborated with, what they collaborated on, how the collaboration process worked, and their perceptions of its benefits to their learning. We also asked the graduates whether they believed that research in mathematics is a collaborative or individualistic endeavour. See Murphy and Meehan (2024) for the full interview schedule.

Template analysis (Brooks et al., 2015) was chosen as the method of data analysis for this study. Initial coding was conducted by the first author before being reviewed and refined collaboratively by both authors. To address RQ1, *mutual engagement*, *joint enterprise*, and *shared repertoire* (Wenger, 1998) were used as *a priori* codes to examine how these characteristics of a CoP manifested in the practices of the participants. To address RQ2, we drew on data analysis undertaken as part of the larger study using reflexive thematic analysis (Braun & Clarke, 2022). Specifically, we examined instances of codes related to participants' perceptions of the collaborative nature of research mathematics in Sullivan and Lynch's interviews and reframed our analysis using Wenger's concepts of *imagination* and *alignment*.

Findings

To address RQ1, we introduce Sullivan, Lynch and their collaborators and substantiate our claim that they can be considered a CoP by illustrating the *mutual engagement*, *joint enterprise*, and *shared repertoire* of the group. In their final year, Sullivan and Lynch collaborated extensively with a group of five or six students. Both participants stated that the main factor that united the group was their mathematical interest and research ambitions:

Lynch: I think the group in fourth year, like, very much what kind of, like, made us friends was the fact we are, like, very into the maths and physics. That was the main thing that drove the group.

Sullivan: We were people with kind of similar ambitions, similar interests. All wanted to do research. From there it kind of naturally progressed to talking about the work and then doing the work together.

The group's *mutual engagement* consisted of their collective work on homework problems. Sullivan explained that through working together the group became friends and enjoyed the social aspect of collaborating on homework:

Sullivan: In kind of academic level we all typically got similar grades which is definitely something that we thought was important because you all want to contribute an equal amount. And I think we're all quite social relative to the typical people who study maths and so we quite enjoyed the aspect of having like a social work together.

As Sullivan and Lynch's community of collaborating peers engaged in the practice of working together on homework, they developed a *shared repertoire* which consisted of a routine and shared discourse. Both participants explained how they routinely met in a room on campus which contained multiple blackboards. Lynch exemplified how they would work on homework problems individually and ask one another questions when stuck:

Lynch: The way we'd work a lot in final year was we'd, it was in one of those rooms with like three sets of double blackboards and we'd literally just stand there, and we'd all work away on the problem sheet on our own on each blackboard. And you'd kind of go like 'Oh what are you doing for this thing here?'. And you're like 'I've tried this thing' and you'd try it yourself. Very much at the high level, not like high level, but yeah, you're not going like 'Here's my solutions, look at this'. It's very much, 'Alright, I'm a bit stuck, what are you thinking?'

As Lynch illustrated, one characteristic of the group's practice was that they restricted their collaboration on homework to high-level discussion. This was important to the members, and we consider it part of their *joint enterprise*. For the remainder of this paper, we focus on one aspect of the CoP's negotiation of their *joint enterprise*. As previously stated, members of the CoP all shared the ambition of pursuing postgraduate mathematical studies and careers in research. We propose that, as undergraduates, they identified with the community of research mathematicians through *imagination*. We address RQ2 by examining how they *aligned* their practice of collaborating on homework with their perceptions of appropriate collaboration in the context of research mathematics.

When asked about their impressions of the collaborative nature of research mathematics, Sullivan reflected that at the beginning of their undergraduate studies they "thought it was the man on his own in a room writing on a piece of paper". However, after completing an undergraduate research project with a collaborative research group they came to realise: "each person can only do so much on their own and each person can only approach a problem in so many ways on their own and really good work only gets done by groups". Similarly, Lynch reflected that the research group that they completed a research project with would "all chat about, like, recent research in the area, and like, it definitely seemed quite collaborative". While both Sullivan and Lynch recognised the potential for mathematical research to be collaborative, they emphasised individual intellectual ability as key to their *imagined* futures as researchers: "if you want to go into research you do need to be able to

work independently” (Lynch). Additionally, both positioned individual academic achievement as intellectually superior to work completed by groups. The stereotype of the lone genius appeared to persist for Sullivan: “there are still some very brilliant people who put papers out on their own and do very, very well”, while Lynch reflected: “you need to be much better than I am to work on your own the whole way through a research career”.

We now examine how Sullivan and Lynch attempted to *align* their practice of collaborating on homework with their perceptions of collaboration in a research context. While both appreciated the merits of collaboration, they emphasised the importance of maintaining one’s independent ability: “I think collaborating is great and I think it’s something that doesn’t get done enough but you can’t do away with individual work” (Sullivan). This emphasis on individual work manifested in the practice of the CoP, with each member initially attempting homework problems independently and only collaborating on problems that multiple members “were really struggling with” (Sullivan). As previously mentioned, the CoP’s collaboration on these difficult problems was restricted to high-level discussion. Sullivan explained: “it’d be sketching ideas on the board. Usually, we would try to avoid doing out explicit things because that didn’t always feel in the spirit of it”. Lynch elaborated that when the group was stuck on a difficult problem, each member wanted to be the one to figure the solution out. The emphasis on individual achievement contributed to a competitive atmosphere, which Lynch reported that the group enjoyed:

Lynch: Nobody wanted to get the answers off. It was, it was like a bit competitive, but in a good way, that you’d always be trying to figure it out, like, the cool idea to figure out the problem.

However, the normalised competition in the CoP and the emphasis on individual academic ability appears to have contributed to Sullivan and Lynch, at times, doubting their own mathematical capabilities. Sullivan reflected on how they did not enjoy feeling reliant on others when stuck on a problem: “It didn’t feel good to be copying what others are doing and so, if you didn’t really get something and you felt like someone just told you what to do, that never felt good”. Lynch expressed similar feelings:

Lynch: It can stress you out sometimes, if you’re in this group of people that are doing really well and you’re just not getting it for a time, or you feel a bit stupid and you’re like, ‘Oh my God, I’m really behind’, like, I can’t understand something.

Lynch questioned their ability to work independently, which they perceived important to their *imagined* future as a researcher: “I was a little bit worried that I was kind of using it too much as a crutch and if you want to go into research you do need to be able to work independently”.

Discussion

In this paper, we illustrated the existence of CoPs of undergraduate mathematics students collaborating on homework. We highlight two main implications. Firstly, our study emphasises the importance of accurately representing the nature of research to undergraduate students. Both participants identified their experience of completing undergraduate research projects as key in informing their perceptions of the collaborative nature of research. We call

for further research on undergraduate students' perceptions of mathematical research and how these perceptions can be accurately informed. Secondly, we wish to highlight the competition that appeared to be a normal part of the CoP's collaborative practice. Research would suggest that the co-existence of competition and collaboration is prevalent in research mathematics (Burton, 2004). However, we note the way in which it caused Lynch and Sullivan to doubt their independent mathematical capabilities. Given that we are reporting on a case study situated in a particular context, we call for further research to examine the co-existence of collaboration and competition in research mathematics and its influence on the perceptions and practices of undergraduate students.

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Assessment in CLIL mathematics: Language-responsive approaches for learners with additional educational needs

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This paper explores the challenges of summative assessment in Content and Language Integrated Learning (CLIL) mathematics classrooms for learners with Additional Educational Needs (AEN). Drawing on a case study in an Irish-medium primary school, we compare outcomes from two summative assessments: a standard, school-approved test and a language-responsive version designed by the researchers. Despite adaptations such as simplified syntax, visual supports, and reduced linguistic complexity, the adapted assessment produced only modest improvements. While these modifications improved task clarity for some learners, they did not fully address the cognitive demands or reflect learners' mathematical competencies. Findings indicate that conventional summative assessments are often ill-suited to AEN learners working in a second language. We argue that formative, language-aware approaches offer a more equitable measure of understanding. This study contributes to ongoing discussions around inclusive assessment in CLIL settings and calls for systemic shifts in evaluating mathematical learning in linguistically diverse contexts.

Keywords: CLIL, assessment, AEN, language-responsive maths, inclusive education

Introduction

The integration of language and content in mathematics education presents significant pedagogical and epistemological challenges, particularly for students with additional educational needs (AEN) in Content and Language Integrated Learning (CLIL) environments. The theoretical underpinnings of this study are grounded in research that emphasises the interdependence of language and conceptual learning in mathematics (Erath et al., 2021). Language proficiency is not merely a medium of access, but a determinant of mathematical reasoning, problem-solving, and participation in disciplinary discourse (Schleppegrell, 2007). CLIL contexts, Irish-medium education in the case of this research project, require students to engage with disciplinary content through a language that is often not their first. This presents unique difficulties for AEN learners whose challenges may span linguistic, cognitive, and affective domains (MacKenzie et al., 2022). While immersion education such as Irish-medium education (IME) has been lauded for promoting bilingualism, it has also been critiqued for insufficiently supporting learners with atypical language development or specific learning needs (Nic Aindriú et al., 2024). Accordingly, mathematics education in CLIL contexts increasingly demands approaches that are sensitive to learners' linguistic and cognitive profiles. Standardised assessments, designed with monolingual and neurotypical learners in mind, often fail to capture the nuanced understanding and potential of these students. This paper responds to this challenge by investigating whether language-responsive adaptations to summative mathematics assessments can offer a more equitable measure of AEN learners' capabilities in IME.

Key Literature

The relationship between language proficiency and mathematical understanding is well-documented in educational research and within multilingual/CLIL contexts where learners are required to access content through an additional language (Erath et al., 2021; Lenz et al., 2024). Mathematics is a linguistically dense subject that demands precise interpretation of abstract concepts, specialized vocabulary, and complex syntax. These challenges are compounded for AEN learners who may already experience difficulties in processing, memory, and executive functioning (MacKenzie et al., 2022). Language-responsive pedagogy offers a strategic approach to mitigating these barriers. It involves deliberate scaffolding of academic language alongside conceptual instruction, and has been shown to support engagement, participation, and comprehension in multilingual classrooms (Erath et al., 2021). However, as Lenz et al. (2024) point out, most studies in this area focus on bi-/multilingual learners without additional needs, thus overlooking the intersectional challenges faced by AEN students in CLIL environments.

The limitations of traditional summative assessments in this context are increasingly evident. Standardised tests often privilege learners with developed Cognitive Academic Language Proficiency (CALP), thereby disadvantaging those with language processing delays or atypical language trajectories (Cummins, 2000). Moreover, as Abedi and Lord (2001) argue, the linguistic complexity embedded in test items can obscure rather than reveal mathematical thinking. This critique is particularly pertinent in IME settings, where assessments may assume not only fluency in Irish but also familiarity with academic language structures that AEN learners have not yet mastered. Despite the theoretical promise of language-responsive pedagogy, its application in summative assessment remains underexplored (Lenz et al., 2024). As Nic Aindriú et al. (2024) and MacKenzie et al. (2022) highlight, IME continues to grapple with resource limitations and a lack of teacher training specific to inclusive bilingual pedagogy. These systemic constraints hinder the development of equitable assessment models that are both linguistically and cognitively accessible. This study seeks to advance the discourse on inclusive assessment by critically examining the extent to which language-adapted summative assessments can more accurately capture the mathematical competencies of AEN learners in a CLIL setting.

The Study

This research adopted a case study methodology to explore the effectiveness of language-responsive summative assessment in an Irish-medium CLIL mathematics classroom. A case study, as defined by Yin (2018), allows for in-depth investigation within real-life contexts, making it particularly suitable for addressing the research questions posed in this study. CLIL research benefits from such context-sensitive approaches. The selected case was a third-class group of five learners, all identified with AEN, who were already receiving targeted support within their school from the teacher-researcher (author 3) working on this project. The intervention was designed in response to the dual challenge faced by AEN learners in immersion education: navigating both complex mathematical content and the demands of instruction in a second language. Over five weeks, participants engaged in four mathematics lessons per week, informed by language-responsive principles. Participants were

withdrawn from their class for these specific mathematics lessons but otherwise participated in their regular classroom lessons. At the end of this period, students completed two summative assessments: the school's standard test (Gafa le Mata, or GLM) and a language-adapted version (RDA) developed by the teacher-researcher.

The instructional design was grounded in the understanding that mathematics is inseparable from the language used to express it. The intervention acknowledged the distinct mathematical registers present in different languages, in this case, Irish and English. In recognition of this, lessons were structured to explicitly scaffold language demands alongside mathematical concepts. The work of Erath et al. (2021) informed the core design features, including: engaging learners in discourse to construct meaning and justify reasoning; establishing routines that promote the use of mathematical language; using multimodal and multilingual supports to bridge conceptual understanding; and sequencing instruction to build from everyday language to academic terminology. Further, Schleppegrell (2007) underscored the need to address the semiotic complexity of mathematics - its reliance on symbols, visuals, and technical vocabulary. Instructional adaptations included simplifying terminology, increasing visual support, emphasizing key terms, using active rather than passive voice, and modifying complex syntactic structures to enhance comprehension. These same principles informed the development of the RDA.

The participants were five pupils aged 9 - 10 years, previously identified as requiring additional support in mathematics based on school assessments. The teacher-researcher had an established instructional relationship with the group, having worked as their special education teacher. Parental consent for continued support had been secured earlier in the academic year, and the intervention was integrated into the students' normal timetable. Two of the five pupils had formal diagnoses for learning difficulties, though it is important to note that these assessments were conducted in English and did not account for their Irish-medium education experience (Nic Aindriú et al., 2024). The mainstream class teacher collaborated in lesson planning and participated in discussions about pupil progress and assessment outcomes throughout the intervention.

Data collection followed a sequential, mixed-methods design. Qualitative data were gathered first through lesson observations and ongoing teacher-researcher reflections, supported by informal feedback from the class teacher at multiple points across the intervention. These insights were captured in field notes and used to monitor individual and group progress. Quantitative data followed in the form of assessment results from both the GLM and RDA. This triangulated approach allowed for a comparison of observed learning during instruction and measured performance in summative assessment (Yin, 2018). Thematic analysis (Braun & Clarke, 2006) was used to examine qualitative data, identifying patterns in how students interacted with language and content during instruction. These themes were then interpreted in relation to assessment performance. Quantitative data from the assessments were analysed descriptively to determine central tendencies and variation across participants. The integration of both data types enabled a holistic understanding of how language adaptations in assessments aligned with students' observed competencies. This comparative strategy allowed for exploration of convergence and divergence in findings, offering insight

into how language-responsive instruction and assessment practices function within a real CLIL mathematics classroom.

Findings and Discussion

Participants completed the summative assessments in varying sequences. Jenny, Mikey, and Saoirse undertook the RDA before the GLM assessment, while Aisling and Caoimhe completed the GLM first. Due to an absence, Caoimhe completed the RDA after a two-week school holiday break. This sequencing allowed an exploration of whether prior exposure or cognitive fatigue influenced performance. Analysis of the results revealed minimal differences in overall performance across both assessments (see Table 1). Notably, Caoimhe scored 18% higher on the GLM than the RDA, while Mikey and Saoirse also achieved better scores on the first test they attempted. The median score for all first assessments was 55, compared to 52 for second attempts. The GLM assessment had a median score of 59 (IQR = 25), while the RDA recorded a slightly lower median of 52 but with a narrower spread (IQR = 14), suggesting greater consistency in participant responses. While overall impact of the language adaptations was limited, patterns observed during lessons help explain the performance dynamics. The tendency for higher first-attempt scores, regardless of format, aligns with research on cognitive load and memory (Lenz et al., 2024), suggesting that learners with AEN may be more vulnerable to fatigue and reduced working memory in sequential assessments. Additionally, the bilingual CLIL context likely amplified cognitive demands during assessments (Abedi & Lord, 2001; Genesee & Fortune, 2014).

Table 1

Participants and results on the summative assessments

Participant	AEN Description	GLM	RDA
Aisling	No formal diagnosis; STEN 3; language skills low average range.	59%	59%
Caoimhe	Diagnosed with dyslexia; STEN 3; language skills low average range.	70%	52%
Jenny	No formal diagnosis; STEN 3; challenges are maths based; language skills average range.	59%	55%
Mikey	Diagnosed with ADHD, Autism and Dyslexia; STEN 2; challenges with mainstream and IME.	24%	28%
Saoirse	No formal diagnosis; STEN 2; low language skills	34%	41%

Aisling's case illustrates the complexity of assessment validity in CLIL settings. Despite her documented difficulties with question comprehension, she performed consistently well across both tests. Teacher observations indicated that mathematical anxiety, a known component of her AEN profile, was alleviated through targeted support during instruction. This underscores the significance of pedagogical context in shaping assessment outcomes. The language adaptations, while having minimal effect on overall scores, did facilitate comprehension in specific tasks. For instance, in a money-related question, all participants

used addition in the RDA, whereas two students had incorrectly subtracted in the GLM version of the same question. This illustrates how linguistic accessibility may support task comprehension even if not consistently reflected in final scores.

Errors in computation were common across both assessments, suggesting persistent conceptual gaps observed during instruction. These may stem from the dual challenge of navigating content and language, where limited proficiency impedes access to mathematical concepts (MacKenzie et al., 2022). Comparison with earlier assessment data offers more nuanced insights with the class teacher reporting notable improvements relative to previous summative assessments, with the median score rising from 41 to 59. This improvement suggests that the language-responsive instructional design positively influenced learners' readiness for summative assessment. The findings also raise questions about the broader role of CLIL in shaping mathematical outcomes for AEN learners. The class teacher queried whether learners, particularly Mikey, might have achieved higher scores in an English-medium context. Challenges during the unit on probability, where mathematical content was highly language-dependent, illustrated how language-specific barriers in Irish may hinder content acquisition. Yet research is divided: while some suggest using L1 to clarify complex content may enhance retention, others argue that low-ability learners in immersion settings perform comparably to monolingual peers, provided adequate support structures are in place (Madrid & Pérez Cañado, 2018).

Further complexity emerged in the topic of time, which the group had less opportunity to explore as the teacher-researcher was unavailable, and small-group language-responsive lessons were suspended. Many of which could not be linguistically adapted due to their visual nature. The absence of targeted instruction appeared to impact performance, with most participants answering few or none of these questions correctly. This finding underscores that assessment design alone is insufficient: learners require explicit, context-rich instruction to engage meaningfully with mathematical concepts (MacKenzie et al., 2022).

Taken together, these results highlight the need for sustained, language-responsive pedagogy and formative assessment practices to accurately gauge and support the mathematical learning of AEN students in CLIL settings. This study corroborates existing literature on the inadequacy of traditional summative assessments for linguistically and cognitively diverse learners (MacKenzie et al., 2022; Nic Aindriú et al, 2024). While language-responsive adaptations improve clarity and reduce misinterpretation, they do not necessarily translate into higher performance if foundational mathematical understanding is lacking, or cognitive overload persists. Moreover, the RDA's limited impact on outcomes underscores the structural limitations of summative testing itself, particularly its reliance on static, decontextualised snapshots of learning.

This reinforces the argument that assessment and instruction should not be treated as discrete entities but rather as interwoven practices. We contend that assessment should be viewed as an ongoing, dialogic process that both informs and is informed by teaching. For AEN learners in CLIL contexts, classroom-based assessments that emerge from rich instructional interactions may offer more authentic insights into learning than static, summative tests. This calls for a paradigm shift toward viewing assessment as partnership

where understanding is built collaboratively and continuously through purposeful, language-sensitive pedagogy. Such practices not only align better with inclusive pedagogy but also provide richer, more accurate insights into learner progression, particularly for AEN students navigating CLIL environments (Madrid & Pérez Cañado, 2018).

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Spatial thinking in primary education in Ireland

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Spatial thinking has been shown to span a multitude of disciplinary areas and is recognised as a core competence related to mathematical achievement and participation in Science, Technology, Engineering and Mathematics (STEM) careers. At this time of intense curriculum change at primary level in Ireland, this paper examines the extent to which spatial thinking is made explicit in the Mathematics and draft Science, Technology and Engineering and Social and Environmental Education (SEE) curriculum documents by analysing curriculum learning outcomes against a typology of spatial skills proposed by Newcombe and Shipley (2015). While many spatial skills are explicit in mathematics curriculum outcomes, they remain implicit in the STEM and SEE curriculum specifications.

Keywords: Spatial thinking, curriculum, STEM, integration, achievement

Introduction

The influential US-based National Research Council (NRC) report describes spatial thinking as a combination of concepts of space, tools of representation, and processes of reasoning (NRC, 2005, p. ix). The report argues that without considered attention spatial thinking will remain extensively relied on across the curriculum but not explicitly or systematically taught in any subject area (p. 6). In Ireland, there is evidence of consistent underachievement in Shape and Space. The area has been called out as a priority for development in the recent Literacy, Numeracy and Digital Literacy Strategy (Government of Ireland [GoI], 2023). Internationally, it has been suggested that spatial reasoning should be considered when “designing curricula, training teachers, setting goals and developing assessments” (Newcombe, 2017, p. 37). This is a time of significant curriculum change in Ireland. The primary curriculum framework (PCF) emphasizes interdisciplinary learning experiences and for the first time includes the curriculum area of STEM (Department of Education [DoE], 2023a). This area will consist of two distinct specifications: the mathematics curriculum specification (DoE, 2023b) and a specification for Science, Technology and Engineering (STE). Spatial ideas pertaining to geography are included in the draft Social and Environmental Education (SEE) specification. This paper presents a short overview of relevant literature, then presents an analysis of the extent to which spatial skills are explicit in the published mathematics and draft STE and SEE curriculum specifications.

Spatial reasoning and achievement

The role of spatial ability as a predictor of success in mathematics and in STEM fields has long been recognised. A UNESCO (2017) research synthesis highlights the foundational role of early spatial abilities for the development of quantitative reasoning, a cognitive domain integral to both scientific and mathematical thinking. While research provides some evidence of gender differences in spatial achievement in favour of boys, it is hypothesised that these arise from the social environment which provides more opportunities for spatial skill development to boys (UNESCO, 2017). Research also demonstrates links between socioeconomic status (SES) and a broad range of cognitive skills in childhood. Spatial skills are among those on which children from lower SES backgrounds perform worse than children from middle or high SES backgrounds (Johnson et al., 2022). In Ireland, the disparity of achievement scores between schools designated as disadvantaged and non-designated schools

is continually noted (e.g., GoI, 2023). Analysis of spatial achievement profiles may be illuminating in terms of developing new ways to tackle this consistent under achievement.

Some researchers advance critiques of a bias toward verbal reasoning and/or mathematics skills in existing educational systems which do not adequately identify and meet the needs of spatially talented students. The US-based analysis of Lakin and Wai (2020) indicates that large numbers of spatially talented students are overlooked in screening procedures and assessments which place a greater value on verbal and mathematical abilities. Additionally, analysis of longitudinal measures showed that spatially talented students had greater academic challenges, including reading difficulties, poor study habits, and behavioural troubles. These students were less likely to complete college degrees compared to other talented students. Such findings may also have relevance in Ireland, where inclusive pedagogy is advocated and the need to enhance outcomes for higher achievers has been noted.

Teaching Spatial Thinking

Research affirms the importance of environmental influences and the malleability of spatial reasoning (Uttal et al., 2013). Spatial thinking may be taught directly or infused in the curriculum. Direct instruction of spatial skills may involve, for example, engaging in play activities that are spatial in nature such as block-building or paper folding (Newcombe, 2017). The second strategy is more indirect and involves ‘spatializing the curriculum’ through the strategic use of tools for spatial thinking during general teaching (Newcombe, 2017, p. 12). Tools may include “spatial language, maps, diagrams, graphs, analogical comparison, physical activity that instantiates scientific or mathematical principles, gesture and sketching” (Newcombe, 2017, p. 37). This approach aims to avoid spatial thinking being seen as an ‘add-on’ to a packed curriculum and instead positions it as a ‘missing link’ that cuts across disciplines (NRC, 2005, p. 7). In the Irish context, Langan (2022) designed an integrated cross-curricular unit of work using the spatial enhancement skills of *visualization instruction, sketching, gesture, spatial comparison* and *spatial language* to target learning outcomes of the Irish Primary Curriculum and spatial thinking skills.

Spatial Skills and the Irish primary curriculum

Uttal et al. (2013) propose that a distinction can be made between spatial thinking related to intrinsic and extrinsic relationships. Intrinsic relationships are understood to mean the “specification of the parts, and the relation between the parts, that defines a particular object” and extrinsic relationships are understood to involve the relationships “among objects in a group, relative to one another or to an overall framework” (Uttal et al., 2013, p. 353). In addition, the research indicates that it is possible to distinguish between reasoning as it pertains to static and dynamic contexts (Uttal et al., 2013). Newcombe and Shipley (2015) expand on the nature of spatial thinking within these distinctions noting, for example, that spatial thinking involves processing both continuous and discrete information, and that scale plays an important role in deciding what will be defined as intrinsic or extrinsic (p. 3). These authors synthesise existing research to identify a list of spatial skills that can be identified in young children (p. 12 - 13). By looking for evidence of the spatial skills identified by Newcombe and Shipley (2015) within relevant areas of the Irish primary curriculum, we sought to identify the extent to which spatial skills are made explicit in curriculum specifications. We examined the learning outcomes of the primary mathematics specification (DoE, 2023b) and the draft specification for Science, Technology and Engineering (STE) (NCCA, 2024a) and Social and Environmental Education (SEE) (NCCA, 2024b). We acknowledge that other conceptualisations of spatial thinking are possible but chose Newton and Shipley’s framework as it specifically considers the spatial thinking of young children.

Table 1 shows Newton and Shipley’s (2015) spatial skills, which encompass dynamic and static contexts, in the far-left column. The intrinsic spatial skills are: *disembedding* (isolating and attending to one aspect of a complex display); *categorisation* (learning categories based on spatial relations); *visualising 3D from 2D*; *penetrative thinking* (visualizing spatial relations inside an object); *mental transformations*; and *sequential thinking* (visualizing the product of a series of transformations). The extrinsic spatial skills are: *locating self and other objects* (e.g., on maps); *alignment* (reasoning about spatial and temporal correspondence); *perspective taking*; *relations among objects in space*; and *updating movement through space* (visualizing movement of an object relative to other objects). Table 1 also shows our analysis of the extent to which these skills are explicit in curriculum learning outcomes. Given the way in which spatial thinking underpins many learning activities, it is possible that classroom activities directed toward various learning outcomes would contain activities that might address the spatial skills. However, ‘explicit’ is only listed if at least one learning outcome addresses the spatial skill directly. The analysis was completed in collaboration between authors, with differences of opinion agreed through discussion.

Not unexpectedly, for the mathematics specification, many of the spatial skills are explicit in the learning outcomes within the Shape and Space strand. While it is positive that many spatial skills are made explicit in the mathematics curriculum, we note that much of this content is a departure from the 1999 mathematics curriculum and teachers will need significant support in reorienting their teaching to address these skills. Skills pertaining to intrinsic spatial relations occur largely in the ‘Shape’ and ‘Transformation’ strand units. We note that the term ‘visualisation’ occurs at the level of the learning outcomes just once, “visualise and model location using symbolic coordinates” (Stage 2, ‘Spatial Awareness and Location’). While opportunities are evident for *penetrative thinking* in some of the progression continua content, e.g., shape dissection at milestone h in ‘Shape’, this remains implicit at the level of the learning outcomes. Many extrinsic spatial skills are evident in learning outcomes associated with the ‘Spatial Awareness and Location’ strand unit. *Perspective taking* involves visualizing the appearance of a scene from a different vantage point (Newcombe & Shipley, 2015, p. 10). Currently this is not present in the learning outcomes of the ‘Spatial Awareness and Location’ strand unit which emphasizes mapping and navigation rather than perspective.

Table 1

Presence and explicitness of spatial skills in the primary mathematics and draft STE and SEE curriculum specifications.

	Mathematics	STE	SEE
<i>Intrinsic Spatial Relations</i>			
1. Disembedding	Explicit	Explicit	Explicit
2. Categorization	Explicit		Explicit
3. Visualizing 3D from 2D	<i>Implicit</i>	<i>Implicit</i>	<i>Implicit</i>
4. Penetrative thinking	<i>Implicit</i>	<i>Implicit</i>	
5. Mental transformations	Explicit	<i>Implicit</i>	
6. Sequential thinking	Explicit	<i>Implicit</i>	
<i>Extrinsic Spatial Relations</i>			

1. Locating self and objects	Explicit	<i>Implicit</i>
2. Alignment	Explicit	<i>Implicit</i>
3. Perspective taking		
4. Relations among objects in space	Explicit	<i>Implicit</i>
5. Updating movement through space	Explicit	<i>Implicit</i>

For STE, *disembedding* is positioned as a key skill with reference to developing ‘STEM eyes’ and observing everyday objects through a STEM lens a learning outcome in the ‘Nature of STEM’ strand unit. Other skills associated with intrinsic spatial relationships are either not present or implicit only. For example, while categorisation (of plants, animals and materials) is included as a learning outcome, the extent to which this may involve *categorisation based on spatial relations* is unclear. In addition, *visualizing 3D from 2D*, *penetrative thinking* and *mental transformations* are core design skills that students should encounter as part of the engineering design process. The stage 4 ‘Engineering’ learning outcome includes reference to the use of sketching and digital tools to plan designs. These representations may facilitate spatial thinking, but the framing of earlier learning outcomes largely focuses on aspects of the design process which may not involve spatial thinking such as assessing user needs, or reflection on the process. We note little evidence of, but huge possibilities for, extrinsic spatial skills in relation to other STE learning outcomes. *Alignment* involves reasoning about spatial and temporal correspondence such as the use of space as a proxy for time, and the use of space as a proxy for other elements on graphs (Newcombe & Shipley, 2015). This is of relevance to science inquiries such as those mentioned in the ‘Materials’ or ‘Forces and Electricity’ strand units. In the same way, *updating movement through space*, seems likely to underpin thinking in such science inquiries though it is not explicitly mentioned. In addition, the potential of digital technologies to support exploration of static and dynamic representations of space and perspective taking is not mentioned.

The draft SEE curriculum specification contains integrated learning outcomes for stages 1 and 2 and history, geography and integrated outcomes at stages 3 and 4. It is possible to identify outcomes which contain references to spatial skills, but much of this remains at the implicit level as outcomes are written in a very broad manner. For example, the following outcome from stage 4 of the strand unit ‘People, places and space’ could be interpreted in ways that may emphasize only historical or social understandings and not draw in any meaningful way on spatial reasoning: “Investigate people's journeys, the events, and motivations for movement, as well as the impact and influence of movement on people and places” (NCCA, 2024b, p. 18).

Implications and recommendations

Spatial reasoning is more prominent in the new mathematics curriculum than the previous version and teachers will need considerable support in understanding and enacting this. The limitations to the explicit attention to spatial thinking in the draft STE and SEE curriculum raises a number of other challenges. There is a danger that teachers will see spatial reasoning as connected primarily, and possibly only, with mathematics. The extent to which it underpins student learning and achievement in other domains may not be grasped by teachers. In initial and continuing professional development, teachers should be made aware of core spatial skills which reach across curriculum areas, such as those identified by Newcombe and Shipley (2015), shown on table 1. In addition, curriculum support materials would be well

advised to address the gaps identified in table 1 above. They should include explicit attention to (i) visualizing 3D from 2D; (ii) penetrative thinking and (iii) perspective taking.

Nationally and internationally, the research which has shown the importance of spatial thinking for equitable outcomes is being used as a foundation for planning pedagogical interventions. The existing research provides strong guidelines for how this might be achieved in ways which do not add to an already packed curriculum. Attention to spatial tools, such as those used in Langan's (2022) work, should be a core part of teachers' preparation and professional development for STEM education. This is important to increase students' spatial reasoning, but also to ensure that teaching reaches those students who may already be better spatial learners. The principles of the PCF include using evidence-based pedagogical approaches which create equity of opportunity and participation (Government of Ireland, 2023). In addition, the PCF principles advocate for meaningful assessment to inform teaching and learning. This is of importance to those with spatial thinking skills and needs which are not being recognised or met within the current environment. While national and international tests suggest issues with achievement in Shape and Space for learners in Ireland (Government of Ireland, 2024), better data on spatial reasoning, at all levels, is warranted. This pertains to assessing and monitoring the development of children's reasoning, and to the spatial thinking capacity of teachers.

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Private tuition in mathematics: What is driving students to seek it out?

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This study explores reasons why students in Ireland pursue private tuition in mathematics at post-primary level. Previous research has established the prevalence of private tuition globally, but little is known about the factors motivating students to seek additional support in mathematics. Using qualitative data from 304 students who had recently completed secondary education, the study identifies students' reasons for engaging in private tuition in mathematics. The reasons cited most frequently include the perceived difficulty of the subject, a desire for higher grades, dissatisfaction with classroom teaching, and the incentive of bonus points. The findings also reveal gender-based differences in the reasons why students seek tuition while different reasons were also noted depending on the level of mathematics they were studying. This research has implications for policy and practice in mathematics education.

Keywords: Private tuition, mathematics, self-efficacy, bonus points

Introduction

Private tuition (PT) is defined as paid tutoring that is supplementary to regular schooling (Bray, 2014). Research has shown that there has been an exponential growth in the uptake of PT in recent years. For example, countries such as Germany, Poland and the UK have all reported significant proportions of students participating in PT across a range of subjects and these figures have been increasing rapidly since the turn of the century (Sutton Trust, 2018; Dlugosz, 2017; Hille et al., 2016). While the findings reported in these studies relate to all school subjects, there has also been research conducted into the incidence of PT specifically for the subject of mathematics. Such research has concluded that mathematics tends to be the subject where PT is most sought after (Hussein, 1987). For example, in England Ireson and Rushforth (2005) found a higher proportion of students from Year 6 through to Year 13 reported receiving PT in mathematics compared to any other subject. Similar findings were reported in Germany, South Korea and Poland.

In addition to research looking at the prevalence of PT, many studies have also considered reasons behind student's decision to pursue PT across a range of different subjects, including mathematics. When considering the uptake of PT across a range of subjects the presence of high stakes examinations; perceptions of inadequacies in mainstream schooling; and the expansion of higher education globally were all considered to be driving the demand for PT (Zhang et al., 2024; Zhang & Bray, 2016; Bray & Lykins, 2012). In addition to this, some research studies have considered why PT in mathematics is particularly popular. According to Bray and Lykins (2012), subjects such as mathematics and national languages, that are needed for educational advancement, tend to be the subjects where PT is most in demand. Additionally, subjects such as mathematics that involve terminal examinations that affect students' educational transition options tend to see higher demand for PT (Bray, 2013).

However, few studies have sought to ascertain, from the perspective of students, why they pursue PT in mathematics. It is this gap in the literature that this study seeks to address.

Private Tuition in the Irish Context

PT in Ireland is colloquially known as “grinds” and, as is the case worldwide, research into the grinds culture here has gathered momentum in recent years. Smyth (2009) discovered that in 1993, 32% of students who completed secondary education reported receiving grinds but by 2003 this figure had risen to 45%. More recently, McCoy and Byrne (2024) reported that by 2016 almost 60% of students received grinds during their final two years of study at secondary level. Both of these studies report on the uptake of grinds across all subject areas. However, O’Meara & Prendergast (In Press) investigated the prevalence of grinds in the subject of mathematics specifically and found that 51% of students in their study availed of mathematics grinds. Furthermore, as was reported internationally, this study found that mathematics was the subject in which grinds were most sought after. At the time of writing no study in Ireland has investigated the reasons behind this, although some have hypothesised that the ‘special status’ attributed to mathematics in Irish secondary schools may be a contributing factor (Prendergast et al., 2022). This is a hypothesis that we will be testing as part of this study. In summary, while the volume of research into PT in Ireland and internationally has grown over the last two decades, there remains a scarcity of studies looking at the reasons given by students as to why they are seeking out PT in such high numbers, particularly in the subject of mathematics. Given the changes to mathematics education curricula worldwide, and the renewed focus on improving the uptake and performance in mathematics in secondary education, a study considering the reasons why additional support is deemed a necessity by many is both necessary and timely. In order to investigate this issue, the authors will seek to address the following research questions:

What are the reasons given by students for participating in PT in mathematics?

Do factors such as gender and level of mathematics studied for Leaving Certificate affect the reasons given for participating in PT in mathematics?

Methodology

In order to address these research questions, an online survey was deemed to be the most appropriate research instrument as it was easy to distribute to a large sample; allowed the research team to ensure participants’ anonymity; was cost effective; and not overly time consuming to complete (Van Selm & Jankowski, 2006). The survey developed for the purpose of this study enabled the authors to gather information in relation to the rationale offered by students for availing of PT in mathematics. The survey yielded qualitative and quantitative data and incorporated dichotomous, multiple choice, Likert-scale and open-ended questions. The survey was piloted with a research advisory group made up of a small number of undergraduate students who were selected using purposive sample. It was adapted from the authors’ previous research investigating mathematics teachers’ perceptions of the grinds culture (Prendergast et al., 2022). For the purpose of this paper, we will report on one of the open-ended questions which asked those who partook in mathematics grinds to outline the reasons why they opted into this practice.

The findings being reported on in this paper emerged from a wider study investigating the prevalence of PT in the subject of mathematics in Ireland and the profile of students who engage in this practice. The target sample for this larger study was graduates of secondary schools in Ireland who had studied mathematics throughout primary and secondary education. Once ethical approval was obtained from the University of Limerick, the authors circulated the survey to graduates who were in their first year of a tertiary programme in three higher education institutes (HEIs). These institutes were selected using purposive sampling and included two universities and one technological university. The online Qualtrics survey link was emailed to first-year course directors in each of the three HEIs and they were asked to share it with their students. The online link was left open over a three-week period in November 2022 and 686 graduates responded to the survey. Of these 686, 350 students reported availing of PT in mathematics and of these, 304 students responded to the question “*List the three main reasons why you took grinds in the subject of mathematics.*” It is these 304 school graduates that constitute the sample for this paper. This sample was made up of 102 male students (33.5%); 199 female students (65.5%); 2 students who identify as third gender/non-binary (0.6%) and 1 student who did not provide details as to their gender. 230 of the 304 students (75.6%) took the higher-level mathematics exam at Leaving Certificate with 74 (24.3%) opting for the ordinary-level exam.

The data from the open-ended question central to this study was transcribed and Braun and Clarke’s (2021) thematic analysis framework was used to analyse the data. An inductive ‘bottom up’ orientation guided the analysis and was used to identify themes generated through the reasons for undertaking mathematics grinds offered by students. Implementing Braun and Clarke’s (2021) six phases in the process of thematic analysis provided a framework for data engagement, coding, and theme development. Both authors engaged in the analysis with repeated movement back and forth as initial codes were generated from individual transcripts and grouped into preliminary themes. The theme development, review, and refining phases involved several iterations of regrouping and renaming themes until both authors felt the final themes and sub-themes uniquely captured participants rationale for availing of mathematics grinds. The key themes to emerge will now be discussed.

Results and Discussion

In order to address the first research question, the authors analysed all students' responses to the statement *List the three main reasons why you took grinds in the subject of mathematics.* In total 22 reasons were provided by students and each of these reasons was cited four or more times. The top five reasons given by students as to why they availed of PT in the subject of mathematics are presented in Table 1.

Table 1

Top five reasons for availing of private tuition in mathematics (n = 304; f = 696)

Reason	f	Sample Response
Perceived difficulty/struggle with mathematics	125	S108: ...very difficult to comprehend some topics...I felt judged when asking questions...a lot of other people seemed to not struggle...maths was one of my weaker subjects...
Desire for higher grades	78	S40: I was receiving grades I was not happy with... I wanted to achieve the best grade possible
Issues with the class teacher	68	S149: Because my teacher disliked me and often singled me out in class which made me uncomfortable and unable to learn. I wanted to prove that I could do well...despite what she said, which I did. I liked maths
Improve knowledge	61	S331: To actually learn the material Understand concepts To go over neglected aspects of the course
To secure bonus points	52	S219: I took grinds because I knew I needed high points for my course and was trying to get the extra 25. I knew I had a better chance of achieving this with grinds my parents gave me the opportunity to take them

As demonstrated in Table 1, the perceived difficulty of the subject is the reason given by most students (41.1%) as to why they participated in PT in mathematics. Research has shown that students from elementary school through to high school find mathematics to be one of the most difficult subjects (Dündar et al., 2014; Brown et al., 2008) and one of the consequences of this is that it is resulting in students seeking extra support in the subject. One of the reasons that mathematics can be perceived as difficult is as a result of students encountering “aversive teaching styles [and] difficulty in following the instruction” (Gafoor & Kurukkan, 2015, p. 4). As such, it is little wonder that a significant proportion of students in this study (22.4%) believed the teaching practices that they were encountering in schools was leading them to pursue extra tuition in mathematics. Finally, the fifth most popular reason, cited by 17.1% of the sample, was that the bonus points on offer for mathematics were resulting in them needing PT. In a study conducted by Prendergast et al. (2020, p.760) one teacher hypothesised that a drawback of the bonus points initiative was that it “promotes a grinds culture where if a parent throws enough money at the problem the problem will be solved” - the findings of this study appear to add weight to this argument. The additional 25 points on offer for achieving 40% or more in the Leaving Certificate higher level examination is motivating students to pursue higher level mathematics (O’Meara et al. 2023), but this study shows that this incentivisation of mathematics is contributing to the surge in uptake of PT in mathematics in Ireland.

In addition to considering the primary reasons offered by students as to why they partake in PT in mathematics the authors also wanted to see if the reasons differed across gender and if the level they studied mathematics at for Leaving Cert had an influence on their reasoning. In relation to gender, of the 22 reasons, the biggest differences were noted in the

reasons *To secure bonus points* (20.1% of females offered this as a reason compared to 11.8% of males); *To improve mathematical knowledge* (17.6% of females offered this as a reason compared to 25.5% of males) and *Fear of failing* (14.6% of females offered this as a reason compared to 6.7% of males). These findings indicate that girls' lower levels of belief in their own mathematical ability is driving female students to seek additional support so that they can achieve a passing grade in mathematics. There were also some differences noted when the reasons offered by those who studied higher-level mathematics were compared with those offered by students who studied ordinary-level for Leaving Certificate. In this instance, the biggest differences were noted in the reasons *Desire for higher grades* (30.0% of those studying higher-level offered this as a reason compared to 12.2% at ordinary level) and *Fear of failure* (8.7% of those studying higher-level offered this as a reason compared to 21.6% at ordinary level). These findings suggest that consequences of failing the Leaving Certificate ordinary-level exam, most notably access to the majority of third-level courses being denied, is a key factor behind ordinary-level students seeking PT in the subject of mathematics. On the other hand, the points race which is central to second level education in Ireland is clearly playing a role in higher-level students' decision to seek additional support in the subject. To conclude, the preliminary analysis of this data allows us to gain a deeper insight into students' reasons for pursuing PT in mathematics and thus this study contributes to a deeper understanding of the educational pressures influencing students' decisions. The differences recorded across gender and level of study also highlight the potential for PT to exacerbate educational inequalities among certain groups and social classes. As such, the study also has implications for policy and practice in mathematics education.

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Scaling up a successful intervention—Teaching toddlers the meaning of numbers in preschool activities

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This paper reports on the development from a micro study on mathematics teaching, in which toddlers were given the opportunity to distinguish necessary aspects of numbers, to a macro study of implementation of these results at large scale. The micro study was a combined research-development project conducted in collaboration between researchers and preschool teachers in three Swedish preschools. The study identified important features of preschool activities in which toddlers are given the opportunity to discern critical aspects of numbers. Based on the positive results, a subsequent study was initiated on how the successful intervention could be implemented on a large scale. In this paper, the focus is on how a successful intervention can be designed and then transformed into a successful implementation.

Keywords: Early mathematics, numbers, preschool, intervention, implementation

Introduction

This paper reports on the development from a micro study on mathematics teaching, in which toddlers (one- to three-year-olds) were given the opportunity to discern necessary aspects of numbers, to a macro study of implementation of these results at large scale. In Sweden, all children are offered and encouraged to participate in preschool, where some hours are free of charge. According to the Education Act (SFS 2010:800), as part of the education system, the preschool is to involve teaching for the youngest learners also. The national curriculum emphasises that this teaching ought to be based on children's exploration, curiosity and desire to play (National Agency for Education, 2018). An evaluation by the Swedish Schools Inspectorate (2017) showed, however, that mathematics teaching practices needed to be improved in many Swedish preschools, as preschool teachers were observed to be uncertain as to how to teach mathematics, especially with the youngest children. Another uncertainty was how to establish a balance between children's experiences, curiosity and interests versus offering them new experiences and challenges leading towards learning goals (Björklund & Palmér, 2019).

Based on these observed challenges, a small-scale combined research-development project was initiated with the aim partly to investigate how ongoing preschool activities could become the starting point for mathematics teaching with toddlers and partly to investigate how such activities could advance toddlers' development of number knowledge. The study yielded both qualitative and quantitative developments in the use of numbers by the toddlers in the intervention group compared to those in the control group (Palmér & Björklund, 2024a). However, research has demonstrated that successful interventions are not easily transferred to a larger community of teachers and that all interventions undergo changes when implemented in new settings (Century & Cassata, 2016). Based on this, a new study was

initiated with the aim of implementing the intervention on a large scale in other preschools. The focus of this paper is on both the intervention and the implementation, that is, on how a successful intervention can be designed and then transformed into a successful large scale implementation.

Number Knowledge

Ordinality, cardinality, part-whole relations and representations have been shown to be essential aspects of the complex number construct that children need to discern to develop their numerical competence (Björklund et al., 2021). However, few studies have used empirical learning outcomes to examine how this development can be facilitated in the early years. Learning about numbers entails discerning their meaning (including ordinality, cardinality and part-whole relations) mediated through different representations. This discernment does not emerge on its own but through communication with others (van Oers, 2010). According to Lesh et al. (1987), learning (e.g. of numbers) is reflected in the ability to make connections between and within the modes of representation used. However, in studies with toddlers, who do not necessarily understand or use any of the commonly used representations, such as verbal notions, symbols or manipulatives, this may be a particular challenge.

Theoretical Framings

In the intervention study, the variation theory of learning (VT) was used as a theoretical framework for both designing teaching activities and analysing learning outcomes. This theory directs attention to how learners experience a phenomenon and, in particular, what they must discern regarding aspects of that phenomenon so as to advance their way of experiencing the phenomenon (Marton, 2015). In every situation, several aspects of a phenomenon may be discerned, and those that are discerned are decisive for how the phenomenon is experienced. Learning then occurs when learners discern new and necessary aspects of a phenomenon, which in this study means children discerning the aspects of ordinality, cardinality and part-whole relations through different modes of representation. Aspects can be discerned when they are encountered as dimensions of variation, that is, when the aspect to be discerned is varied against an invariant background. This means, for example, that cardinality is discerned when cardinality is varied in a setting of otherwise similar features (e.g. three blue blocks are compared to two identical blocks, making the numerosity stand out and be discerned). This basic conjecture regarding how learning new meaning happens offers a theoretical principle for how to direct children's attention to an aspect of a phenomenon and thus offers strategies for designing tasks that offer children the best conditions under which to specifically discern necessary aspects that advance their ways of experiencing the intended object of learning.

In the implementation study, a framework focused on the quality of the implementation was used (Coburn, 2003). This framework comprises the four interrelated dimensions of depth, sustainability, spread and shift in ownership. The framework was developed as an alternative to implementation studies that focus only on the numbers of teachers, schools or districts involved. The argument for the alternative framework was that a

focus only on numbers of teachers, schools or districts provides limited insight into the nature of the envisioned or enacted changes as well as into the degree to which any observed changes are sustained.

Methods

In the intervention study, two researchers and three preschool teachers from three preschools in two Swedish municipalities collaborated for three semesters. At the start of the study, the 27 children participating in the intervention were 12–27 months old. A comparison group was included in the study consisting of 10 toddlers from the same preschools who did not participate in the intervention. In line with the Swedish regulations for good research ethics (Swedish Research Council, 2017), the children’s guardians consented to their children’s participation.

To make the children’s experiences, needs and interests the starting point of the activities to be developed, observations were conducted before the intervention was initiated. By categorising the observations and comparing their frequency, the activities in which the children at all three preschools showed high spontaneous interest were selected as a starting point for further development within the project. Five main activities were developed that differed in nature to correspond to the variety of observed activities: book reading, motoric play, singing, lotto and memory games. The activities were implemented individually and in small group settings by the three preschool teachers during the three semesters of the intervention programme. Teaching principles informed by VT (see above) were embedded in both the choice of materials and the teachers’ ways of directing attention to aspects of numbers verbally, through actions and gestures. Through an iterative process of trialing, evaluating and re-designing the activities, the critical aspects of numbers were made possible to discern for the children. Cardinality was the prominent aspect in all of the activities but also ordinality and part-whole relations became possible to emphasise through embedded patterns of variation. Modes of representations were constantly assessed to offer best opportunities for the children to experience the meaning of numbers. To identify different ways in which children understand and use numbers, play-based interviews were designed to capture all kinds of expression, particularly non-verbal (see Palmér & Björklund, 2024b). These interviews were conducted five times with the children over four semesters, also providing an opportunity to follow the development of their numerical understanding and number use. All interviews were conducted by the preschool teachers, who were trained in conducting research interviews with young children (Palmér & Björklund, 2022). Based on video recordings of teaching activities (78 hours) and interviews (60 hours), analysis using VT was conducted based on coding in NVivo. The VT analysis focused on the qualitatively different ways in which the children experienced numbers’ meanings and on how the teaching act made it possible for them to discern new aspects of numbers.

The scaling-up of the intervention programme was conducted in line with educational design research, which implies implementation in cycles, whereby new preschools are successively included in an increasingly broad implementation. The first step was to identify the core elements of the above described intervention, that is, to identify the innovation to be implemented (Century & Cassata, 2016; Coburn, 2003). These core elements then constituted

the content of the guiding material used in the implementation. This guiding material was developed in collaboration between researchers and the same preschool teachers involved in the first intervention. In a first cycle, the guiding material was implemented in two preschools. In a second cycle, six new preschools were involved, and finally, in a third cycle, all preschools from one municipality implemented the guiding material. The data generated were individual interviews (12), focus group interviews (28) and video documentations (219) of the teaching conducted at the preschools. Also, delayed interviews (after 6 and 12 months) were conducted. The interviews focused on the guiding material and how it was used and experienced by the preschool teachers. The video documentations made visible how the teaching was adapted by the preschool teachers. These video documentations were also used during the interviews as a form of stimulated recall. These empirical data were analysed with a focus on whether the core elements were visible in the teaching and, if not, how the guiding material needed to be developed. This analysis was based on depth, sustainability, spread and shift in ownership in accordance with Coburn (2003). Based on the findings from each cycle, the guiding material was revised.

Results

As mentioned, the aim of the intervention was twofold: partly to investigate how ongoing preschool activities could become the starting point for mathematics teaching with toddlers and partly to investigate how such activities could advance toddlers' development of numerical skills. For activities to become a starting point for mathematics teaching, the study revealed that they ought to be designed in accordance with three design principles: (DP1) The context of the activities ought to be based on children's experiences, needs and interests, and they should be familiar so that the children can participate in, relate to and reason about the content based on their previous social and cultural experiences. At the same time, in line with VT, interference from irrelevant elements ought to be reduced. (DP2) The activities ought to make it possible for the children to discern essential aspects of numbers (i.e. representations, cardinality, ordinality and part-whole relations) through carefully planned patterns of variation. (DP3) The activities ought to allow the children to express different ways of understanding and be open to a variety of experiences and expressions.

To investigate the extent to which it is possible to advance toddlers' tendency to attend to numbers when encountering numerical tasks, the intervention group was compared with the control group. The results of the task-based interviews show that there was more frequent use of counting words in the intervention group (almost three times as often) than in the control group. There was furthermore a distinct qualitative difference in how the two groups of children used numbers. For example, all use of counting words observed in the control group were emphasising the counting sequence, whereas those observed in the intervention group also emphasised cardinality and the relational features of numbers. This implies that the intervention group had developed a broader range of ways to use counting words, reflecting an advanced way of experiencing numbers.

As mentioned, the aim of the implementation study was to determine how a successful intervention could be transformed into a successful implementation. Based on the three design principles from the intervention, the guiding material consisted of five parts, with each part

focused on one of the core elements identified: part 1, mathematising; part 2, contrast and generalisation; part 3, representations; part 4, cardinality; and part 5, part-whole relationship of numbers. The final guiding material consisted of one text to read, one video illustrating the content of the text in authentic teaching situations, questions to discuss with colleagues, one scripted activity to be carried out with children and, finally, one activity to be planned and carried out by the preschool teacher. The analysis of the interviews and video documentations of the conducted teaching revealed a broad variety of ways to conduct the teaching, some in line with the provided material as well as some with adaptations. A closer analysis of the character of these adaptations showed examples of misinterpretations of the intended principles to be implemented, whereas others revealed a critical feature: adaptations, when they are productive, indicate depth, spread, sustainability and ownership and can therefore be considered empirical evidence of successful implementation. This insight made it possible to address non-productive adaptations in the guiding material as examples of how not to teach the innovation. Altogether, the analyses reveal certain features that seem critical for the success of the intervention in terms of positive learning outcomes and features that turned out to be critical for the success of the implementation at large scale.

Discussion

The focus of this paper was on how a successful intervention on early number knowledge could be designed and then transformed into a successful implementation. The design principles and the core elements as well as the positive view on productive adaptations presented in this paper must, however, be understood within its national educational context, that is, in connection to the Swedish preschool. As mentioned, the national curriculum emphasises that teaching in preschool ought to be based on children's exploration, curiosity and desire to play (National Agency for Education, 2018). Further, as Swedish preschool teachers have a three-year university education and a significant degree of autonomy in their teaching practice, they are both qualified and expected to make adaptations in relation to the children in their preschool. It is within this context that the design principles and core elements as well as the positive view on productive adaptations presented in this paper must be understood. That is, in a context with a different preschool curriculum upholding other priorities and other views on learning and teaching, the design principles for the intervention may have been different and thus also the guiding materials developed. Likewise, in a context where teachers have a different teacher education, the ability to make productive adaptations may be either greater or less.

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Mathematics teachers' views on classroom-based assessments as part of the junior cycle

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This paper explores the perspectives of mathematics teachers on the implementation and impact of Classroom-Based Assessments (CBAs) as part of the Junior Cycle Mathematics Specification. Reflecting international trends, Ireland's recent Junior Cycle reform introduced CBAs to complement traditional state examinations, creating a dual assessment model. Drawing on data from an online survey of 196 respondents, preliminary analysis identifies both perceived advantages and notable challenges. While some teachers acknowledged the value of CBAs in promoting independent learning and linking mathematics to real-life contexts, widespread concerns were identified regarding the substantial time demands, low levels of student engagement, and the potential for increased educational inequities. These preliminary findings suggest that, despite the intended pedagogical benefits, the current implementation of CBAs presents significant obstacles in mathematics classrooms. Implications for policy and practice are discussed.

Keywords: Classroom-based assessments, mathematics, secondary level, Junior Cycle, teachers' perspectives.

Background to the Study

A gradual rollout of specific curriculum reform measures at Junior Cycle (lower secondary education) began in Irish schools in September 2014 and continued through to September 2019. Curriculum reform in education is inherently complex and often presents a range of challenges, resulting in curricula rarely being implemented exactly as intended by their designers (Prendergast & Treacy, 2018). To conceptualise this further, we draw on Glatthorn et al.'s (2012) curriculum model which defines the 'intentional curriculum' as "the set of learnings that the school system consciously intends" (p.6). Glatthorn and colleagues identify four different pillars of the intentional curriculum. These pillars are the *written curriculum*, that is the curriculum as it is presented in policy documents; the *supported curriculum*, that is the resources that support the curriculum; the *taught curriculum*, that is the curriculum delivered by teachers in the classroom; and the *tested curriculum*, that is the aspects of the curriculum which appear in key assessment tools. While the reforms at Junior Cycle permeate all four pillars of Glatthorn et al.'s model, this paper focuses specifically on teachers' perspectives of changes to the *tested curriculum*. More especially, it focuses on their views towards the introduction of Classroom-Based Assessments (CBAs) at Junior Cycle.

To address concerns about the overly summative and limiting nature of previous Junior Cycle assessments, CBAs were introduced as a means to diversify assessment methods. Their implementation is consistent with assessment practices in education systems such as those in Finland, Hong Kong, New Zealand, Scotland, and Queensland, Australia (Murchan, 2018). CBAs represent a form of school-based assessment that can be classified as

formative in nature. According to Black and Wiliam (1998), formative assessment involves activities designed to gather evidence of student learning, which can then be used by teachers, students, or others to guide future teaching and learning. While research consistently underscores the benefits of formative assessment for student learning, it remains a difficult approach for many teachers to adopt, and some are reluctant to implement it in practice (Yan & King, 2023). In the Irish context, the introduction of school-based assessment aimed to shift the focus away from summative assessment and empower teachers with more autonomy in evaluating their students (Murchan, 2018). The initial plan proposed that teachers would both design and grade the CBAs and a final summative assessment. However, this led to strong opposition from teacher unions (Murchan, 2018), resulting in industrial action. As a compromise, it was proposed that the National Council for Curriculum and Assessment (NCCA) would develop the CBA tasks, which students would complete during class time within a designated national timeframe. Teachers would evaluate the work using standardised descriptors and the outcomes would be shared with students and parents via the Junior Cycle Profile of Achievement (JCPA), though they would not contribute to official state certification. Formal certification would instead consist of: (1) a revised version of the traditional final examination, created and graded by the State Examinations Commission (SEC), and (2) a written Assessment Task related to one of the CBAs, completed in class during third year but also graded by the SEC. These arrangements were finalised following negotiations between the Department of Education and Skills (DES) and teacher unions, just prior to the first revised English examination in June 2017.

In summary, students typically complete two Classroom-Based Assessments (CBAs) for each subject—one during second year and the other in third year—with guidance and support from their classroom teacher. After finishing the second CBA, third-year students complete a written Assessment Task that encourages them to reflect on their learning, as well as the skills and competencies developed during the CBA. This task accounts for 10% of the total subject grade in most cases, with the remaining 90% assigned to the final state examination held in June. The new Junior Cycle Mathematics Specification (JCMS) was introduced to all schools in September 2018. Similar to other subjects, there are two CBAs (see Table 1) which are undertaken by students in the second and the third year of the Junior Cycle.

Table 1

Overview of CBAs in Mathematics (NCCA, 2018)

Type	Detail	Time Taken	Completion
CBA1: Mathematical Investigation	Students will follow the problem-solving cycle to investigate a mathematical problem.	Three-week period	End of second year

CBA2: Statistical Investigation	Students will follow the statistical enquiry cycle.	Three-week period	End of first term of third year
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Although introduced in 2018, the full implementation of the *Junior Cycle Framework* in Mathematics— particularly around the completion of two CBAs and the subsequent Assessment Task—has yet to be fully realised due to disruptions and revised arrangements caused by the COVID-19 pandemic. Thus, the implementation of the JCMS is still in its early stages, with teacher perspectives continuing to evolve. However, with the Leaving Certificate mathematics curriculum currently under review and expected to introduce elements of continuous assessment by 2027 (NCCA, 2024), it is timely to examine mathematics teachers’ experiences with these classroom-based assessments. This paper study therefore seeks to explore the following research question: What are mathematics teachers’ views on the introduction of CBAs as part of the Junior Cycle Mathematics Specification?

The Study

As part of a broader research project looking at the implementation of the JCMS as a while, the authors developed an online survey to capture mathematics teachers’ perspectives on the key changes introduced to the subject at Junior Cycle since 2018. This paper addresses questions concerning the implementation of CBAs, focusing specifically on responses where participants were asked to describe both the advantages and challenges of CBAs using two open-ended prompts:

- Please describe the advantages of the inclusion of CBAs in the JCMS.
- Please describe the disadvantages of the inclusion of CBAs in the JCMS.

The intended sample included all teachers of Junior Cycle mathematics in Ireland’s secondary sector. After obtaining ethical approval from the Education and Health Sciences Research Ethics Committee at the University of Limerick, the survey—administered via Qualtrics—was distributed using a non-probability sampling method. The link was shared with all national branches of the Irish Mathematics Teachers’ Association (IMTA), who were asked to circulate it among their members. The survey remained open for three weeks in November 2024, ultimately yielding 196 responses.

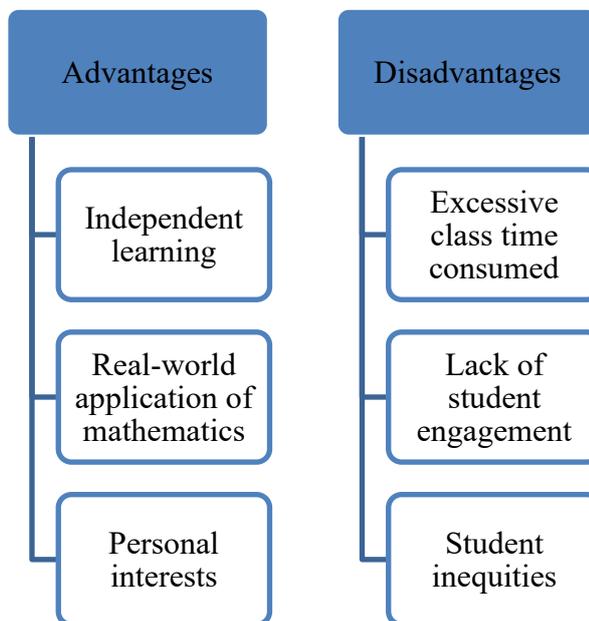
Qualitative data from open-ended responses were transcribed into Microsoft Word and analysed using an inductive thematic analysis approach guided by Braun and Clarke (2022). The process was iterative and reflexive, involving repeated coding, theme development, and cross-checking. While the first author conducted initial coding, both authors collaborated to refine and finalise the themes, sometimes merging categories or introducing sub-themes.

Findings

The preliminary findings presented in this paper will highlight the key themes that were generated from the data, outlining three advantages and three disadvantages of CBAs as reported by the responding teachers. These themes are discussed in order of frequency of occurrence within the data (see Figure 1).

Figure 1

Mathematics Teachers' Perspectives on the Advantages and Disadvantages of CBAs



From a positive perspective, a commonly reported advantage of CBAs is their role in fostering ‘independent learning’. This emerged as the most frequently mentioned positive theme across the responses. Numerous teachers observed that CBAs promote student autonomy, encouraging learners to take charge of their own work and develop the ability to work independently. One respondent emphasised this point, stating: “Students are encouraged to take responsibility for their project, which builds independence” (T68). Another widely acknowledged benefit relates to the ‘real-world application of mathematics’. Many participants pointed out that CBAs help bridge the gap between classroom learning and everyday life. For instance, one teacher remarked: “Demonstrating application of maths is positive” (T147), while another noted: “It does connect to real life, and I do think that CBA 2 is worth doing as stats are so relevant in life” (T171). The opportunity for students to pursue ‘personal interests’ was also a recurring theme. Teachers valued how CBAs allow students to explore topics they find meaningful, which can lead to higher engagement and more personalised learning experiences. As one teacher described: “They give students a greater opportunity to shape their own learning and investigate things that interest them” (T75). While these three themes showcased some of the benefits in relation to CBAs, it must be noted that the data from the open-ended question on the advantages also highlighted widespread concerns. Indeed, of the 120 teachers who answered the question on advantages, only 46 provided an exclusively positive response. The others either mentioned both advantages and disadvantages or gave solely negative responses. Such negatives are further emphasised in the responses to the question on the disadvantages of CBAs in the JCMS

Among the challenges associated with CBAs, the most frequently cited by teachers was the issue of ‘excessive class time consumed’. Many participants expressed concern that the time devoted to CBAs significantly interferes with syllabus coverage. One teacher emphasized this impact by stating: “Six weeks (nearly 10% of 2nd and 3rd year) of teaching

time lost, yet course content was not reduced” (T107). Several teachers viewed the time requirement for CBAs as disproportionate to their educational value, particularly since students are expected to complete CBAs in numerous subjects. As a result, teachers reported feeling the strain of meeting both curriculum demands and CBA requirements, which often led to hurried instruction and learning gaps. As one respondent noted: “Fitting CBAs into the timetable forces us to rush through other core topics, leaving gaps in students' understanding” (T135). Another recurring concern was the ‘lack of student engagement’. Many teachers indicated that since CBAs do not contribute to students’ final grades, they are often not taken seriously by learners. One teacher observed: “Does not count for a grade, so students don’t care about it.” (T15). Others described how students tend to perceive CBAs as obligatory rather than meaningful, with one remarking: “The CBAs are viewed as a chore that needs to be completed by students” (T94). Teachers also expressed that CBAs exacerbate ‘student inequities’. This broader concern emerged through two interrelated sub-themes, namely, ‘disparities in access to resources’ and ‘differences in student ability’. In terms of resources, respondents highlighted inequities in access to technology and varying levels of support from home and school. For example, one teacher commented: “Different kids have different resources to avail of, and different schools have different levels of resources. We’ve 15 laptops for 120 kids doing a 2nd-year maths CBA and we are a no-phone campus now” (T139). These disparities often result in an uneven experience of CBAs across different student groups. Regarding ability, many teachers felt that CBAs tend to favour high-achieving students while posing significant challenges for others. Lower-achieving students, in particular, were reported to find CBAs stressful and difficult to complete independently. As one teacher explained: “Stronger students tend to enjoy CBAs, but weaker students get incredibly stressed or minimally engage” (T3).

Conclusion

To sum up, this study found that although teachers expressed a variety of opinions on the implementation of CBAs as part of Junior Cycle mathematics in Ireland, their overall sentiment was predominantly negative. However, a number of benefits were recognised, many of which align with the original goals of introducing school-based assessment. Specifically, teachers noted that CBAs support the development of ‘independent learning’, helping students see the ‘real-world application of mathematics’, and enabling them to explore areas aligned with their ‘personal interests’. These themes align with existing research on the advantages of formative assessment, underscoring its capacity to improve student learning, boost motivation, and foster greater student ownership over their educational progress (Black and Wiliam, 1998).

Despite the positives, the overall responses from participants in the study reflected a more negative perception of the CBAs in general. The most commonly reported challenge related to CBAs was their tendency to take up a significant amount of class time, which hinders teachers from sufficiently covering the necessary syllabus content. In 2022, the Irish Mathematics Teachers Association (IMTA) conducted a survey among its members and students to gather general feedback on the JCMS. Regarding time constraints, 67% of the teachers surveyed indicated that they did not have sufficient time to cover the course content

(IMTA, 2022). This is a particular issue in the subject of mathematics where concern has long been expressed regarding the workload and timeframe of the mathematics curriculum (Prendergast & Treacy, 2018).

The findings point to a multifaceted and evolving situation that warrants continued examination and critical reflection. From the beginning, assessment reform, and particularly CBAs, has remained the central point of debate within the *Framework for Junior Cycle* (MacPhail et al., 2018). This study confirms that they remain a source of concern for teachers. Although the JCMS and its associated CBAs were introduced in 2018—seven years ago—the impact of the COVID-19 pandemic has undeniably disrupted their rollout. Nevertheless, the results of this research indicate that significant apprehensions persist among educators. Byrne and Prendergast (2019) observed that while concerns around curriculum reform may lessen over time, they must be addressed through sustained support and possible refinements to the reform.

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The impact of mathematics teacher shortages in Irish secondary schools: Perspectives from consecutive ITE graduates

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The significance of mathematics in the Irish secondary school curriculum is undisputed. However, the persistent challenge of teacher shortages in the subject has garnered substantial attention over the past fifteen years. There has been a change in the numbers entering the mathematics teaching profession, with the number of newly qualified mathematics teachers from the consecutive initial teacher education route particularly affected. A shortage of qualified mathematics teachers has serious implications for effective teaching and student learning outcomes in classrooms. This paper investigates perspectives on the repercussions of ongoing teacher shortages in Irish secondary schools, drawing insights from semi-structured interviews with eight mathematics teachers who graduated from consecutive initial teacher education (ITE) programmes in recent years. We employed thematic analysis and found that their considerations of the main impacts centred around the prevalence of 'out-of-field' teachers, larger class sizes, a lack of continuity in teaching, and ensuing gaps in students' knowledge. Implications for practice are discussed.

Keywords: Teacher shortages, mathematics, secondary level, impact.

Background to the Study

Teacher shortages are having a profound effect on education systems globally (UNESCO, 2023). To meet the target of universal primary and secondary education by 2030, as outlined in Sustainable Development Goal 4, an estimated 44 million additional teachers are needed (UNESCO, 2023). Countries such as Australia, Canada, the UK, the USA, and Ireland are grappling with this shortage, resulting in unfilled teaching positions that affect class sizes and limit subject availability - issues that disproportionately affect schools in lower socio-economic communities (Black, 2017; Blackmore et al., 2023; Mills et al., 2025). A common response to these shortages has been the deployment of unqualified or 'out-of-field' teachers. In Ireland, for instance, research by Ní Ríordáin and Hannigan (2011) found that 48% of those teaching mathematics at secondary level were not specifically qualified to do so. Although this figure had decreased to 25% by 2018 (Goos et al., 2023), mathematics has remained one of the focus areas in the Department of Education and Skills 2019 Teacher Supply Action Plan.

While the causes of global teacher shortages are varied, several recurring factors emerge across different national contexts. Studies have highlighted that challenges related to teacher pay, limited career advancement opportunities, the perceived status of the profession, and insufficient access to meaningful professional development have all negatively influenced teacher recruitment and retention (Blackmore et al., 2023; Darling-Hammond, 2017; UNESCO, 2023). In addition, financial constraints often deter prospective candidates from

enrolling in initial teacher education programmes, particularly those pursuing postgraduate routes (Mills et al., 2025; White et al., 2024).

In Ireland routes into the secondary teaching profession have traditionally followed two main pathways: (1) concurrent via an undergraduate programme and (2) consecutive via a postgraduate qualification following a degree in a specific subject area(s). The concurrent route typically consists of a four-year Level 8 degree that integrates subject-specific content with education and pedagogy. In contrast, the consecutive route delivers professional training in pedagogy and teaching after candidates have already completed an academic degree in a relevant subject area. Since 2014 this consecutive route has transitioned from a one-year Postgraduate Diploma in Education (PDE) to a two-year Professional Master of Education (PME) programme. While this reform has undoubtedly strengthened teacher preparation by deepening pedagogical engagement, it has also created new barriers. Recent graduates navigating this route often face financial burdens, reduced income opportunities during study, and a lack of systemic supports, all of which contribute to declining enrolment and retention (Ní Ríordáin et al., 2025). Indeed, graduate numbers from this route have fallen nationally by more than a third since the two-year programme was introduced (Sahlberg, 2019). This fall in graduate numbers has been felt across all curricula areas, but the impact is more pronounced in subjects such as mathematics which have traditionally experienced teacher supply issues.

Considering the well-established link between effective teaching and student learning (Prendergast & O'Donoghue, 2014), the lack of supply of highly qualified mathematics teachers has serious implications. Harford and Fleming (2023) critique the Irish state for not effectively tackling the worsening shortage of qualified teachers, especially in light of rising demand and the growing complexity of the education system. The aim of this paper is to investigate perspectives on the repercussions of ongoing mathematics teacher shortages in Irish secondary schools, and to address the following research question: According to consecutive ITE graduates, what impact do mathematics teacher shortages have on teaching and learning in secondary mathematics classrooms?

The Study

While this paper focuses specifically on the impact of mathematics teacher shortages, it is part of a larger study which examines why fewer mathematics undergraduates are choosing to pursue secondary teaching careers in Ireland through the consecutive route. As part of the overall research, we conducted semi-structured interviews with eight mathematics teachers who had recently graduated from two of the six universities in Ireland offering the consecutive ITE route. These participants were recruited using a purposive sampling strategy. There were four graduates from each of the two universities selected and the sample included teachers with varying undergraduate backgrounds in mathematics and other teaching subjects. Participants identified as male ($n = 5$) and female ($n = 3$) and had graduated from the PME programme at each university between 2016 and 2024.

Semi-structured, one-to-one, interviews were conducted by the lead author between January and March 2025. At the beginning of each interview the author explained the purpose of the study and the interview structure. With specific reference to this paper, the intention was to probe the lived experiences and perspectives of teachers regarding the impact of

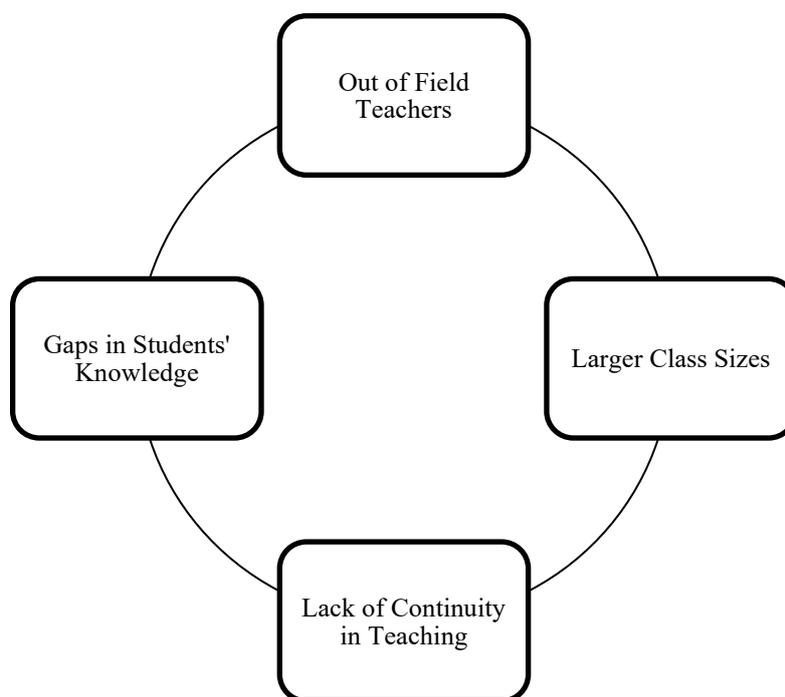
mathematics teacher shortages in their schools. All eight interviews took place online via Microsoft Teams, which offered the benefit of allowing participants to engage in a comfortable and convenient setting. After providing informed consent, each semi-structured interview lasted approximately 30 minutes. The recordings were transcribed word-for-word using online transcription software and then cleaned and edited by the authors. Braun and Clarke's (2022) thematic analysis framework was used to analyse the transcribed data. An inductive "bottom up" orientation guided the analysis and was used to identify themes generated through the views of mathematics teachers' in this study. Implementing Braun and Clarke's (2022) six phases in the process of thematic analysis provided a framework for data engagement, coding and theme development. It was a flexible and recursive process, with repeated movement back and forth as codes were identified and initial themes were generated. For example, codes such as 'teacher turnover', 'multiple teachers' and 'disruptions in learning' were categorised into the theme 'Lack of Continuity'.

Findings

In terms of impacting teaching and learning, there were four main themes generated from the analysis (see Figure 1). These themes are discussed in order of frequency of occurrence within the data.

Figure 1

Impact of Mathematics Teacher Shortages on Teaching and Learning



The main impact of mathematics teacher shortages identified in the data was the assignment of 'out of field teachers' who are not specifically qualified to teach the subject. This trend was noted as a significant issue, with several participants pointing out that teachers with backgrounds in other subjects, such as business or science, are often asked to teach mathematics despite lacking the specialised content and pedagogical knowledge required.

Barry: I know it has been well highlighted down through the years but there continues to be a problem with teachers being assigned to teach maths classes even though it might not be one of their qualified subjects. And it is an issue, like for someone to try and go in and teach maths, their knowledge of the subject is going to be limited to what they have done themselves in school, rather than having a degree in the subject and learning about specific ways to teach it and help students learn, and what works and what doesn't.

Sean: It's often that case that teachers from other subjects, for example business, will be asked to fill in and teach first year maths. But they wouldn't have the experience of learning how to teach maths. I've no doubt that they have the maths ability, but they haven't been trained in how to use the most effective approaches to teach the subject. Just like I wouldn't know the best way to go about teaching accounting.

Another impact of mathematics teacher shortages was 'larger class sizes'. Several respondents noted that when there are not enough qualified mathematics teachers, schools are often forced to combine or reconfigure classes, leading to larger student groups. This increase in class size results in less individualised attention for students, which, in turn, affects the quality of instruction.

Mary: We end up having to split classes, like as in distribute students of the teachers who are out amongst the other classes, which is better because at least they're in a classroom. But that has an impact on a whole year group then you know in terms of the class sizes.

Paul: What you're looking at is things like bigger class sizes, to try and split the class cohort across all the available maths teachers. You know schools are good at keeping the numbers manageable, but they can only do so much with what they have. So, a lot of times you can lose that kind of individualized student attention maybe to a certain extent, because of having to take on bigger class sizes.

The third issue stemming from the shortage of qualified mathematics teachers was the 'lack of continuity in teaching'. When teachers leave for reasons such as maternity leave, illness, or other absences, finding qualified replacements becomes difficult, often resulting in multiple teachers being assigned to the same class over the course of a term or school year. This lack of consistency disrupts the students' learning process and can be especially problematic for students at critical stages of their mathematical development, where foundational concepts are being built. Colm and Julie highlighted this issue of lack of continuity.

Colm: There is a real issue with a lack of continuity of teachers for class groups in maths through a junior or senior cycle. It does have an effect. You can minimise it as much as possible in a school by having clear schemes of works you know and effective handovers..... but you know

a class might have a teacher in 1st year, and then a different teacher in 2nd year and it could even happen again in the third year.

Julie: I think a lot of chopping and changing between teachers really has a huge effect on the students.

The fourth and final theme generated from the data highlighted how mathematics teacher shortages often result in ‘gaps in students’ knowledge’. This directly links to the consequences of a lack of continuity and having ‘out of field’ teachers, as students’ foundational skills are not developed adequately, which negatively affects their ability to handle more advanced topics.

Edel: And then as a maths teacher in 3rd year and senior cycle, you are inheriting a lot of students with problems in maths due to a lack of understanding of the basics.

Colm: A junior cycle class will have had three different teachers for their three years and just that discontinuity at a fundamental or like a critical point of their development, you know a step up from primary school and then the introduction of new concepts like algebra and things like that. And to have that discontinuity in there can just create issues that didn't need to be there and can actually take a while to fix.

Conclusion

The analysis of the data highlights several critical consequences of the shortage of qualified mathematics teachers. The prevalence of 'out-of-field' teachers, larger class sizes, and a lack of continuity in teaching leads to a diminished learning experience for students. These issues contribute to gaps in students' knowledge, hindering their understanding of fundamental concepts, and negatively affecting their overall academic performance. Given the importance of mathematics as a cornerstone subject within the Irish secondary school curriculum, addressing these challenges is crucial to ensuring that students are well-prepared for future academic pursuits and careers, particularly in STEM fields. While the transition from a one-year PDE to a two-year PME programme in Ireland has brought substantial benefits (Ní Ríordáin et al., 2025), it has also coincided with a decline in the number of mathematics teachers graduating through the consecutive route. To tackle this issue, it may be helpful to examine countries with well-established teacher recruitment and retention strategies—such as Finland, where teacher preparation is fully funded by the government, and trainees receive a salary during their studies (Darling-Hammond, 2017). Countries that have successfully addressed teacher supply challenges explicitly recognise the critical role of education in a prosperous society and implement coherent policies to recruit, develop, and retain skilled educators in order to ensure that every school is staffed by effective teachers (Darling-Hammond, 2017). In this context, the issue of mathematics teacher supply must be addressed in a focused and coordinated manner to mitigate the detrimental impacts identified in this study and to ensure that all students receive the high-quality mathematics education to which they are entitled. Crucially, efforts to recruit new mathematics teachers into the profession must not be undermined by the current challenges faced in schools (Harford &

Fleming, 2023); rather, these efforts should be accompanied by clear strategies to support early-career teachers and foster a positive and sustainable professional environment.

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‘Playful and engaging’: Redeveloping the Drumcondra tests of early numeracy

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The Drumcondra Tests of Early Numeracy (DTEN) are undergoing redevelopment and embracing the vision outlined in the 2023 Primary Mathematics Curriculum for a more “playful and engaging” approach to primary mathematics. The revised DTEN seeks to embody the ‘spirit’ of this curriculum while producing useful and high-quality assessments for Senior Infants and First Class children. The revised DTEN involves an expansion of curriculum coverage; an Irish language version; parallel forms; ‘scenario’ context items; child-friendly materials; and improved materials for the diagnostic assessment. Teacher questionnaires and child focus groups have also been conducted. The pilot of the screening booklet has concluded among First Class children and analysis is underway. This paper will outline the rationale for redeveloping the DTEN, the impact of the 2023 curriculum on its design, and an exploration of its novel features.

Keywords: Early numeracy, assessment, play, screening

Redeveloping the Drumcondra Tests of Early Numeracy

The Drumcondra Tests of Early Numeracy (screening and diagnostic) are well-established resources for identifying children in Senior Infants and First Class who are at risk of difficulties in mathematics. Such children are identified by a predetermined cut-off point in the raw score (number of items correct) on the screening test, a group-administered activity booklet. These children then complete the individually-administered diagnostic test, which covers similar content in more detail so that particular areas in which the child may need support can be determined.

The first DTEN (Screening and Diagnostic) was validated in 2008-2009. Redevelopment of the DTEN began in 2023, 15 years after its initial validation. These types of assessments tend to become easier over time as their content and format become more familiar, and they should be re-validated periodically to ensure the cut-off points continue to identify children at risk of difficulties. Therefore, the most basic aim of the new DTEN is to reflect up-to-date achievement levels. Further, the educational landscape in Ireland has changed considerably since the DTEN validation. Changes relevant to the redevelopment of the DTEN assessments include, but are not limited to: increases in achievement levels in mathematics of primary school pupils; changes to early childhood education resulting in children being older, on average, when they start school; the development and introduction of a new *Primary Mathematics Curriculum* (PMC) in September 2023; increased recognition of the need for more assessment resources for Irish-medium schools; and demographic shifts in the population of Ireland.

Teacher Feedback

In the initial stages of redevelopment, feedback was sought from teachers regarding the existing assessments (screening and diagnostic), including how they are used in the real world, and any recommendations for the new versions. A questionnaire was circulated in May 2023 to primary schools that had purchased or administered the DTEN in recent years and 109 teachers completed at least half of the questionnaire (238 started the online questionnaire).

Comments were very positive overall but highlighted a few areas to be addressed. For example, almost as many teachers reported that the tests were too easy to provide useful information as those who disagreed. A sizeable minority of respondents were concerned that the tasks were not engaging enough (S: 25%, D: 33%) and that the pupils struggled to concentrate (S: 24%, D: 38%). Worryingly, 28% agreed that pupils were nervous or intimidated by taking the tests. Teachers also showed concern for children with hearing difficulties and speakers of English as an additional language. Responses also indicated that operating in a context of staff and room shortages presented significant challenges for using these assessments.

Based on feedback from teachers, the redeveloped DTEN aims to improve the test experience for the children and ensure the length of the assessment is age-appropriate. Teachers also requested a parallel booklet for the screening test to reduce opportunities for copying, therefore increasing the validity of results, and to allow teachers to re-administer the test if needed. Several teachers explicitly requested that the assessments be made available in Irish, and for the first time, the revised DTEN will provide early numeracy assessments that have had input from Irish language experts and been piloted and validated in Irish.

The 2023 Primary Mathematics Curriculum

The revised *Primary Mathematics Curriculum* (PMC) (DoEb, 2023) was published after an extended development and consultation process and in the context of the earlier publication of the broader *Primary Curriculum Framework* (PCF), which was the first of its kind (DoEa, 2023). Notably, the PCF includes ‘being mathematical’ as one of the seven key competencies for primary school learners. This competency involves children using logic and mathematical thinking to solve a range of problems both inside and outside the classroom, and aligns with the PCF, the *Framework for Junior Cycle*, and Aistear (*Early Childhood Curriculum Framework*). The link with Aistear in the PCF is further strengthened by the focus on the importance of play and playfulness in primary education (NCCA, 2009).

The PMC (DoEb, 2023) is based on the principles that every child is capable of engaging with mathematical ideas and can develop over time. It describes mathematics as a tool for understanding the world and argues that creative problem-solving in everyday contexts is a key component of developing critical mathematical thinking. Since mathematics is a human and social phenomenon, the school environment should promote collaboration and communication, an “essential” pedagogical practice (DoE, 2023b, p. 26). Nic Mhuirí (2011) noted the discourse present among mathematicians involves making conjectures, examining and justifying or “mathematical thinking”, which should be brought into the classroom as

thinking aloud or *Maths Talk*. An egalitarian environment, where children feel safe to express their thinking and ideas, should be encouraged.

The PMC departs from the prescriptive content objectives of its predecessor, the 1999 *Primary School Mathematics Curriculum* (PSMC), and instead presents broad learning outcomes that can be tailored towards the children and allow for greater teacher autonomy. Mathematical Concepts, which are the *Big Ideas* (Charles, 2005) underpinning the Learning Outcomes, and Progression Continua or detailed suggested learning pathways, are provided in the toolkit.

The main curricular changes are primarily focused on the ‘how’ of primary school maths, rather than the ‘what’. There is an increased emphasis on playful, engaging and meaningful mathematics in the PMC and these priorities have been used as a guiding principle in developing the pilot. The revised DTEN embraces the ‘spirit’ of the new PMC by using child-friendly illustrations, relevant real-world contexts, and a playful approach to administering the “activity booklets.”

What is measured by the new DTEN?

In terms of curriculum matching, the DTEN was redeveloped with both the 1999 PSMC and the 2023 PMC in mind. This is because the timeline for the roll-out of the PMC intersects with the validation process for the revised DTEN, as the assessments will be validated before the curriculum has been fully embedded. As a result, the DTEN does not contain content areas that are not in the PSMC, but the items were developed to embody the playfulness of the PMC, with a particular emphasis on creative, engaging and meaningful mathematics.

Like the 2008/2009 DTEN, the revised assessment has a focus on the *Number* strand. It has been argued that “number provides the foundation of mathematics in primary school” (Philpot et al., 2021, p.8). Clements and Sarama also state that “in early childhood, number and operations is arguably the most important area of mathematics learning” (2007, p.466). The pilot therefore retains a strong coverage of the *Number* strand unit (*Numeration & counting, sets & operations, etc.*). However, some experts have raised that an overfocus on *Number* in screening and diagnostic tests for young children may lead to the exclusion of other important domains of mathematics (Dunphy et al., 2014). As a result, the revised DTEN seeks to slightly expand the curricular coverage, while also ensuring that the assessment is not too long for this young age group.

Early pattern work has been highlighted as a key constituent of mathematical development, given its role in the formulation of algebraic thinking (Clements & Sarama, 2007). The revised DTEN expands coverage of the *Algebra* strand by using patterns with a range of characteristics and more engaging contexts to add playfulness and explore the connections between patterns inside and outside the classroom. The original DTEN included elements of the *Shape and Space* strand as the context for other skills, such as classifying simple 2D shapes or extending shape patterns, while the pilot tests included items more explicitly targeting *Shape and Space* content. Similarly, items involving comparing and re-ordering objects based on physical characteristics (*Measuring*) and classifying objects and

sets according to specific attributes (*Data*) now feature in both the screening and diagnostic, the latter of which uses illustrated cards and manipulatives. Lastly, an informal understanding of *Time* has been piloted in the diagnostic for the first time.

How are these components measured in the new DTEN?

The screening test booklet features a range of item types, including short-answer, open-ended items with numeric answers, drawing shapes, selecting part of an image, or matching related objects. Multiple choice items were used with numbers or images as answer options, and distractors (incorrect answer options), where possible, were based on common errors or misconceptions.

Given its format as a one-to-one interview, the diagnostic test allows for more flexibility in item types – from verbal, to interacting with a display book, to placing cards, cubes or objects. This more tactile experience in the assessment supports the child’s engagement and promotes enjoyment of the tasks (Baroody, 2017).

It is important to note that the purpose of these tests is to identify children at risk of struggling with numeracy and clarify the content areas in which they may be experiencing difficulties. The tests are therefore designed for children who may benefit from additional learning supports or have weaknesses in specific areas of numeracy that can be addressed by the class teacher.

Mathematics items in context

As has already been discussed, the PMC has a strong focus on creative, playful and engaging mathematics, and the DTEN seeks to embody the ‘spirit’ of the new curriculum. In a consultation with children commissioned by the NCCA as part of the development of the PMC (Leavy et al., 2023), the authors found that the children themselves highlighted the importance of meaningful and engaging mathematics with rich real-world contexts. A novel approach was also piloted in which three items relating to the same large image were presented on a double-page spread. These items were mixed in terms of the mathematical content area but connected by a contextual theme, e.g. a nature scene. One of these “scenario” pages was included in each of the sets of booklets.

Inclusive and child-friendly presentation

To further promote an engaging and enjoyable child experience, a professional illustrator was engaged to provide clear and appealing images for the test materials. The illustrations were commissioned to represent the diversity of children in Irish classrooms, such as including images of children with a range of skin colours, avoiding gender stereotypes, and featuring children using hearing aids or wheelchairs. Crucially, the ability to distinguish between colours was avoided in order to be inclusive for colour-blind children.

As with the earlier DTEN, instructions for the screening booklets were read aloud to pupils in full and a detailed script was provided for the test administrator. This helped to minimise the linguistic load and therefore the confounding effect of linguistic competency, since this is a numeracy assessment. It also improved accessibility for children who were not taking the test in their native language, as they received both written and oral instructions.

Parallel Forms

Parallel forms of the screening booklet were piloted. This means that one set of oral instructions applies to multiple forms of the booklet so that they can be administered simultaneously in the same classroom. The different versions of the booklet vary in terms of the numbers, images and positions of the correct answers. This has already been implemented successfully in the New Drumcondra Primary Mathematics Test (DPMT) at Levels 1 and 2 (First and Second class) but represents a new approach for Senior Infants.

To trial a larger number of items, the pilot involved two sets of booklets (schools were assigned either X or Y) with three forms in each set (A, B, C), in English or in Irish translation. Both sets measured the same content areas but contained different items and contexts. These six different booklets (per language) at the pilot stage will be consolidated into two booklets (per language) for validation and publication.

Concrete materials

As noted earlier, the use of concrete materials in the DTEN-D allows for the assessment of skills and concepts outside those that can be addressed by a paper-and-pencil test. However, it is also in keeping with the vision of the PMC, as well as the recommendations of the NCCA consultation with children (Leavy et al., 2023). In the latter, children expressed a clear preference for interactive, hands-on experiences of mathematics. As a result, the DTEN-D seeks to provide an interesting, meaningful and enjoyable test-taking experience for all children who complete it.

Data Collection

Schools were stratified by medium of instruction, DEIS status, size and Gaeltacht location where relevant. 100 schools were randomly selected and their First Class classes were invited to participate. Information letters and parental consent withdrawal forms were distributed by schools in advance and schools were advised to excuse from testing children who may become distressed or would not be able to meaningfully engage with the tests (e.g. due to a disability or insufficient level in the testing language). However, we encouraged schools to otherwise include as many children as possible, including those with special educational needs.

Teachers were provided with comprehensive administration manuals containing general guidance and a detailed script and the research team were available for support. We recommended that screening booklets be administered to groups of 8 children either with a break at the midway point or over two days, although we encouraged teachers to use their professional discretion and knowledge of their pupils to take extra breaks if needed. To capture the pupil perspective, eight focus groups of four children were conducted shortly after they had completed the tests. Due to the young age of the participants, they were asked at some points to place stickers on spectra from, for example, from fun to boring. Opt-in consent was obtained from parents. All materials were returned to the ERC and open format items were coded in-house before the booklets were sent for data entry. SPSS and Iteman will be used for analysis.

Based on the scores in the screening pilot, preliminary cut points were used to identify the children most suitable for the diagnostic pilot. A separate information letter and consent form was sent to parents. Diagnostic tests were administered one-to-one by class teachers or SETs in the school and the interview was divided into three parts to avoid pupil fatigue. Due to delays beyond the control of the research team, materials for the diagnostic tests were sent to schools in May, resulting in a slightly lower pupil response rate than anticipated due to the pressure in schools at this time.

Results

78 schools took part in the screening pilot, comprised of 115 classes and 2326 children, and a few children in 75 of these schools went on to pilot the diagnostic assessment. Teacher questionnaires have yielded positive responses to the new test materials and format of the tests, saying that the children enjoyed the activities and that the tests appeared to provide useful information. Teachers have also provided feedback about specific items and made recommendations as to how some items can be improved.

Analysis of the First Class scores has begun and it seems that the difficulties of Set X and Y are acceptable for a pilot screening test (X: 75.1%, n=1040, Y: 81.2%, n=1266), although it is worth noting that this large sample was not nationally representative and has not been weighted. Initial analysis suggests that response patterns from children in Scoileanna Lán Ghaeilge are different from those in English medium schools. Analysis of individual items (the primary purpose of the pilot) is forthcoming and more findings will be available for presentation in the autumn.

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More than a routine: How talk moves support mathematical discourse during a number talk

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This paper investigates how teacher talk moves during a Number Talk (NT) support students' participation in mathematical discourse. Drawing on Gee's (2005, 2011) conception of discourse and Sfard's (2007) framework for mathematical discourse, we analyse a transcribed NT in a Grade 5 classroom using qualitative discourse analysis. Our findings identify three overlapping discursive functions of talk moves: making student thinking significant, enacting mathematical practices, and building connections between ideas. We show how specific talk moves—prompting elaboration, modeling strategies, and verifying understanding—work simultaneously across these functions to position students as doers of mathematics in sharing, reasoning and connecting ideas. We highlight the potential efficacy of NTs as generative spaces for doing mathematics when teachers use talk moves intentionally to foster discourse.

Keywords: Number talks, discourse, talk moves

Introduction

Although Number Talks (NTs) have gained popularity in elementary mathematics classrooms as a way to promote mental math and student discourse, there remains a lack of research examining their efficacy. Matney et al. (2020) describe this gap as a "black hole" (p. 247) in the literature—a space where practice has outpaced empirical investigation. This absence of evidence-based clarity raises important questions about how, and how well, NTs function in classrooms. Additionally, there is considerable variability in how teachers implement NTs, often adapting the format to suit their own teaching style or classroom needs. Despite this variability, teachers often employ talk moves—such as revoicing, waiting, and prompting—to support student participation. These strategic moves contribute to a dynamic discourse environment where mathematical meanings can emerge. Yet, without analyzing the nature of these discursive interactions, the efficacy of NTs remains uncertain. To understand more fully what happens in these brief, but potentially rich, classroom moments, we recorded and transcribed a NT to examine the discourse patterns present. The purpose of this paper is to share a NT from an elementary classroom that illustrates the teacher's discursive moves. Our research question asked: *In what ways can the teacher's use of talk moves during a Number Talk support students' participation in mathematical discourse?*

Promoting Mathematical Discourse through Number Talks

NTs are short, 10–12-minute, structured discussions designed to promote mental math, mathematical reasoning, and student discourse (Parrish, 2021). They focus on how students solve an arithmetic question mentally (e.g., 8×6) and share their strategies with the class. NTs provide a platform for students to articulate their thinking, listen to others, and build on collective understanding (Woods, 2022). Teachers facilitate these sessions using strategic talk moves—such as revoicing, asking orienting or evaluative questions, and encouraging peer-to-peer clarification—to develop a discourse-rich environment (Joswick et al., 2025). These

moves not only help students become better explainers and listeners, but also shift responsibility for learning toward students, encouraging participation and a sense of ownership over mathematical ideas (Raymond & Campbell, 2024). The National Council for Curriculum and Assessment's curriculum objectives support NTs because they promote *maths talk* as a key pedagogical practice (NCCA, 2023).

A growing body of research suggests that when implemented purposefully, NTs foster meaningful mathematical discourse. The teacher's talk moves create space for students to engage in reasoning, critique strategies, and learn from mathematical errors—features aligned with inquiry-based and student-centered learning (Pak et al., 2025). Studies show that talk moves play a pivotal role in this process, influencing the depth of student thinking and the development of strategy use (Murata et al., 2017). However, findings also caution that many NTs remain at the level of eliciting strategies rather than probing to make mathematical connections, pointing to the need for professional development that enhances teachers' questioning practices (Joswick et al., 2025). Despite these variations, the routine consistently enables teachers—especially beginning ones—to rehearse key practices such as scaffolding student thinking, monitoring discourse, and leveraging errors as learning moments. These findings emphasize the promise of NTs to provide discursive spaces where the teacher and students think about mathematical ideas in relation to others and use language to communicate understandings.

Theoretical Framework

Our theoretical framework draws on Gee's (2005, 2011) conception of discourse, building on Sfard's (2007) work, which provides a mathematics-specific interpretation of Gee's broader theory. From Gee's perspective, discourse (lowercase "d") encompasses language-in-use and ways of being, acting, valuing, and interacting that align with particular social and cultural practices, while Discourse (capital "D") highlights how language is intertwined with identity, power, and broader societal structures. Sfard's conception of mathematical discourse aligns closely with three of Gee's building tasks of language: making things significant, enacting practices, and connecting. Specifically, Sfard's emphasis on the use of mathematical words and visual mediators (e.g., diagrams and symbols) parallels Gee's notion of making things significant, as language highlights and renders certain ideas meaningful within the practice. Sfard's concept of routines—recurrent, recognizable patterns of mathematical action—mirrors Gee's idea of enacting practices through discourse. Finally, Sfard's notion of narratives, through endorsing or rejecting mathematical claims, corresponds to Gee's building task of constructing connections among ideas, people, and activities. By grounding our use of Gee's conception of discourse in Sfard's detailed account of mathematical communication, we analyze our NT transcript in terms of how teacher and students develop and negotiate specific features of mathematical discourse.

Methodology

Our study employed a qualitative discourse analysis methodology to examine how language in classrooms constructs meaning, shapes social interactions, and supports the development of mathematical reasoning (Herbel-Eisenmann et al., 2011). This methodology

is well suited to our study because it emphasizes the functions of language—specifically, how talk makes certain ideas significant, enacts socially recognizable practices, and builds connections among ideas and participants. By focusing on the form and function of language during classroom interactions, qualitative discourse analysis allowed us to investigate how teacher and student talk during a NT contributed to students' engagement with mathematical discourse. In the context of mathematics education, we drew on Herbel-Eisenmann et al.'s (2013) research on teacher talk moves and Parrish's (2021) design principles for NTs to guide our analysis. We were interested in how combinations of talk moves could work together to support students' participation in mathematical thinking and communication. Our analytic approach was iterative and thematic, guided both by theory and by emerging data patterns.

Methods

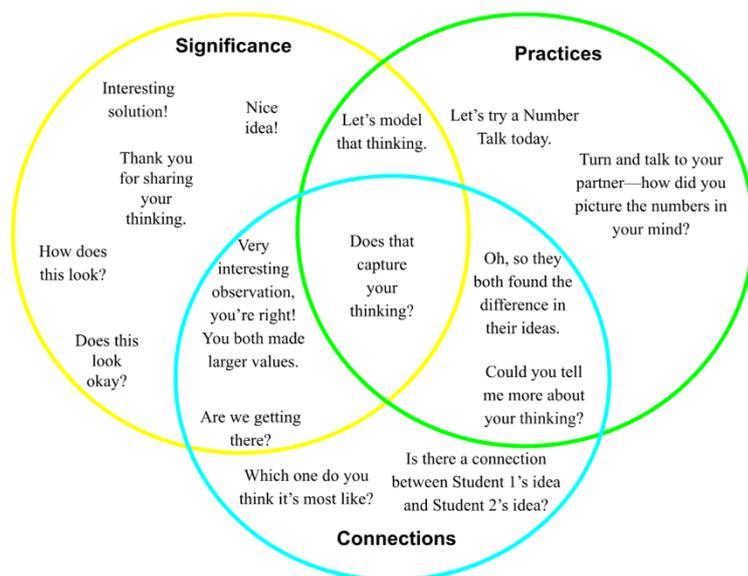
We undertook the study in a Grade 5 classroom in a Regional Centre for Education (RCE) on the east coast of Canada. Participants included one classroom teacher, 24 students, and two mathematics mentors who provide ongoing support to the RCE's mathematics program. We obtained ethics approval from the RCE and the University prior to data collection. Data sources included audio and video recordings of NT sessions, along with line-by-line annotated transcripts of classroom interactions. Following each NT, the research team, teacher, and mentors reviewed the recordings independently and collaboratively to identify and discuss the teacher's use of talk moves. To triangulate our findings through multiple review rounds and to increase inter-rater reliability, we each analyzed, verified, and discussed how we viewed the classroom discourse unfolding, with a particular focus on how talk moves supported students into mathematical reasoning and communication. While research suggests ambiguity regarding the efficacy of NTs (Matney et al., 2020), in what follows, we detail our analysis of a specific NT in which we asked students, "What is 8×6 ?" to explore its efficacy in terms of how the teacher used talk moves to orchestrate the NT and support students in the discourse of mathematics.

Findings

Our analysis identifies three interrelated ways in which discourse shaped participation in the NT: making things significant, enacting practices, and building connections. While each category reflects distinct functions of language—drawing respectively from Gee's (2005, 2011) and Sfard's (2007) frameworks—they also overlap in practice (see Figure 1). For example, talk moves that highlight a student's idea (significance) often occur within familiar routines (practices) and are used to draw links among ideas (connections). What follows is our analysis of each category, with illustrative excerpts from the NT transcript.

Figure 1

Overlapping discourse purposes of teacher talk moves



Making Mathematical Ideas Significant

The teacher made student ideas publicly visible and meaningful through both language and representation. For example, when one student explained their strategy, the teacher responded: *“Interesting solution... we’re just going to model that thinking... is that okay?”* This verbal and visual validation positioned the student’s idea as worthy of discussion. While making this student’s strategy visible and asking, *“How does this look? Are we getting there?”* the teacher affirmed the student’s contribution by using a visual mediator (e.g., board representations), which made the strategy visible for public reasoning. This is illustrated in Figure 2, where Maya’s and Leo’s strategies have been recorded side by side on the board. Maya’s strategy used compensation by starting with 10×6 and subtracting 12 (2×6), while Leo’s strategy began with 8×8 and subtracted 16 (2×8).

Figure 2

Maya and Leo’s Strategies for 8×6 as Represented on the Board

Maya’s Strategy	Leo’s Strategy

Enacting Practices of Number Talks

The NT was enacted through repeated, recognizable moves that constituted the activity itself. The teacher’s phrases like *“Turn and talk to your partner”* appeared multiple times, each with a different discursive function: *“How did you picture the numbers in your*

mind?” “*What’s another way to solve it?*” “*What’s the same or different between Maya’s idea and Leo’s?*” Opportunities for students to talk with a peer marked the NT as a space for collaborative reasoning and comparison. The teacher also used prompts like “*Could you tell me more about your thinking?*” and “*Does that capture your thinking?*” to elicit reasoning and confirm understanding — characteristics of mathematical practice within NTs.

Making Connections Between and Among Ideas

The teacher drew connections between students’ strategies and constructed shared mathematical narratives by asking: “*Is there a connection between Maya’s idea and Leo’s idea?*” When one student observed, “*Maya and Leo both used minus in theirs,*” the teacher elaborated: “*Yes, they both found the difference... they both made two groups and then found the difference.*” Revoicing the student’s connection elevated their comment into a broader narrative about the underlying mathematical concept that was similar between the strategies—both strategies used a known fact that was larger before subtracting to find the answer to 8×6 . This compensation strategy was observed by one student who remarked, “*We both used a higher number to get to the equation.*” The teacher affirmed this observation as an insightful mathematical generalization by saying: “*Very interesting observation, you’re right! You both made larger values to find the answer.*” These responses show how students connected two strategies, elicited and made visible for discussion by the teacher.

Discussion

This study illustrates how teacher talk moves within a NT support students in mathematical discourse. Drawing on Gee’s (2005, 2011) building tasks and Sfard’s (2007) features of mathematical discourse, our analysis shows that talk moves do more than facilitate participation—they shape the nature of the discourse. Phrases like “*Let’s model that thinking*” and “*Does that capture your thinking?*” operated simultaneously across multiple discursive purposes: they made student thinking visible, enacted mathematical practices such as representation and verification, and built connections between strategies. These overlapping functions positioned students as doers of mathematics—listening to, comparing and analyzing peers’ mathematical ideas. The findings suggest that the efficacy of NTs lies in the intentional discursive work embedded in teacher talk moves. When used purposefully, talk moves transform NTs from procedural routines into generative spaces for doing mathematics—eliciting student thinking, making it visible and visual, and fostering connections to broader mathematical ideas. From this perspective, there are implications for professional learning: rather than limiting NTs to eliciting strategies, teachers may benefit from learning how to layer moves that emphasize reasoning, representation, and connection-making. Although based on a single episode, our analysis contributes to the emerging literature on the role of discourse in NTs. Additionally, our study sheds light on what makes NTs effective and emphasizes the need for professional learning that supports teachers in leveraging talk moves to support students in mathematical discourse. Our on-going research inquiries in the elementary mathematics classroom will continue to unpack the intentional talk moves occurring across teacher–student discourse during a NT.

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Oral performance assessment in an undergraduate calculus module for pre-service secondary mathematics teachers

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Oral performance assessments are commonly utilised in university mathematics modules in countries such as Germany and Italy but are rarely integrated into assessment methods for mathematics in Ireland or the UK. This type of assessment is dynamic and dialogic in nature as it challenges students to demonstrate their mathematical ability through combined written and verbal explanations in response to questions posed by an examiner. This paper discusses the experiences of undergraduate pre-service mathematics teachers in Ireland who have engaged in oral performance assessments as part of a calculus module. The participants in this study typically had a positive experience of the assessment, alluded to enhanced depth of mathematical understanding as a result of the assessment type, and highlighted the value of support from the examiner during the assessment.

Keywords: Oral performance assessment, university mathematics, teacher education

Introduction

Assessment of mathematics in Ireland at tertiary level tends to rely heavily on summative closed-book written examinations (Fitzmaurice & Ni Fhloinn, 2021). This is not uncommon in this region as universities in the UK tend to adopt a similar approach to the assessment of mathematics (Iannone & Simpson, 2022). An alternative to the closed-book written assessment in mathematics is the oral performance assessment (OPA). This type of assessment is characterised by its dynamic and dialogic nature as the assessor poses questions while the student responds through verbal explanations supplemented by written solutions on a whiteboard or a piece of paper (Iannone et al., 2020). The questions posed are often applied in nature as the student may be asked to solve completely (or in part) a problem posed. Similarly, questions can take on a theoretical nature, e.g., explaining a proof (or part thereof) for a given theorem (Iannone et al., 2020). Utilisation of the OPA at tertiary level is common in countries such as Germany and Italy but rare in Ireland or the UK. This paper will examine a case study in an Irish university within which the OPA was integrated into the assessment of undergraduate pre-service teachers completing a calculus module.

Literature Review

Research related to assessment in mathematics commonly cites the need for educators to utilise assessment to inform their teaching and guide their students' learning. Over-dependence on timed, closed-book examinations tends to 'narrow the curriculum' as students often just focus on the knowledge and skills which enable successful performance within the assessment (William, 2001). Varied methods of assessment, such as OPAs, can enable a more complete approach to assessing students' knowledge and skills while also encouraging students to develop knowledge and skills they might otherwise neglect (Iannone & Simpson, 2015).

Research on OPAs in mathematics is sparse but promising. A case study of 19 undergraduate students of probability and financial mathematics in the UK concluded that these students were more likely to apply revision strategies more conducive to deep learning when preparing for OPAs as compared to preparation for closed-book examinations (Iannone et al., 2020). This change in revision strategy appears to be linked to the belief that enhanced conceptual knowledge was perceived by these students to be more important in this context. Similarly, these students indicated they were more active and engaged in lectures than they otherwise would have been. Iannone & Vondrová (2024) found that university students in the Czech Republic who are accustomed to OPAs often equate them with assessing their conceptual understanding of mathematics, while viewing written examinations as assessments of procedural understanding. These students also tended to rate OPAs more highly as a learning experience when compared to closed-book written examinations.

Research Aims and Methods

This research aims to better understand undergraduate pre-service teachers' experiences of preparing for and engaging in OPAs in an Irish university. Given that such assessment would be novel to the pre-service teachers involved, it is of great interest to gain insights into their experiences in terms of how they prepared for this type of assessment and the learning they perceived to have gained from the experience. Similarly, an enhanced understanding of how this experience was different to the typical written assessment to which they have been accustomed is of interest.

This research study adopted an instrumental case study approach. The case in this instance – a cohort of undergraduate pre-service mathematics teachers in Ireland experiencing OPAs in mathematics for the first time – was chosen in order to better understand the experiences of this group in this scenario in relation to their assessment preparation, assessment engagement, and views on alternatives to closed-book examinations. Each OPA was ten minutes in duration and took place at the end of the 2023/24 Spring semester. The assessment focused on methods of integration and applications of integration.

Twenty-one undergraduate pre-service mathematics teachers within a particular cohort in an Irish university were contacted by e-mail and asked to complete an online survey (hosted on MS Forms) which included multiple choice items, Likert scale items, and text responses. This survey was anonymous in nature. The questionnaire utilised elements of the Assessment Preferences Inventory (Iannone & Simpson, 2015) and the Assessment Experience Questionnaire (Gibbs & Simpson, 2003), which have been applied in numerous studies in the UK and beyond. Twelve participants completed the questionnaire in full. Qualitative data underwent thematic analysis in order to identify key themes which would best characterise the collective responses of the participants.

Findings

Responses by the participants in this study ($n = 12$) to questionnaire items will be presented in this section. The following item provides a suitable overview of the respondents' general views in relation to OPAs. This item (outlined below) utilised an adapted Likert scale and is summarised in Figure 1.

Figure 1

Responses to statements which enable comparison between written examinations and oral examinations

Now compare written exams to oral exams. Please rate the statements below where:

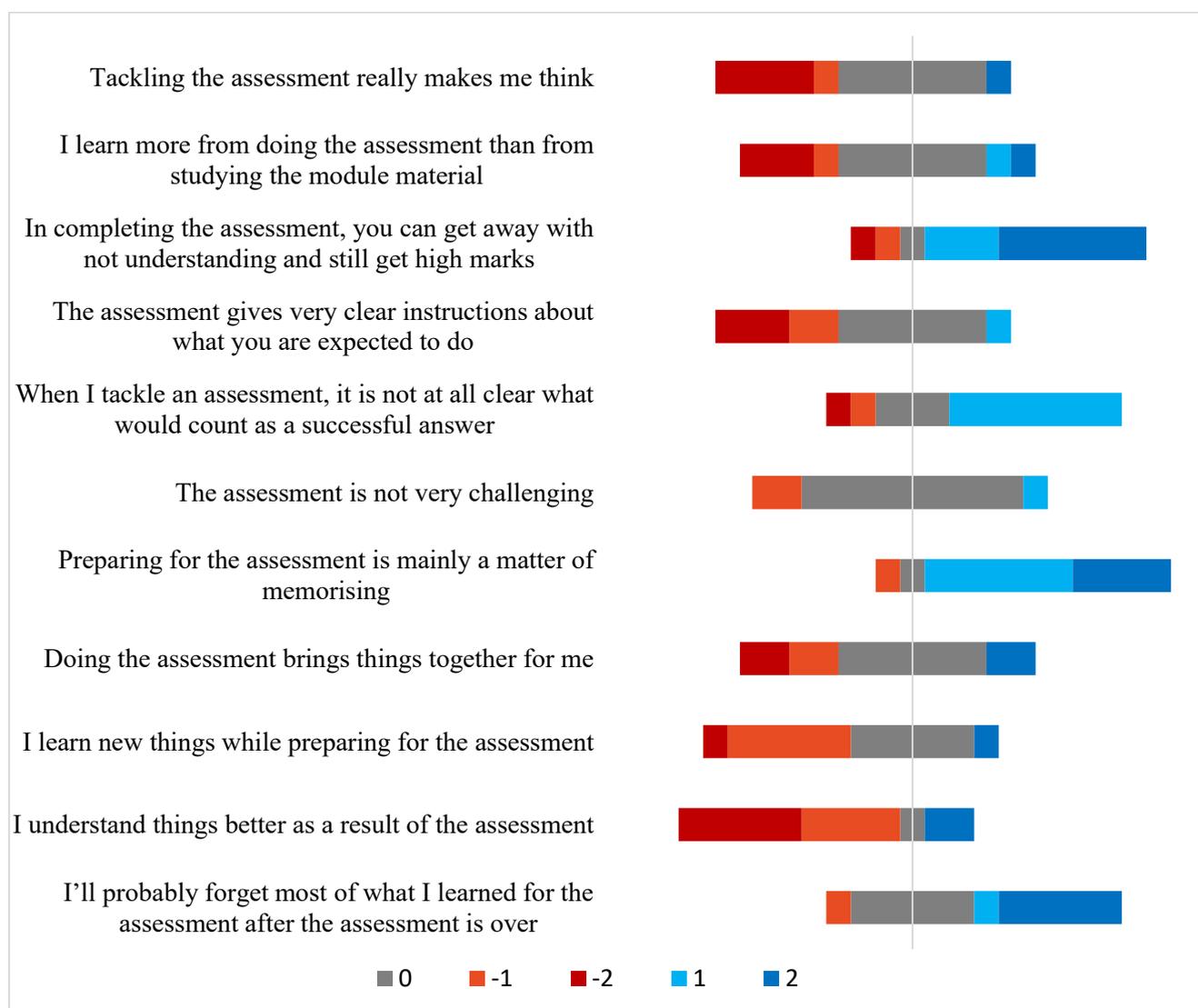
– 2: Much more accurate of oral assessment than written exams.

– 1: More accurate of oral assessment than written exams.

0: About the same of written exams and oral assessment.

+ 1: More accurate of written exams than oral assessment.

+ 2: Much more accurate of written exams than oral assessment.



Interestingly, the participants generally rated the need for memorisation and ‘getting away’ without having a full understanding of the content to be much more accurate of written examinations than oral assessments (see Fig. 1). Understanding the content better as a result

of the assessment, learning new things during preparation for the assessment, and being challenged to think were generally more accurate of oral assessment than written examinations according to the participants of this study. The comparison in retention of learning was also notable as respondents indicated that it was more accurate of written examinations than oral assessments that they would forget most of what they learned for the assessment.

There were two items in the questionnaire which requested a text response, these were as follows:

- What are your overall thoughts regarding oral assessments as a means of assessment for mathematics modules at Tertiary Level (University Level)?
- Was your preparation for the oral examination different in any way to your preparation for a standard written mathematics examination? Please elaborate.

Analysis of the responses to these two items resulted in the following over-arching themes: preparation for the OPA; emotional response & support; understanding of mathematics; and links to future career. These themes are discussed in further depth in the following section

Preparation for the OPA

Responses to the second text response item highlighted above were somewhat mixed. A small minority indicated that their preparation did not change, e.g. “No I just practiced the questions on the tutorial sheet” (S5). Others seemed to alter their approach in order to better understand the underlying mathematical concepts so that they could explain these aspects in sufficient depth during the assessment. For some, this alteration to preparation was moderate, while for others, it was significant:

For the oral exam my focus was on my understanding of the topic and questions and being able to explain why I was doing what I was doing whereas for written exams sometimes my focus can be on learning the questions off but not fully understanding steps I am taking. (S12)

This shift from mainly focussing on development of procedural knowledge to incorporating the development of conceptual knowledge was regularly evident in participant responses.

Emotional Response and Support

Participants in this research project tended to have different reactions to the preparing for and engaging in OPAs. In general, despite some participants feeling apprehension, the overall experience appeared to be positive. Respondents alluded to worries regarding being put ‘on the spot’ during the assessment and having to verbalise their mathematical understanding which would be an uncommon mathematical assessment challenge for these students. Each of the respondents that alluded to these worries indicated an overall positive experience in the end. There were mentions of the assessment being more relaxed than a typical written examination and the feeling that the OPA was more like a conversation.

Understanding the mathematical content and links to future careers

As discussed previously, students tended to believe that preparing for OPAs challenged them to better understand the relevant mathematical content when compared to their typical preparation for closed-book written examinations:

Oral Assessment has been key in understanding the content thoroughly and meticulously, particularly the nomenclature and how to describe the sequences accurately through spoken language. (S3)

The comment above also highlights an important aspect of the contrast between OPAs and written examinations as verbalising their understanding of mathematics is often neglected in traditional classrooms (or lecture halls) and their associated examinations. For this cohort of respondents, this perspective is even more important due to the potential impact on their future careers as teachers as they will be challenged to demonstrate their mathematical understanding verbally on a daily basis.

Overall views of mathematical assessment

Participants in this research project held a favourable view of OPAs as a means to assess their mathematics achievements. A significant majority wanted OPAs as part of the means by which they would be assessed in mathematics. Written examinations with support materials (e.g. relevant mathematics textbook) and continuous assessment through the submission of weekly tutorial sheets also proved popular. Such a response, particularly for a novel assessment type, indicates quite a positive experience with regards to OPAs.

Discussion and Conclusions

The findings outlined above indicate that students generally had positive experiences of OPAs and most wanted them as part of their mathematics assessments. Given that these assessments were novel for this cohort, it would be interesting to track their opinions with further experience of OPAs. Typically, students focussed more on enhancing their conceptual knowledge in preparation for OPAs when compared to written assessments. This is in keeping with the conclusions drawn in a comparable study in the UK by Iannone et al. (2020). Similarly, a significant majority indicated that they were more likely to retain the learning gains made in preparation for the OPA as compared to a written examination. Identifying assessments which encourage students to develop more in-depth and durable understanding of the concepts underpinning the mathematics content being studied can only be a positive development. Given that the balance is often skewed towards students focussing on maximising their procedural knowledge in order to be successful in assessments, this change is welcome.

Respondents in this study mentioned the value of support during the assessment which again contrasts with the typical closed-book written examination. Traditionally, students would not have any assistance in overcoming certain barriers to solving mathematical problems which may have limited their ability to demonstrate their full range of mathematical knowledge and skills within these assessments. Having a supportive resource such as the examiner as an aid, or even just as a potential aid, can provide a reassuring presence during

the assessment process. This type of assessment also links well to pre-service mathematics teachers' future careers. The challenge to explain solutions to problems and/or explain proofs of theorems not only assess a student's level of understanding but also their ability to verbalise this understanding as well as communicating effectively in writing. As such, it can offer advantages in this regard over written assessments.

There are, of course, limitations to OPAs. Assessments can be time-consuming for examiners, particularly when they have large cohorts. Similarly, time limitations can impact on the complexity of tasks posed – ten minutes (the length of each OPA in this study) is not long enough to engage fully in a complex problem-solving task. In light of these limitations, utilising OPAs as a complement to other forms of assessment (e.g. written assessments) seems like a logical way forward, particularly if OPAs need to be kept to a relatively short time period for examiner workload reasons.

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An exploration of how gender interacts with other factors in predicting achievement in mathematics

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Gender gaps persist in international assessments of mathematics, and in most cases, differences are in favour of boys. In Ireland, concern has been raised about lower numbers of girls than boys attaining at the highest levels of mathematics. The research described in this paper examined data from a longitudinal government study in Ireland. Relationships were explored between attainment in mathematics at school completion and socioeconomic status (SES), general anxiety, attainment in mathematics at age 9, and enjoyment of mathematics at age 9. Gendered differences were found in the strength of the correlation in all cases. The completion of mathematics-rich subject(s) was also explored as a potential contributing factor to variations in attainment in mathematics, but no gendered difference was found.

Keywords: Gender, socioeconomic status, mathematics attainment

Gender Differences in Mathematics Attainment

The Trends in International Mathematics and Science Study (TIMSS) 2023 at 8th grade found that of 42 participating countries, 17 had no significant difference between boys and girls, 21 countries, including Ireland, had a significant difference in mean scores that favoured boys, and 4 countries had a significant difference that favoured girls (von Davier et al., 2024). In this paper we will refer to boys and girls as the data we are working with uses these binary categories. We would like to acknowledge that this categorisation excludes those children who do not fit within the binary classification, as highlighted by Cox and Pinheiro (2024). Baye and Monseur (2016) caution that focusing solely on mean scores obscures the male dominance of mathematics, and advocate for a focus on the two tails of the achievement distribution, where a persistent gender gap becomes evident at the highest proficiency level. In Ireland a significantly higher percentage of boys than girls achieved the High and Advanced International Benchmarks at 8th Grade (McHugh et al., 2024).

Gendered perceptions of mathematics manifest in a variety of ways as girls progress through education. Gendered perceptions of girls as less capable of mathematics reduce the expectations on girls to succeed, thereby allowing girls to more comfortably opt out of mathematics (Lee et al., 2022). Conversely, a perception of being an imposter in a male domain also contributes to greater self-evaluation and self-doubt among those who choose to opt in. This heightened judgement contributes to greater mathematics anxiety which has been seen to impact both on attainment as well as long term perseverance with the subject, for example into third level (c.f. Lee et al., 2022).

Mathematics anxiety has implications for both attainment and perseverance in mathematics where higher levels of mathematics anxiety have been found to be associated with poorer outcomes for children (Szczygieł, 2020). Considering the sources of mathematics anxiety and the disparity between genders, Szczygieł (2020) examined the general anxiety of the children in their study, and found that gender differences in general anxiety may explain

gender differences in mathematics anxiety. This finding resonates with the findings of Xie et al. (2019) who attested that mathematics anxiety in girls may be a “a generalization of general anxiety and test anxiety” (p. 242).

Socioeconomic Status (SES) is a comparative measure that typically reflects the education, employment and income levels of parents within the home, and thereby seeks to identify the family’s access to the resources that are critical to thriving within society (Muñez et al., 2021). High SES has been consistently associated with high levels of attainment in mathematics in many jurisdictions, including Ireland (Duggan et al., 2023). While the home environment can be seen to mediate SES in some situations, the role of SES in predicting outcomes for children is persistent and consistent internationally (Muñez et al., 2021).

The TIMSS 2023 study found that there was no gender disparity in mathematics attainment at 4th Grade (Fourth Class in Ireland) in Ireland and a significant disparity at 8th Grade (Second Year in Ireland), in favour of boys (McHugh et al., 2024). Many factors impact on children’s progress in mathematics during this time, including the transition to secondary school. On entering secondary school in Ireland children select typically nine subjects to study for Junior Cycle (a three-year program in junior secondary school). Four of the available subjects are Applied Technology, Wood Technology, Engineering and Graphics. The curriculum for each of these subjects includes “The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning” as the first ‘Statement of Learning’ (National Council for Curriculum and Assessment, 2024). Typically, more boys than girls select these subjects for study, and we theorise that this additional time to consolidate and apply mathematical concepts may contribute to boys achieving at a higher level than girls in mathematics.

Many factors impact on whether children persevere and attain at a high level in mathematics and in this paper we seek to explore whether there are gender-related differences in how socioeconomic status, anxiety, enjoyment, and subject choice predict children’s attainment in mathematics.

Methods

Growing Up in Ireland (GUI) is a government-funded longitudinal study of children in Ireland that has been running since 2006, and aims to facilitate elucidation of key aspects of the lives of children and adolescents in Ireland. In 2007, nationally representative samples of 9-year-old children and 9-month-old children were selected (Thornton, et al., 2010), and in our research we drew from the data collected on the 9-year-old cohort who were selected through their primary schools. Within this paper, we analyse the 4958 participants who returned surveys at all four waves, including 2580 boys and 2378 girls, representing 8.8% of all 9-year olds in Ireland in 2007 when sampling took place.

To compare the impact of the variables on each gender, the data was split by gender and regression models were generated. A model was created for each gender with the dependent variable of attainment at school completion and the independent variables of a) SES, b) enjoyment of mathematics at age 9, c) attainment in mathematics at age 9, d) freedom from anxiety; and e) whether the participant studied additional subjects with high

mathematical content. Following the regression analyses, t-tests and correlations were conducted to help further explain and understand the findings in the models.

Variables Included in the Analysis

Attainment at school completion. On completion of secondary school in Ireland, students undertake ‘Leaving Certificate’ examinations, and their results are converted into points for a competitive university entry system. We are using these points as a measure of the participants’ mathematics attainment on completion of secondary schooling.

SES. The Growing Up in Ireland research team derived an SES variable that reflected solely the occupations of the primary and secondary caregiver who are present in the household for each participating child (Quail et al., 2014). Applying a dominance model, the SES assigned reflected the higher social class of the primary and secondary caregiver.

Enjoyment of Mathematics. Children’s affective response to mathematics has been seen to contribute to their engagement and attainment over time (Hannula et al., 2014). In seeking to consider how engaged each child was with mathematics we have drawn on the GUI Survey Item where the children responded “always”, “sometimes” or “never” to “I like Maths”. For the purposes of the regression analysis, we numerically coded this item so that the higher value indicated the higher enjoyment of mathematics.

Attainment. Assessments of mathematics that were standardised across the Irish population of primary school children were used to measure the attainment in mathematics of the participant children at Waves 1 and 2. Depending on each child’s class level, different assessments were used and a ‘logit score’ was calculated that incorporated the score obtained, the class level of the assessment and the difficulty of the items that the children answered correctly (Irish Social Science Data Archive, n.d.).

Freedom from Anxiety. The Growing Up in Ireland longitudinal study utilises the Second Edition of the Piers-Harris Children’s Self-Concept Scale (PH2) in order to measure the participants’ overall self-concept (25 items) along with Freedom from Anxiety (14 items), and five other constructs. For the purposes of this paper, we are using the PH2 measures for Freedom from Anxiety from Wave 2, when the participants were 13 years old.

Mathematics-rich subjects. The GUI dataset includes information about all subjects studied by each participant in junior secondary school. We generated a variable to identify whether a participant had studied any or none of the mathematics-rich subjects listed above. Within the cohort involved in this study, 4% of girls completed at least one of these subjects and 33% of boys.

Results and Discussion

In Table 1 we present the Standardized Coefficients from both regression analyses, and whether each variable is a statistically significant predictor of the outcome variable, attainment in mathematics at school completion.

Table 1

Regression analyses for male and for female participants where attainment on school completion is the dependent variable.

	Boys Standardized Coefficients Beta	Sig.	Girls Standardized Coefficients Beta	Sig.
SES	.130	<.001*	.192	<.001*
<i>I like Maths</i> at age 9	.108	<.001*	.048	.011
Attainment at age 9	.368	<.001*	.381	<.001*
Freedom from Anxiety	.035	.071	.111	<.001*
Mathematics-rich subjects	-.075	<.001*	-.074	<.001*

Note. * correlations are statistically significant at $p < .01$ level.

As can be seen from Table 1, attainment at age 9 was the largest predictor of attainment at school completion, when SES, freedom from anxiety, completion of mathematics-rich subjects and enjoyment of mathematics at age 9 are taken into consideration. Among the participants in this study, there was a statistically significant difference in favour of girls ($t(df) = 6.559, p < .001$) in attainment at age 9 and it may be reasonable to assume, that these two factors would result in a significant difference in favour of girls in attainment on school completion. However, in attainment on school completion the difference is in favour of boys ($t(df) = 6.722, p < .001$). The relationship between attainment at age 9, gender and attainment on school completion is therefore complex and would require further investigation.

The opportunity to study one or more of the mathematics-rich subjects describe above is availed of by a larger proportion of boys than of girls and we theorised that this engagement would support attainment in mathematics on school completion, but the regression model indicates that the engagement negatively predicts attainment. The relationship between subject choice and attainment in mathematics is complicated due to the tendency of less academic students to select the mathematics rich subjects which are perceived as practical in nature. Within this cohort, 35% of the participants who were in the lowest 15% on overall achievement at school completion completed a mathematics-rich subject, and only 8.8% of the 23% who achieved the highest marks overall did so.

SES, attainment at age 9 and freedom from anxiety positively predict attainment on completion of secondary school, and all are significant for girls, where SES and attainment at age 9 are statistically significant predictors for boys but freedom from anxiety is not. The standardized coefficients on these three variables are larger for girls than boys. This indicates that girls are more vulnerable to under-achievement due to SES, anxiety and attainment at age 9, as each contributes more to the variation in attainment on school completion for girls than for boys when the other variables in the model are controlled for.

Conclusion

The findings of this study indicate that girls' attainment on school completion may be more vulnerable to the impacts of SES, general anxiety and early attainment in mathematics. We contend that gendered perceptions of mathematics described above and the associated higher levels of anxiety in relation to mathematics may contribute to this vulnerability. As gendered perceptions are very challenging to address, interventions targeted at intersectional vulnerabilities are warranted, for example interventions designed for girls in the lower SES categories, or girls who are experiencing general anxiety.

As attainment at age 9 is the most substantial predictor of attainment at school completion, early and sustained intervention would seem to be critical, but interventions that target attainment will not be sufficient to address girls' underachievement longitudinally, given the complexity of this relationship, wherein the difference in mean attainment favours girls at age 9 and favours boys at school completion. The role of enjoyment of mathematics in girls' attainment at school completion was not significant among the cohort in this study. This may indicate that interventions targeting girls' enjoyment of mathematics may be limited in their success.

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Learning to teach algebra: an analysis of student-teachers' mathematical knowledge for teaching functional thinking

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Significant strides have been made in the research field of early algebra, but more attention to the Mathematics Knowledge for Teaching (MKT) of teachers within this domain is warranted (c.f. Wilkie, 2016; Kieran, 2007). The study described in this paper engaged student-teachers in Germany, Ireland and South Africa in the role of student when they attended a preparation programme for the teaching of functional thinking, and in the role of novice teacher when they taught functional thinking in classrooms. The participants completed pre and delayed-post questionnaires designed to measure their mathematical knowledge for teaching functional thinking. While considerable progress was made by the cohort between the pre- and post-assessment, challenges persisted on some items. In this paper, we present quantitative evidence of the development in students' thinking between the pre- and post-assessment and discussion of the relatively challenging items.

Keywords: teacher-education, algebra, patterning, functional thinking

Mathematical knowledge for teaching functional thinking

Seeking to address the gatekeeper role played by algebra in many education systems (c.f. Mason, 2008), patterning activities have established prominence as an accessible context through which children may develop functional thinking by exploring relationships between quantities. Warren and Cooper (2008) suggest that when children succeed in identifying the relationship between a cardinal aspect of a figure (such as the quantity of components of an element) and the ordinal aspect of the figure within the pattern, they are developing the ability to think of the pattern as a function.

The many decisions that teachers make in the preparation, delivery and assessment of learning rely on their knowledge for teaching, and strong MKT has been identified as a key driver of high-quality teaching in classrooms (Hill et al., 2008). Wilkie (2016) highlights the relevance of MKT in the domain of functional thinking where teachers need proficiency in understanding children's thinking and the challenges they might encounter. In this paper, we present results from an assessment of Mathematical Knowledge for Teaching within the domain of functional thinking (MKT_FT) that was used in an intervention designed to develop student-teachers' MKT_FT. The intervention involved a preparation programme, comprising of three seminars and an accompanying Teacher Manual focused on the delivery of four lessons, which we entitled the ZADIE lessons in functional thinking, where 'ZADIE' reflects the three participating countries of South Africa (ZA), Germany (D) and Ireland (IE) (Twohill et al., 2023). The preparation programme and lessons focused on key ideas of functional thinking such as the potential perspectives children might adopt in their observations of patterns. Such perspectives include recursive thinking where children compare consecutive figures; covariational thinking, where one figure is scaled up to produce

another; and explicit thinking where a correspondence relationship is identified between a figure and its position in the pattern (c.f. Smith, 2008).

The overarching research question for this study asked to what extent the ZADIE intervention supported the student-teachers in teaching functional thinking. As such, the research sought to examine the MKT_FT of the student-teachers before and after the intervention, and in this paper we present an analysis of the questions which were particularly challenging for the participants, both before and after the intervention. Indication of these challenging questions may act as a guide for future reviews of the ZADIE materials, and any accompanying follow-on materials.

Methodology

Hill et al. (2008) parse Mathematical Knowledge for Teaching into multiple components, including ‘knowledge of content and students’ (KCS), ‘knowledge of content and teaching’ (KCT), ‘common content knowledge’ (CCK) and ‘specialised content knowledge’ (SCK), which refers to mathematical content knowledge that is specific to teaching, rather than common to all users of mathematics. Participants in the ZADIE project were asked to complete pre- and post-assessments of their MKT_FT, to complete structured reflections after their teaching of each lesson and to take part in a focus group interview. The pre- and post-assessments consisted of a 19-item paper and pencil assessment where items were clustered across questions 1 to 8 and labelled as such, for example, 1a, 1b, 2a, 2b. In developing the assessment of MKT_FT, items were designed to test distinct components of MKT_FT as presented in Table 1.

Table 1

Assessment items designed to test components of MKT_FT

<i>Component of MKT_FT</i>	<i>Assessment items</i>	<i>Example</i>
KCS	6a, 6b, 8a, 8c	An example of a child’s thinking is provided and the participant is asked to explain the child’s thinking.
SCK	1a, 1c, 2a, 4, 5a, 7a, 7b, 7c	The participant is asked to provide a function rule for a given pattern.
KCT	1b, 2b, 3, 5b, 8b, 8d, 8e	The participant is asked for representations for a given function (other than the given pattern) that would be appropriate for primary school.

In April 2019 students in the 2nd, 3rd or 4th year of their 4-year undergraduate initial teacher education degrees, in the affiliated universities of this paper’s authors, were invited to participate in a research project focusing on algebraic thinking. Of the participants who completed the preparation programme and participated in all data collection points, fifteen participants, including five participants from each of South Africa, Germany and Ireland, were randomly selected for analysis. Students’ responses to each item of the assessment were marked on a 5-point scale, from 0 to 4, where 0 was awarded for no response, and 4 for an accurate and complete response. Each assessment item had an individual 5-point scoring

rubric designed to best assess the students' thinking on that specific item. An example item with scoring rubric is presented in Figure 1.

Figure 1

Example of an SCK item and accompanying scoring rubric, from the assessment of Mathematical Knowledge for Teaching Functional Thinking

Item 5 (a): Draw the first four figures of a geometric pattern that would match the function rule $2n + 2$.

4	Both constant and variable represented in a valid way
3	Error with constant or variable but not both
2	Correct total of elements but no distinction between variable and constant
1	Incorrect (including no structure apparent)
0	No answer

In this paper we discuss total scores for each participant along with percentages of the maximum score available of 76 (4 on each of 19 items).

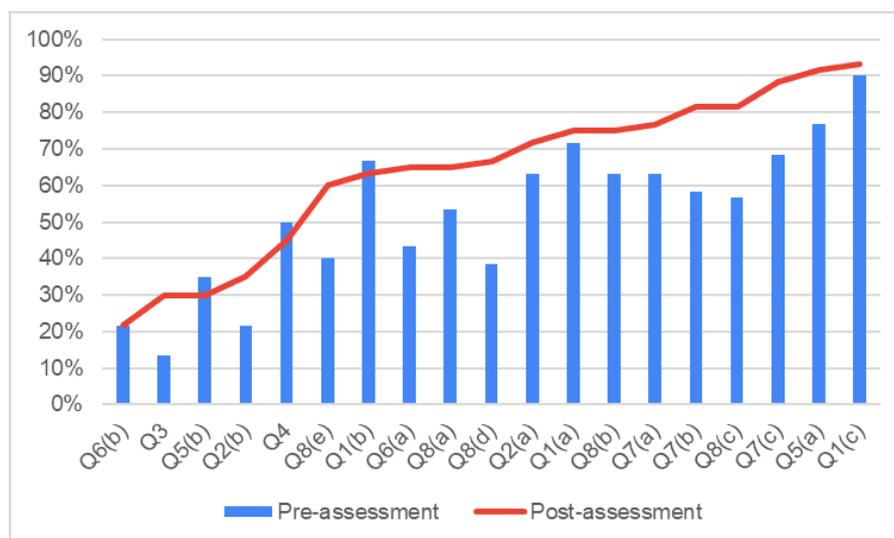
Results

Overall, the MKT_FT as measured by the assessment improved for the cohort of participants, where the mean score of all participants on the assessment increased from 39.8 (52%) on the pre-assessment to 48.7 (64%) on the post-assessment. There is notable variation in the percentage improvement of the participants who completed the pre- and post-assessments. Two students had lower scores on the post-assessment than on the pre-assessment, four students achieved post-assessment scores between 2% and 18% higher than their score on the pre-assessment, and nine students achieved a score on the post-assessment that was more than 20% higher than that achieved on the pre-assessment. We were satisfied that these statistics demonstrated the value of the ZADIE project, and that the mathematical knowledge for teaching functional thinking of most students had improved as a result of their participation in the preparation programme and in the teaching of the ZADIE lessons.

Seeking to inform future iterations of the Teacher Preparation Programme, we examined the scores achieved on each item of the assessment and the progress made by participants between the pre- and post-assessments. In Figure 2, we present the percentage of the full marks that were achieved on each item, ordered by the percentages achieved by participants on the post-assessment.

Figure 2

The percentage of the full marks achieved on each item, on both the pre-and post- assessment



From these results, we focused our attention on the items where participants continued to experience challenge following the preparation programme, 2b, 3, 4, 5b and 6b. In Table 2, for each item, we present the mean percentage achieved on the pre- and post-assessments, a brief description of the item and the relevant component of MKT.

Table 2

Percentages achieved, description and MKT component for each of the challenging items

Item	Mean percentage achieved		Description	Component of MKT Assessed
	On pre-assessment	On post-assessment		
6b	22 %	22 %	Analysis of a child’s response to a patterning question	KCS
3	13%	30%	Language used in teaching patterning	KCT
5b	35%	30%	Alternative representations of patterns	KCT
2b	22%	35%	Use of linear and non-linear patterns in primary school	KCT
4	50%	45%	Matching expressions to a tooth-pick representation of a pattern	SCK

Of these five challenging items, four assessed KCS and KCT, and it was not surprising that these items would be challenging for student-teachers with limited, or no, experience of teaching this content.

6b and 3 assessed participants’ language in the domain of functional thinking. Full marks were awarded for item 3 if participants selected which of a list of terms was relevant to functional thinking, for example ‘generalising’, without including any listed terms that were not relevant, for example ‘tangent’. Item 6b required participants to answer “What type of

thinking is Laura using (recursive, covariation, explicit)? Why do you think this?”. As such, these questions assessed participants’ familiarity with and understanding of key terminology within the domain. Participants made considerable progress on item 3 between the pre- and post-assessment but no progress on item 6b.

Items 2b and 5b asked open questions where participants were asked “How would you use linear and non-linear geometric patterns in lessons for primary school?” (2b) and “When working in a classroom, are there other ways to represent” a given pattern (5b). In order to obtain full marks on these questions, participants needed to include a number of possible elements, for example tables, graphs and functions for 5b and examples of both linear and non-linear patterns for 2b. We suggest that the open nature of these questions, which did not include guidance about how comprehensive a response might be, led to very few participants achieving full marks on these questions. Again, participants made no progress on item 5b between the pre- and post-assessment and moderate progress on item 2b

The percentage correct achieved on item 4, which assessed SCK, was higher than the challenging items assessing KCS or KCT. This item, which had the lowest score achieved of the SCK items, asked participants to identify, of four expressions, which one did not match a given pattern and to explain why. While no student achieved zero on this item, three students identified the incorrect expression, and no student gave a fully convincing description of why they selected the expression that did not match. Participants’ mean score on this item was lower on the post-assessment. In a similar manner to questions 2b and 5b this item included an open question that required a comprehensive response to achieve full marks.

Discussion and Conclusion

Following the ZADIE preparation programme and the teaching of the ZADIE lessons, 13 of the 15 participants achieved increased scores on their post-assessment of MKT_FT. While challenging to speculate about how easily the participants will generalise this MKT_FT to other functional-thinking tasks beyond this single assessment, we are confident that they provided evidence of understanding of key ideas of functional thinking and of appropriate pedagogical approaches in this domain. For example, Wilkie (2015) states that identification of a functional relationship between two variables, and expressing this relationship mathematically is a key aspect of both learning and teaching algebra. In the post-assessment described above over 70% of the cohort successfully identified a functional relationship and expressed a functional relationship mathematically.

However, there exist a number of items on the assessment of MKT_FT which proved consistently challenging on both the pre- and post-assessment, and we contend that these items merit further attention in future reviews of the ZADIE teacher preparation programme. The most challenging items related to pedagogical content that required fluency with language that is specific to this domain or asked open questions (KCT or SCK) with a scoring protocol that rewarded comprehensive answers. While the scoring protocol may appear overly stringent, we took the position that the highest quality teaching will draw on multiple representations and multiple perspectives to be inclusive of all learners, and as such responses to these items that are less than comprehensive do not merit full marks.

To support participants in overcoming the challenges described here, we intend to include a glossary of terminology in future editions of the ZADIE teacher preparation programme, and facilitate participants in using the terminology in meaningful contexts, for example in the analysis of children's thinking. We also plan to include further opportunities to consider broadly the many ways in which patterns may be represented, as the creation of meaningful representations and their role in the building of generalisations is key to successful teaching and learning of algebra (Smith, 2008).

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Illustrating short-term expansions in children's thinking about fractions

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Evidence shows the teaching and learning of fractions to be particularly challenging. Despite an extensive research base, the extended time needed to develop fraction sense makes it difficult for teachers to have appropriate expectations for student learning in the shorter term. This paper reports on data arising from an ongoing partnership project which employed a fraction intervention in two classes. We illustrate one child's expanded fraction reasoning by drawing on the learner-generated examples apparent in pre- and post-intervention interview data. This data shows a greater command of formal fraction language, a greater willingness to use diagrams to support reasoning and explanations, and a more competent and nuanced approach to the use of fraction procedures.

Keywords: Fractions, learner-generated examples, repertoire, representations

Introduction

Extensive evidence points to difficulties for students and teachers in the learning and teaching of fractions, noting an overemphasis on the part-whole fraction sub-construct and, within this, on area representations (Charalambous et al., 2010), and further, the disconnected teaching and learning of fraction computation procedures. There are instances of intervention studies that have addressed these challenges with teachers and students, but one acknowledged challenge is the extended timeline involved in appropriating a strong grounding in fraction sense (Lamon, 2020). There is a literature base on trajectories of fraction learning (Simon et al., 2018), and impetus to begin fraction instruction with a focus on the fraction as measure sub-construct to build a strong sense of fraction magnitudes that can then underpin later learning of concepts and procedures linked with fractional equivalence and operations (Cortina et al, 2012). However, the extended timeline remains within this work coupled with a narrower lens on fractions than is typical in many state and national curricula. Both these aspects make it hard for primary teachers working with children over the course of an academic year to get a sense of what expansions and extensions in children's repertoires they could be aiming for within this timeframe.

It is these smaller expansions and extensions that are our focus in this paper. Our illustrations of change are drawn from a school-led collaborative intervention project set in one Dublin primary school. The school approached the university in 2022 with interest in trying to expand children's fraction repertoires beyond procedural calculations and suggested that working on approaches emphasising reasoning and mathematical talk could be a useful route into this goal. The result was a school-university collaboration, now in its third year, that has used and adapted tasks drawn from the literature base that focus on fraction magnitude comparisons. Teacher feedback from the first short (four lesson) intervention cycle, run across four weeks, was that while reasoning and talk had improved in the intervention lessons, children reverted to procedural approaches on more traditional fraction tasks. The following cycles have therefore tried to build more explicit connections between

the ideas developed in fraction magnitude comparison settings and the work on fraction operations that the children cover as part of the curriculum.

In 2025, our third cycle, two teachers (one 5th class and one 6th class) elected to work on an extended fractions-based intervention lesson sequence with their classes. Pre- and repeat post-tests were developed and administered and pre- and post-task-based interviews were then conducted with a cross-attainment student sample identified from the written tests. The 6th class intervention is now complete. In this paper, we use data from one 6th class interview sample student to illustrate our early routes into understanding the changes in her repertoire linked with a small set of fraction magnitude comparison, and fraction operation tasks. Our two research questions are combined in our presentation of findings:

- What tools are useful for illustrating expansions and extensions in children's fraction repertoires in the course of shorter-term fraction interventions?
- What expansions and extensions in children's fraction repertoires can be illustrated using these tools?

Data sources

Twenty lessons - focused on fraction as measure, fraction addition and subtraction, multiplication and division, and ratio - with emphasis on representations and reasoning, were enacted in the 6th class intervention. The teacher planned the lessons in collaboration with the university team and monitored progress to decide the content of subsequent lessons. The class contained 25 children with matched pre- and post-test data from 24 children. This test included items focused on fractional measures, fraction comparison, equivalence and operations, and ratio. The written test data indicated a mean gain of 14 percentage points. Most class work was collaborative and interspersed with some individual tasks. The intervention was bookended by pre- and post- interviews. Six cross-attainment range children were selected for interview. We video-recorded the interviews in 'talking hands' format, i.e. with focus on children's talk (which was audio captured), their gestures and inscriptions on paper. The interview data was transcribed verbatim with screen captures and descriptions of the children's oral, written and gestural responses to tasks across the pre- and post-interviews.

In this paper we focus on one interview sample student, identified as a high attainer on the basis of her pre-test mark, to illustrate the expansions seen in her pre- and post-interview responses. A pseudonym is used to respect student anonymity.

Useful tools

Watson and Mason (2005) have noted that studying learner generated examples in response to teacher questions and prompts can provide a window into children's understandings of mathematical ideas. A root prompt from Watson and Mason's work that was widely used in the pre- and post-interviews carried out in this study was: 'Can you give me an example of ?' This was often followed by a request for another example. The interview was framed in terms of the school-university team's interest in understanding children's thinking about fractions, so questions about fraction examples and operations were accompanied with requests to children to explain their choices and approaches.

Watson and Mason (2005) make use of the construct of ‘dimensions of variation’ to understand the nature and scope of learner generated examples. In Marton and Booth’s (1997) variation theory, critical aspects are aspects that need to be varied within an object of learning for students to perceive this aspect. Critical aspects are subsequently combined to apprehend the idea in focus. In variation theory, each aspect is defined as a ‘dimension of variation’. Watson and Mason (2006) extend this idea in the context of mathematics teaching and learning to the terminology of ‘dimensions of possible variation’ and ‘range of permissible change’. For Mason (2011) the word ‘possible’ serves as a reminder that teacher and students’ conceptions may not align. The range of permissible change refers to the values or examples within a dimension that a child offers in response to a prompt. Mason notes that ‘students and teacher may vary considerably on the universe of discourse, on the objects being considered’ (p.108). Changes – ideally expansions - in a child’s ‘universe of discourse’ thus offers a key marker of learning.

In our focus on fractions, children’s responses pointed to the following as salient dimensions of variation:

- Choices of fractions given in response to specific prompts
- The ‘benchmarks’ implied in children’s responses for how they considered the size of fractions, e.g. as bigger than, or smaller than $\frac{1}{2}$ or 1, as fixed benchmarks, but also comparing fractions based on numerator varying with denominator invariant, or of numerator invariant and denominator varying
- Procedures or approaches used for particular fraction operations
- Representations used within reasoning and problem-solving involving fractions

In our presentation of findings, we use these dimensions of variation to consider the differences seen in one student’s responses across the pre- and post-interviews

Findings

The focus student Cara made a 13% gain on her written post-test assessment. In line with teacher observations in classroom activities, the interviewer (second author) found that Cara was at all times pleasant and appeared eager to please. The opening questions for both the pre- and post-interview asked students to provide an example of a fraction that was greater than $\frac{1}{2}$ but less than 1. Cara’s initial response to this question in the pre-interview was to offer what can be considered the prototypical example of $\frac{3}{4}$ (Goldenberg & Mason, 2008). She explained that this answer was appropriate because, “two quarters is equivalent to a half and then one more quarter is bigger than a half because four quarters is a whole”. This shows evidence of a sense of the relative size of $\frac{1}{2}$, $\frac{3}{4}$ and 1. When asked to generate further examples, her reasoning about the relative size of unit fractions appeared to be solid but reasoning about non-unit fractions was not as fluid. She could correctly identify fractions such as $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$, which are one unit fraction away from the whole as larger than $\frac{1}{2}$ and could reason about the relative size of these with reference to their respective unit fractions. In seeking to identify a fraction between $\frac{11}{12}$ and 1, she identified that the denominator would have to be larger than 12, but was rather hesitant about this. After introducing this idea, she returned to discussing examples that she was familiar with. She stated that she knew $\frac{9}{9}$

was larger than $\frac{11}{12}$ because $\frac{9}{9}$ is a whole. She also stated that $\frac{1}{13}$ would be smaller than $\frac{1}{12}$, and justified this with reference to the example of $\frac{1}{10}$ being larger than $\frac{1}{12}$ “by a little bit” because the denominator is larger. It was only after this extended discussion, which also included a reference back to the example of $\frac{3}{4}$ being smaller than $\frac{11}{12}$, that she directly asked the interviewer: “Could there be numbers, that the denominator is, like, bigger than 12? Could that work?” After the interviewer acknowledged this possibility, she generated $\frac{12}{13}$ as a possibility. She proceeded to also suggest that $\frac{13}{14}$ and $\frac{14}{15}$ would also work as fractions between $\frac{11}{12}$ and 1, indicating that $\frac{13}{15}$ might also work, as “it’s not smaller by that much”.

Throughout this conversation, there was evidence of some non-standard fraction language: “nominator” for “numerator” and “irregular fraction” for “improper fraction”. When asked if she ‘had a picture in her head’ or whether she normally drew anything to help her figure things out, she sketched a simple fraction wall, showing a unit, halves and quarters and stated, “I basically kind of made like this, um, like number chart in my head, and the units, um, like a fraction wall thing in my head”. This was constructed post-solution of the tasks and only at the prompt of the interviewer (Figure 1). On a related question, later in the pre-interview, Cara used a formal equivalent fractions procedure to compare $\frac{5}{8}$ and $\frac{1}{2}$. That is, she generated a list of multiples of each denominator to identify an appropriate common denominator and proceeded to multiply the numerator and denominator of $\frac{1}{2}$ by 4 to produce the equivalent fraction of $\frac{4}{8}$ (Figure 1). Alongside this procedural approach, she was able to use $\frac{1}{2}$ as a benchmark to reason about their relative size. She compared $\frac{2}{5}$ and $\frac{1}{3}$ in the same way, despite being able to immediately state 15 as common denominator. Across Cara’s comments in the pre-intervention interview, we see indications of an ability to work with half and 1 as benchmarks for simpler fractions but not for fractions with more awkward denominators, where her preferred approach was to compare the numbers only with no reference to the logic of an underpinning image.

Figure 1

Sample inscriptions from the pre-intervention interview – comparison tasks



Further questions probed estimates and methods for completing fraction calculations. Despite being prompted to offer estimates, in the pre-interview, Cara did not offer them. For the fraction addition question, $\frac{5}{8} + \frac{1}{2}$, she successfully used the common denominator procedure. For the multiplication question, $\frac{3}{4} \times \frac{1}{2}$, she completed this successfully multiplying the numerator by the numerator, and denominator by denominator noting that you: “don’t have to worry about the different denominators cos its multiplying”. When prompted she could not offer any images or other ways to decide if her answer was sensible. For the division, question $(8 \div \frac{4}{5})$, she recalled the KCF (Keep, Change, Flip) method which she had learnt at an online class. She offered no images or estimation of this calculation.

The same, or similar, questions were posed in the post-intervention interview. In general, improved fraction language and fluency of reasoning were evident. She used diagrams spontaneously and showed a more judicious use of procedures. When, asked for a fraction that lies between $\frac{1}{2}$ and 1, she offered $\frac{5}{8}$, stating “one half needs another half to make a whole, while $\frac{5}{8}$ only needs $\frac{1}{8}$ to make a whole. I know $\frac{1}{8}$ is smaller than a half”. When she was asked to compare $\frac{5}{8}$ and $\frac{1}{2}$, Cara stated that she knew $\frac{5}{8}$ is larger “by looking at it”. Unprompted, she sketched a mini diagram (Figure 2) which shows a quarter equivalent to $\frac{2}{8}$, and said, “which means four eighths will be a half. It’s only bigger by one eighth”. Significantly, she noted that she might employ a different strategy involving getting common denominators for more complex examples: “if it was a harder one, I’d probably use the one that’s making them the same fraction. It’s easier to draw it because you can see it more”.

Figure 2

Sample inscriptions from the post-intervention interview- comparison and multiplication tasks

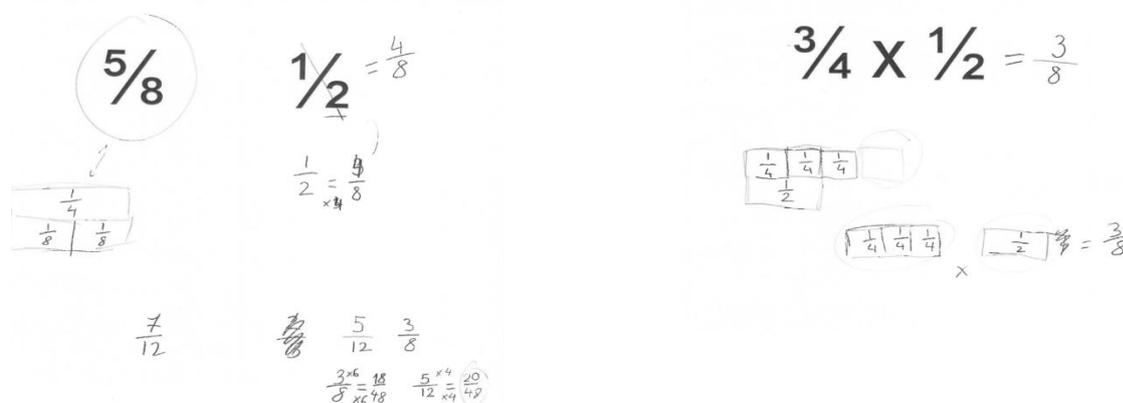


Figure 2 also shows some of Cara’s responses when asked to identify a fraction between $\frac{5}{8}$ and $\frac{1}{2}$. She considered $\frac{2}{3}$, then rejected it with the justification that being a $\frac{1}{3}$ away from a whole is less than $\frac{3}{8}$ away from a whole. She surmised that a larger denominator would work, and suggested twelfths, noting, “Has to be bigger than six-twelfths, so ... seven-twelfths?” While she was somewhat hesitant to proffer definitive claims, her persistence in investigating this was evident in the data, where she generated equivalent fractions using common denominators and compared with a whole to check that $\frac{5}{12}$ is larger than $\frac{3}{8}$ as the pieces needed to make up the whole, and therefore, that $\frac{7}{12}$ is further away from the whole than $\frac{5}{8}$.

In each of the calculation questions post-intervention, she offered an estimation before attempting the calculation. For the addition question, $\frac{5}{8} + \frac{1}{2}$, she quickly offered that it would be more than 1 and less than $1\frac{1}{2}$, and noted that she would use the ‘denominator’ way to get the answer but could check with a drawing afterwards. She quickly and correctly decided on 8 as the lowest common denominator, produced $\frac{4}{8}$ as equivalent to $\frac{1}{2}$, and added to get $\frac{9}{8}$. She stated: “it’s bigger than denominator” and proceeded to write $1\frac{1}{8}$. Following this she drew an abbreviated diagram showing a $\frac{1}{8}$ strip x 9. She also worked from a procedure-based answer back to a representation in the multiplication and division calculations. On the task, $\frac{3}{4} \times \frac{1}{2}$,

she initially guessed that it would be bigger than one, then self-corrected, saying, “Yeah but now I realise that’s not true, cos you don’t add them. You multiply them”. She correctly produced $\frac{3}{8}$ as the answer. Though her attempt at diagramming this does not constitute an accurate representation of the mathematical operation (see figure 2), her explanation gives evidence of her understanding: “You take up three quarters half times cos your multiplying. And that will make it $\frac{3}{8}$... because, three of .. three quarters .. and you .. cos you don’t take it like a whole number times, you only do it by a half a .. like a half and a whole.”

Conclusion

In the post-interview, an expansion in Cara’s ‘universe of discourse’ (Mason, 2011) is evident. She generated representations for comparison and for addition (and subtraction- not discussed due to space limitations) and showed competent use of $\frac{1}{2}$ and 1 as benchmarks. Her abbreviated diagrams evidence her strong visual sense of the relationships between the sizes of different fractions. For multiplication and division, Cara’s ‘universe of discourse’ remains restricted to the procedure, but we note a new willingness to try and make sense of operations with diagrams and estimation - both of which were absent in the pre-interview.

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Symposium

Reimagining pedagogical practices in primary mathematics through the lens of the new curriculum

This symposium explores how core pedagogical practices from the new Primary Mathematics Curriculum (NCCA, 2024) are being interpreted and enacted in Irish primary classrooms. Four papers examine broader curriculum enactment, purposeful maths talk, mathematical modelling, and cognitively challenging tasks - each central to the curriculum's vision of learner agency, conceptual depth, and inclusive engagement. Paper 1, presented by curriculum developers at the NCCA, explores how curriculum vision and pedagogical principles—especially playfulness and productive disposition—are designed to support meaningful, equitable learning experiences and how these are being translated from policy into classroom practice. Paper 2 analyses how teachers facilitate meaningful mathematical dialogue, navigating tensions between traditional and dialogic norms. Paper 3 presents a case study of teachers using Model-Eliciting Activities to support abstraction and agency through modelling. Paper 4 introduces the F-PosE framework, developed through design-based research, to support teachers in selecting and designing rich, challenging problems. Together, the papers offer a timely and grounded account of curriculum reform in action.

The *primary mathematics curriculum* in practice: Realising the vision

Gráinne Higgins, Tracy Curran

National Council for Curriculum and Assessment

The new *Primary Mathematics Curriculum* (PMC) (Department of Education [DE], 2023a) in Ireland sets forth a fresh vision for the learning, teaching and assessment of Mathematics in primary and special schools. This paper offers an overview of the vision of the PMC including the curriculum rationale and the overarching aim of mathematical proficiency. In addition, it will explore aspects of the curriculum specification that support the transition from policy into practice, shining a spotlight on the key pedagogical approaches promoted in the curriculum with particular emphasis on fostering productive disposition and encouraging playfulness (DE, 2023a). Consistent with the theme of the symposium, the paper offers an insight into the significance of such pedagogy, as informed by contemporary research, in the provision of learning experiences that are optimum for the mathematical learning of children in primary and special schools.

Keywords: Curriculum development, primary mathematics, curriculum enactment

Introduction

The National Council for Curriculum and Assessment (NCCA) is a statutory body of the Department of Education and Youth (DEY). NCCA advises the Minister for Education and Youth on matters relating to curriculum and assessment for early childhood, primary and post-primary education.

In 2023, a new *Primary Mathematics Curriculum* (PMC) (DE, 2023a) was published. This new curriculum builds upon the vision of and approaches to learning, teaching and assessment captured in the overarching *Primary Curriculum Framework* (PCF) (DE, 2023b). The PMC presents Mathematics as a worthwhile, meaningful and playful pursuit for all children (DE, 2023a) and is underpinned by an extensive body of research, as well as being informed by sustained deliberations, networking and consultation with key education stakeholders.

Fundamental to the enactment of the PMC is the adoption of five key pedagogical approaches – fostering productive disposition, encouraging playfulness, emphasising mathematical modeling, using cognitively challenging tasks and promoting maths talk (DE, 2023a). The inclusion of these approaches stems from an extensive body of contemporary research. Included in this are *Mathematics in early childhood and primary education (3-8 years): Research report no. 17* (Dunphy et al., 2014), *Mathematics in early childhood and primary education (3 – 8 years): Research report no. 18* (Dooley et al., 2014) as well as *Learning and teaching primary mathematics: An addendum to NCCA research reports 17 and 18* (Dooley, 2019). These offer theoretical perspectives and empirical findings of pedagogical approaches that are most effective in the development of mathematical proficiencies.

In this paper, the new vision for children’s mathematical learning that is encapsulated in the PMC will be outlined. Furthermore, key components of the PMC will be discussed with a focus on the role that pedagogy plays in the provision of optimum learning experiences for

children. In particular, two of these pedagogical approaches will be explored, namely encouraging playfulness and fostering productive disposition.

Enacting the Primary Mathematics Curriculum

The PMC outlines important features of learning and teaching that support the development of children's mathematical proficiency (DE, 2023a). This term encompasses the web of knowledge, skills, abilities and beliefs that children develop in becoming more confident and competent mathematical learners. Mathematical proficiency comprises of five aspects - adaptive reasoning, strategic competence, conceptual understanding, procedural fluency and productive disposition (DE, 2023a; Dunphy et al. 2014). These are interwoven and interdependent, with each aspect complementing and strengthening the others. Within the curriculum specification, five key pedagogical approaches that support the provision of quality mathematics learning are spotlighted. These pedagogical practices – fostering productive disposition, encouraging playfulness, emphasising mathematical modeling, using cognitively challenging tasks and promoting maths talk – engender the type of learning experiences that research shows support the development of mathematical proficiency. The five practices naturally link with each other and aim to foster an inclusive learning environment and culture where children engage in rich and meaningful learning processes. This paper will explore in more depth two pedagogies in particular – encouraging playfulness and fostering productive disposition.

Encouraging playfulness

Encouraging playfulness is an essential pedagogical practice for providing quality mathematical learning experiences for children (DE, 2023a; Dooley et al., 2014). It is noteworthy that playful learning is appropriate for children across all stages of primary and special education (DE, 2023b; Dooley, 2019). The inherent playfulness of children, and indeed the playful dispositions of teachers, can be capitalised upon in the learning at teaching of mathematics (Perry & Dockett, 2007).

Playfulness holds strong significance in the principles and pedagogical approaches of the PCF and the PMC. Its importance is evidenced through its explicit inclusion in the stem of each Learning Outcome (DE, 2023a). Across all five Strands (algebra, data and chance, measures, number, shape and space), children work towards Learning Outcomes “through appropriately playful and engaging learning experiences” (DE, 2023a, p. 18). These promote the provision of learning experiences that are open, motivating and accessible, and that also encourage children to explore the creativity, challenge and beauty of Mathematics. The inclusion of practical examples of playful mathematics within the PMC (DE, 2023a) supports a shared understanding of what this pedagogical approach looks like in the classroom. Playfulness incorporates creativity, imagination and exploration, but also encompasses children and teachers' playful dispositions, and the use of children's interests and curiosities to inspire meaningful mathematical learning.

As outlined in the PMC, pedagogical practices do not occur in isolation (DE, 2023a). Playful learning provides teachers with a powerful context through which they can facilitate and encourage maths talk in ways that are both engaging and meaningful for children.

Through carefully designed or chosen playful tasks, children are empowered to develop their understanding in creative and explorative ways, while also being provided with an appropriate level of cognitive challenge. When provided with appropriate supports and encouragement, children can model mathematical learning in ways that resonate with them.

Fostering productive disposition

Productive disposition relates to the belief that mathematics is useful and worthwhile, and that the process of learning – including struggle and perseverance - is to be valued (Dunphy et al., 2014; DE, 2023a). Teachers' own attitudes towards Mathematics, and to its relevance in our world, are powerful in nurturing children's dispositions. Creating a learning environment in which mistakes are an essential part of the learning process further contributes to the productive disposition of children. The pedagogical practice of fostering productive disposition is interwoven with other key pedagogical approaches. Cognitively challenging tasks that offer a 'low threshold, high ceiling' encourage children to engage in productive struggle, to take risks, and to persevere (NCCA, 2022). This supports children to recognise that success in Mathematics is achievable for all with suitable support, effort and attitude (Boaler, 2016). Furthermore, providing opportunities for children to engage in maths talk and mathematical modeling encourages children to focus on the process of learning, rather than the product alone.

As with encouraging playfulness, the significance of a productive disposition towards Mathematics is strongly communicated across the PMC and to date is well embedded in Irish classrooms (DEY, 2024). It is both a facet of mathematical proficiency, as well as one of the five key pedagogical practices outlined in the PMC (DE, 2023a). Productive disposition is reinforced through the rationale of the PMC by presenting Mathematics as a worthy pursuit for all, laying the foundation for the development of a positive identity as a mathematical learner.

Conclusion

This paper, in line with the aims of this symposium, presents a reimagining of pedagogy through the lens of the PMC (DE, 2023a). With the PMC now being enacted in all primary and special schools, a focus on the move from policy to practice is imperative. The PMC lays out a clear vision for mathematical learning that is accessible for all, and that recognises the pervasiveness of Mathematics in our lives and the wider world (DE, 2023a). While the underpinning rationale is clearly outlined in the PMC, change in curriculum requires change in attitudes and in practice. The explicit inclusion of key pedagogical approaches supports teachers to embrace this change, providing clear guidance on the kind of learning experiences that research shows support children as they strive towards mathematical proficiency.

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Purposeful Maths Talk: Intentions, Enactments and Tensions

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This paper focuses on Maths Talk as a core pedagogical practice of the Primary Maths Curriculum (PMC) for promoting reasoning, sensemaking, and collaboration. Drawing on sociocultural theories of classroom discourse and existing research in the field, this paper explores how teachers facilitate purposeful talk in classrooms and how their questioning and framing of contributions influence the mathematical quality of dialogue. The paper highlights promising shifts in teacher practices but also identifies enduring challenges, such as managing competing discourses of correctness and efficiency.

Keywords: Primary Maths Curriculum, maths talk, dialogue, pedagogical practices, teacher questioning

Introduction

How teachers teach is as important as ‘what’ they teach. Maths Talk is one of the five pedagogical practices embedded in the PMC in Ireland to enhance and support learning (NCCA, 2024). These pedagogical practices are a series of interconnected meta – practices designed for use by teachers and are recognised as fundamental to delivering high-quality mathematical experiences for pupils (NCCA, 2024). Rooted in international research, the practices embody the PMC’s fresh vision for children’s mathematical learning that is characterised by playfulness, creativity, challenge, risk-taking, collaboration and opportunities for reasoning and solving real-life problems (NCCA, 2024).

Maths Talk as a pedagogical practice of the PMC

Maths Talk is defined in the PMC as a ‘collaborative process where children’s thinking, strategies and ideas are expressed, shared and/or exchanged’ (NCCA, 2024, p. 32). In this sense, learning the language of mathematics does not entail just learning new words. The curriculum presents a much broader vision of Maths Talk which is based on inclusivity, playfulness, collaboration and agency (NCCA, 2024).

Constructivist and sociocultural theories of learning highlight the central role of language in fostering young children's mathematical understanding (Piaget, 1952; Vygotsky, 1978). More recent theoretical developments further underscore the significance of mathematical discourse as a medium through which learners construct mathematical knowledge (Sfard, 2007). Sfard introduces the notion of “commognition,” a blend of communication and cognition, to highlight how thinking is formed through participation in specific discursive practices. Taken together, these perspectives converge on the view that structured, purposeful talk in mathematics classrooms is essential for developing both conceptual understanding and mathematical identity.

While the term ‘Maths Talk’ is widely referenced in both academic literature and practitioner resources, notable differences exist between the conceptualisation of Maths Talk in the PMC (NCCA, 2024) and those in other literature. For instance, the widely cited framework developed by Hufferd-Ackles et al. (2004) conceptualises Maths Talk through the

lens of classroom discourse progression. The framework identifies a trajectory from teacher-directed exchanges to student-led dialogue across four dimensions: questioning, explanation of thinking, source of ideas, and responsibility for learning. Although this model aligns with the PMC's (NCCA, 2024) emphasis on student voice, it adopts a more narrowly defined focus on the structural and developmental aspects of discourse within individual classrooms, whereas the PMC offers a more holistic and pedagogically integrated perspective.

On a similar note, Mercer and Sams (2006) conceptualise *Maths Talk* as a form of peer dialogue that supports joint reasoning and problem-solving. Their work highlights how shared language practices support cognitive development, with an emphasis on the social construction of meaning through collaborative dialogue. While this aligns with the sociocultural foundation of the definition of Maths Talk in the PMC (NCCA, 2024), Mercer and Sams (2006) conceptualisation of Maths Talk focuses more specifically on peer interactions and group reasoning than on curriculum-wide pedagogical integration.

Teacher as facilitator of Mathematical Discourse

Dooley et al (2014, p.37) highlight the important role the teacher plays in assisting children in using and articulating mathematical language in their 'descriptions, explanations and justifications'. The Primary Mathematics Toolkit (NCCA, 2024) identifies several core teaching practices which teachers can utilise to support the development of both teacher led and pupil led Maths Talk in the classroom. These include the strategic use of 'talk moves' to engage pupils in dialogue for example revoicing, where the teacher restates a student's contribution to clarify or extend meaning as well as the use of effective questioning, the explicit connection of mathematical ideas and the incorporation of children's thinking to advance discussion, particularly through the identification and exploration of misconceptions (Chapin et al, 2009).

The pivotal role of the teacher in facilitating Maths Talk, as outlined in the PMC (NCCA, 2024), marks a notable departure from the more traditional, didactic models of mathematics instruction in earlier curriculum iterations where the teacher was largely positioned as the provider of "appropriate mathematical language" to pupils (NCCA, 1999, p. 6). This approach, while acknowledging the importance of language in mathematics, often situated the learner in a more passive role, with limited opportunities for dialogic engagement or shared meaning-making. In contrast, the latest edition of the PMC (NCCA, 2024) redefines the teacher's role as a facilitator of mathematical dialogue—one who cultivates a classroom culture where children's ideas are valued, reasoning is encouraged, and all learners are supported.

Challenges to the Teacher facilitating Maths Talk within the PMC

While the availability of resources to support teachers in facilitating Maths Talk is beneficial, one must consider the challenges which arise when trying to develop Maths Talk in classrooms. In analysis of mathematics lessons, NicMhuirí (2011, cited in Dunphy et al, 2014, p. 63) found that 'important opportunities for engaging in mathematical dialogue, including mathematical reasoning, may be overlooked'. This and similar work (e.g., Dooley, 2011) point to a need to support teachers to reflect on their classroom dialogue and provide children

with more opportunities to engage in mathematical thinking, along the lines described earlier. Specifically, it is claimed that there is an ‘urgency’ in promoting more interactive mathematical discourse in learning settings. (Dunphy et al, 2016, p.64).

Furthermore, Mathematics, perhaps more than any other subject, has historically been associated with a “chalk and talk” pedagogy, which prioritises teacher-led instruction and rote procedures over meaningful dialogue and conceptual understanding (Boaler, 2016). This traditional mode of instruction can lead to what Boaler (2009, p. 35) describes as “learning without talk” and “learning without thought”. In such environments, students are often positioned as passive recipients of knowledge rather than active participants in constructing mathematical dialogue. As a result, opportunities for developing Maths Talk are significantly diminished.

What’s more Boaler (2009) makes the point that attempts to change our approaches to the teaching of Mathematics often co- exist with entrenched expectations that privilege efficiency, correct answers and procedural fluency. This tension can create a pedagogical dilemma: teachers may feel compelled to prioritise efficient coverage of curriculum content and standardised test preparation over the development of Maths Talk which by its nature is a slow, open – ended process. Moreover, if the dominant discourse of correctness and efficiency prevails, Maths Talk risks being viewed as an ‘add on’ or supplementary activity rather than a core pedagogical practice of the PMC (NCCA, 2024).

Conclusion

Maths Talk as presented in the PMC (NCCA, 2024) conceptualises discourse not merely as a strategy but as a foundational element of a reimagined mathematics education in Irish primary classrooms. The success of Maths Talk will not be instant: it will take time for both teachers and pupils to grow accustomed to their changing role in the classroom – the teacher to managing the discussion rather than directing or dictating, the pupils to articulating their thinking. For *Maths Talk* to move from policy to practice, the purposes of Maths Talk, as well as the conditions under which it thrives needs to be understood.

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Curriculum meets classroom: A case study of teachers' engaging with MEAs in the Irish primary school classroom

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This paper investigates participants' understanding of mathematical modeling, and how it evolved through implementation of Modeling Eliciting Activities (MEAs) in the Irish primary school classroom. It explores the teaching practices engaged with, and the strengths and challenges encountered. A case study methodology was used, where five participants (primary school teachers from third - sixth class) formed a Professional Learning Community and engaged in three cycles of action research over a six-month period. Participants investigated research in the field, reflected on and implemented their learning in the classroom. The paper also reflects on how modelling practices interacted with teachers' mathematical beliefs and confidence.

Keywords: Mathematical modeling, complex problem solving, pedagogical practice

Introduction

The introduction of the Primary Curriculum Framework early in 2023 established a vision for teaching, learning, and assessment, supporting high-quality provision that is inclusive and grounded in evidence. In the *Primary Mathematics Curriculum* (PMC; 2023), 'how' children learn is as important as 'what' they learn. Mathematisation is a key aim of the *PMC* (2023) to promote mathematical proficiency, where emphasis is placed on expressing real-world problems in mathematical terms in order to problem-solve (OECD, 2013). Pedagogical practices are "central to the provision of high-quality mathematical learning experiences" (p.2) and mathematisation. This research concerns professional development and guidance in implementing mathematical modeling in the primary classroom.

Model Eliciting Activities (MEAs) are utilised in this research as a problem-solving task that is designed to reveal learners' mathematical thinking and reasoning. It is grounded in the real world, which requires the learner to engage in the process of mathematical modeling (Lesh & Doerr, 2003). Mathematical modeling is a process where pupils engage in realistic problem-solving, relevant to them, and generate mathematical ideas through enquiry. As a result of engaging in mathematical modeling, pupils generate artefacts or conceptual understandings (models) that can be shared to explain the problem situation. A mathematical model is the representation of a real-world situation using mathematical concepts, structures and language. It is expected that models become more sophisticated through engagement in multiple modeling cycles. The *PMC* (2023) states that accuracy is valued; however, "mathematical modeling places more importance on exploration, sense-making, conceptual understanding and flexibility in thinking" (p.30).

A gap in the literature field exists in the primary education context to support the implementation of mathematical modeling, especially in professional development. Three research questions were posed, investigating how participants' understanding of mathematical

modeling evolved during and after the research, the teaching practices engaged with, and the strengths and challenges encountered.

Methods

A case study approach was employed to investigate how teachers' understanding of mathematical modeling evolved through engagement in multiple cycles of action research. Five participants teaching third to sixth class formed a Professional Learning Community with the common goal of understanding and implementing mathematical modeling in their classrooms. Action research (McNiff, 2017) was adopted as the professional development model, underpinned by the TRU Framework (Schoenfeld, 2016). The TRU Framework is a research-based approach to designing and evaluating learning environments that promote deep understanding and engagement. It supports educators to reflect on and improve their practices. It explores five dimensions of powerful classrooms: content, cognitive demand; equitable access to content; agency, ownership and identity; and formative assessment (Schoenfeld, 2016). The TRU Framework was used in the design of data collection tools, but also as a tool for reflection for the participants as part of the study. Data collection tools comprised transcripts of action research meetings, a reflective diary, and participants' reflections on learner artefacts or models. An exit interview was conducted at the end of the research to established participants' understanding of mathematical modeling at the end of the research.

Participants began by documenting their understanding of mathematical modeling and mathematical models at the beginning of the research in their reflective diary. They were then introduced to some definitions of mathematical modeling, the TRU Framework and action research principles. Participants set a long-term learning goal from the TRU Framework that they focused on with their class throughout the research. Each action research cycle was broken down into five steps: observation, reflection, action, evaluation, and modification (McNiff, 2017). During the observation phase, the participants engaged with research articles from Lyn English's longitudinal study on complex problem-solving (English & Watters, 2004; English & Watters, 2005; Doerr & English, 2001). Participants reflected on key learning once the articles were engaged with. They implemented the MEAs inherent in the article with their class and then evaluated how the lesson went at the next action research meeting. Teachers discussed modifications they would make and applied these changes to the next cycle of action research. The researcher observed the third action research cycle lesson using the TRU Framework to support participant reflection. Braun and Clarke's (2022) Thematic Analysis was conducted to generate themes that emerged from the research.

Findings

Teacher Understandings of Mathematical Modeling

Participants generated new understandings of mathematical modelling through engagement in the professional development model of action research combined with the TRU Framework. Participants reflected on mathematical models from their class, including multiple representations of informal and formal models. These models were meaningful for the learners in the classroom, and therefore, participants believed they were more inclusive

than traditional problem-solving tasks. Participants began to analyse different stages of the mathematical modeling cycle. For example, they discussed mathematisation of the problem situation and the conceptual understandings explored: “The idea of averages was too complex for them, and they tended to rely on more simple procedures such as repeated addition/subtraction” (P5, ARC 1, Reflective Diary). This class had not explored the concept of averages before engaging with this task, but were still able to mathematise the situation using mathematical concepts that made sense to them. The research illustrated participants growing understanding of mathematical modeling, thus demonstrating the effectiveness of the professional development model.

Interdependencies of Teaching Practices Engaged With

The participants' teaching practices were central to the research, as they engaged with the TRU Framework. Four teaching practices emerged strongly from the dataset. This included engaging with group dynamics, the dialogic classroom, ascertaining prior knowledge of the learner, and agency (finding the balance between teacher and learner agency). Participants experienced challenges and strengths when focusing on group dynamics when implementing mathematical modeling in the classroom. P3 had picked pupils who were perceived to be good at mathematics to lead the groups in action research cycle two and found that the dynamics were not conducive to developing mathematical models. In action research cycle three, P3 looked at pupils' personalities instead of mathematical ability and found that the groups worked much better when engaging in the mathematical modeling cycle. The dialogic classroom was key for all participants in the research and played a central role in the modeling cycle. For example, the pupils “had great conversations and discussions and were able to discuss their ideas and try different approaches” (P1, ARC 1, Reflective Diary). Participants introduced planned pauses in the modeling cycle to support the learner “to allow for a whole-class discussion, and then also, a kind of midway interruption. . . because if there is something that you think that they might be missing” (P3, ARC2, Transcript). P4 discussed the value of the dialogic classroom at the end of the modeling cycle, where the learners “could agree or disagree and have an open discussion. The children gave really good feedback. Very confident in their answers. Some noticed similarities to each other and questioned how they did it” (ARC 1, Reflective Diary). Participants commented on the importance of argumentation in the modeling cycle, “highlighting how an activity should encourage conflict, discussion and difference of opinion” (ARC 2, Reflective Diary). Prior knowledge of the learner was contentious for the participants in the research. P2 described a situation where a child had strong prior knowledge of farming within the group when engaging with the *Butterbean task*, but did not have the ability to convince the group of this understanding. The learner needed support in building consensus in the group dynamic. For other pupils, prior knowledge supported them in approaching the problem and gave them “greater confidence and comfort” (P2, ARC 1, Reflective Diary). P3 described trying to find the balance of teacher and child agency like a “tightrope between assisting and stepping away” (ARC 2, Reflective Diary). P4 found that they stepped too far back at times when they reflected on their practice “I’ve been struggling with the change in the style of teaching. I need to stop worrying about doing it right and being afraid to guide when needed” (ARC 1, Reflective Diary). Participants found that questioning supported agency in the mathematical modeling

process. From engaging in these four teaching practices, participants highlighted the interdependencies that existed between them and its importance in implementing mathematical modeling in the classroom.

Strengths and Challenges in the Implementation of Mathematical Modeling in the Irish Primary Classroom

Sharing of solutions worked very well in this research where multiple representations were utilised. These included letter writing to support verification of models, three reasons to support their solutions, posters to prove the effectiveness of their solutions, and interviews with the groups to establish thinking inherent in mathematical models. Incorrect assumptions were made by pupils in the modeling process and as a result mathematical models were inaccurate. Pupils often failed to realise their mistakes. Participants introduced a pause in the lesson (as outlined above) to support pupil discussion in hope that pupils would recognise incorrect assumptions made. However, this was not always successful. It was hoped that groups would realise inaccurate models in the verification stage of the modeling cycle but they often seemed oblivious to it. Lack of solution pathways frustrated the participants when most of the class engaged with the same conceptual models. The models utilised were basic, but there were more efficient and effective models that the groups could have pursued. One example from the research exemplified the lack of solution pathways in the *Aeroplane task*. Pupils had to come up with a system to select a winner from a set of data showing distance, time in the air and if the paper aeroplanes had scratches. Most groups just added all the values together and selected the winner based on that model. Participants wondered why the pupils hadn't thought about systems that would be used in the Olympics etc... when choosing their models. Through discussion they realised that pupils in this class were probably too young the last time the Olympics were on due to Covid 19. Participants found it challenging to spotlight more accurate models.

Conclusion

By engaging in the professional development model adopted in this research, participants demonstrated beliefs in process-oriented goals where the focus was on learner thinking. Participants reflected on the types of word problems they typically explored with their class and the limited opportunities they offered to the learners. Engagement in MEAs presented opportunities for “critical thinking”, “perspective taking” (P1, Transcript, ARC 2) and the opportunity “to work together to problem-solve in groups” (P1, Reflective diary, ARC 3). Features of MEAs that supported the learners' engagement in complex problem-solving included the inclusion of real-life situations (P1, ARC2, Reflective Diary), variables within the task (Group 3, ARC 1 Transcript), and the “variety of ideas” (P4, ARC 1, Reflective Diary). Overall, participants reflected on pupil “exploration, sense-making, conceptual understanding and flexibility in thinking” (PMC, p.30) demonstrating strong engagement with the mathematisation stage of the modeling cycle striving for accuracy.

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The F-PosE framework: Supporting primary teachers in unpacking cognitively challenging mathematics tasks

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This paper introduces the *Framework for Posing Elementary Mathematics Problems* (F-PosE), a research-informed tool designed to support primary teachers in identifying and designing cognitively challenging mathematics tasks. Developed through design-based research involving pre-service teachers, in-service teachers, children and teacher educators, the framework responds to the pedagogical emphasis on rich problem-solving within the *Primary Mathematics Curriculum* (NCCA, 2024). F-PosE comprises eight indicators that reflect the key features of worthwhile mathematical tasks, including cognitive demand, openness to multiple strategies and solutions, and contextual relevance. Each indicator is supported by focus questions to guide teachers' critical engagement with existing problems or the design of new ones. This paper outlines the development of the framework and discusses its potential to enhance task selection and adaptation, particularly for generalist teachers. Positioned within the broader context of curriculum reform, F-PosE contributes to the enactment of ambitious pedagogies that promote mathematical thinking and learner agency.

Keywords: F-PosE, worthwhile mathematical problems, cognitively challenging tasks

Introduction

The Primary Mathematics Curriculum (NCCA, 2023) outlines five key pedagogical practices essential for providing quality mathematical learning experiences. These practices include Fostering productive disposition, which cultivates a positive view of Mathematics as worthwhile and useful, and Encouraging playfulness, integrating playful and engaging activities into learning across all stages. Additionally, teachers should be Emphasizing mathematical modeling for children to describe problems and find solutions, Using cognitively challenging tasks that stretch conceptual understanding, and Promoting maths talk to facilitate the collaborative sharing of ideas and strategies.

The inclusion of cognitively challenging tasks can be seen as a central pedagogical commitment of the Primary Mathematics Curriculum (PMC) (2024), which emphasises the importance of providing “interesting, relevant and appropriately challenging experiences” for primary children (p. 3) and highlights their role in promoting conceptual understanding, encouraging discourse, and fostering perseverance and resilience (NCCA, 2024, p. 30–31). Cognitively challenging tasks—often referred to as “low-threshold, high-ceiling”—are designed to be accessible to all learners while supporting meaningful mathematical engagement. While the term “low-threshold, high-ceiling” is often used synonymously with cognitively challenging tasks, the two are not entirely analogous. A task can be cognitively demanding without being accessible to all learners, particularly if it lacks entry points or assumes prerequisite knowledge. The F-PosE framework (described below) encourages

attention to both challenge and accessibility, helping teachers to ensure that tasks invite all learners into deep mathematical engagement.

Cognitively challenging tasks align with the curriculum's vision for learner agency and productive disposition, and are supported by international research for their potential to foster higher-order thinking and problem-solving. They enable learners to engage with mathematics as dynamic and connected, offering opportunities to grapple with complex ideas, make conjectures, and justify reasoning—essential for mathematical proficiency. This emphasis necessarily foregrounds the critical role of teachers in selecting and posing rich mathematical problems. The ability to effectively pose problems—that is, to select or design tasks that engage learners in desirable mathematical practices—is recognised as a key element of high-quality mathematics teaching (Watson & Ohtani, 2015).

Current research, however, highlights ongoing challenges for pre-service teachers when attending to cognitive challenge during mathematical problem posing. This includes difficulty anticipating student thinking and a tendency to revert to procedural or low-demand tasks (Crespo & Sinclair, 2008; Crespo, 2003; Silver et al., 1996; Singer & Voica, 2013; Leavy & Hourigan, 2020; 2022a). In response, the researchers developed a pedagogical framework to guide pre-service teachers in analysing, selecting, and adapting problems that reflect the characteristics of cognitively challenging tasks.

Methods

The development of the Framework for Posing Elementary Mathematics Problems (F-PosE) was situated within a design-based research methodology and evolved through a three-phase process: preliminary, prototyping, and assessment (Cobb et al., 2003; Plomp, 2007). The study was underpinned by an extensive review of the literature on problem posing and cognitively demanding tasks, and by document content analysis of mathematics problems designed for elementary learners. Central to the framework's development was a participatory research approach involving a community of educators, pre-service primary teachers, in-service classroom teachers, children and mathematics teacher educators. This collaborative effort aimed to support the development of pre-service primary teachers' capacity to pose rich mathematics problems appropriate for primary classrooms. The three-phase design research approach included preliminary (Leavy and Hourigan, 2020), prototyping and assessment phases (see Leavy & Hourigan, 2022b)

In the **preliminary phase**, a quasi-experimental pre/post design with 415 pre-service teachers was used to identify desirable characteristics of mathematics problems. This informed the creation of an initial set of problem features. The **prototyping phase** involved iterative classroom-based testing of these characteristics through five microcycles of problem posing. This phase engaged 28 pre-service teachers, three in-service primary teachers, and 56 fifth-class children. Data collected included 150 unique mathematics problems, pre- and post-problem appraisals, samples of pupil work, feedback from classroom teachers, field notes, and survey responses. The final **assessment phase** entailed a retrospective analysis of the entire data corpus using a grounded theory approach, with constant comparative methods (Ralph et al., 2015) used to refine and categorise emerging indicators. These indicators ultimately

informed the F-PosE framework, designed to support pre-service teachers in evaluating and selecting cognitively challenging tasks that align with curriculum goals and foster deep mathematical thinking.

The grounded analysis was instrumental in surfacing features not consistently emphasised in prior frameworks, such as the importance of *opportunity for success* and *curriculum coherence*, which were added to F-PosE to reflect the need for both accessibility and alignment with learning goals identified through participant responses and problem appraisals.

Findings

The data analysis process culminated in the development of a framework that encapsulated the characteristics of high-quality mathematics problems (see figure 1). The resulting *Framework for Posing Elementary Mathematics Problems* (F-PosE) comprises eight indicators: (1) use of a motivating and engaging context, (2) clarity in language and cultural context, (3) curriculum coherence, (4) attention to cognitive demand, (5) an appropriate number of solution steps to support reasoning, (6) a variety of solution strategies, (7) facilitation of multiple solutions, and (8) opportunity for success. Each indicator is accompanied by guiding questions to support pre-service teachers in critically evaluating the extent to which a problem reflects these features. In doing so, the framework informs their decision-making processes around the selection, adaptation, or creation of rich and cognitively demanding mathematical tasks (cf. Leavy and Hourigan, 2022b).

Conclusions

The F-PosE framework is grounded in research on problem posing and emerged from an iterative design process shaped by the collective practices of educators and children. This collaboration led to a practical tool to support pre-service teachers in designing worthwhile mathematics problems that promote deep engagement and understanding. Within the *Primary Mathematics Curriculum* (NCCA, 2024), which highlights cognitively challenging tasks as a key pedagogical practice, F-PosE addresses a clear need by helping both pre-service and practising teachers recognise the features of rich mathematical tasks. While F-PosE was developed primarily to support pre-service teachers in learning to identify and design cognitively challenging tasks, its application extends beyond the ITE context. The framework is also well-suited to newly qualified teachers and practicing generalist primary teachers, particularly those seeking structured support in implementing the pedagogical aims of the new curriculum. By offering clear indicators and guiding questions, F-PosE serves as a practical tool for any educator aiming to deepen mathematical engagement through rich task design.

Through focused indicators and guiding questions, the framework supports the selection and adaptation of problems aligned with the curriculum's vision of learner agency, problem-solving, and conceptual understanding. Particularly for generalist primary teachers, F-PosE can complement textbook materials, ensuring children engage with meaningful, challenging tasks that foster mathematical proficiency. In line with the aims of this symposium, the framework illustrates how structured support for problem posing can bring

curriculum principles to life and enhance teacher agency in creating rich mathematical learning environments.

Figure 1

The Framework for Posing Elementary Mathematics Problems (F-PosE)

Indicator I. Use of a Motivating and Engaging Context

To what extent does the problem use a motivating and engaging context?

Absent Weak Adequate Good Excellent

Does the problem:

- Establish a connection between mathematics and students' real or imaginary worlds?
- Allow students to draw on their personal experience and knowledge to make sense of the situation?
- Present a context with a meaningful and compelling purpose?
- Engage and motivate all students?

Indicator II. Clarity in Language and Cultural Context

To what extent is there clarity in the problem presentation?

Absent Weak Adequate Good Excellent

Does the problem:

- Incorporate language and terminology that makes it accessible to the reading level of target students?
- Reflect the mathematics cultural systems of the students? Do the measurement (metric/imperial) and currency (Euro/Dollar/Yen etc.) systems align with those of students?
- Draw on contexts that are culturally accessible to all students?

Indicator III. Curriculum Coherence

To what extent does the problem incorporate or require the use of appropriate mathematics content?

Absent Weak Adequate Good Excellent

Does the problem:

- Require the use of mathematical content understandings that are grade-level appropriate (as determined by local or national curriculum standards)?
- Require students to engage in mathematics processes (e.g., understanding, connecting, communicating, reasoning, applying, and problem solving) that support the development of mathematical proficiency to arrive at a solution?

Indicator IV. Attention to Cognitive Demand

To what extent does the problem present an appropriate level of cognitive challenge?

Low Medium High

Does the problem:

- Present a situation where neither the solution nor the solution strategy is immediately apparent?
- Require more than recall, memorisation, or performance of routine procedures to reach the solution?
- Require the student to make connections to underlying concepts and mathematical meanings?
- Engage the student in complex thinking and reasoning strategies such as meta-analysis, justifying, conjecturing, reasoning, and problem solving?

Indicator V. Appropriate Number of Solution Steps to Promote Reasoning

How many steps are required to solve the problem?

One Step Multiple steps

Does the problem:

- Present a situation where the solution is not immediately apparent?
- Require more than one step to arrive at a solution?
- Constitute more than the application of routine operational procedures involving rational or whole numbers?
- Engage the student in reasoning about mathematics

Indicator VI. A Variety of Solution Strategies

Does the problem structure support the use of multiple different strategies to arrive at a solution?

No Yes

Does the problem:

- Present a situation where the solution strategy is not immediately apparent?
- Require students to figure out a strategy to arrive at the answer?
- Present a situation where multiple different approaches are possible or, in some cases, explicitly required?

Indicator VII. Facilitates Multiple solutions

Does the problem have multiple correct solutions?

No Yes

Does the problem:

- Have more than one correct solution?
- Encourage students to seek correct alternative solutions?

Indicator VIII. Opportunity for success

To what extent does the problem incorporate opportunities for the student to experience success?

Absent Weak Adequate Good Excellent

Does the problem:

- Present an opportunity for the student to demonstrate a sense of autonomy as a mathematics learner through requests to represent the problem using a diagram? Explain their strategy selection? Describe their mathematical thinking?
- Contain a component that is within reach of students? Provides a 'warm up' problem of low cognitive complexity? Provides a simpler version of the problem?
- Incorporate the provision of supports or hints if the student experiences (non-productive) struggle?

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Symposium

Mathematics in Context: Disciplinary Integrity and Interdisciplinary Possibilities

In response to the MEI 10 theme Mathematics for a Connected World: Integration, Challenges, and Possibilities, this symposium explores the role of mathematics in interdisciplinary teaching and learning, with particular attention to how mathematics is situated—and sometimes overshadowed—in integrated curricular contexts. Across three papers, we examine how mathematics education can remain robust, meaningful, and connected to real-world challenges in integrated STEM, data science, and computational thinking initiatives at primary level.

The drive toward interdisciplinary learning in education has opened up exciting opportunities to make mathematics more relevant and engaging to students. However, this integration also poses a critical tension: how to maintain the disciplinary integrity of mathematics while leveraging the affordances of cross-curricular approaches. This symposium aims to illuminate this tension by presenting three studies that examine mathematics in context—its visibility, its educational potential, and the pedagogical challenges and innovations needed to support its integration across STEM fields.

Foregrounding mathematics in integrated STEM education

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This paper investigates how mathematics is represented and enacted within integrated STEM learning environments. Drawing on a study that examined interdisciplinary STEM tasks in Irish primary classrooms, it explores the challenges of maintaining mathematical depth within rich, hands-on contexts often dominated by science or engineering content. The findings suggest that while STEM tasks provided fertile ground for developing 21st-century skills and science understanding, mathematics was often under-leveraged. The paper advocates for more intentional design processes that embed curriculum-appropriate mathematics within STEM activities. Possibilities to ensure that mathematics is not only present but central to integrated STEM learning, allowing students to engage in authentic mathematical thinking and reasoning, are explored.

Keywords: Primary mathematics education, integrated STEM, initial teacher education

Introduction

The rapid rise in national and international STEM policy discourse has been spurred by increasing calls for a more STEM literate society. Recent studies have highlighted the role of STEM for active citizenship, recognising that all citizens, regardless of career paths, need STEM literacy to navigate current and emerging challenges (Mohr-Schroeder et al., 2020). Real-world problems cannot be neatly siloed by subject area. They are messy, wicked problems that demand the use and synthesis of multiple disciplines. There is growing conviction, therefore, that classroom practices should reflect the cross-disciplinary nature of STEM and that, consequently, STEM disciplines should not be taught in isolation. This is reflected in the new Primary Curriculum Framework (NCCA, 2023) which positions STEM education as one of the five broad curriculum areas. While welcomed, this also poses many challenges for classroom enactment. Although there is general agreement on the benefits of high-quality integrated STEM experiences, such as the development of problem-solving skills, the strengthening of productive dispositions and the enhancement of key competencies, there is also evidence to suggest that poor integrated STEM teaching “damages prospects for both disciplinary learning and learning of transversal skills” (Murphy et al., 2023, p.8). Despite teachers’ reported enthusiasm for STEM education, many feel ill-prepared for integrated approaches. A key area of concern relates to maintaining a balance between the disciplines in integrated STEM, with research reporting significant difficulties in incorporating authentic interdisciplinary mathematics (Tytler et al., 2023).

Research on interdisciplinary approaches to mathematics highlights its tendency to be used as an auxiliary discipline, playing a service role to the other disciplines within integrated STEM. Indeed, Fitzallen (2015) suggests that positioning mathematics as the ‘underpinning discipline’ sets it up in a supporting role in an integrated context. This leads to a devaluing of the discipline, in which teachers view mathematics as a support for teaching science and engineering within integrated STEM curricula (Forde et al., 2023). Designing and

implementing authentic interdisciplinary mathematics experiences is challenging work. Teachers have reportedly struggled to incorporate meaningful interdisciplinary mathematics in ways that challenge and extend mathematical thinking (Tytler et al., 2023). In fact, teachers have identified mathematics as the most challenging discipline to integrate (Arnone & Hanuscin, 2018). A recent US study of over 2000 integrated STEM observations found that, when mathematics content was present in a task, it was generally of low cognitive demand (Forde et al., 2023). Given the recent changes to our curriculum landscape, we need to prepare tomorrow's teachers for the challenges of interdisciplinary mathematics. As Tytler et al. (2023) note, "balancing disciplinary with interdisciplinary learning depends on the imagination and thoroughness of planners of STEM projects who are dedicated to integrating mathematics into projects in ways that engage students" (p. 1312).

Methodology

This paper reports on a series of integrated STEM tasks facilitated in fifth and sixth classes as part of a larger action research inquiry into teacher learning for STEM education during ITE. Participants were two consecutive cohorts of preservice teachers (PSTs) (cohort 1, n=30 and cohort 2, n=28) undertaking a mathematics education specialism as part of their undergraduate programme. Each intervention comprised a 12-week, 6-credit module in Year 3. STEM task design and the facilitation of these lessons in authentic classroom settings were central to both modules. In the first iteration of the module, cohort 1 designed and facilitated five individual STEM workshops centred around engineering design tasks (such as designing a rollercoaster, detailed in Fitzpatrick et al., 2024). In line with our action research approach, we aimed to be responsive in our teaching by capitalising on the successes and redesigning to address the shortcomings of the previous iteration. As findings from cohort 1 highlighted the ease in which mathematics became silenced in these classroom STEM tasks, the second iteration sought to foreground mathematics within integrated STEM. In the second module, cohort 2 designed and facilitated a statistics- focussed STEM unit, comprising five sequential lessons based on the plight of the honeybee (detailed in Leavy et al., 2025). Data generation methods included pre-post intervention surveys, field notes, post-teaching focus groups, and reflective journal entries. A grounded approach to analysis was adopted. All ethical considerations were adhered to.

Findings

Cohort 1: Mathify-ing STEM

Participants in the first iteration of the module reported initial confidence in their abilities as mathematics teachers and learners. However, as a teacher educator, I made assumptions about the ease with which the PSTs would apply this disciplinary knowledge and skills developed in their discrete disciplinary modules to more integrated settings. While findings suggest that the five design-based STEM tasks were successful in promoting STEM learning (most notably science, engineering and 21st century skills), they failed to leverage authentic, age-appropriate, challenging mathematics learning one would expect to find in the senior classes. Furthermore, the PSTs in cohort 1 struggled to recognise the incidental and unambitious nature of the mathematics within the task and required significant support from

the teacher educators in retrospectively identifying opportunities to support and promote mathematics within these tasks. By placing an emphasis on engineering design and practices (the discipline that PSTs were least familiar with and reported least confidence in), I shifted my gaze from mathematics and failed to sufficiently support the PSTs in identifying meaningful opportunities for rich mathematics teaching and learning. The task design lacked specific curriculum-based mathematics learning outcomes and overlooked the importance of incorporating task criteria and parameters that specifically stimulated mathematics thinking and reasoning. Through careful scaffolding and prompting during post-teaching discussions and subsequent tutorial work, the PSTs were supported in incorporating appropriate and meaningful mathematics into these integrated STEM lessons, as well as recognise opportunities to build upon discrete mathematics lessons to develop STEM inquiry cycles.

Cohort 2: STEMify-ing mathematics

The second iteration of this module turned this idea of *Mathifying* a STEM lesson on its head and instead sought to *STEMify* mathematics. In recognising how easily the M in STEM fell silent (Shaughnessy, 2013), the second intervention attempted to foreground mathematics within a STEM inquiry cycle. Situated within a rich STEM context (local honeybee data), statistics was the central discipline and the starting point in planning the STEM unit. With one eye firmly on the data at all times, PSTs were supported in creating a five-lesson STEM inquiry around a societally relevant dataset. The authentic real-world problem (colony collapse) drove the statistical analysis and engaged students in age-appropriate, challenging statistical reasoning. CODAP (Common Online Data Analysis Platform) was utilised to carry the procedural load that often impedes advancement, in turn freeing up instructional time to engage in cognitively challenging tasks. PSTs scaffolded the children's analysis of a complex data set (beehive conditions collected by sensors in local hives) through the exploration of landmarks and measures of centre. They challenged them to compare data and explore relationships between variables, and supported higher-order thinking by encouraging children to draw data-informed conclusions. Furthermore, close links and interdependence between the disciplines of science, technology and mathematics were evident. The science content provided necessary background information needed to engage meaningfully with the data by developing an understanding of the honeybee and highlighting the characteristics of a healthy hive. Meanwhile, technology fostered interest in, and supported analysis of the data, while the data itself raised interest in the innovate technologies that captured it. This unit, therefore, provides an example of mathematics-centred STEM, that promotes rich disciplinary learning, while also advancing other STEM literacies.

Conclusion

This study supports the findings of previous studies in which mathematics succumbed to a service role, while serving as an example of possibilities for maintaining the spotlight on mathematics in integrated STEM tasks. Upholding the disciplinary integrity of mathematics within integrated STEM demands explicit and intentional planning for mathematics outcomes from the outset. Given that much of the literature on STEM classroom innovation comes from the field of science, further evidence of classroom initiatives foregrounding mathematics in integrated STEM is greatly needed.

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Data science education for civic engagement and social justice

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This paper explores the role of mathematics—and specifically data science—as a tool for interdisciplinary learning, civic engagement, and social justice in primary and lower-secondary education. Drawing on the EU-funded *DataScEd4CiEn* project, it presents a conceptual framework grounded in situated learning theory and developed through systematic literature review and iterative collaboration. The framework supports the integration of data science within STEAM education, promoting students’ capacity to critically engage with complex, real-world data. A series of interdisciplinary STEAM scenarios, co-designed using design-based research, demonstrate how statistical reasoning can be mobilised to investigate issues such as drought, fast fashion, and food waste. Leveraging open-access datasets and digital tools like CODAP, these scenarios foster data literacy and empower students to use mathematics for inquiry and civic action. The paper argues that such approaches reposition mathematics as a powerful tool for understanding and addressing pressing societal challenges.

Keywords: Data science, civic engagement, STEAM

Introduction

In today’s data-saturated world, the ability to interpret, evaluate, and use data has become a foundational skill for active citizenship. The rapid proliferation of data—driven by digital technologies, media platforms, and algorithmic systems—has transformed how we engage with information in every domain of life. As a result, data science, broadly defined as the practice of extracting value from data (Wing, 2019), has emerged not only as a technical field but as a powerful lens through which to understand and address societal challenges such as climate change, public health, and social inequality (Tanaka et al., 2022). While democratic societies place high value on open and accessible information, the ability to navigate the increasingly complex and dynamic information landscape is not yet widely developed. The COVID-19 pandemic exposed this gap with stark clarity: making sense of data related to infection rates, risk levels, and vaccine efficacy became a matter of daily concern for citizens across the globe. However, evidence suggests that many young people lack the capacity to critically interpret relatively straightforward datasets (Konold et al., 2015), struggle to evaluate the credibility of scientific and political information (Kahne & Bowyer, 2017) and fail to detect bias in online sources (Wineburg et al., 2016).

The EU-funded project *DataScEd4CiEn—Data Science Education in STEAM for Civic Engagement and Social Justice* [<https://datasc4ed.euc.ac.cy/>] responds directly to these concerns. It seeks to embed data science within primary and lower-secondary education in ways that promote critical engagement, interdisciplinary inquiry, and democratic participation. Framed around the idea of “data self-empowerment,” it views data science education as a means of enabling young people to act as informed and thoughtful contributors to society. Recent studies suggest that primary children can engage with complex, messy datasets in ways that foster curiosity, critical thinking, and civic awareness (Leavy et al., 2024; Makar et al., 2023). When embedded in meaningful curricular contexts, data science

serves as a bridge across STEAM disciplines, supporting the development of statistical reasoning and active citizenship.

Method

The *DataScEd4CiEn* conceptual framework was developed through a systematic process that drew from existing research and educational practice across multiple disciplines. The first phase involved the targeted identification and analysis of 35 frameworks related to data science education, STEAM education, statistical investigative cycles, and integrated STEM. This analysis aimed to uncover common components and recurring principles that could inform the foundation of a scientifically grounded framework. Following this, a comprehensive scoping review was conducted to investigate five core thematic areas central to the project: civic engagement and active citizenship, social justice in education, STEAM learning in connection with social justice issues, data science in education, and data practices in education. Systematic searches across academic databases (including Web of Science, ERIC, and ScienceDirect), grey literature, and key journals yielded 231 relevant sources, encompassing research articles, national policy documents, and curriculum frameworks.

Project partners engaged in focused reviews of literature aligned with each of the five components. Their task was to clarify definitions, establish a rationale for inclusion, explore examples of implementation in educational settings, extract pedagogical principles, and identify illustrative datasets and civic issues. Following four iterative cycles of reflection and refinement, the researchers critically evaluated the coherence and utility of the emerging framework, ensuring its relevance to both educational theory and classroom practice. The resulting conceptual framework integrates the identified components into a coherent structure to guide data science education for civic engagement and social justice in primary and lower-secondary education (see figure 1).

Framing Data Science Education within STEAM and Society

The *DataScEd4CiEn* conceptual framework is grounded in situated learning theory, which emphasises authentic, socially embedded learning experiences. It positions data science not only as a technical skillset but as a tool for exploring real-world, interdisciplinary problems—particularly those connected to civic life and social justice. Through engagement with complex, large, and often messy datasets using innovative digital tools, students are supported in developing core STEAM competencies alongside broader critical thinking and problem-solving skills. At the heart of the framework is the recognition of data science as inherently interdisciplinary, intersecting with science, technology, engineering, the arts, and mathematics. This intersectionality enables learners to use data science techniques to address authentic societal challenges that require insights from multiple domains. While data science is typically introduced at later stages of education or in adult learning, *DataScEd4CiEn* explicitly seeks to build data literacy from an early age, highlighting its relevance and accessibility for primary and lower-secondary students.

Figure 1

Conceptual Framework

The visual representation of the framework (Figure 1) reinforces the collaborative and inclusive ethos of data science education.



The image of people working together reflects the framework's emphasis on participation, dialogue, and shared inquiry. In this model, data science is not just a set of computational tools—it is a means for young learners to engage as thoughtful, critical, and active citizens. In equipping students with the capacity to interpret and act on data, schools can help foster the next generation of engaged leaders and informed decision-makers (Witte et al., 2025).

The conceptual framework informs the development of professional development and curricular materials shaping both content and pedagogical approaches, ensuring alignment with the project's interdisciplinary and ethical focus. It also guides the design of STEAM Scenarios—interdisciplinary teaching units rooted in authentic, real-world contexts. These scenarios provide practical examples of how the framework's principles can be applied in classrooms and serve as resources for teachers, teacher educators, and other stakeholders seeking to embed data science education in meaningful, socially engaged ways.

STEAM Scenarios: Interdisciplinary Inquiry through Data

To support the application of the *DataScEd4CiEn* framework, a series of interdisciplinary STEAM scenarios are being developed using a design-based research approach. This iterative methodology enables the refinement of each scenario through cycles of classroom implementation, teacher feedback, and analysis of student engagement and learning outcomes. The scenarios are designed to be both pedagogically sound and socially meaningful, offering rich contexts for the integration of mathematics and data science in real-world inquiry. A central feature of these scenarios is the use of publicly available big data sources from platforms such as the **World Bank** (e.g., *Poverty headcount ratio*), and **Gapminder** (on health, economics, and development, <https://www.gapminder.org/>) to ground student investigations in authentic, up-to-date global and local issues. These sources provide students with opportunities to engage in real data exploration using CODAP, a digital environment that supports visualisation, filtering, and statistical interrogation of large data.

Current scenarios being piloted include one on **drought**, where students explore local water scarcity through environmental and usage data; **fast fashion**, in which students interrogate global industry data to ask “*What is the true cost of my clothes?*”; the exploration of **economic disparities and social justice** through data analysis and **food waste**, where students analyse per-capita waste data across Europe to explore the impacts of dietary habits. Each scenario challenges students to pose questions, explore patterns, and develop arguments using data—fostering both mathematical proficiency and critical civic awareness. These scenarios exemplify how mathematics, as part of an interdisciplinary inquiry process, can

empower students to understand and respond to complex societal challenges. Strategic use of open, real-world data provides opportunities to see maths as relevant and transformative.

Conclusion

This paper highlights the potential of data science education, grounded in civic engagement and social justice, to transform how mathematics is taught and experienced. The conceptual framework and its application in real-world STEAM scenarios, supports students in developing statistical reasoning and data literacy while engaging meaningfully with complex societal issues. In doing so, mathematics becomes a tool not only for analysis, but for informed action and responsible citizenship.

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Beyond the sums: Integrating mathematics and computational thinking in primary education

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The final paper considers the intersection of mathematics and computational thinking in light of the rollout of the new primary STEM curriculum in Ireland (September 2025). It presents findings from a ten-week Scratch programming initiative involving students in third, fifth, and sixth class, designed to develop computational thinking through creative coding. The paper analyses students' learning across Brennan and Resnick's (2012) three dimensions of computational thinking: concepts, practices, and perspectives. While students showed strong conceptual understanding in areas such as synchronisation and parallelism, challenges remained in areas like conditional loops and variable initialisation. Through thick description of classroom experiences, the study highlights the pedagogical approaches that supported student learning—including scaffolding, project design, and peer interaction—and proposes a refined framework for computational thinking in primary education. The paper also emphasises how computational thinking can be a fertile context for deep mathematical learning when pedagogically framed with intention.

Keywords: Primary Mathematics education, computational thinking, programming

Introduction

Over the past decade, the world has witnessed unprecedented technological evolution, fundamentally reshaping how societies interact, work, and learn. As emerging technologies increasingly permeate all facets of life and work, there has been a growing imperative for education systems to adapt, ensuring learners are equipped with the skills necessary for a digitally driven future (NCCA, 2024). In response, the Irish education system has initiated significant reforms, most notably by formally incorporating STEM (Science, Technology, Engineering and Mathematics) as a distinct curricular area within the *Primary Curriculum Framework* (NCCA, 2023). The draft specification was published in March 2024, and the introduction of the finalised STEM curriculum is set to commence from the 2025/2026 school year. The STEM curriculum (NCCA, 2024) provides a dedicated space for digital learning, facilitating a more comprehensive exploration of programming concepts and computational thinking methodologies, which have been to the forefront of discussions to capture and define new ways of thinking that are increasingly important in the digital age. Additionally, 'being a digital learner', was identified as a key competency that would be integrated across all subjects of the primary school curriculum (NCCA, 2024).

Computational thinking rose to prominence in 2006 when Jeanette Wing wrote a seminal article advocating for its inclusion in education curricula worldwide (Wing, 2006). Computational thinking draws on the fundamentals of computer science, applying problem solving processes in a way that allows computers to assist us (NCCA, 2024). Several researchers have highlighted the challenges in capturing the complex, multidimensional nature of computational thinking (Lye & Koh, 2014; Kafai & Burke, 2013). In defining

computational thinking for the Irish context, Millwood et al. (2018) identified three distinct aspects of computational thinking; knowledge (cognitive), craft (kinaesthetic) and character (affective). These three domains of competence are represented in Brennan and Resnick's (2012) framework for studying and assessing computational thinking development. Computational concepts represent the cognitive component. These are the concepts that designers employ as they program. Computational practices represent the kinaesthetic component. These are the design practices designers develop while working with these concepts. Computational perspectives represent the affective component. These are the views that designers form about the role of technology in the world and their relationship to it. In the new primary STEM curriculum, programming has been promoted as a medium to build students' computational thinking skills, particularly as they progress through primary school (NCCA, 2024). Initial efforts focussed on embedding computational thinking and programming in the new primary mathematics curriculum (NCCA 2018). However, while recognising the intrinsic connection between mathematics and these concepts, it was deemed more appropriate to integrate these concepts across the curriculum rather than framing them as merely a mathematical discipline.

Methodology

This paper reports on the findings of a ten-week Scratch programming initiative conducted in an all-girls primary school in the South of Ireland. An instrumental case study approach was adopted to provide insights into how computational thinking can be developed within formal primary school settings. The participants were sixty-seven students from third (n=23), fifth (n=21) and sixth (n=23) class. The majority of students had little to no programming experience, and they worked in pairs or threes on the programming activities. The first five weeks focussed on developing the students' basic programming skills. During the next four weeks there was no explicit instruction, instead, a learning on demand model (Kafai & Ching, 2001) was adopted as they students worked on creating their self-directed projects. In the final week the students worked on debugging projects designed to encourage the use of specific programming concepts. Various data collection methods were adopted to capture the multidimensional nature of computational thinking, including artefact analysis, artefact-based interviews, questionnaires and an observation diary. Brennan and Resnick's (2012) three computational thinking dimensions (concepts, practices and perspectives) were used to interpret the data, before deductive and inductive analyses were adopted to conceptualise the nature of the dimensions. The verification of a code required that it be evidenced across five or more participants/groups and two or more data sources. Following analysis, computational thinking was conceptualised in accordance with the revised framework presented in Table 1.

Table 1

Computational Thinking Framework (adapted from Brennan and Resnick 2012)

Computational Concepts	Synchronisation	
	Parallelism	
	Thinking Logically	
	User Interactivity	
	Flow Control	
	Abstraction	
	Data	<ul style="list-style-type: none"> • Attribute Variables • Programmer Declared Variables
Computational Practices	Abstracting and Modularising	
	Experimenting and Iterating	
	Testing and Debugging	
	Reusing and Remixing	
Computational Perspectives	Expressing	
	Connecting	
	Questioning	
	Motivation to Engage	

Findings

Computational Thinking

Dr. Scratch, the automated project analysis tool, was used to evaluate the students' final projects and assign a programming score. Each computational thinking concept is given a score between 0-3, depending on the competence demonstrated in the project. The findings indicated that, on average, the students best understood synchronization and parallelism, demonstrating proficient use of both concepts. Flow control, user interactivity and data representation scored slightly lower and were evaluated as developing. The lowest scores were achieved in abstraction and logical thinking, with students only demonstrating a basic proficiency with respect to these concepts. Abstraction was not included as a concept in Brennan and Resnick's (2012) conceptualisation of computational thinking, but during this research, the potential for abstraction concept development afforded by Scratch became evident. In addition, this research advocates for a separation of data into variables declared by the programmer and attribute variables. This recommendation is derived from the particular difficulty that students displayed in comprehending attribute variable initialisation.

The findings of this study illustrate that students engaged in all four computational practices included in the Brennan and Resnick (2012) framework, abstracting and modularising, experimenting and iterating, testing and debugging, and reusing and remixing. The data indicated that these practices were often inextricably linked with students frequently engaging in two or more practices simultaneously. This was particularly evident for the first three practices. The poor broadband connectivity in the school, curtailed the opportunities for students to engage in reusing and remixing by restricting their access to the global Scratch community. Hence, this practice was not as prevalent as has been reported in previous studies. The exploration of how students engage in these practices, provided insights into several pedagogical factors which positively impacted practice development. These included, the use of storyboards, the implementation of 'gallery walk' and 'learning walk' strategies, and the suitability of Scratch as a programming language for novice programmers. The immediate

feedback provided by Scratch and being able to trial the function of certain blocks within the command palette were particularly helpful in encouraging students to experiment and iterate, test and debug, and reuse and remix.

Students welcomed the opportunity to explore the creative affordances of the Scratch programming environment. The wide walls of Scratch ensured that the students felt they could express themselves in meaningful ways, with abundant choice (project types, backdrops, sprites), ability to incorporate their personal interests (importing favourite songs and images) and explore their creative talents (composing images and sounds). The students were also positive about the potential of ‘creating with’ and ‘creating for’ others (Brennan & Resnick, 2012). They recognised the benefits from working as part of a group, while also acknowledging that their intended audience, their peers, became a central consideration for them during the design process. Upon completion of the programming initiative, students held varying perspectives on their relationship with technology from a functional perspective to an exploratory perspective all the way through to an empowered perspective. This illustrates that while programming in schools won’t necessarily make all students want to be programmers, it does encourage them to consider and develop their ‘relationship’ with technology. None of the aforementioned perspectives captured students’ motivational beliefs with respect to learning programming, something which came across strongly in the data. Therefore, it is proposed that an additional dimension, motivation to engage, be added to the dimensions already included in computational perspectives of Brennan and Resnick’s (2012) framework.

Mathematical Insights

This research offered insights into how students’ mathematical proficiency can at times impede the development of their computational thinking, while also highlighting instances where both mathematical proficiency and computational thinking could be developed simultaneously. The logic concept was the computational concept for which the final projects in this study achieved the lowest scores. The inclusion of logical operators in Scratch requires an understanding of inequalities, and without direct instruction none of the students were able to successfully build a script incorporating this concept. A solid understanding of coordinates and negative numbers is essential for effective programming in Scratch, as it enables students to accurately position, move, and animate sprites within the two-dimensional stage environment. Both mathematical concepts are considered to be more challenging concepts and are not introduced in the curriculum until stage 4 (fifth and sixth class). However, despite not having previously encountered coordinate geometry or negative numbers, the third-class students readily grasped these concepts within the context of Scratch. This illustrates that programming can provide valuable opportunities to explore mathematics concepts in a meaningful way.

Conclusion

This paper explored the development of each of the concepts, practices and perspectives conceptualised in the Brennan and Resnick (2012) framework. The data provided evidence of additional computational concepts and perspectives that were not included in this conceptualisation. Throughout this research the close associations between programming,

computational thinking and mathematics were reinforced. As work continues on the development of learning trajectories for technology, it is important to consider mathematical concepts that students will encounter when developing their understanding of computational concepts.

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Symposium

Understanding and Enhancing Mathematical Knowledge for Teaching in South African Primary Education: Research in Progress

Primary education in South Africa faces persistent challenges. The majority of learners fail to meet grade-level expectations in early mathematics and teachers appear to lack the pedagogical tools necessary to bridge the gap. This symposium presents interim findings of a Research Ireland funded project aiming to enhance the mathematical knowledge for teaching (MKT) of South African primary teachers.

Three papers which examine different facets of MKT will be presented. These papers draw on baseline data collected from teachers participating in an after-school mathematics programme across four South African provinces. Professional development to support the afterschool programme involves innovative WhatsApp-based professional learning in addition to in-person training sessions. The findings reported in this symposium arise from data gathered through pre-tests focused on foundational number operations and video-based tasks which asked teachers to comment on a range of instructional explanations.

The international research team presenting in this symposium brings together diverse perspectives on mathematics education and professional learning. While rooted in the South African context, the findings have relevance for teacher education more broadly, specifically for mathematics teaching in multilingual contexts, and for professional development in under-resourced and remote learning environments—challenges shared across many global education systems.

Clustered Profiles of Primary School Teachers' Mathematical Knowledge: Insights from a South African Study

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CASTeL, Dublin City University

The Common Content Knowledge (CCK) domain of Mathematical Knowledge for Teaching is an essential component of the knowledge required to teach mathematics (Ball et al., 2008). An NGO-led project in South Africa recruits practising teachers to teach in after-school mathematics clubs and provides them with Continuous Professional Development (CPD) delivered through a WhatsApp platform. Seeking to evaluate the impact of the CPD on the teachers' CCK, a longitudinal study was established. This paper presents findings from pretest data collected from 121 primary school teachers. Cluster analysis was used to identify distinct performance profiles. These preliminary insights highlight variations in teacher knowledge that could inform the design of differentiated support strategies in subsequent professional development sessions.

Keywords: MKT, assessment of teacher knowledge, CCK

Background and context

Mathematical Knowledge for Teaching (MKT) has long been established as a productive framework for understanding the specialised knowledge required by teachers for the effective teaching of mathematics. Building upon Shulman's (1986) categories of subject matter knowledge (SMK) and pedagogical content knowledge (PCK), Ball et al. (2008) articulated MKT as a domain-specific form of professional knowledge, distinct from both general mathematical knowledge and general pedagogical knowledge. While Shulman also introduced Curricular Knowledge as a third dimension, our study focuses specifically on SMK and PCK due to their direct relevance to the MKT framework. The MKT framework emphasises the unique cognitive demands placed on teachers as they navigate the complexities of classroom mathematics.

An essential and foundational element of MKT is Common Content Knowledge (CCK), which Ball et al. (2008) identified as a component of Subject Matter Knowledge (SMK). CCK refers to the general mathematical knowledge used by anyone practising mathematics, distinct from Specialised Content Knowledge (SCK) and Horizon Content Knowledge, which focus on deeper disciplinary understanding and connections. In the context of this study, participants must demonstrate CCK by accurately solving addition and subtraction problems.

In South Africa, limited teacher mathematical knowledge is widely acknowledged as a key barrier to primary school mathematics success (Taylor, 2021). Continuous Professional Development (CPD) interventions are common and are often funded by Non-Governmental Organisations (NGOs). A number of these interventions are adopting a research-informed approach, which is beginning to show positive effects (Taylor, 2021). One such intervention is a professional development programme facilitated through an after-school mathematics club programme (Bowie et al., 2022). The programme runs after-school mathematics classes for children (aged 10-12) in grades 4 and 5 across South Africa. The teachers on the programme are qualified and practising teachers, but on the programme, they may be teaching children who attend a grade other than the grade of the class that they typically teach.

This paper reports results from a pre-test of the teachers' CCK, undertaken as preliminary data collection in a longitudinal project aimed at designing interventions to support teachers in improving their MKT and informing future CPD. As such, the study described in this paper aims to answer the following research questions:

1. What CCK do teachers participating in an after-school mathematics club programme demonstrate on a baseline assessment focusing on addition and subtraction?
2. How is the distribution of teachers' CCK scores characterised across performance clusters within the cohort?

Methods

This paper presents preliminary findings from a larger study exploring experiences and MKT-related progressions among tutors participating in a CPD. Building on a pilot by Bowie et al. (2025), it reports on pre-test scores from 121 primary school teachers across four provinces: Free State, Gauteng, KwaZulu-Natal, and Western Cape. As mentioned earlier, all teachers in the sample are involved in teaching Grade 4 or 5 within the said programme, although they may teach different grades and subjects in their regular school assignments.

The assessment measured teachers' CCK in addition and subtraction, including number and word sentences and connections with representations. Test items were aligned with the early grade mathematics curriculum of the South African Education System and validated through an exploratory pilot study (Bowie et al., 2025) to ensure relevance to classroom instruction. The test was administered via pencil-and-paper before training. The test consisted of nine questions, with a number of sub-questions in each question (see Figure 1 for samples of test items). Items were scored as either correct or incorrect, with a maximum score of 48. Nonetheless, question nine was not assessed solely on the basis of correctness, as it required qualitative insights. It was scored out of four marks: one mark was awarded for providing only an answer with no calculations; two marks for a response that included a visual or symbolic representation; three marks for clearly articulating the steps taken to arrive at the answer; and four marks for demonstrating all three elements along with a rationale explaining why the chosen method is effective.

Cluster analysis using the K-means model in JASP's open-source machine-learning tool identified three performance groups: low, moderate, and high. The boundaries for these clusters were determined by the algorithm, rather than predefined thresholds. Given this automated clustering process, significance testing was unnecessary, as distinct groupings are an inherent outcome of the method. Additionally, item-level analysis examined variations in CCK between clusters, informing potential support needs in future training. Figure 1 illustrates sample items administered to teachers.

Figure 1

Sample items from the pre-test assessment

<p>Question 6: Circle the number sentence that represents the word problem</p> <p><i>Note: You do not need to provide the numerical answer to the question</i></p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left; padding: 2px;">Word problem</th> <th style="text-align: center; padding: 2px;">$51 - 18 = \square$</th> <th style="text-align: center; padding: 2px;">$51 + 18 = \square$</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px;">Zinhle has 51 sweets. That is 18 more sweets than Thapelo has. How many sweets does Thapelo have?</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> <tr> <td style="padding: 2px;">Maria has 51 sweets. That is 18 less sweets than Busi has. How many sweets does Busi have?</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> <tr> <td style="padding: 2px;">Shelton has 18 less sweets than Lindo. Lindo has 51 sweets. How many sweets does Shelton have?</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> </tbody> </table>	Word problem	$51 - 18 = \square$	$51 + 18 = \square$	Zinhle has 51 sweets. That is 18 more sweets than Thapelo has. How many sweets does Thapelo have?	<input type="checkbox"/>	<input type="checkbox"/>	Maria has 51 sweets. That is 18 less sweets than Busi has. How many sweets does Busi have?	<input type="checkbox"/>	<input type="checkbox"/>	Shelton has 18 less sweets than Lindo. Lindo has 51 sweets. How many sweets does Shelton have?	<input type="checkbox"/>	<input type="checkbox"/>	<p>Question 3: True or False</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left; padding: 2px;">Statement</th> <th style="text-align: center; padding: 2px;">T or F</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px;">a) 7 is 2 more than 5</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> <tr> <td style="padding: 2px;">b) 5 is 2 less than 7</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> <tr> <td style="padding: 2px;">c) 7 is larger than 5 by 2</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> <tr> <td style="padding: 2px;">d) 2 more than 7 is 5</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> <tr> <td style="padding: 2px;">e) 2 and 5 more is 7</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> <tr> <td style="padding: 2px;">f) 7 is smaller than 2 by 5</td> <td style="text-align: center; padding: 2px;"><input type="checkbox"/></td> </tr> </tbody> </table>	Statement	T or F	a) 7 is 2 more than 5	<input type="checkbox"/>	b) 5 is 2 less than 7	<input type="checkbox"/>	c) 7 is larger than 5 by 2	<input type="checkbox"/>	d) 2 more than 7 is 5	<input type="checkbox"/>	e) 2 and 5 more is 7	<input type="checkbox"/>	f) 7 is smaller than 2 by 5	<input type="checkbox"/>
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<p>Question 9:</p> <p>Learner: "Teacher please help me I am stuck with this question"</p> <p style="text-align: center;">$\square - 23 = 14$</p> <p>Show how you would help this learner.</p>	<p>Question 1: Fill in the gaps</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px;">a) $80 + 7 = \underline{\quad}$</td> <td style="padding: 2px;">f) $\underline{\quad} - 23 = 14$</td> </tr> <tr> <td style="padding: 2px;">b) $\underline{\quad} + 4 = 60$</td> <td style="padding: 2px;">g) $53 - 27 = \underline{\quad}$</td> </tr> <tr> <td style="padding: 2px;">c) $\underline{\quad} = 47 - 9$</td> <td style="padding: 2px;">h) $26 + \underline{\quad} = 33$</td> </tr> <tr> <td style="padding: 2px;">d) $30 = 37 - \underline{\quad}$</td> <td style="padding: 2px;">i) $304 - 145 = \underline{\quad}$</td> </tr> <tr> <td style="padding: 2px;">e) $26 + 74 = \underline{\quad}$</td> <td style="padding: 2px;">j) $296 + 348 = \underline{\quad}$</td> </tr> </tbody> </table>	a) $80 + 7 = \underline{\quad}$	f) $\underline{\quad} - 23 = 14$	b) $\underline{\quad} + 4 = 60$	g) $53 - 27 = \underline{\quad}$	c) $\underline{\quad} = 47 - 9$	h) $26 + \underline{\quad} = 33$	d) $30 = 37 - \underline{\quad}$	i) $304 - 145 = \underline{\quad}$	e) $26 + 74 = \underline{\quad}$	j) $296 + 348 = \underline{\quad}$																
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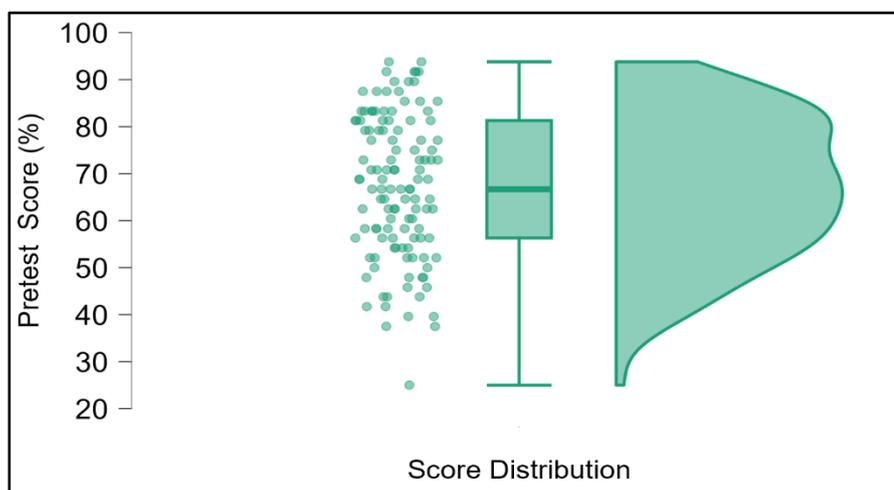
Results

Scores achieved and distribution by participant

The overall score distribution ($M = 67.2$, $SD = 15.2$) of the 121 primary school teachers revealed a wide range of performance, ranging from 25 to 93.8. This suggests a significant variation in participants' mathematical knowledge. The Raincloud plot (Figure 2) illustrates this variability, with noticeable clustering around the mid-to-high score range, alongside a spread of lower-performing individuals.

Figure 2

Overall score distribution



No significant differences were observed across provinces, grades taught, their role or teaching experience in the programme. K-means clustering analysis was used to gain further insights into variations in performance, resulting in three performance brackets. Descriptive statistics and the cut-off points associated with each of the three clusters (low, moderate, and high) are given in Table 1.

Table 1.

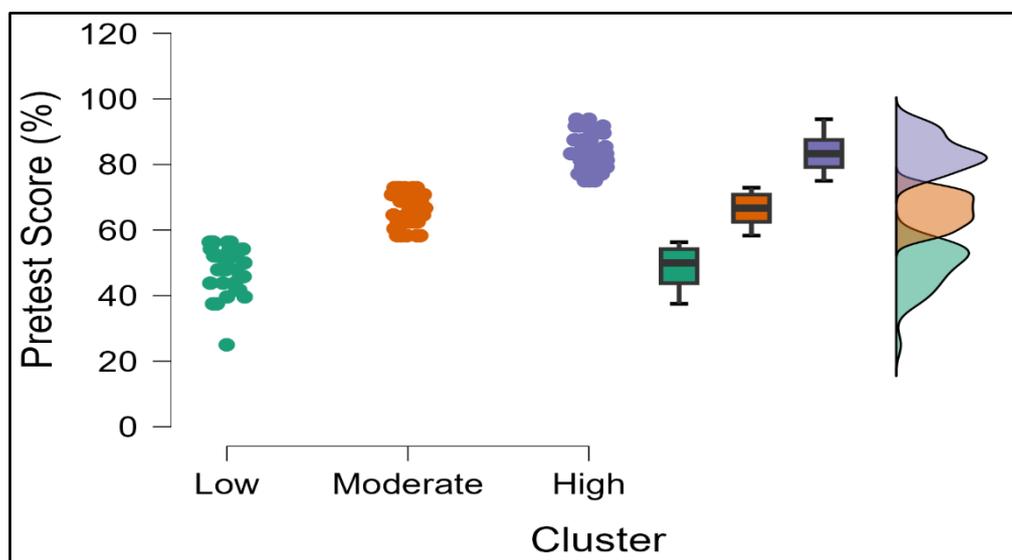
Descriptive statistics by clusters

Cluster	N	Mean	Std. Deviation	Minimum	Maximum
Low	34	48.4	7.19	25.0	56.3
Moderate	44	65.9	4.85	58.3	72.9
High	43	83.6	5.29	75.0	93.8

Based on the information displayed in Table 1, it is evident that the three clusters vary in size, with the low-performing cluster (n = 34) being the smallest, followed by the high-performing (n = 43) and then the moderate-performing (n = 44) clusters. The standardised cluster means confirm a clear separation: The low-performing cluster (-1.25) is far below zero, the moderate-performing cluster (-0.091) is close to zero, and the high-performing cluster (1.08) is well above zero. The explained proportion of within-cluster heterogeneity suggests that the low-performing cluster has the highest internal variation, while the moderate cluster is the most homogeneous, as displayed in Figure 3.

Figure 3

Score distribution within clusters



Scores achieved and distribution by participant

Item-by-item analysis revealed that high-performing teachers exhibited notable procedural fluency in addition and subtraction tasks. On average, they consistently recognised number relationships, interpreted comparative phrasing accurately, and navigated between symbolic and visual representations, such as part-part-whole structures and number lines. In contrast, low-performing teachers struggled with borrowing, frequently misapplied operations in structured number sentences, and often misinterpreted comparative language (e.g., “more than...by”, “18 less sweets than...”). This indicates gaps in both basic skills and conceptual knowledge. For instance, in Question 1 (see Figure 1 above), 55.8% of high-performing teachers achieved full marks (10/10), compared to only 8.8% of the low-performing group who achieved this. On Item 1(i) (304 – 145), common errors among low performers, such as answers like 259 or 169, suggested confusion with place value and regrouping procedures.

Questions 3 and 6 (see Figure 1 above) reveal sensitivity to mathematical language as a key distinguishing factor. Items with unfamiliar comparative phrasing (e.g., “smaller than...by”) were generally challenging, and even more challenging for low-performing

participants. In Item 3(f) (“7 is smaller than 2 by 5”), 49.6% of all participants answered incorrectly, 58.1% of whom came from the low-performing group. Similarly, in Item 6.2 (“Maria has 51 sweets. That is 18 less sweets than Busi has”), only 33 participants (27.3%) answered correctly, with most correct responses coming from high-performing ($n=26$) and moderate-performing ($n=6$) clusters, while just one low-performing participant succeeded. Conversely, 32 out of 33 low-performing participants misinterpreted the problem, often associating “less” with subtraction, which reflects a misunderstanding of a comparative context depicted in the problem.

Moderate performers appear to be at a transitional stage in developing connected understanding. While not as conceptually grounded as the high performers, they possess foundational strengths, particularly in routine tasks, and show early signs of understanding number structures. However, they were less consistent and struggled to explain or justify their methods. Importantly, it should be noted that not all high-performing teachers displayed consistently well-connected understanding, which may suggest that strong procedural performance did not equate to conceptual depth.

Overall, a notable pattern emerged: while many teachers were able to perform procedures correctly, very few could explain or justify their methods when prompted. The data revealed a heavy reliance on rote procedural knowledge, with limited evidence of conceptual understanding. For instance, in Question 1, where participants were only required to provide the answer to $\square - 23 = 14$, 104 out of 121 (85.9%) produced the correct response. However, when asked in Question 9 to explain how they would assist a learner struggling with the same problem, only four participants (3.3%) provided a justification that included a clear rationale. Over 60% of participants did not attempt to explain their reasoning at all.

Discussion

The participants in this research were drawn from a mathematics programme that seeks to provide after-school mathematics classes for children, while simultaneously supporting the teaching of mathematics and the CCK of the teachers who take part. As such, the purpose of answering the research question lies in the potential of the analysis to support targeted and relevant CPD for these teachers. The analysis presented in this paper demonstrates a need for CPD, which is highly differentiated, as a one-size-fits-all approach would be inappropriate given the breadth of the distribution and the differences between the clusters. The cluster analysis revealed distinct performance brackets, with low-performing teachers requiring specific support in place value understanding, while high-performing teachers demonstrated some level of procedural fluency but not necessarily deeper conceptual understanding. This variation in performance aligns with Skemp’s (1978) distinction between instrumental and relational understanding. Many teachers in the study appear to rely on instrumental understanding, where they apply rules without necessarily grasping the underlying concepts (Kinach, 2002). This is particularly evident in the error analysis, where procedural fluency was observed, but conceptual reasoning, such as interpreting comparative phrasing in word problems, as well as justifying their reasoning, was often lacking.

The findings indicate that CPD should not only focus on producing a correct answer but also on justifying why particular methods or strategies work. Ball et al. (2008) highlight the necessity for teachers to fully understand how procedures work, enabling them to explain methods accurately and select appropriate representations. Given the challenges observed in interpreting comparative language in word problems, future CPD efforts should also consider linguistic factors that may impact teachers’ ability to convey mathematical concepts effectively. By addressing these gaps, CPD can move teachers beyond instrumental

understanding, fostering relational understanding that enhances their ability to support learners in developing both procedural fluency and conceptual depth.

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Negotiating mathematical norms: Teacher perspectives on explanation

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This paper reports on a task from a design-based research project investigating a hybrid WhatsApp-based mathematical knowledge for teaching (MKT) course for South African primary teachers working in an after-school mathematics programme. Teachers evaluated different explanations for solving $\square - 23 = 14$. A grounded analysis of the teachers' justifications for their preferences suggests difference between the sociomathematical norms of the course designers and most of the teachers. Whilst the course designers valued explanations with clear mathematical justifications, the teachers tended to prioritise simplicity and correct procedures.

Keywords: Mathematical knowledge for teaching, sociomathematical norms, explanations

Introduction

The research reported on in this paper forms part of a larger design-based research project in which we seek to design, implement and study teachers' experiences and outcomes in a hybrid WhatsApp-based course. This course was aimed at supporting a cohort of teachers teaching on an after-school mathematics programme working with grade 4 and 5 learners in South Africa. Research shows that primary school mathematics teachers in South Africa display poor content knowledge (Taylor, 2021). The course thus focuses on the development of the teachers' mathematical knowledge for teaching (MKT), incorporating both content knowledge and knowledge of how to present mathematical ideas in coherent, well-connected and responsive ways.

The research was sparked by an interaction, reflected in the vignette below, between a training facilitator and one of the teachers during an in-person training session conducted during the initial pilot of the course. Teachers were discussing how they would help learners figure out what number should go in the empty box when given $\square - 23 = 14$:

Teacher: I would tell them to work out 4 and 3 is 7, 2 tens and 1 ten is 3 tens so the answer is 37.

Facilitator: OK. And if the learner asks you why you are doing that...

Teacher: Because adding the units and adding the tens is easier

Facilitator: No, I mean if the learner wants to know why you are adding

Teacher: Because you need to add to get the right answer

Facilitator: But why does adding get the right answer?

Teacher (turning to fellow teachers, looking perplexed): Cultural differences!

This exchange was one of the experiences in the initial pilot that suggested that the course designers and teachers might not share mathematical and pedagogical norms and motivated a need to understand the norms that teachers bring to the course. Morrison et al.

(2021) argue that sociomathematical norms and their effect on the learning of early number have been largely unexplored in South Africa. Their small-scale study indicated that close attention to building desired sociomathematical norms in learners had positive effects on learners' progress in early number. This study, like much of the work on sociomathematical norms, was done in classrooms and investigated the co-creation of norms between learners and teachers. Some studies in the international context have turned their attention to sociomathematical norms of teachers and their construction in teacher education (McNeal & Simon, 2000, Güven & Dede, 2017). However, we could not find work that specifically looked at the sociomathematical norms of teachers in the South African context. This suggested it would be fruitful to investigate this and led to our research question:

What norms do the pattern of teachers' responses to pedagogical explanations of a mathematical problem suggest and how do they compare with those of the course designers?

In this paper we present the findings from a task designed to elicit information about the mathematical and pedagogical norms valued by the teachers on the course and consider the implications of this for course design.

Literature on sociomathematical norms

Yackel & Cobb (1996) explain that sociomathematical norms are the shared standards and expectations within a classroom that guide mathematical activity. Through the analysis of video recordings of classroom episodes taken over the course of a year in a second grade classroom they show how these norms are interactively developed through classroom discourse and influence how students engage in mathematical reasoning and argumentation. Much of the literature contrasts norms in traditional versus reform-oriented classrooms (see, for example, McNeal & Simon, 2000, McClain & Cobb, 2001, Cobb et al., 1992). Cobb et al. (1992) use their analysis of two elementary school mathematics classes to argue that in reform contexts, norms favour explanations involving actions on mathematical objects rather than procedural instructions alone. Kang & Kim (2016) observed 13 sessions of a grade 4 mathematics class and interviewed both the teacher and students in that class. In their analysis they considered the teachers' beliefs about the nature of mathematics and about the nature of mathematical teaching and learning and investigated the link between these and the sociomathematical norms constructed in the class. They used this analysis to argue that the teacher's beliefs had a strong influence on the content and methods they presented to the class and thus on the sociomathematical norms that were created. In both these studies the understanding of what constitutes an acceptable mathematical explanation is seen as a key sociomathematical norm. Cobb et al. (1992) showed that in the inquiry-based classroom in their study, students evolved from evaluating mathematical explanations on the basis of social status of the explainer to examining the actions on mathematical objects in the explanations. The explanations became more conceptual rather than calculational and ultimately students became aware of the need to make their explanation understandable to others.

Sociomathematical norms are forged and continuously adapted in the classroom interaction between the teacher and learners. In the context of a largely remote MKT course for teachers the opportunities for dialogue are limited which makes negotiating shared norms

difficult. The teachers are active participants in other arenas (e.g. their own classrooms, official Education Department structures) in which, possibly competing, sociomathematical norms are being constructed. In addition, they bring with them experience of sociomathematical norms constructed in the classrooms in which they were learners or pre-service teachers which can have strong and long-lasting effects on their beliefs (Westaway & Graven, 2019). Understanding the norms that the teachers bring to the course is an important initial step in ensuring we do not end up with incommensurable discourses.

Methodology

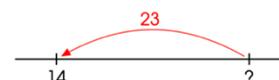
The teachers involved in the MKT course teach across 16 schools located in four different provinces of South Africa. Only two days of in-person training are possible per year and so the ongoing support of the teachers is delivered via weekly videos and quizzes delivered via WhatsApp. To investigate the research questions, we focused on the mathematical problem from the vignette: How would you help a learner who is stuck trying to figure out what number needs to go in the empty box when given $\square - 23 = 14$? We created four brief “talking hand” videos each modelling a possible teacher explanation. These explanations were purposefully created to reflect different underlying mathematical norms. This paper focuses on three of the videos, summarised in Table 1.

Teachers viewed each video and were asked whether they liked the explanation, and why. After watching all the videos, they were asked which explanation(s) they would use in their classroom, and whether there were any they would not use, and why. This task was completed at the in-person training session at the start of the 2025 school year. A total of 76 teachers consented to having their responses analysed. We first counted the number of teachers who like or disliked each video and then conducted a grounded analysis of the reasons given, through iterative coding. In doing the grounded analysis, we initially provided a face-value summary code for each reason e.g. “simple” or “uses a number line to show what is happening”. Thereafter one of the researchers grouped related codes together and provided a descriptor indicating the inclusion criteria for that group e.g. “States that explanation is clear/straightforward/quick with no other reason given”. These over-arching codes were cross-checked by the other two researchers and were refined and improved in that process.

Table 1

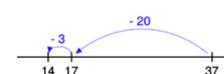
Description of the explanations given to help learners to solve for \square in $\square - 23 = 14$

Video A: Teacher illustrates on number line that we start at the unknown and jump back 23 to get to 14. She uses that illustration to show we can calculate the unknown by jumping forward 23 steps from 14 and then does that calculation by saying 14 plus 20 is 34 and then plus 3 is 37.



Video B: Teacher just states that to get the answer we need to add. She then writes $23 + 14$ and says 23 plus ten is 33 plus another 4 is 37.

Video C: Teacher states that we can use a number line to help us and we can start at 37 and jump back 23. She shows a jump back of 20 to 17 and then a jump back of 3 to 14 and concludes that what goes in the empty box is 37.



The mathematical and pedagogical norms privileged by the course designers would suggest that video A is a good explanation in that it uses a relevant mathematical model (the number line) to justify why adding would be the appropriate operation to use to get the correct answer and then illustrates an efficient procedure to calculate the answer. Video B would be seen as an inadequate explanation as it contains no justification for adding and thus, although offering a correct procedure, offers a “rules without reason” view of mathematics and runs the risk of overgeneralisation to cases where addition might not be the correct operation. Video C would be seen as a bad explanation (or non-explanation) in that it starts with the answer and thus offers no method for getting to the answer.

Findings and discussion

Table 2 indicates the number of teachers who liked each video, the videos they felt would be best to use in their classes and those they would not use in their class.

Table 2

Teachers’ evaluation of the explanations provided in the videos

The number of teachers who ^a	Video A	Video B	Video C
Liked the explanation	62	64	40
Did not like the explanation	14	11	33
Would be best to use in your class	36	41	28
Would not use in your class	15	11	35

Note: ^aSome teachers did not state whether or not they like video B or video C. Some teachers chose more than one video as giving the best explanation and some teachers said there were no explanation they would not use.

In contrast to the course designers, the explanations in videos A and B seemed to be more or less equally popular with the teachers. Along with the course designers, 33 teachers did not like the explanation in video C and 35 said they would not use it in their class. However more than half the teachers liked video C and more than a third of the teachers considered it would be best to use in their class. In addition, of those who did not like the explanation, only 20 provided some indication that starting with the answer was problematic. The remaining 13 who did not like the explanation rejected video C for reasons like “the explanation is long therefore only a few will be able to do it correctly” or “it is complicated”.

The divergence between what the course designers see as a good mathematical explanation and what the teachers viewed as a good mathematical explanation raises the question: what key themes emerged from the coding of the reasons teachers gave for liking an explanation? In the grounded analysis of reasons teachers gave for liking the explanation four key categories emerged.

Table 3

Four key reasons teachers gave for liking an explanation

	Video A	B	C
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States that explanation is clear/straightforward/quick with no other reason given.	19	21	5
Likes the fact that one just adds the two numbers to get the solution	4	26	0
Likes that a number line or visual representation is used	22	0	12
Likes the fact that the numbers are broken down into tens and ones to do the calculation	4	10	17

Teachers appreciated clarity and simplicity, for example praising explanations for being “short and easy for learners to understand” or “the quickest and simplest method to find the answer”. Although clarity and simplicity in explanation would be a norm shared by the course designers, it would not be considered sufficient for a good explanation. 21 teachers like the explanation in video B for this reason and a further 26 appreciated the simplicity of just adding to get the answer with statements like “it is easy and simple to explain to learners. I think learners will enjoy it. Learners will be able to answer this question faster”. It appears that the majority of the teachers value procedural clarity and simplicity over justification. This appreciation of the ease or speed contrasts with Cobb et al.’s (1992) findings of a move towards privileging a mathematical basis for an acceptable mathematical explanation.

The number line in video A is used as a model to help think about the mathematical problem and understand the operation needed to get to the answer. 22 of the teachers liked the use of the number line, but their justifications were stated more as a general appreciation of the use of number lines (e.g. “Number lines help learners with addition and subtraction problems”) than as model of the problem situation.

For a proportion of the teachers their focus on the explanation was on the calculation method and in particular on the fact that it was made easier by breaking down the 23 into tens and ones, thus valuing the procedural calculation over the mathematical integrity of the solution. Again, this is in direct contrast to Cobb et al.’s (1992) findings of a move towards valuing the conceptual over the calculational in mathematical explanations suggesting a strong divergence between the sociomathematical norms of the teachers and those the course designers would have anticipated the teachers having.

Conclusion

This analysis suggests a mismatch between the norms of the course designers and those of most of the participating teachers. While both groups value clarity and efficient calculation, the course designers emphasize sense-making and justification whereas the teachers prioritize straightforward procedures to get to the solution and have a focus on the calculation strategies. The challenge for the course design is to consider how to open a dialogic space for exploration of these norms and to create the possibility for developing shared sociomathematical norms in the context of a course that is almost entirely remote. In the current iteration of the course we will be trialling using the themes that have emerged from this research to present video vignettes where mimicking the debates that might happen about the norms in an in-person setting and creating tasks that surface different norms and offer opportunities to reflect on them.

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Adapting instrumental and relational descriptors to study comments on instructional explanations in early mathematics

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In this paper, we share details of our process of adapting Kinach's (2002) 'frames of knowledge' indicators, developed for considering the quality of instructional explanations in middle and higher-grade mathematics in the United States of America, to early mathematics instructional explanations in South Africa. We detail the thinking and the literature underlying our adaptations based on a dataset that consists of South African primary school teachers' comments on instructional explanations as part of a baseline assessment of their mathematical knowledge for teaching. Examples of teacher comments from our dataset are then coded and commented on using our adapted descriptors. Our hope is to develop these descriptors into a model that can be used to understand the gap seen in broader South African data between teachers of early grade mathematics who can solve the problems they teach themselves, while leaving the majority of children unable to meet grade-level curricular expectations.

Keywords: Instrumental/relational knowledge, subject matter knowledge, pedagogical content knowledge, instructional explanations

Background

At primary level, much more attention has been given to mathematical and pedagogical content knowledge from the middle grades onwards in comparison to the early grades. In contexts where the majority of students meet early grade-level expectations, this deferral of attention is understandable. However, South Africa is an example of a 'majority marginalised' context – where the majority of children in the early grades fail to meet core grade-level outcomes. Spaul et al. (2022) found that by the end of Grade 1 (aged 6-7), just 31% of learners in the Eastern Cape province were able to subtract single-digit numbers, with even lower proficiency levels reported in Limpopo province. Schollar (2008) found that a significant percentage of Grade 5 and 7 learners still relied on unit-counting, demonstrating that weak early numeracy skills persist into later grades and hinder mathematical progression.

While testing of teachers' mathematical knowledge in South Africa has tended to focus on the middle grades and beyond, classroom observation studies have noted that early grades' teachers can, in most cases, solve the problems they set for students themselves, and particularly so with number problems in the 1-100 range that predominate in Grades 1 and 2. Nonetheless, these studies highlight issues with teachers' ability to assemble instructional explanations for learners, leading to gaps in coherence and understanding. Prior research has documented specific challenges, such as the mixing of given and unknown quantities (Mathews, 2021) and difficulties in effectively coordinating instructional explanations with representations (Venkat & Askew, 2018).

Bowie et al. (2022), in recognising that poor number sense in high-school learners was limiting their ability to engage with mathematics, shifted their attention to primary schools and building learners' facility with number from the start of their schooling. Evidence arising

from the subsequent intervention study pointed to the acute need to improve models for supporting and studying early mathematics instruction.

Methods

One route into doing this is to consider instructional explanations linked to early mathematical tasks. In this paper, we use data gathered within a baseline assessment that was set as part of an intervention project that seeks to improve primary teachers' mathematical and pedagogical content knowledge linked to the early mathematics that these teachers are focused on as part of an after-school Maths Clubs programme. This baseline assessment included early mathematics problems and tasks with a more pedagogic slant, for example, moving between number or word problems and models of these problem situations in number lines and part-part-whole diagrams, diagrams widely described as particularly useful for supporting early number problem solving (Beishuizen, 1993; Xin, 2012). One baseline assessment task was focused on teachers' responses to four short video clips, each offering a different explanation in response to a student's question: "*Teacher, please help me. I am stuck with this question: $__ - 23 = 14$.*" The videos consisted of the following responses:

- **Video 1:** The teacher starts by using the known numbers (23 and 14), trying subtraction first: $23 - 14 = 9$. She tests this: $9 - 23 \neq 14$, so it is incorrect. Then she tries addition: $23 + 14 = 37$. Testing this: $37 - 23 = 14$ works. The teacher emphasises using what you know, trying a method, and checking the result.
- **Video 2:** The teacher uses a number line and plots a box (mystery number) on the right. She explains that subtraction means jumping backward, and that a -23 jump lands on 14. To find the mystery number, she starts at 14 and jumps forward 23, reaching 37. She concludes that the mystery number is 37 and places it in the box.
- **Video 3:** The teacher states that such problems are easy because you just need to add the numbers: $23 + 14 = 37$. This is confirmed by checking: $37 - 23 = 14$. The teacher reinforces this as a general rule: if you see a question like this, just add the numbers.
- **Video 4:** The teacher uses a number line but starts at 37, the unknown. She explains subtraction as jumping back: $37 - 20 = 17$, then $17 - 3 = 14$. So, jumping back 23 from 37 lands on 14, confirming that 37 goes in the box.

Videos 1, 3, and 4 were based on explanations seen in earlier studies and in teachers' explanations from the pilot rollout of the mathematical knowledge for teaching course in this project in 2024. After viewing each video, participants were asked to indicate whether they liked it, would use it in class, or would avoid it, and to give reasons for their choices.

Studying Instructional Explanations

With the data in, the research team started to explore routes into understanding the comments offered in response to each video clip in terms of what the teacher valued or emphasised mathematically. At this point, we dipped into the literature on instructional explanations to understand concepts developed in earlier studies that we could use. A key model of interest was Kinach's (2002) presentation of different 'frames of knowledge' that

could feature within instructional explanations. Kinach's work is a modification of an earlier model focused on levels of subject-matter understanding, which was originally proposed by Perkins and Simmons (1988). Kinach distinguishes between instrumental and relational understandings, drawing on Skemp's (1978) conceptualisation. In Kinach's (2002) model, the 'content' frame aligns with instrumental understanding, which is typically characterised by procedural explanations that emphasise algorithms, vocabulary, and rote skills, and are linked by Kinach to traditional school learning of mathematics (p. 55). In contrast, the 'concept', 'problem-solving', and 'epistemic' frames are associated with relational understanding, reflecting deeper conceptual reasoning, strategic thinking, and consideration of disciplinary warrants for certain actions, with all of these latter categories seen as important parts of pedagogic content knowledge and teaching that extends beyond communicating procedures. Kinach's descriptions of these frames of knowledge in the paper are brief (see table 1). They are also applied to higher level mathematics content – the addition and subtraction of negative numbers in her paper, with Perkins and Simmons earlier work developing the original concepts at higher levels also and looking across mathematics, science, and computing.

Table 1

Kinach's Frames of Knowledge Descriptors

Frame	Kinach's description
Content	Algorithms, vocabulary, facts, and rote skills that is usually taught in schools
Concept	Knowledge about and experience with the generalized ideas that define, bound, and guide inquiry in a discipline
Problem-solving	General and domain-specific strategies and heuristic schemas for monitoring one's own thinking
Epistemic	Warrants for evidence in a discipline

These concepts appeared useful to us as routes into considering the extent to which our teacher group, the majority (87.5%) of whom were able to answer the same item correctly in an earlier section of the test, would reflect the content frame that Kinach described as prevalent amongst the pre-service teachers in her study. To do this though, we needed to develop descriptors better suited to the much simpler mathematics of the task at hand, but retaining the echoes of Kinach's ranging across different aspects of the mathematical terrain.

In the next section, we present our initial thinking about adapted descriptors and the thought processes that fed into their constitution. We then go on to share some illustrative examples of each of our adapted categories. We comment on some of the slants that we are seeing in the South African dataset, drawn from a context of teachers' expressions in English, which, for the majority, is not their first language, and where problems with coherent translations of subject matter knowledge into instruction have already been noted.

Tailoring Kinach's Categories to Early Mathematics

In Table 2, we present adapted descriptors for Kinach's categories and comment on the thinking behind them. Where descriptors are linked to particular video clip instructional explanations, these are noted.

Table 2

Adapted descriptors

Frame	Adapting Kinach's descriptors for early mathematics
Content	<p><i>Algorithms, vocabulary, facts, and rote skills that is usually taught in schools</i></p> <p>Valuing the communication of the steps or sequence of steps needed to solve the problem.</p>
Concept	<p><i>Knowledge about and experience with the generalised ideas that define, bound, and guide inquiry in a discipline</i></p> <p>Communication of sense of the broader terrain in which the focal task is located - e.g.:</p> <ul style="list-style-type: none"> - knowledge of the 'reach' of the solution procedure that can be applied to $_ - 23 = 14$; - awareness of examples where the same solution procedure cannot be applied; - awareness of how inverse property can feature within solution of this task; - awareness of working from givens to unknowns as a <u>general</u> feature of mathematical working.
Problem-solving	<p><i>General and domain-specific strategies and heuristic schemas for monitoring one's own thinking</i></p> <p>Metacognitive monitoring of solution process offered in the video clip, e.g.:</p> <ul style="list-style-type: none"> - aware that the missing number in $_ - 23 = 14$ must be bigger than 23 and 9 in order for the subtraction to be carried out; - (Video 1): aware that trial and error approach shows why subtracting the given numbers won't work - (Video 2): Aware that reversing direction of travel on number line leads to missing number 37 - (Video 3): Aware that adding the given numbers follows from awareness that the missing number must be bigger than 23, as subtracting given numbers will give a value smaller than 23 - (Video 4): Aware that starting explanation with 37 can only work if students already know how to produce 37 as the answer
Epistemi	<i>Warrants for evidence in a discipline</i>
^c	Aware of the need for reasons for selections and actions

A range of literature underpinned our initial devising of adapted indicators. While the content frame descriptor was largely similar, the emphasis on ‘generalised ideas’ in the concept frame was applied in terms of awareness of the terrain of tasks related to $_ - 23 = 14$, and of how problem-solving procedures play out across this terrain. This thinking drew from Goldenberg & Mason’s (2008) writing on dimensions of possible variation and range of permissible change, in which the reach of procedures across an example space is considered.

In the problem-solving frame, we followed Kinach’s descriptor’s focus on strategies, schemas and self-monitoring to metacognitive awareness (Schoenfeld, 1987) guiding the problem-solving procedure, ensuring more efficient problem-solving. In the epistemic frame, Kinach’s descriptor was retained, but worded in terms of awareness of the need to offer reasons, or rationales, for selections and actions.

Coding Data Exemplars

In this section, we present and discuss the codes and comments we associated with a small selection of responses given by teachers in our baseline sample. We found several instances that contained allusions to particular frames, but with limited specific detail and, in some cases, incorporating mathematical errors or misconceptions. These examples are marked with an asterisk to denote problematic or partial presence of that frame in Table 3.

Table 3

Coding Responses – Examples and Commentary

Teacher code	Video #/response	Code/Commentary
G35	V1: ‘showed or proved that the number we’re looking for is bigger than 9. Learners will see that they have to add two numbers to get a number bigger than the one provided.’	Problem-solving: Focus is on the specific example, with a vantage point on what the trial-and-error procedure does offer: visible evidence that subtraction did not work, and thus, leading into the need to add the given numbers
G18	V2. ‘easy to do and explain ... just add given numbers’	Content: Focus is solely on the calculation to be carried out.
WC18	V3: ‘The teacher does not explain why the learner should add. Thus, not teaching the concept/skill. Would the learner know when to apply this rule? What if the operator changes?’	Epistemic: Focuses on the absence of a rationale for adding the given numbers. There are hints here that the teacher is aware of examples where adding the given numbers won’t apply, but this is not communicated explicitly, so we have not added the concept frame here.

KZN2	V4: ‘It does not make any sense. We need to use the numbers that are given to us, so 37 was not given. So, what will I tell a learner about 37, when he/she asks where I got it from? This explanation will definitely confuse learners’	Concept The ‘need to use numbers that are given’ is stated here in general terms and then followed with the specifics from the task.
F10	V1: ‘It clears misconceptions of whether we should use the same sign to find the missing value.’	Content Focus is on the steps to calculate missing value, but what ‘it’ refers to is not specified.

Conclusion

Our initial sense is that our adaptation of frames is potentially useful. However, the fragmented nature of some of the responses means that there is likely more work to do to produce a robust and distinct set of descriptors that we can use to study our dataset. Our goal then is to see if we can develop our adapted indicators into a model that is robust enough for us to use to get insight into the profile of knowledge frames that these teachers are drawing on in their consideration of instructional explanations. The forward goal is then to understand whether more robust profiles based on knowledge frames allow us to develop professional learning content that can address any limitations that we find.

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Doing it by the book: The role of textbooks in Irish primary mathematics classrooms

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Introduction

Concerns about student outcomes in mathematics give rise to recommendations about what to teach in schools and how to teach it. The new Primary Mathematics Curriculum (NCCA, 2023) identifies five key pedagogical practices as essential to the provision of high quality learner experiences in Mathematics: Fostering productive disposition, using cognitively challenging tasks, promoting maths talk, emphasising mathematical modelling and encouraging playfulness. A consideration of the role of textbooks is essential in this endeavour because, as many advocates of reform attest, the potential of textbooks in influencing classroom practice and subsequent student achievement is considerable (Rezat et al., 2021). Research conducted by Duffy et al. (2022) report that teachers appear to relinquish their responsibility for lesson planning to textbooks which provide a routine and time saving approach to teaching and learning mathematics. The authors further purport that teachers were found to rely on textbook schemes to inform what happens in mathematics lessons choosing to talk about mathematics in relation to chapters in textbooks with little or no conceptual framework for the subject. While textbooks are useful in supporting the teacher's goal of introducing students to the practices and language of the mathematics community, a major failing of textbooks arises when teachers try to cover all aspects of the text, hindering or ignoring the application of suitable methodologies for teaching and learning.

Methodology

In Irish primary schools, children are seldom consulted or given the opportunity to formally express or document their experience of learning in school. The pervasive social lens through which children's learning is examined is an adult one. This study employs a mixed methods design approach across eight primary school classrooms at second (7 and 8 year olds) and fifth class level (10 and 11 year olds). In this paper, I focus on data drawn from a series of three observations of mathematics lessons, children's drawings and questionnaires and focus group interviews with teachers, children and school leaders.

Findings and Discussion

It should be more the teachers explaining because the text books just give you the questions...they can't answer your question. It is not the real person (Declan, Mount Eagle, 5th Class, Middle Class).

In the schools that participated in this study, the textbook is a very influential factor in determining the what, when and how of actual teaching. This research found that learning mathematics is still largely based on the assumption that it is learned from a textbook. An examination of the mathematical tasks and activities encountered by the children in this research, particularly in the senior classes, found that children worked on routine problems broken into discrete steps and divorced from their real-world experiences. These tasks served

to test children's knowledge and understanding that they acquired through repetitive drill and practice. The teachers in this study found that the textbook provided a routine and time saving approach to teaching and learning mathematics. Teachers discussed mathematics in relation to chapters in the textbook with little or no conceptual framework for the subject. Strong adherence to the textbook by four of the eight teachers in this study revealed how teachers allowed the textbook to 'frame' the curriculum. In the classroom teachers presented academic knowledge in the form of the textbook, which was the major pedagogic device. Children's drawings and focus groups with children revealed that mathematical knowledge and competence is something only to be gained through conscientious application to solitary written work defined through the directives of the teacher, textbook, worksheet and whiteboard. Children's drawings showed that Mathematics was presented as finding the right answer to routine questions and tasks. Strong framing in the majority of mathematics classrooms manifested itself through repetitive, individually orientated, fast paced, sequenced tasks at both second and fifth class level. Such strong framing in classrooms provided less opportunity for self-directed, independent, flexible learning as the management of selection, organisation, pacing and timing of knowledge presented to students became more constrained.

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Observing and Documenting Early Mathematical Skills: Beliefs and Reported Practices of Early Years Professionals

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Keywords: Early mathematics, beliefs, reported practice, observing and documenting

Introduction

In order to support sustainable growth in mathematical competences for all children, early years professionals should support mathematical development in an adaptive way (Clements & Sarama, 2024). Achieving this requires knowledge of the child's mathematical skills, for instance, through observing and documenting. In many countries, early mathematics education mostly takes place in everyday (play) situations. Therefore, observing is usually done spontaneously during these everyday activities but may also be applied in specifically designed activities and planned one-on-one situation (Benz et al., 2024). As there are hardly any studies on the perspectives of early years professionals towards observing and documenting mathematical skills, this study set out to examine both beliefs and reported practices of early years professionals.

Methods

99 German early years professionals ($n=93$ female, $n=5$ male, $n=1$ diverse) with an average age of 42 years ($min=22$ years, $max=65$ years; 16 years of average work experience) participated in an online survey evaluating their (i) *affective attitude*, (ii) *instrumental attitude*, and (iii) *self-efficacy* towards observing and documenting as well as (iv) their *reported practices*. The questionnaire was adapted from Yan and Cheng (2015) for the context of early (mathematical) education. The subscale *affective attitude* addressed early years professionals' emotions towards observing and documenting (10 items). In the subscale *instrumental attitudes*, professionals' views on the perceived benefits of observing and documenting were addressed (9 items), while the subscale *self-efficacy* targeted professionals' perceived ability and contextual conditions for observing and documenting (11 items). For all three subscales, early years professionals had to indicate their agreement on a 6-point Likert scale (i.e., 1=strongly disagree, 6=strongly agree). The professionals' *reported practices* were gauged by two items, one for observing and one for documenting mathematical skills (i.e., "Please estimate, in the past six months, how often you have implemented observation/documentation of children's mathematical skills in your institution"). Professionals could choose between 6 response options: 1=never, 2=rarely (on less than 3 days), 3=seldomly (about 5 times), 4=sometimes (every 2 weeks), 5=frequently (once a week), 6=very frequently (everyday/almost every day).

Results

Results showed that professionals had an overall positive *affective attitude* towards observing and documenting ($M=5.23$; $SD=.57$). This is consistent with results observed for their *instrumental attitude* ($M=5.06$; $SD=.64$), suggesting that they on average agree that observing and documenting improve the quality of professional support and promote children's learning. Furthermore, results revealed that professionals estimated their *self-efficacy* regarding observing and documenting mathematical skills ($M=3.67$; $SD=.88$) descriptively lower than their *affective* and *instrumental attitude*. As regards *self-efficacy*, the items "I have sufficient supporting materials" and "I have enough time" received the lowest level of approval, while "I can integrate observation/documentation into everyday daycare", and "I have sufficient skills" were rated the highest. This indicates that professionals are rather confident with respect to their abilities compared to how they perceive and evaluate contextual conditions. Contrary to their reported attitudes, the descriptive analysis of the two reported practices items revealed, that they observed children's mathematical competencies not frequently over the past six months ($M=3.61$; $SD=1.46$) – which means every two weeks or less for 69% of the professionals. Documentation of the observed competencies occurred even less often ($M=2.68$; $SD=1.17$) with 72% of professionals indicating that they documented mathematical competencies about 5 times or even less in the past sixth months.

Discussion

All in all, the survey revealed a discrepancy between attitudes (affective and instrumental) and reported practice and highlights a potential gap between belief and actual implementation with respect to observing and particularly documenting children's mathematical skills (for similar findings in primary school teachers see Yan and Cheng, 2015). According to the results on professionals' self-efficacy regarding observing or documenting mathematical skills, it is likely that other, external factors, like time or materials are the reason for the less frequent implementation. In further analyses, connections between possible influencing factors will be examined.

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Curricular Reform: Considering the role of the growth mindset of School leaders, through the lens of the Primary Mathematics curriculum 2023

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Introduction

Curricular change involves both structural shifts and changes in mindset. This study explores whether fostering a growth mindset in curriculum leaders supports the successful implementation of the Primary Mathematics Curriculum (PMC), (NCCA, 2023), with a focus on promoting productive disposition. Defined by Dweck (2006) as the belief that abilities grow through effort and feedback, a growth mindset may help educators view reform as an opportunity rather than a threat.

Growth Mindset and Productive Disposition

Individuals with a growth mindset show greater ability to cope with change (Dweck, 2006, a trait that can extend to institutional cultures and promote adaptability (Yeager & Dweck, 2012). Resistance decreases when educators are supported to reflect and learn (Oreg, 2006), and responsiveness to feedback is a key feature of growth-minded individuals (Kluger & DeNisi, 1996). Productive disposition, central to the PMC (NCCA, 2023) aligns closely with growth mindset, encompassing a belief in the value of mathematics and the ability to improve through effort (Kilpatrick et al., 2001). This is reinforced by Bandura's (1993) concept of self-efficacy, which links belief in one's competence to greater engagement and perseverance.

Planned Study

This doctoral study, guided by Bandura's (1993) conception of collective efficacy and Fullan's (2007) idea of visionary leadership, investigates how curriculum leaders who model a growth mindset can build a culture of trust and openness. Teacher autonomy and collaboration are key factors. Drawing on the Finnish model, where local curriculum adaptation boosts engagement (Hargreaves, 2000; Sahlberg, 2011), the study also explores how leadership and pedagogy shape students' productive disposition, supporting engagement and mathematical identity (NCCA, 2023).

Challenges and Limitations

While growth mindset offers valuable insights, overreliance can obscure systemic inequities by placing responsibility on individuals and overlooking institutional barriers (Brown, 2017; Ladson-Billings, 2014). Sustainable cultural change requires strong leadership, coherent policy, and long-term professional learning (Hargreaves & Fullan, 2012); without these, efforts may remain at the surface-level (Dweck, 2015). An individualised focus may also overlook the collaborative, systemic nature of educational change (Yeo & Neal, 2004). This study explores whether curriculum leaders modelling a growth mindset, supported by aligned policy and CPD can drive local curriculum reform. In the context of the PMC, growth mindset aligns with promoting productive disposition and learner agency. However, mindset

alone is not enough; it must be embedded within a wider framework that acknowledges systemic challenges, resource needs, and school culture complexity.

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Developing capacity for agency: Supporting pre-service teachers' enactment of the *Primary Mathematics Curriculum*

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National Council for Curriculum and Assessment

This paper will explore the role of teacher agency in the redeveloped Primary School Curriculum, and in particular, within the *Primary Mathematics Curriculum* (PMC) (Department of Education [DE], 2023a). It will provide an overview of agency in recent curriculum redevelopment and will highlight the role of the Primary Mathematics Toolkit in assisting teachers' decision-making within their own unique contexts as they enact the PMC. As part of a workshop session, participants will be invited to examine and discuss the role of Initial Teacher Education (ITE) providers in developing pre-service teachers' capacity to develop and demonstrate agency as they navigate the initial stages of their professional careers as teachers. Participants will be asked to consider how the current model of school placement might be reimagined and refined to align with the vision of agency outlined in the *Primary Curriculum Framework* (PCF) (DE, 2023b) and the implications of this vision for agency in supporting pre-service teachers' engagement with school placement, and for fostering their professional capacity for decision-making as they begin their teaching careers.

Keywords: Curriculum development, primary mathematics, curriculum enactment, teacher agency

Introduction

The National Council for Curriculum and Assessment (NCCA), comprising twenty-six members, is a statutory body of the Department of Education and Youth. NCCA advises the Minister for Education and Youth on curriculum and assessment for early childhood education, primary and post-primary schools, as well as assessment procedures used in schools and examinations on subjects which are part of the curriculum.

NCCA develops this advice by drawing on four key pillars: research, deliberation, consultation and networks. Feedback from stakeholders is gathered through rigorous consultation processes and is deliberated upon in developing curriculum and assessment advice. The Council is supported by three boards (early childhood and primary, junior cycle and senior cycle) and a range of development groups that contribute to the development of specific curriculum specifications and support.

The Primary School Curriculum in Ireland has undergone significant change and redevelopment in recent years. The *Primary Curriculum Framework* (PCF) (DE, 2023b) presents the vision, principles and key competencies that underpin the redeveloped curriculum. Stemming from this is the new *Primary Mathematics Curriculum* (PMC) (DE, 2023a) which was published in 2023, with enactment in schools from September 2024. The PMC is characterised by playful and engaging learning experiences that support and celebrate the learning journey and progression of all children in primary and special schools in Ireland.

In developing the PMC, NCCA engaged in an extensive consultation process (NCCA, 2018; NCCA, 2023), drawing on feedback of a wide range of stakeholders. In addition, developments are underpinned by an extensive body of research, including *Mathematics in*

early childhood and primary education (3-8 years): Research report no. 17 (Dunphy et al., 2014) which focuses on the theoretical underpinnings of mathematics education, and *Mathematics in early childhood and primary education (3 – 8 years): Research report no. 18* (Dooley et al., 2014) which outlines some of the related implications. A background paper and brief for the development of the PMC was also developed (NCCA, 2016).

This paper provides insights into the significance of the concept of teacher agency within the PMC and the overarching PCF. Furthermore, a workshop session will be facilitated in which participants will have the opportunity to explore the implications of teacher agency on the pre-service teachers' initial teacher education.

Teacher agency in the redeveloped curriculum

A key feature of the redeveloped curriculum is the significance and prevalence of teacher agency. Teacher agency refers to the capacity of teachers to make informed decisions around learning, teaching and assessment, with such decisions centred on their knowledge of children and their prior learning, strengths, needs and interests (DE, 2023b). In addition, knowledge of the curriculum and pedagogy contribute to a comprehensive approach to learning, teaching and assessment with due account taken of schools' unique contexts (DE, 2023a; DE, 2023b; Burke & Lehane, 2023; Biesta et al., 2015). Both the PCF and the PMC empower and support teachers and schools to demonstrate agency and flexibility (DE, 2023a; DE, 2023b).

Agency within a Learning Outcomes based curriculum

Central to teacher agency within the PMC is the inclusion of Learning Outcomes, as distinct from the more prescriptive learning objectives of the 1999 Primary School Curriculum (DES, 1999; DE, 2023a). Learning Outcomes are intentionally broad and flexible, in recognition of the diverse range of learners in primary and special schools. They provide teachers with the scope to make professional judgements around the kind of learning experiences that will best support children's mathematical progression (NCCA, 2016). In the context of the PMC, Learning Outcomes describe the expected mathematical learning of children at the end of a two-year stage. For example, within the Strand Unit '*Sets and operations*', the Learning Outcome for Stage 1 (junior and senior infants) states that "through appropriately playful and engaging learning experiences, children should be able to recognise and understand what happens when quantities (sets) are partitioned and combined" (DE, 2023a, p. 22) and by Stage 4 (fifth and sixth classes), children should "build upon, select and make use of a range of operation strategies" (DE, 2023a, p. 22). While a curriculum that empowers teachers to be "skilful and agentic professionals" (DE, 2023b, p.20) can contribute to improved educational outcomes, it is also important that curriculum structures enable this agency (Manyukhina & Wyse, 2025; Burke & Lehane, 2023). As a mechanism for supporting teacher agency in enacting the PMC, NCCA have developed the online Primary Mathematics Toolkit. The role of the Toolkit in fostering teachers' capacity for agency is outlined in the next section.

Supporting agency through the Primary Mathematics Toolkit

As teachers seek to enact the PMC with agency and through playful and engaging learning experiences as promoted within the PMC (DE, 2023a), NCCA has developed an online Toolkit which hosts a range of supports materials and resources that teachers can draw upon to inform practice and professional decision-making. The Toolkit is designed to respond to questions that teachers may pose as they consider the unique learning experiences that are most relevant to their context, thus providing scope for the demonstration of agency. For example, in considering the mathematical knowledge that children ought to acquire as they work towards a particular Learning Outcome, teachers may refer to mathematical concepts, that are available as part of a suite of support materials for each of the fifteen Strand Units of the PMC (see Figures 1 and 2). These mathematical concepts capture the key ideas that underpin each Learning Outcome and may serve as a reminder to teachers of essential mathematical knowledge at each stage. In considering what playful and engaging experiences might look like in the classroom, teachers may explore the Strand Unit support materials ‘Suggestions for children’s learning’ (see Figure 3). Importantly, the Toolkit provides educative *suggestions* for teachers, offering supports to draw and build upon in ways that are meaningful to them, and that can be adapted to best suit the needs of the children in their class, and their unique school context. The Toolkit is in continued development and as such is well placed to respond to the emerging and evolving needs of schools, teachers and children.

Figure 1

Support Materials for the Strand Unit ‘Measuring’

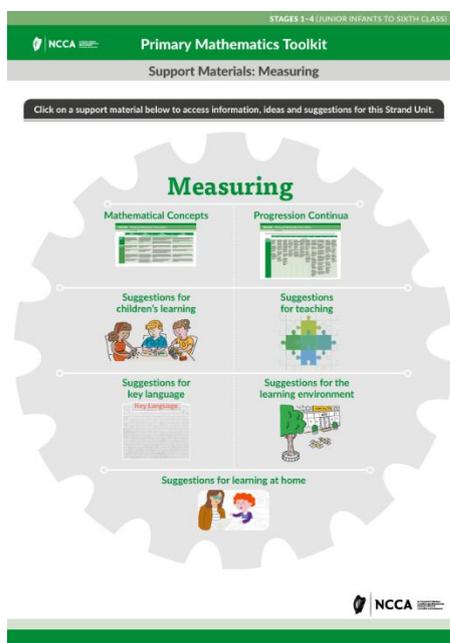


Figure 2

Mathematical concepts for the Strand Unit ‘Measuring’

NCCA Primary Mathematics Curriculum				
Measuring				
	Stage 1 (Junior & Senior Infants)	Stage 2 (1st & 2nd Class)	Stage 3 (3rd & 4th Class)	Stage 4 (5th & 6th Class)
Learning Outcomes	demonstrate an awareness that attributes such as length, weight, capacity and area can be measured and compared.	compare, approximate and measure length, weight, capacity and area using appropriate instruments and record using appropriate units of measurement.	compare, estimate and measure length, weight, capacity, area and volume using appropriate instruments and record and communicate appropriately.	determine and calculate units of measurement in fractional and/or decimal form to solve practical problems.
Mathematical concepts	Objects have attributes that can be measured such as length, weight, capacity and area. One of the purposes of measurement is to compare. We can compare and order things by how much of a particular attribute (physical quantity) they have relative to each other. Attributes are compared and ordered using units of measurement.	Common base units of measurement are useful to make and test comparisons. The size of the unit chosen affects the number of units needed to measure an object. We can compare, measure and order physical quantities by selecting an appropriate unit and determining how many units the thing has/holds. Measurement instruments (e.g., rulers) are tools for measuring physical quantities or attributes such as length, weight and capacity.	Metric units help us to interpret, communicate and calculate measurements with increasing accuracy and precision. Measurements can be made more precise by selecting metric units (multiples or subdivisions of base units e.g., km or cm), while realising that all measurements have an inherent degree of approximation. The metric system is based on multiples of ten. Any measurement given in one metric unit (e.g., kilogram) can be converted to and renamed as another metric unit (e.g., gram). The relationships between metric pre-fixes can be understood and applied in a similar way across different units of measurement.	Purpose and practicality are important to consider when measuring attributes and selecting units and instruments for measuring. Purposeful descriptions and comparisons often involve the measurement of more than one attribute. The relationship between equivalent units in the metric system helps us to judge attributes, move flexibly between units and do calculations. Measurement sense develops as we anchor the meaning of measurement units to measurement benchmarks in the everyday world.

Figure 3
Suggestions for children’s learning for the Strand Unit ‘Measuring’

STAGES 1-4 (JUNIOR INFANTS - SIXTH CLASS)

NCCA Primary Mathematics Toolkit – Support material

Measures: Measuring – Suggestions for children’s learning

The child has opportunities to...

Understand and connect

- explore the conservation of length, weight, capacity and area and challenge possible misconceptions through practical activities, e.g., if you break a 100g piece of playdough into two pieces, the combined weight of the two halves is still 100g.
- identify how to measure using base units for length (metre), weight (kilogram), capacity (litre) and area (square metre) using a variety of measuring instruments.
- make links between base ten to move flexibly between units of measurement, e.g., 1000m = 1km, 1000g = 1kg, 1000ml = 1l.
- apply and connect prior knowledge of 2-D and 3-D shapes to estimate and measure, e.g., the perimeter and area of regular and irregular 2-D shapes, the surface area and volume of 3-D shapes.
- use knowledge of number to compare and order metric units of measurement in fractional and decimal form, e.g., compare and order $\frac{1}{2}$ l, 0.75l, 350ml.

Reason

- analyse the need for units in measuring through engaging in hands-on measuring with non-standard and standard units.
- create and justify conjectures based on personal benchmarks, e.g., if I am 140cm, then the door must be at least 180cm.
- identify and validate the appropriate measurement instruments and units for a given situation, e.g., 10ml spoon or 1 litre jug.
- engage in practical activities that require smaller units of measurement for a more accurate measurement.
- use estimation to calculate sums, differences, products and quotients of measurements (e.g., we have a two litre bottle of orange juice to share among the class, approximately how much juice will each child get?) and when required or useful use formulae generated from experience.

Communicate

- describe the process of measuring and justify the selection of units for measurement, e.g., it is difficult to measure the length of a room in cubes, it is impractical to measure the length of the school yard using centimetres.
- listen to, compare and discuss other children’s estimations and measurements using base units and symbols.
- use concrete, pictorial and abstract recording when estimating and measuring.
- explore how the use of digital technology can represent measurements, e.g., annual rainfall, how logging measurements can generate simulations and models such as the volume of a 3-D shape.
- develop systematic approach to recording measures over time to communicate changes/developments, e.g., height of a plant.

Apply and problem-solve

- explore and compare measurable attributes of objects, surfaces and containers in contexts that are meaningful for the children, e.g., baking activities, measuring the distance from the school to buildings of interest in the locality.
- investigate how to read a variety of common measuring instruments using increasing accuracy.
- use repetitions of the same size unit to make approximate measurements.
- apply knowledge of measurement to real world situations, e.g., which is better value for money?
- conduct investigations to solve problems and practical tasks involving more than one attribute, e.g., designing floor plans to suit criteria involving length and area, planning how to pack for a trip abroad with restrictions on size and weight of luggage.

Developed in collaboration with Sarah Keane, Primary School Teacher, Co. Clare

Goals of the Workshop

It is important that pre-service teachers, through the course of their initial teacher education, are supported to develop a positive sense of self as teachers of Mathematics (Dooley et al., 2014), as well as the ability and confidence to make professional decisions that serve the mathematical learning needs of the children in their context. The role of teacher agency is particularly nuanced when it comes to pre-service teachers. The PMC recognises that “teachers are best placed to determine the learning needs and strengths of the children in their class” (DE, 2023a, p.18). However, in making these determinations certain conditions are required for effective agency to truly come to the fore (Burke & Lehane, 2023). These include the capacity of the individual teacher, as well as contextual considerations, such as school culture and environment (Biesta et al., 2015). Both the former and the latter may present challenges for pre-service teachers as they work towards developing and exercising

their agency. Often, their knowledge of school context and the prior learning and experiences of children may be limited, thus impacting their ability or confidence to be agentic in their teaching.

In this workshop session, participants will be invited to consider the role of ITE providers in developing pre-service teachers' capacity for agency as they enact the PMC on school placement and through their scholarship of teaching as a profession. Participants will be invited to discuss the following: 1) What content and pedagogical knowledge do pre-service teachers need to build during their study and what does this mean in terms of content that is currently delivered? 2) How might the school placement model be reimagined to provide for greater opportunities and capacity building for agency? 3) How might ITE providers build capacity for agency in teaching mathematics in the context of early teaching experiences with the new PMC?

It is envisaged that these discussions will provide a stimulus for continued and sustained professional conversations within ITE and beyond. Through these conversations, pre-service teachers can be better supported to promote and support curriculum with agency, and importantly, in a way that is responsive to the bespoke context in which they teach whilst also ensuring the provision of learning experiences that reflect the vision and ambition of the PMC.

Conclusion

The introduction of the new PMC (DE, 2023a) presents opportunities and possibilities for ITE providers as they prepare and support pre-service teachers to enact the curriculum. Teachers' professional agency plays a crucial role in curriculum enactment, as they respond to the needs of children and the unique contexts of their schools. The workshop discussions outlined will offer a valuable opportunity to consider how ITE can actively support the ambition for teacher agency expressed in the PCF (DE, 2023b). By examining the content, experiences, and structures that shape pre-service teachers' professional learning, participants can reflect on how agency might be meaningfully developed and sustained from the outset of teacher preparation. In doing so, this paper aims to contribute to ongoing professional conversations about how existing models of teacher education and professional development might be refined to better reflect and support the vision for learning, teaching, and assessment, as set out in the PCF.

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