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Sales or Profits? Managerial Decision-Making in a Duopoly With Symmetric and Asymmetric Costs

Malcolm Brady

Business School, Dublin City University, Dublin, Ireland

Correspondence: Malcolm Brady (malcolm.brady@dcu.ie)

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ABSTRACT

The paper examines the performance of firms when managers can choose sales or profit maximization strategies and when costs can be symmetric or asymmetric. Three competitive scenarios are examined: both firms choose profit maximization strategies, both firms choose sales maximization strategies, and one firm chooses profit and its rival chooses sales maximization. The paper uses theoretical and numerical analysis to examine a number of symmetric and asymmetric cost scenarios and identifies six different types of industry equilibrium. It confirms the well-known prisoners' dilemma behavior but demonstrates that this occurs only at relatively low levels of cost.

1 | Introduction

While it may appear counterintuitive that a strategy of sales revenue maximization should outperform a strategy of profit maximization, it can be the case in certain circumstances. This paper examines the performance of sales maximizers and profit maximizers as they compete against each other under different cost conditions.

Objectives for firms other than profit maximization have long been suggested by authors. Several authors focus on growth as an objective. Baumol (1962) suggests that firms may pursue sales growth maximization; Koutsoyiannis (1979, ch. 15) suggests sales revenue maximization as an objective; Marris (1963) suggests that firms set out to maximize a balanced rate of sales growth. The managerial incentive literature suggests that firms pursue a combination of sales and profits (Fershtman and Judd 1987). Yet another stream of research suggests that managers are motivated to increase their perquisites (Jensen and Meckling 1976; Fama 1980). Behavioral theorists suggest that firms have several different goals: "profit maximization ... is either only one among many goals of business firms or not a goal at all" (Cyert and March 1992, 7).

This paper examines and compares strategies of sales maximization and profit maximization for two firms competing in a duopoly. In the first stage of a two stage game, managers simultaneously select a strategy of either profit maximization or sales maximization. In the second stage, managers simultaneously select the quantity to produce. From a modeling point of view, the paper draws on the seminal managerial incentives literature (Fershtman and Judd 1987; Sklivas 1987; Vickers 1985) and a more recent revival of this literature (Brady 2007; Colombo, 2019 and 2021; Delbono et al. 2016; Fang and Zhao 2021; Sen and Stamatopoulos 2015; Zanchettin 2006). Fershtman and Judd (1987) show that it may be in the interests of owners to incentivize managers to maximize a combination of sales and profits rather than to simply maximize profits. Brady (2007) shows that a weakly governed firm may outperform a strongly governed firm. Sen and Stamatopoulos (2015) show that a less efficient firm may outperform a more efficient firm. Delbono et al. (2016) show that a prisoners' dilemma arises if both sets of owners incentivize managers to maximize a combination of sales and profits as this leaves both firms worse off than if they had both chosen to be pure profit maximizers. Colombo (2019) demonstrates that the prisoners' dilemma can be avoided if firms are sufficiently different, with the more cost-efficient firm performing better than its less efficient rival. Colombo (2021) shows that vertical differentiation between firm products is another means by which firms can avoid the prisoners' dilemma. Fang and Zhao (2021) conclude that under cost asymmetry strategic delegation does not always lead to a prisoners' dilemma and that at high levels of cost asymmetry horizontal differentiation can have opposite effects on the two firms in the duopoly. Taking a different approach, Fanti et al. (2017) show that the prisoners' dilemma can be avoided if managers negotiate incentives with owners.

This paper takes the view that managers are the strategic decision makers in an organization. Managers are responsible for setting the objectives of the firm while the board of directors has oversight of this process (Thompson et al. 2020, 40). This contrasts with the economics literature which tends to assume that managers have little agency, with owners delegating to managers their strategic objective function (Fershtman and Judd 1987). This paper agrees with Sen and Stamatopoulos (2015) that a strategy with the aim of maximizing a linear combination of sales and profits, while acceptable in theory, is difficult to implement in practice; motivating an organization to maximize a linear combination of sales and profits would be a challenging task for managers. This paper therefore assumes that managers have a binary choice available to them at the strategy stage - they must choose to either maximize sales revenue or maximize profits; managers are not given the choice of maximizing a combination of both. This contribution is different to that of Colombo (2019, 2021), Delbono et al. (2016) and Fang and Zhao (2021) who compare the performance of entrepreneurial firms (pure profit maximizers) and incentive firms (where managers are delegated the objective of maximizing a linear combination of profit and sales). The paper extends and elaborates on the three cost levels identified in Brady (2007, 151).

Three scenarios are examined reflecting different combinations of manager choice: a scenario where both firms adopt a profit maximizing approach, a scenario where both firms adopt a sales maximizing approach, and a scenario where one firm adopts a profit maximizing approach and the other firm adopts a sales maximizing approach. From an analysis of these three competitive scenarios, three distinct symmetric cost levels are identified—high, medium, and low. The paper then numerically examines industry outcomes at each of these three levels of cost. Low-cost levels lead to both sets of managers maximizing sales and the industry outcome is the familiar prisoners' dilemma. High-cost levels lead managers to select profit maximization and the prisoners' dilemma is avoided. Mid-level costs lead to an anti-coordination game providing the opportunity for one firm to perform substantially better than its rival; but this embodies a coordination difficulty as to which firm of the two firms will reap the benefit, and the risk of a substantially worse outcome for both firms should they mis-coordinate. The paper then considers cost asymmetry and demonstrates that asymmetric costs can alter industry and firm outcomes. At high-cost levels a cost asymmetry can result in the cost leader eschewing profit maximization in favor of sales maximization. At mid-level costs the "chicken" situation can be avoided with the more cost-efficient firm choosing sales maximization and performing better than its less efficient rival who chooses profit maximization. At low levels of cost a cost asymmetry does not change the nature of the equilibrium attained and the prisoners' dilemma continues to hold.

2 | The Model

We assume that two firms exist in the industry and that price is set according to the inverse demand function:

$$p = a - b\,Q$$

that is,

$$p = a - b\left(q_1 + q_2\right) \quad (1)$$

where price p is assumed to be an inverse linear function of quantity q. The variable a is the reservation price which can be thought of as the highest price that the good is likely to achieve; for a firm to be viable a must be greater than unit cost c, that is, $a > c_1, c_2$. The variable b represents the behavior of the market, that is, as quantity increases price will decrease linearly according to b, the market response. Variables a, b, c, p, and q are assumed always to be positive, that is, $a, b, c, p, q > 0$. Subscripts 1 and 2 refer to firm 1 and firm 2, respectively. We note that the inverse demand function creates an interdependence between the two firms in the industry.

Competition is assumed to be Cournot: each firm chooses a quantity to bring to the market and the market then determines the price at which the total quantity brought by all firms is sold (Qin and Stuart 1997). Following the standard Cournot equilibrium analysis approach, we obtain the partial derivative of the objective function with respect to quantity and set the result to zero to obtain a maximum. We carry out this procedure for three scenarios: where both firms act as profit maximizers, where one firm acts as a profit maximizer and its rival acts as a sales maximizer, and where both firms act as sales maximizers.

2.1 | Scenario One: Both Firms Act as Profit Maximizers

In this case, we assume that managers of the two firms in the duopoly act to maximize profits. We assume that variable cost is a linear function of quantity and that there are no fixed costs. Both firms therefore pursue the following objective function:

$$O_i = \left(p_i - c_i\right) q_i \qquad i = 1, 2 \quad (2)$$

where p represents unit price, c represents unit cost, q represents quantity sold and the subscript i represents firms one and two.

The above procedure gives the following expressions, known as reaction functions, for quantities at equilibrium for the two firms:

$$q_i = (1/(2b))\left(a - c_i - bq_j\right) \qquad i = 1, 2 \quad j = 3 - i \quad (3)$$

Solving for q_1 and q_2 by substitution gives

$$q_i = (1/(3b))\left(a - 2c_i + c_j\right) \qquad i = 1, 2 \quad j = 3 - i \quad (4)$$

Inserting these expressions for quantities into the inverse demand function (1) gives the following expressions for price at the Cournot equilibrium:

$$p = (1/3)\left(a + c_i + c_j\right) \qquad i = 1, 2 \quad j = 3 - i \tag{5}$$

Inserting the above expressions for price and quantity into the objective function (2) gives the following expressions for firm profit at the Cournot equilibrium:

$$\Pi_{i|P,P} = (1/(9b))\left(a - 2c_i + c_j\right)^2 \qquad i = 1, 2 \quad j = 3 - i \tag{6}$$

where the subscripts P,P indicate that both firms are profit maximizers. To ensure that quantities remain positive, expression (4) requires $a > 2c_1 - c_2$ and $a > 2c_2 - c_1$; when costs are identical this reduces to $a > c$. Note that the Cournot equilibrium is a Nash equilibrium (Qin and Stuart 1997, 502) in that neither firm can improve its situation by changing its decision.

2.2 | Scenario Two: One Firm Acts as a Profit Maximizer and Its Rival Acts as a Sales Maximizer

In this scenario, we assume that firm one is a profit maximizer and firm two is a sales maximizer. The objective functions that managers act to maximize are as follows:

$$O_1 = \left(p - c_1\right) q_1 \tag{7}$$

and

$$O_2 = p\, q_2 \tag{8}$$

where subscript one refers to firm one, the profit maximizer, and subscript two refers to firm two, the sales maximizer. Following the procedure given above, we obtain the following reaction functions:

$$q_1 = (1/(2b))\left(a - c_1 - bq_2\right) \tag{9}$$

and

$$q_2 = (1/(2b))\left(a - bq_1\right) \tag{10}$$

Solving for q_1 and q_2 by substitution gives

$$q_1 = (1/(3b))\left(a - 2c_1\right) \tag{11}$$

and

$$q_2 = (1/(3b))\left(a + c_1\right) \tag{12}$$

We note that q_2 is always greater than q_1 when costs are positive, that is, the sales maximizer always produces a greater quantity than the profit maximizer. Price at the equilibrium point is determined from Equation (1) as follows:

$$p = (1/3)\left(a + c_1\right) \tag{13}$$

and profits for the two firms are

$$\begin{aligned}\Pi_{1|P,S} &= \left(p - c_1\right) q_1 \\ &= (1/(9b))\left(a - 2c_1\right)^2\end{aligned} \tag{14}$$

$$\begin{aligned}\Pi_{2|P,S} &= \left(p - c_2\right) q_2 \\ &= (1/(9b))\left(a + c_1 - 3c_2\right)\left(a + c_1\right)\end{aligned} \tag{15}$$

where $\Pi_{1|P,S}$ represents profits of firm one when firm one chooses to maximize profit and firm two chooses to maximize sales. Profits of the sales maximizer will be greater than profits of the profit maximizer when $\Pi_{2|P,S} > \Pi_{1|P,S}$, that is, when

$$\left(a + c_1 - 3c_2\right)\left(a + c_1\right) > \left(a - 2c_1\right)^2$$

that is, when

$$a\left(2c_1 - c_2\right) > \left(c_1^2 + c_1 c_2\right) \tag{16}$$

When costs of both firms are identical, that is, when $c_1 = c_2 = c$, this condition reduces to $a > 2c$. We also note from the reaction function (11) above that positive q_1 requires $a > 2c_1$. This implies that in the case of identical costs and when one firm is a sales maximizer and the other is a profit maximizer, sales maximization yields greater profits than profit maximization for all feasible equilibria (i.e., where both quantities are positive).

The intuitive argument is as follows: profit for both the profit maximizer and the sales maximizer is quantity multiplied by price less cost. If we refer to price less unit cost as unit margin, profit is equal to quantity multiplied by unit margin. Where costs are the same, unit margin will be the same for both firms; this implies that the firm that sells the greater quantity—the sales maximizer—will make the greater profit. This is so because, in a duopoly, price results from the combined action of both firms, not the action of one firm alone.

Now let us examine the conditions under which profit for firm two, when it acts as a sales maximizer while firm one is a profit maximizer, would be greater than the profit it would earn if it too acted as a profit maximizer. This occurs when $\Pi_{2|P,S} > \Pi_{2|P,P}$, that is, when

$$\begin{aligned}&\left(a + c_1 - 3c_2\right)\left(a + c_1\right) > \left(a + c_1 - 2c_2\right)^2 \\ &\text{i.e.} \quad c_2 < \left(a + c_1\right)/4\end{aligned}$$

This can be written in a more general form for the two firms as

$$c_j < \left(a + c_i\right)/4 \qquad i = 1, 2 \quad j = 3 - i \tag{17}$$

When costs are equal these expressions reduce to $c < a/3$. Clearly, $c = a/3$ is a change point when costs are symmetric: above this point, when $c > a/3$, that is, when costs are relatively high, it is in the best interests of the firm to act as a profit maximizer; below this point, when $c < a/3$, that is, when costs are relatively low, it may be in the interest of the firm to act as a sales maximizer when its rival is a profit maximizer.

2.3 | Scenario Three: Both Firms Act as Sales Maximizers

Let us now consider the case where both firms adopt a sales maximization strategy. In this case, the objective functions for both firms are similar:

$$O_i = p\, q_i \qquad i = 1, 2 \tag{18}$$

Following the same analysis procedure as outlined in scenario one above, we find expressions for quantity, price, and profit:

$$q_i = a/(3b) \qquad i = 1, 2 \tag{19}$$

$$p = a/3 \tag{20}$$

$$\Pi_{i|S,S} = (1/(9b))\left(a\left(a - 3c_i\right)\right) \qquad i = 1, 2 \tag{21}$$

As one would expect, quantities produced are larger for both firms and price is lower than when both firms were profit maximizers. Profit for sales maximizers is less than the profit that the firm would make if both firms were profit maximizers when $\Pi_{1|S,S} < \Pi_{1|P,P}$, that is, when

$$a\left(a - 3c_1\right) < \left(a - 2c_1 + c_2\right)^2 \tag{22}$$

For identical costs, that is, when $c_1 = c_2 = c$, this condition becomes $a > -c$, which is always true. Therefore, when costs are identical, profit for both firms when both are sales maximizers is always less than profit for both firms when both are profit maximizers.

In a similar fashion, we can show that profit for a sales maximizer, the other firm also being a sales maximizer, is greater than the profit it would get if it were a profit maximizer when $\Pi_{1|S,S} > \Pi_{1|P,S}$, that is, when

$$a\left(a - 3c_1\right) > \left(a - 2c_1\right)^2 \tag{23}$$

which reduces to $c_1 < a/4$, that is, when firm cost is relatively low. Expressions (17), (22), and (23) provide the conditions for the prisoners' dilemma.

We can now put the above results for the three cases together in the form of a payoff matrix and game decision tree for the various combinations of managerial strategies. Figure 1a provides expressions for payoffs for the two firms for each strategy combination, with the upper expression in each cell corresponding to the payoff for firm one, and the lower expression in each cell corresponding

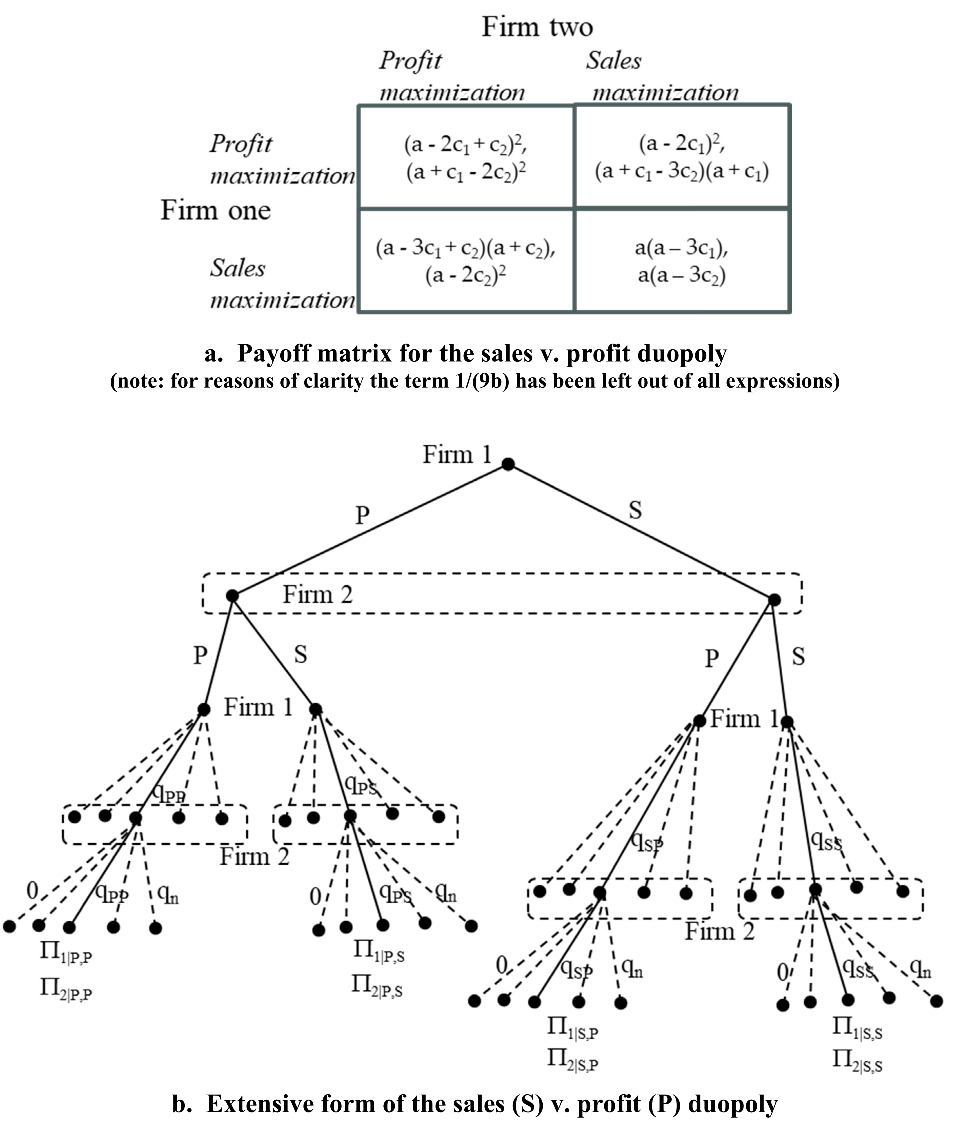

FIGURE 1 | (a) Payoff matrix for the sales vs. profit duopoly. (*Note:* for reasons of clarity the term 1/(9b) has been left out of all expressions). (b) Extensive form of the sales (S) vs. profit (P) duopoly.

to the payoff for firm two. The strategy of profit (sales) maximization implies that the manager chooses to maximize profit (sales) at the first stage of the game and chooses the relevant Cournot quantity at the second stage. The full two player two stage extensive form of the game is given as a decision tree in Figure 1b. (Information sets are represented as dotted cushion rectangles; for clarity reasons, only one of the choice sets available to firm 2 is shown at each of the four information sets at the second stage). In the next section, we examine three scenarios where costs are symmetric and high, medium and low. Costs are deemed high when $c > a/3$, medium when $a/4 < c < a/3$, and low when $c < a/4$.

2.4 | Three Symmetric Cost Scenarios: High, Medium, and Low

We now examine the scenario where costs are identical are relatively high, that is, $c_1 = c_2 = c > a/3$. Under these conditions inequality (17) does not hold, and therefore, $\Pi_{1|P,P} > \Pi_{1|S,P}$ and $\Pi_{2|P,P} > \Pi_{2|P,S}$, making (P,P) a Nash equilibrium. Inequality (23) does not hold either, and therefore, $\Pi_{1|P,S} > \Pi_{1|S,S}$ and $\Pi_{2|S,P} > \Pi_{2|S,S}$, making (P,P) the only Nash equilibrium. Profit maximization is a dominant strategy[1] for both firms. This situation is summarized as scenario 1 in Table 1.

To illustrate this scenario with a numerical example, we let reservation price a have value 25 and cost c for both firms have value 9. These values could represent for example products such as a tin of paint or a box of chocolates. In this scenario, costs are identical and relatively high compared with reservation price. Payoffs, determined from the expressions in Figure 1a, are given for the two firms in Figure 2. Managers have a straightforward decision to make. Profit maximization dominates sales maximization, and therefore, managers of both firms will choose profit maximization as a strategy. A unique Nash equilibrium, depicted by a dotted ellipse, exists in the top left cell where both firms choose profit maximization strategies.

Now we examine the situation when costs are identical and at a medium level, that is, $c_1 = c_2 = c$ and $a/4 < c < a/3$. Here, inequality (17) holds, and therefore, $\Pi_{1|S,P} > \Pi_{1|P,P}$ and $\Pi_{2|P,S} > \Pi_{2|P,P}$. Inequality (23) does not hold, and therefore, $\Pi_{1|P,S} > \Pi_{1|S,S}$ and $\Pi_{2|S,P} > \Pi_{2|S,S}$. This scenario therefore yields two Nash equilibria, at (P,S) and (S,P) and is summarized as scenario 4 in Table 1.

To illustrate this scenario with a numerical example, we reduce cost from 9 to 8 with reservation price remaining at 25, making $c < a/3$. The payoff matrix for this duopoly situation is shown in Figure 3. This payoff matrix is of the hawk-dove archetype, also known as the "chicken" game, with two Nash equilibria occurring at the positions: profit maximizer/sales maximizer and sales maximizer/profit maximizer. This is a type of anti-coordination game: each firm is better off being a sales maximizer provided the other firm is not; however, if the firms mis-coordinate and *both* choose sales maximization strategies then they end up in the bottom right cell where both will be worse off.

The final symmetric scenario is that of identical but low costs, that is, $c_1 = c_2 = c$ and $c < a/4$. If $c < a/4$ then $c < a/3$ is also true, and so inequality (17) holds; therefore, $\Pi_{1|S,P} > \Pi_{1|P,P}$ and $\Pi_{2|P,S} > \Pi_{2|P,P}$. Inequality (23) holds, and therefore, $\Pi_{1|S,S} > \Pi_{1|P,S}$ and $\Pi_{2|S,S} > \Pi_{2|S,P}$. Sales maximization is a dominant strategy for both firms with a unique Nash equilibrium at (S,S). Inequality (22), which is always true for identical costs, tells us that $\Pi_{1|S,S} < \Pi_{1|P,P}$ and $\Pi_{2|S,S} < \Pi_{2|P,P}$, making a prisoners' dilemma. This situation is summarized as scenario 6 in Table 1.

We illustrate this low-cost scenario with a numerical example. We reduce cost for both firms to 6 with reservation price remaining at 25, making $c < a/4$. The resulting payoff matrix is shown in Figure 4. Here, a single Nash equilibrium exists with both firms adopting a sales maximization strategy. Note that an alternative exists where both firms would be better off, but this alternative is not a Nash equilibrium and therefore difficult to achieve in a one-shot game. This matrix represents the prisoners' dilemma familiar from the literature; however, in this paper we show that the dilemma occurs only when costs are relatively low and does not occur when costs are at medium or high levels.

These three scenarios show that the decision for managers as to how to compete in a Cournot duopoly is not straightforward and varies according to the cost situation of the two firms. When costs are symmetric and relatively high, managers of both firms will choose profit maximization strategies. This situation reflects the standard economic approach with all firms adopting a profit maximization strategy. When costs are symmetric and relatively low, managers of both firms will choose sales maximization strategies and positive payoffs are achieved by both firms. However, this is a prisoners' dilemma as a better outcome for both firms is available at (P,P) but difficult to achieve in a one-shot game. When costs are symmetric at medium level, one firm will choose profit maximization as a strategy and its rival will choose sales maximization. The problem here is that a mis-coordination in choices could lead to the undesirable situation of both firms choosing sales maximization and both firms achieving poor payoffs and possibly losses.

In this section, we have examined competitive scenarios where costs were the same for the two firms. In the following section, we adjust these scenarios to allow costs to become asymmetric, that is, when one firm has a cost advantage over the other.

2.5 | Asymmetric Costs

We now introduce a cost asymmetry. Let firm two be a low-cost operator with a cost advantage over firm one, that is, $c_2 < c_1$. At the upper end of costs, three situations arise: when inequality (17) fails for both firms, when inequality (17) holds for the lower cost firm but not for the higher cost firm, and when inequality (17) holds for both firms. At the lower end of costs, three situations arise: when inequality (23) holds for both firms, when inequality (23) holds for the lower cost firm but not for the higher cost firm, and when inequality (23) fails for both firms. This provides six scenarios which we now examine in more detail.

TABLE 1 | Cost scenarios, payoffs, and equilibrium strategies.

Scenario	Costs $\hat{c}_1 = (a + c_2)/4$ $\hat{c}_2 = (a + c_1)/4$ $\check{c} = a/4$	Payoffs	Equilibria	Strategy
1. High costs	$c_1 > \hat{c}_1$	$\Pi_{1\|P,P} > \Pi_{1\|S,P}$	Unique Nash equilibrium (P,P)	Profit maximization dominant for both firms
	$c_2 > \hat{c}_2$	$\Pi_{2\|P,P} > \Pi_{2\|P,S}$		
	$c_1 > \check{c}$	$\Pi_{1\|P,S} > \Pi_{1\|S,S}$		
	$c_2 > \check{c}$	$\Pi_{2\|S,P} > \Pi_{2\|S,S}$,		
2. High/mid asymmetric costs	$c_1 > \hat{c}_1$	$\Pi_{1\|P,P} > \Pi_{1\|S,P}$	Unique Nash equilibrium (P,S)	Profit maximization is dominant for higher cost firm. Lower cost firm maximizes sales (but not a dominant strategy)
	$c_2 < \hat{c}_2$	$\Pi_{2\|P,P} < \Pi_{2\|P,S}$		
	$c_1 > \check{c}$	$\Pi_{1\|P,S} > \Pi_{1\|S,S}$		
	$c_2 > \check{c}$	$\Pi_{2\|S,P} > \Pi_{2\|S,S}$		
3. High/low asymmetric costs	$c_1 > \hat{c}_1$	$\Pi_{1\|P,P} > \Pi_{1\|S,P}$	Unique Nash equilibrium (P,S)	Profit maximization dominant for higher cost firm; sales maximization dominant for the lower cost firm
	$c_2 < \hat{c}_2$	$\Pi_{2\|P,P} < \Pi_{2\|P,S}$		
	$c_1 > \check{c}$	$\Pi_{1\|P,S} > \Pi_{1\|S,S}$		
	$c_2 < \check{c}$	$\Pi_{2\|S,P} < \Pi_{2\|S,S}$		
4. Mid-level costs	$c_1 < \hat{c}_1$	$\Pi_{1\|P,P} < \Pi_{1\|S,P}$	Two Nash equilibria (Hawk–Dove) (P,S) (S,P)	One firm maximizes profit; its rival maximizes sales. Neither firm has a dominant strategy
	$c_2 < \hat{c}_2$	$\Pi_{2\|P,P} < \Pi_{2\|P,S}$		
	$c_1 > \check{c}$	$\Pi_{1\|P,S} > \Pi_{1\|S,S}$		
	$c_2 > \check{c}$	$\Pi_{2\|S,P} > \Pi_{2\|S,S}$		
5. Mid/low asymmetric costs	$c_1 < \hat{c}_1$	$\Pi_{1\|P,P} < \Pi_{1\|S,P}$	Unique Nash equilibrium (P,S)	Sales maximization is dominant for lower cost firm. Higher cost firm maximizes profit (but not a dominant strategy)
	$c_2 < \hat{c}_2$	$\Pi_{2\|P,P} < \Pi_{2\|P,S}$		
	$c_1 > \check{c}$	$\Pi_{1\|P,S} > \Pi_{1\|S,S}$		
	$c_2 < \check{c}$	$\Pi_{2\|S,P} < \Pi_{2\|S,S}$		
6. Low costs	$c_1 < \hat{c}_1$	$\Pi_{1\|P,P} < \Pi_{1\|S,P}$	Unique Nash equilibrium (prisoners' dilemma) (S, S)	Sales maximization dominant for both firms
	$c_2 < \hat{c}_2$	$\Pi_{2\|P,P} < \Pi_{2\|P,S}$		
	$c_1 < \check{c}$	$\Pi_{1\|P,S} < \Pi_{1\|S,S}$		
	$c_2 < \check{c}$	$\Pi_{2\|S,P} < \Pi_{2\|S,S}$		

Firm one \ Firm two	*Profit maximization*	*Sales maximization*
Profit maximization	256, 256	49, 238
Sales maximization	238, 49	-50, -50

FIGURE 2 | Payoff matrix when costs are symmetric and high, $c > a/3$ ($a = 25$, $c = 9$).

Firm one \ Firm two	*Profit maximization*	*Sales maximization*
Profit maximization	361, 361	169, 403
Sales maximization	403, 169	175,175

FIGURE 4 | Payoff matrix when costs are symmetric and low, $c < a/4$ ($a = 25$, $c = 6$).

Firm one \ Firm two	*Profit maximization*	*Sales maximization*
Profit maximization	289, 289	81, 297
Sales maximization	297, 81	25, 25

FIGURE 3 | Payoff matrix when costs are symmetric and mid-level, $a/4 < c < a/3$ ($a = 25$, $c = 8$).

Firm one \ Firm two	*Profit maximization*	*Sales maximization*
Profit maximization	225, 324	49, 340
Sales maximization	198, 81	-50, 25

FIGURE 5 | Payoff matrix when costs are asymmetric and high ($a = 25$, $c_1 = 9$, $c_2 = 8$).

From inequality (17), we can determine the upper threshold value for the lower cost firm as $\hat{c}_2 = (a + c_1)/4$. When the low-cost firm is above this threshold, that is, $\hat{c}_2 < c_2 < c_1$, the situation is similar to the symmetric case depicted in Figure 2 and summarized as scenario 1 in Table 1, with profit maximization being a dominant strategy for both firms.

When the low cost firm goes below the upper threshold, that is, $c_2 < \hat{c}_2$, inequality (17) now holds for firm two and $\Pi_{2|P,S} > \Pi_{2|P,P}$. For the higher cost firm inequalities (17) and (23) fail and so $\Pi_{1|P,P} > \Pi_{1|S,P}$ and $\Pi_{1|P,S} > \Pi_{1|S,S}$ making outcome (P,S) a unique Nash equilibrium. The lower threshold value, determined from inequality (23) is $\check{c} = a/4$. When $c_2 > \check{c}$ then inequality (23) fails with respect to firm two and $\Pi_{2|S,P} > \Pi_{2|S,S}$ and therefore sales maximization is not a dominant strategy for the lower cost firm. This situation is summarized as scenario 2 in Table 1.

We illustrate the high but asymmetric cost scenario with a numerical model. Let us assume that firm one's cost remains at 9 and firm two's cost is reduced to 8, that is, $c_2 = 8 < \hat{c}_2$, giving firm two a cost advantage over firm one; reservation price remains at 25 as before. Payoffs, determined from the expressions in Figure 1a, are given in Figure 5. The top right-hand cell (P,S) is a Nash equilibrium, achieved when the low-cost operator chooses a sales maximization strategy and its higher cost rival chooses a profit maximization strategy. Introducing a cost advantage has led to a change in industry competition with the cost leader switching from a profit maximization strategy to a sales maximization strategy (compare Figure 5 with Figure 2).

Note that when $c_2 < \check{c}$, that is, when its costs are very low, then inequality (23) holds for firm two and $\Pi_{2|S,S} > \Pi_{2|S,P}$, making sales maximization a dominant strategy for the lower cost firm. This situation is summarized as scenario 3 in Table 1.

We now examine the mid-level scenario where costs for both firms drop below the upper threshold level, that is, $c_1 < \hat{c}_1$ and $c_2 < \hat{c}_2$ but remain above the lower threshold level, that is, $c_1 > c_2 > \check{c}$. This means that $\Pi_{1|S,P} > \Pi_{1|P,P}$ and $\Pi_{2|P,S} > \Pi_{2|P,P}$ and that $\Pi_{1|P,S} > \Pi_{1|S,S}$ and $\Pi_{2|S,P} > \Pi_{2|S,S}$. Together these conditions lead to Nash equilibria at (S,P) and (P,S) where one firm chooses to maximize sales and the other firm chooses to maximize profits in a Hawk-Dove game archetype and is similar to the situation depicted in Figure 3 and summarized as scenario 4 in Table 1.

When costs of the low cost firm drop further and go below the lower threshold then $c_2 < \check{c}$ and the relevant payoff becomes $\Pi_{2|S,P} < \Pi_{2|S,S}$ leading to a unique Nash equilibrium at (P,S) with sales maximization a dominant strategy for the lower cost firm. This situation is summarized as scenario 5 in Table 1. We illustrate this asymmetric cost scenario with a numerical example. Setting cost for firm one at 7, that is, $c_1 = 7 > \check{c}$, and firm two's cost at 6, that is, $c_2 = 6 < \check{c}$, we obtain the payoffs shown in Figure 6. There is a unique Nash equilibrium at (P,S) where firm one chooses a profit maximization strategy and its lower cost rival chooses its dominant strategy of sales maximization. The possibility of mis-coordination does not arise here as there is only one Nash equilibrium (compare Figure 6 with Figure 3); the cost asymmetry has eliminated the coordination issue that arose when costs were symmetric.

Where costs are low and asymmetric, inequality (23) holds for both firms and the situation is similar to that for low but symmetric costs as depicted in Figure 4 and summarized as scenario 6 in Table 1. Sales maximization is a dominant strategy for both

		Firm two	
		Profit maximization	*Sales maximization*
Firm one	*Profit maximization*	289, 400	121, 448
	Sales maximization	310, 169	100, 175

FIGURE 6 | Payoff matrix when costs are asymmetric and mid-level (a = 25, $c_1 = 7$, $c_2 = 6$).

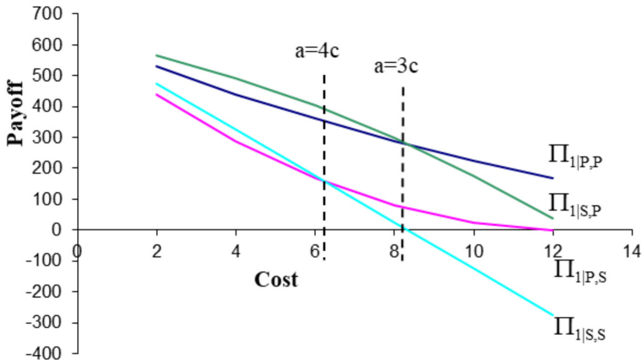

FIGURE 7 | Payoff vs. Cost for firm 1 for symmetric costs (a = 25, $c_1 = c_2$).

firms and the familiar prisoners' dilemma outcome occurs at (S,S).[2]

3 | Discussion

This paper finds that, when costs are identical, if one firm adopts a strategy of sales maximization and the other one of profit maximization then the sales maximizer obtains higher profits than the profit maximizer for all feasible equilibria. It also finds that the profits of the sales maximizer are higher than its profits would be if both firms were profit maximizers when $c < a/3$, that is, when costs are relatively low, but worse when $c > a/3$, that is, when costs are relatively high.

The paper shows that a duopoly where both firms adopt a sales maximization strategy yields worse results for both firms than if both firms adopt profit maximization. However, for $c < a/4$, that is, when costs are low, the sales maximizer, where both firms are sales maximizers, will perform better than it would have done as a profit maximizer, where the other firm was a sales maximizer.

A broad overall conclusion is that where both firms have identical and relatively high costs ($c > a/3$) both firms will adopt strict profit maximization policies. Where both firms have identical and mid-range costs ($a/4 < c < a/3$) one of the firms will adopt a sales maximization strategy and equilibrium considerations will force the other to adopt a profit maximization strategy, even though it will underperform its rival. This creates an anti-coordination problem (Sen and Stamatopoulos 2015) as it is unclear which of the two equilibria will be attained. However, when costs are sufficiently asymmetric the coordination problem is resolved as only one equilibrium exists, with the lower cost firm choosing to maximize sales and its less efficient rival choosing to maximize profit.

For identical and low costs ($c < a/4$) a sub-optimal equilibrium exists where both firms choose a sales maximization strategy resulting in a prisoners' dilemma. A contribution of this paper is to show that, while a prisoners' dilemma occurs, it only does so when costs for both firms are relatively low with respect to reservation price.

Trends in profit figures under different strategies as symmetric costs change are shown in Figure 7. As expected, where the duopoly contains a profit maximizer and a sales maximizer, profit levels for the sales maximizer always exceed those of the profit maximizer. To the left of the line a = 4c (i.e., for $c < a/4$), a sales maximization strategy may be in the best interests of the firm: if the other firm recognizes the Nash equilibrium and chooses sales maximization then profits are made; and if the other firm selects profit maximization then even larger profits are made. To the right of the line a = 4c (i.e., for $c > a/4$), sales maximization is risky because if the other firm also chooses sales maximization then both firms will make poor profits and may even incur losses. To the right of the line a = 3c (i.e., for $c > a/3$), profit maximization is clearly the best strategic choice for managers: if the other firm recognizes the Nash equilibrium and chooses profit maximization then both firms make profits; if the other firm chooses sales maximization then the firm will still make some, albeit low, profit.

An incentive still exists for managers to adopt a sales maximization strategy when $a/4 < c < a/3$. If firms are measured not only on the absolute position achieved but also on the relative position, then the hawk-dove Nash equilibria positions are more favorable to the winning manager as this manager significantly outperforms his or her rival, that is, the difference in levels of firm profits is large as shown in Figure 7. To a manager being significantly ahead of your rival in market share and profit may be as important, or even more important, than the absolute value of profit. Focal point considerations may apply in determining which of the two firms will choose sales maximization and which will choose profit maximization to maintain the Nash equilibrium: for example, if managers of one of the firms had a reputation for sales maximization this may lead the other firm to choose profit maximization.

4 | Conclusion

This paper presents a model of duopoly, framed for a managerial audience, with three parameters—a, b and c—only one of which—cost—is allowed to vary. It shows that a choice of a profit maximization strategy by both firms does not always lead to an equilibrium solution. When costs are high and symmetric both firms will choose profit maximization strategies. However, as costs decrease but remain symmetric it is in the interests of one of the firms to choose sales maximization; mis-coordination, however, can lead to worse outcomes for both firms. As costs decrease further but remain symmetric a choice of sales maximization by both firms becomes an equilibrium, but prisoners' dilemma, solution. Introducing a cost asymmetry changes these equilibria: at high but asymmetric cost levels the low-cost firm will chooses sales maximization over profit maximization. At medium but asymmetric cost levels managers may avoid the mis-coordination issue. At low but asymmetric cost levels the prisoners' dilemma outcome largely remains. Avenues for future research are to determine empirically whether cost leaders tend towards sales revenue maximization, and to determine empirically whether firms in lower margin industries are more likely to tend towards sales revenue maximization than are firms in higher margin industries. An avenue for future theoretical research is to consider evolutionary stable equilibria for profit and sales maximizers.

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Conflicts of Interest

The author declares no conflicts of interest.

Data Availability Statement

The author has nothing to report.

Endnotes

[1] A dominant strategy is a strategy that outperforms other strategies irrespective of the choice made by the other party.

[2] Note: for relatively extreme combinations of high reservation price and low, asymmetric costs condition (22) may not hold making (S,S) a Nash equilibrium but not a prisoners' dilemma.

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