Resilience of Green Bonds in Portfolio Diversification: Evidence from Crisis Periods

Abstract

This study employs the spillover index methodology by Diebold and Yilmaz (2009, 2012, 2014) and the time-varying parameter vector autoregression (TVP-VAR) approach by Antonakakis et al. (2019, 2020) to analyze returns connectedness between the green bond index (GRBI) and other major financial indices across three sub-periods: Pre-Covid, During-Covid, and Post-Covid (Russia-Ukraine war). Two portfolios are compared: a base portfolio (without GRBI) and a delta portfolio (with GRBI). Results show that EQWI is the largest net transmitter of shocks, while GRBI is the largest net receiver. Introducing GRBI reduces the total connectedness index (TCI), indicating its role as a spillover absorber. During crisis periods, the delta portfolio demonstrates superior performance in downside-risk measures and adjusted return ratios. The hedging effectiveness analysis highlights GRBI's natural hedge properties, reducing significant negative HE values in other assets. This comprehensive evaluation suggests that adding GRBI to a portfolio enhances resilience against systemic shocks, offering better downside-risk adjusted returns during crises. These findings are critical for investors seeking to optimize portfolio performance through enhanced diversification and risk management strategies.

Keywords: Green Bonds, Portfolio Diversification, Systemic Risk, TVP-VAR, Downside-Risk Adjusted Returns.

JEL Classification: G11, G15, C58, Q56.

1. Introduction

Sustainable finance is crucial for driving initiatives that mitigate the severe impacts of the climate change crisis. Among various financial instruments, green bonds (GBs) have emerged as a major source of funding for projects aimed at fostering a low-carbon economy. Their pivotal role in this transition has been extensively documented (Sartzetakis, 2021). The climate targets set by the Paris Agreement and the UN Sustainable Development Goals (SDGs) have catalyzed the development of green financial instruments, including sustainability-linked loans or bonds, social impact bonds, transition bonds, and green loans or bonds. Data from the Climate Bonds Initiative shows a consistent increase in GB issuance until the first half of 2023[1]. Despite a 22% drop in 2022, attributed to a broader slowdown in corporate bond issuance due to exceptionally high borrowing costs from significant monetary tightening by global central banks, this trend is expected to reverse. More supportive policies and a stable interest rate environment are anticipated to foster growth. In the first quarter of 2023, GB issuance reached an unprecedented USD 164 billion, making it the largest category of sustainable debt, despite recent banking sector turmoil[2]. By mid-2023, GB issuance had already surpassed 2019 levels, with Europe, the US, and China leading as the largest issuers.

Several studies have explored the determinants of the GB market and issuance (García et al., 2023; Cicchiello et al., 2022). Other research has examined the linkages between GBs and various asset classes, discussing their portfolio implications (Abakah et al., 2022). For instance, Naeem et al. (2023) found that GBs offer significant hedge effectiveness for precious metals but not for cryptocurrencies. Conversely, Abakah et al. (2022) concluded that GBs are strongly connected with treasury and corporate bond markets but weakly connected with stocks and energy markets, aligning with findings by Ferrer et al. (2021). Moreover, various studies have substantiated the spillovers from cryptocurrencies (Haq et al., 2023), oil prices (Azhgaliyeva et al., 2022), equities (Pham, 2021), bonds (Umar et al., 2023; Abakah et al., 2022), commodities (Tsagkanos et al., 2022), volatilities (Long et al., 2022; Pham and Do, 2022), carbon markets (Gabauer et al., 2022), and economic uncertainty (Tang et al., 2023; Sohag et al., 2022) on GBs. These studies indicate that the diversification benefits of GBs vary significantly with different asset classes. In this context, this study makes three significant methodological contributions to the existing literature on green bonds and their role in financial markets:

First, it extends the analysis of spillover effects by employing the time-varying parameter vector autoregression (TVP-VAR) model, as proposed by Antonakakis et al. (2019, 2020). This advanced econometric model allows for the dynamic assessment of interconnectedness between the green bond index and other global investable asset indices, including the dollar currency index, world equity index, gold price index, global oil index, government bond index, carbon index, bitcoin index, alternative energy index, volatility index, CDS North America Investment Grade index, and CDS Emerging Markets Investment Grade index. The novelty of this approach lies in its ability to capture the evolving nature of spillover dependence structures, especially during periods of market stress, thus providing a more accurate and comprehensive analysis compared to static models.

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[1] https://www.reuters.com/business/sustainable-business/green-bonds-are-set-drive-corporate-esg-debt-out-slump-2023-barclays-2023-01-04/

[2] https://www.bloomberg.com/news/articles/2023-04-06/green-bonds-post-record-quarter-as-issuers-pounced-before-tumult?utm_source=website&utm_medium=share&utm_campaign=copy

Second, the study introduces a comparative portfolio analysis using the minimum connectedness portfolio (MCoP) approach, as proposed by Broadstock et al. (2022). This method is applied across three sub-periods: Pre-Covid, During-Covid, and Post-Covid (coinciding with the Russia-Ukraine war). By comparing a base case (without GBs) and a delta case (with GBs), the study offers insights into building connectedness-resilient portfolios and evaluating performance across different crisis periods. This methodological innovation highlights the importance of GBs as an effective hedging tool within a portfolio that includes credit default swaps (CDXs), marking the first investigation of its kind to the authors' knowledge.

Third, the study employs a range of downside-risk measures, downside-risk-adjusted returns, and hedging effectiveness metrics to assess portfolio performance. By focusing on these measures, the research provides a nuanced understanding of the risk-return trade-offs associated with including GBs in investment portfolios. This comprehensive evaluation helps investors refine their hedging strategies by identifying the optimal asset mix during similar crisis periods, thereby enhancing portfolio predictions and performance.

In light of the recent increase in systemic events, both financial practitioners and academicians have emphasized the need to analyze the time-varying interconnectedness between various asset classes. Investors, facing economic, financial, and geopolitical uncertainties, must carefully balance their portfolio allocations. This study provides crucial insights for investors to make informed portfolio allocation decisions, considering the inclusion of CDXs alongside GBs. Policymakers and financial regulators can also benefit from understanding the interdependence between asset classes to develop appropriate macro-prudential policies, monitor financial system compliance, and mitigate systemic risk. This study aids in better scrutiny and monitoring of systemic stress and dependence during globally destabilizing events.

The paper proceeds as follows: Section 2 describes the dataset, Section 3 describes the empirical methods used, Section 4 presents the results & discusses the results, and Section 5 concludes the study.

2. Data

The dataset comprises daily prices for the Bloomberg MSCI Global Green Bond Index (GRBI) and eleven other financial asset indices, spanning from October 10, 2014, to October 3, 2023. This period provides 2,343 observations per index, resulting in a balanced panel dataset of 28,116 data points, all sourced from the Bloomberg database.

The Bloomberg MSCI Global Green Bond Index (GRBI) has garnered significant recognition, winning the “Environmental Finance Bond Award for Index of the Year” seven consecutive times. This accolade underscores the index's credibility and the trust it has earned within the investment community. Various studies, such as those by Khalfaoui et al. (2023) have used this index to proxy for green assets. According to MSCI documentation, the GRBI is a multi-currency benchmark that includes 28 local currency-denominated debt markets, tracking the performance of green bonds that meet the Green Bonds Principles (GBP) as set by the MSCI ESG Research Group. These bonds are investment-grade, have a minimum outstanding issue size, and are priced daily until maturity. For the analysis, data on the performance of other asset classes was also collected using the following indices:

- **Stock Market (EQWI):** The MSCI ACWI Index captures large and mid-cap representation across 23 Developed Markets (DM) and 24 Emerging Markets (EM) countries, covering approximately 85% of the global investable equity opportunity set with 2,948 constituents.
- **Bond Market (GTBI):** The FTSE World Government Bond Index measures the performance of fixed-rate, local currency, investment-grade sovereign bonds, providing a broad benchmark for the global sovereign fixed income market.
- **Commodity Market (COMI):** The Bloomberg Commodity Index consists of 24 exchange-traded futures on physical commodities, representing 22 commodities, weighted for economic significance and market liquidity.
- **Oil Market (OLGI):** The S&P GSCI Brent Crude Oil Index serves as a benchmark for investment performance in the Brent crude oil market.
- **Currency Market (DOSI):** The Bloomberg Dollar Spot Index tracks the performance of a basket of 10 leading global currencies versus the U.S. Dollar, dynamically updated to reflect trade and liquidity considerations.
- **Carbon Market (CAGI):** The IHS Markit Global Carbon Index is the first benchmarking and liquid investable index to track global carbon credits markets.
- **Cryptocurrency Market (BTCI):** The Bloomberg Galaxy Bitcoin Index measures the performance of Bitcoin traded in USD, chosen for this analysis due to its dominance and suitability during the study period.
- **Alternative Energy Market (ALEI):** The MSCI Global Alternative Energy Index includes companies that derive 50% or more of their revenues from alternative energy products and services.
- **Volatility Index (VIXI):** The CBOE VIX Index represents the market's 30-day forward-looking expectations of S&P 500 index volatility.
- **Credit Default North American Investment Grade Index (CXNI):** The IHS Markit CDX.NA.IG Index includes the 125 most liquid North American investment-grade entities in the CDS market.
- **Credit Default Emerging Markets Investment Grade Index (CXEI):** The IHS Markit CDX.EM.IG Index consists of 18 sovereign reference entities from the CDS market, selected based on criteria to maintain regional diversity.

To conduct the analysis, the logarithmic returns ($r_{i,t}$) of all indices' price series ($r_{i,t}$) and the 1-day lag value price series ($p_{i,t-1}$) were calculated. The returns were defined as $r_{i,t} = log\left(p_{i,t}/p_{i,t-1}\right) * 100$. The Covid-19 pandemic and the Russia-Ukraine war significantly impacted financial markets, causing increased uncertainties, disrupted supply chains, and rising energy costs. Recent studies, such as those by Umar et al. (2024), and Ha (2023) have examined the impacts of these events on green bond and renewable energy spillovers. The World Health Organization declared Covid-19 a pandemic on March 11, 2020 (Cucinotta and Vanelli, 2020), and Russia invaded Ukraine on February 24, 2022 (Ohikhuare, 2023). To analyze and derive insights on green bond spillover patterns and portfolio benefits with other asset classes during these global events, we divided our sample into three sub-periods: Pre-Covid (October 10, 2014, to March 10, 2020), During-Covid (March 11, 2020, to February 23, 2022), and Post-Covid (corresponding to the Russia-Ukraine War period) (February 24, 2022, to October 3, 2023).

2.1 Descriptive Statistics

Figure 2 presents the price series on the primary axis and the return series on the secondary axis for each asset index over the entire period. It is important to note that the scales for the primary and secondary axes differ for each asset class. The analysis reveals that the Volatility Index (VIXI), Bitcoin Index (BTCI), and CDX North American Investment Grade (CXNI) exhibit the highest volatility levels among all asset classes. In contrast, the CDX Emerging Markets Investment Grade (CXEI), Dollar Index (DOSI), Government Bond Index (GOBI), and Green Bond Index (GRBI) are the least volatile, in that order. The data indicates significant volatility across all indices during the Covid-19 event window, marked by extreme return movements. Additionally, the price series for the Equity Index (EQWI), Bitcoin Index (BTCI), Oil Index (OLGI), Commodity Index (COMI), Alternative Energy Index (ALEI), Carbon Index (CAGI), Volatility Index (VIXI), and both CDX indices (CXNI & CXEI) show notable changes around the peak of the Russia-Ukraine war tensions, with most indices experiencing a decline, except for VIXI, which rose during this period.

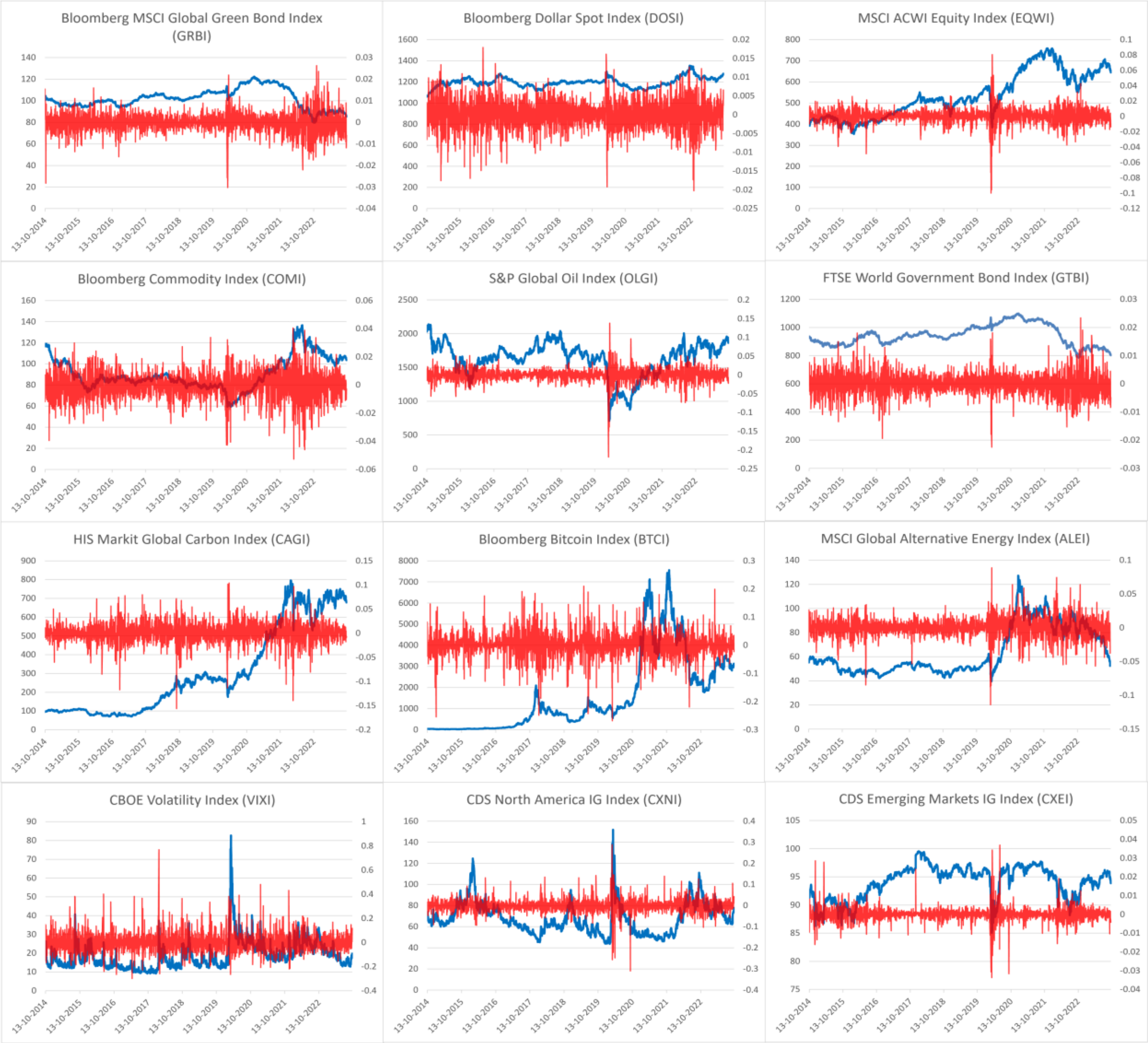
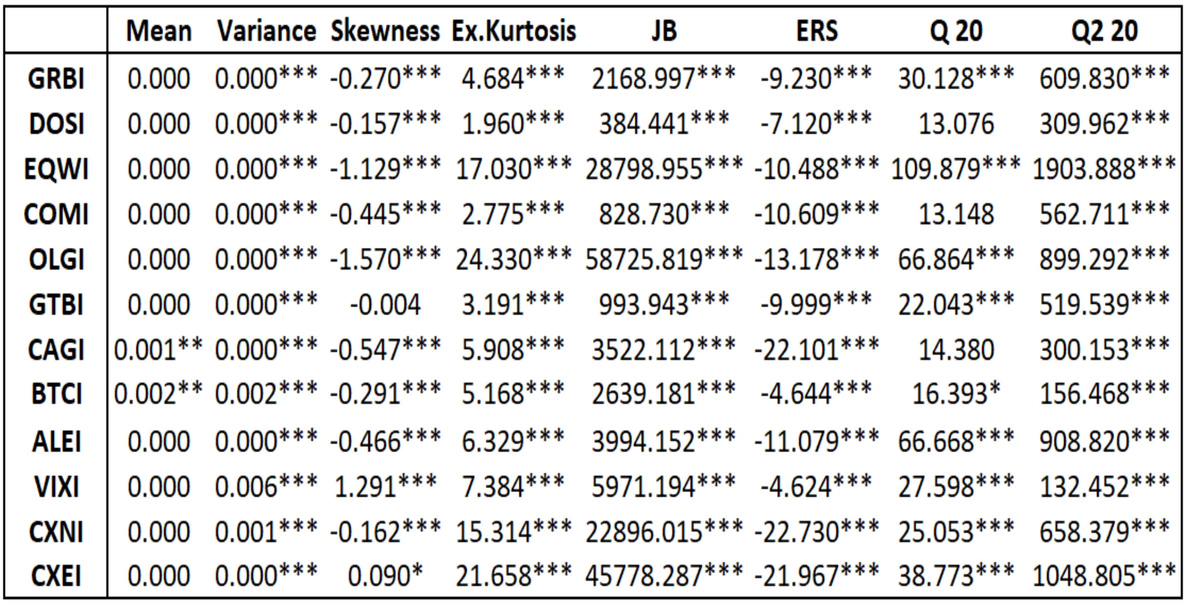


Figure 2: Daily Index Prices and Returns (Full Sample)

Figure 3 presents the descriptive statistics for the asset classes over the full period. The Jarque-Bera test results indicate that the return series is not normally distributed for all asset indices.

This conclusion is further supported by the skewness and excess kurtosis values. The Elliot-Rothenberg-Stock (ERS) efficient test for an autoregressive unit root is statistically significant at the 1% level, indicating that the return series is stationary. Additionally, the Ljung-Box Q-test results for the residuals of the return series demonstrate statistically strong autocorrelation for most asset indices, with the exceptions of the Dollar Index (DOMI), Commodity Index (COMI), and Carbon Index (CAGI), which do not exhibit significant autocorrelation.



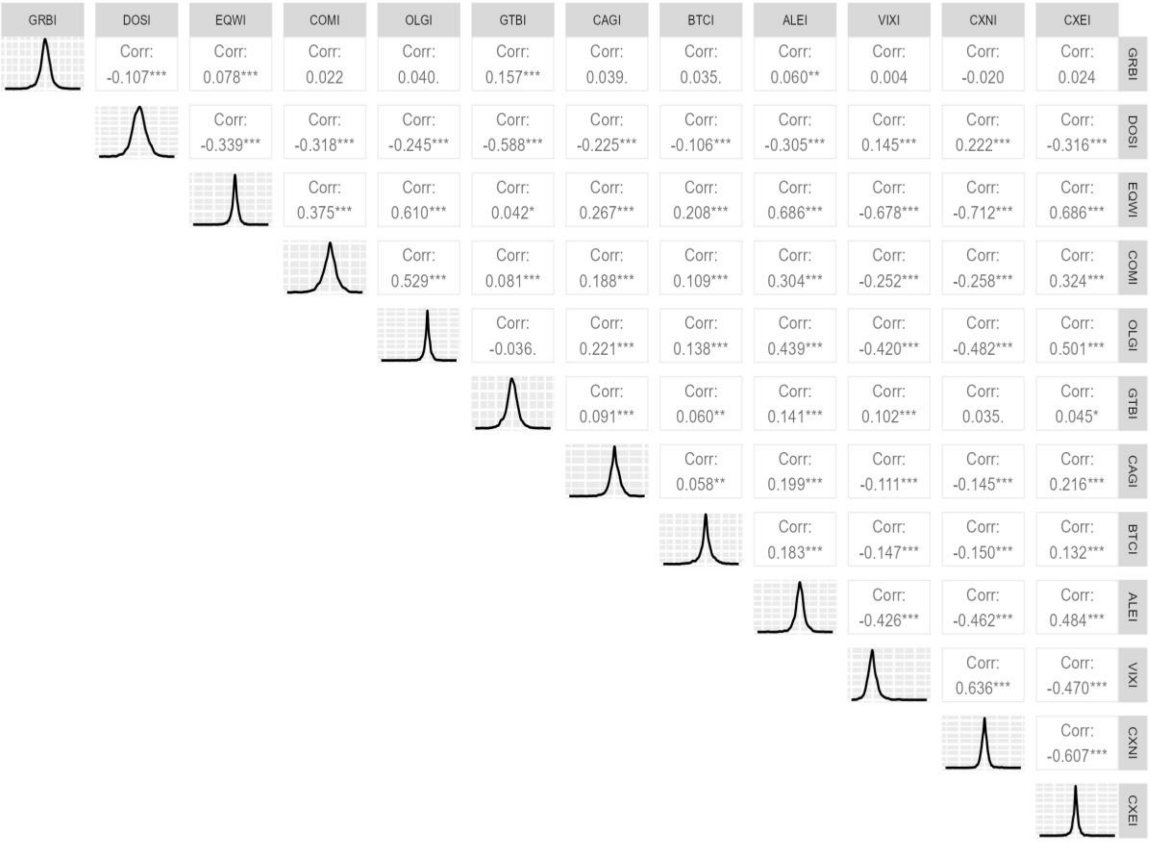
	Mean	Variance	Skewness	Ex.Kurtosis	JB	ERS	Q 20	Q2 20
GRBI	0.000	0.000***	-0.270***	4.684***	2168.997***	-9.230***	30.128***	609.830***
DOSI	0.000	0.000***	-0.157***	1.960***	384.441***	-7.120***	13.076	309.962***
EQWI	0.000	0.000***	-1.129***	17.030***	28798.955***	-10.488***	109.879***	1903.888***
COMI	0.000	0.000***	-0.445***	2.775***	828.730***	-10.609***	13.148	562.711***
OLGI	0.000	0.000***	-1.570***	24.330***	58725.819***	-13.178***	66.864***	899.292***
GTBI	0.000	0.000***	-0.004	3.191***	993.943***	-9.999***	22.043***	519.539***
CAGI	0.001**	0.000***	-0.547***	5.908***	3522.112***	-22.101***	14.380	300.153***
BTCI	0.002**	0.002***	-0.291***	5.168***	2639.181***	-4.644***	16.393*	156.468***
ALEI	0.000	0.000***	-0.466***	6.329***	3994.152***	-11.079***	66.668***	908.820***
VIXI	0.000	0.006***	1.291***	7.384***	5971.194***	-4.624***	27.598***	132.452***
CXNI	0.000	0.001***	-0.162***	15.314***	22896.015***	-22.730***	25.053***	658.379***
CXEI	0.000	0.000***	0.090*	21.658***	45778.287***	-21.967***	38.773***	1048.805***

Figure 3: Descriptive Statistics (Full Sample)

NOTE:

1. ., *, **, *** indicates 10%, 5%, 1% and 0.1% significance level.
2. Column JB presents Jarque-Bera test statistic, where the null hypothesis is that the data is normally distributed.
3. Column ERS presents Elliot-Rothenberg-Stock efficient test for an autoregressive unit root which is a modification of the Augmented Dickey-Fuller (ADF) test. The null hypothesis of the ERS test statistic is that the time series has a unit root i.e. the series is non-stationary.
4. Column Q 20 presents Ljung-Box ("portmanteau") Q-test with null hypothesis that the residual of series exhibits no autocorrelation for a fixed number of lag (L=20).
5. Column Q2 20 presents Ljung-Box ("portmanteau") Q-test with null hypothesis that the squared residual of series exhibits no autocorrelation for a fixed number of lag (L=20).

Figure 4 displays the cross-correlation among the indices along with the significance values. As expected, the Green Bond Index (GRBI) shows a strong positive correlation with the Government Bond Index (GTBI) and a lesser extent with the Equity Index (EQWI). Notably, GRBI has very low cross-correlation coefficients with other asset indices such as the Alternative Energy Index (ALEI), Oil Index (OLGI), Carbon Index (CAGI), and Bitcoin Index (BTCI). Furthermore, it is important to highlight the statistically significant and strongly negative cross-correlation of the CDX North American Investment Grade (CXNI) with ALEI, OLGI, CAGI, and BTCI. In contrast, the CDX Emerging Markets Investment Grade (CXEI) exhibits statistically significant and strongly positive cross-correlation with the same indices (ALEI, OLGI, CAGI, and BTCI). These observations suggest that GRBI could be an effective portfolio diversification substitute for these asset indices. However, further analysis of nonlinear and systemic connectedness is necessary to fully understand and quantify the diversification benefits.



	GRBI	DOSI	EQWI	COMI	OLGI	GTBI	CAGI	BTCI	ALEI	VIXI	CXNI	CXEI
GRBI		Corr: -0.107***	Corr: 0.078***	Corr: 0.022	Corr: 0.040.	Corr: 0.157***	Corr: 0.039.	Corr: 0.035.	Corr: 0.060**	Corr: 0.004	Corr: -0.020	Corr: 0.024
DOSI			Corr: -0.339***	Corr: -0.318***	Corr: -0.245***	Corr: -0.588***	Corr: -0.225***	Corr: -0.106***	Corr: -0.305***	Corr: 0.145***	Corr: 0.222***	Corr: -0.316***
EQWI				Corr: 0.375***	Corr: 0.610***	Corr: 0.042*	Corr: 0.267***	Corr: 0.208***	Corr: 0.686***	Corr: -0.678***	Corr: -0.712***	Corr: 0.686***
COMI					Corr: 0.529***	Corr: 0.081***	Corr: 0.188***	Corr: 0.109***	Corr: 0.304***	Corr: -0.252***	Corr: -0.258***	Corr: 0.324***
OLGI						Corr: -0.036.	Corr: 0.221***	Corr: 0.138***	Corr: 0.439***	Corr: -0.420***	Corr: -0.482***	Corr: 0.501***
GTBI							Corr: 0.091***	Corr: 0.060**	Corr: 0.141***	Corr: 0.102***	Corr: 0.035.	Corr: 0.045*
CAGI								Corr: 0.058**	Corr: 0.199***	Corr: -0.111***	Corr: -0.145***	Corr: 0.216***
BTCI									Corr: 0.183***	Corr: -0.147***	Corr: -0.150***	Corr: 0.132***
ALEI										Corr: -0.426***	Corr: -0.462***	Corr: 0.484***
VIXI											Corr: 0.636***	Corr: -0.470***
CXNI												Corr: -0.607***
CXEI												

Figure 4: Cross-Correlation (Full Sample)

NOTE: ., *, **, *** indicates 10%, 5%, 1% and 0.1% significance level.

3 Research Methodology

3.1 TVP-VAR based connectedness approach

Building on the work of Koop and Korobilis (2013), Antonakakis et al. (2020) extend the connectedness approach initially proposed by Diebold and Yilmaz (2014). This extension allows for a varying variance-covariance matrix via Kalman filter estimation to establish a Time-Varying Parameter Vector Autoregression (TVP-VAR) model. This model outperforms its homoscedastic counterpart as demonstrated by Koop and Korobilis (2013). The TVP-VAR(k) model can be mathematically expressed as:

$$Y_t = \phi_t u_{t-1} + \epsilon_t \qquad \epsilon_t \mid \Omega_{t-1} \sim \mathbb{N}\,(0, \Sigma_t) \tag{1}$$

$$vec(\phi_t) = vec(\phi_{t-1}) + \xi_t \qquad \xi_t \mid \Omega_{t-1} \sim \mathbb{N}\,(0, \Xi_t) \tag{2}$$

with

$$u_{t-1} = \begin{pmatrix} Y_{t-1} \\ Y_{t-2} \\ \vdots \\ Y_{t-k} \end{pmatrix},$$

$$\Phi_t' = \begin{pmatrix} \Phi_{1t} \\ \Phi_{2t} \\ \cdots \\ \Phi_{kt} \end{pmatrix}$$

where Ω_{t-1} denotes past information set up to $t-1$, Y_t and u_{t-1} denotes vectors of order $h \times 1$ and $hk \times 1$ respectively, Φ_t and Φ_{it} represent $h \times hk$ and $h \times h$ dimensional matrices, respectively, ϵ_t is an $h \times 1$ vector, and ξ_t is an $h^2k \times 1$ dimensional vector, whereas the time-varying variance-covariance matrices Σ_t and Ξ_t are $h \times h$ and $h^2k \times h^2k$ dimensional matrices, respectively. Moreover, $vec(\Phi_t)$ is vectorisation of Φ_t which is an $h^2k \times 1$ dimensional vector.

To estimate the parameters of the TVP-VAR model, it is transformed into its vector moving average (VMA) representation. This transformation is crucial for calculating the connectedness index introduced by Diebold and Yilmaz (2012). The connectedness index relies on the generalized impulse response function (GIRF) and the generalized forecast error variance decomposition (GFEVD), as developed by Koop et al. (1996) and Pesaran and Shin (1998).

The pairwise directional connectedness from variable j to variable i, as represented by the generalized forecast error variance decomposition, is denoted as $\Psi^g_{j,t}(J)$. To calculate the share of the forecast error variance of one variable that is attributable to another, the following equation is used:

$$\Pi^g_{j,t}\,(J) = \frac{\sum_{t=1}^{J-1} \Psi^{2,g}_{ij,t}}{\sum_{j=1}^{N} \sum_{t=1}^{J-1} \Psi^{2,g}_{ij,t}} \tag{3}$$

Note that $\sum_{j=1}^{N} \Pi^N_{i,j=1}\,(J) = 1$, and $\sum_{i,j=1}^{N} \Pi^N_{j,t}\,(J) = N$.

Next, to evaluate how a shock to any one parameter causes spillover to other parameters, a total connectedness index (TCI) is formulated, allowing the assessment of interconnectedness within the network.

$$H^g_t\,(J) = \frac{\sum_{i,j=1,\ i \neq j}^{N} \Pi^g_{ij,t}\,(J)}{N} \times 100 \tag{4}$$

The total directional connectedness **from** others which allows one to estimate the amount of directional spillover that a node i **receives from** all other nodes j is given by:

$$H^g_{i \leftarrow j,\ t}\,(J) = \frac{\sum_{i,j=1,\ i \neq j}^{N} \Pi^g_{ij,t}\,(J)}{\sum_{j=1}^{N} \Pi^N_{ij,t}\,(J)} \times 100 \tag{5}$$

Similarly, the directional spillover that node i transmits to all other nodes j is defined as the total directional connectedness to others as:

$$H^{g}_{i\rightarrow j,\ t}(J) = \frac{\sum^{N}_{i,j=1,\ i\neq j} \Pi^{g}_{ji,t}(J)}{\sum^{N}_{j=1} \Pi^{N}_{ji,t}(J)} \times 100 \tag{6}$$

Lastly, the net pairwise directional spillover is calculated as the difference between "TO" and "FROM." Thus, "NET" represents the residual impact a particular node iii makes on the entire network.

$$H^{g}_{i,t}(J) = H^{g}_{i\rightarrow j,\ t}(J) - H^{g}_{i\leftarrow j,\ t}(J) \tag{7}$$

If $H^{g}_{i,t}(J) > 0$, it implies that the node i propagates disturbances to the network whereas if $H^{g}_{i,t}(J) < 0$ then node i receives disturbances on a net basis.

Chatziantoniou and Gabauer (2020) found interpretation of high TCI to be subjective and adjusted TCI accordingly:

$$H^{g}_{t}(J) = \frac{\sum^{N}_{i,j=1,\ i\neq j} \Pi^{g}_{ij,t}(J)}{N-1}\ ,\ 0 \leq H^{g}_{t}(J) \leq 1 \tag{8}$$

The pairwise connectedness index (PCI) measures the connectedness across i and j as shown by Gabauer (2021):

$$H^{g}_{ij,t}(J) = 2\left(\frac{\Pi^{g}_{ij,t}(J) + \Pi^{g}_{ji,t}(J)}{\Pi^{g}_{ii,t}(J) + \Pi^{g}_{ij,t}(J) + \Pi^{g}_{ji,t}(J) + \Pi^{g}_{jj,t}(J)}\right)\ ,\ 0 \leq H^{g}_{ij,t}(J) \leq 1 \tag{9}$$

3.2 Minimum Connectedness Portfolio

To assess the benefits an investor may accrue from adding green bonds to an investable asset portfolio, the minimum connectedness portfolio construction technique is employed. This approach, based on the pairwise connectedness index proposed by Broadstock et al. (2022), aims to minimize the connectedness between assets. By assigning appropriate weights to the assets, the entire portfolio becomes less susceptible to systemic risk shocks within the network. The optimal portfolio weights are calculated using the following formulation:

$$w_{C_t} = \frac{PCI_t^{-1}\, I}{I\, PCI_t^{-1}\, I} \tag{10}$$

where PCI_t is the pairwise connectedness index matrix, and *I* is the identity matrix.

3.2.1 Hedging Effectiveness

Additionally, the portfolio's hedging effectiveness (HE) is evaluated to highlight the degree of risk reduction in a portfolio relative to a single asset. This measure helps investors gauge the portfolio's performance from a hedging perspective. The hedging effectiveness (HE) indicates how much the portfolio reduces risk compared to holding a single asset. A higher HE value suggests better risk reduction, thus enhancing the portfolio's hedging capabilities. This measure is crucial for investors seeking to optimize their portfolios by incorporating green bonds, as it provides a clear indication of the risk mitigation benefits achieved through diversification. The hedging effectiveness is calculated using the following formula:

$$HE = 1 - \frac{Variance_{portfolio}}{Variance_{asset}} \tag{11}$$

3.2.2 Portfolio Performance Measures

Grootveld and Hallerbach (1999) demonstrate that certain downside-risk measures outperform both-ways risk measures. Therefore, this study focuses on downside risk measures and tail-risk measures to assess portfolio performance. Additionally, the evaluation of portfolios includes downside-risk adjusted returns and tail-risk adjusted return measures. In portfolio theory, semi-deviation provides an effective measure of downside risk. Similar to standard deviation, semi-deviation considers only those periods where the portfolio's return was below the target or average level. This metric allows investors to understand the potential losses from a portfolio, rather than just its overall fluctuations. While standard deviation is the most widely used measure of investment risk, it has limitations, such as treating all deviations from the mean equally—whether positive or negative. However, investors are generally more concerned with negative divergences, or downside risk, than positive ones. Downside deviation addresses this issue by focusing solely on downside risk (Sortino and Van Der Meer, 1991; Johansson et al., 1999). By concentrating on downside risk and tail-risk measures, along with their adjusted return counterparts, the study provides a more accurate and investor-relevant assessment of portfolio performance. These measures, defined herein below, help in identifying the true risk and return characteristics of a portfolio, particularly in the context of potential losses and extreme market movements.

Semi-Deviation

Semi-Deviation is defined as:

$$Semi\ Deviation = \sqrt{\frac{1}{n}\sum_{r_t<Average}^{n}(Average - r_t)^2} \tag{12}$$

where, $n = total\ number\ of\ observations\ below\ the\ mean$, $r_t = the\ observed\ value$, and $Average = the\ mean\ or\ target\ value\ of\ a\ data\ set$.

Annualized Downside Risk

Annualized Downside Risk is the annualized value of the downside risk measure as calculated by semi-deviation.

$$Annualized\ Downside\ Risk = Semi - Deviation_T \times \sqrt{T} \tag{13}$$

Maximum Drawdown (MDD)

Maximum Drawdown (MDD) is a measure of a portfolio's maximum loss during a peak-to-trough decline before a new peak is achieved. It is typically expressed as the percentage difference between the peak and the trough. MDD serves as an indicator of downside risk over a specified period (Magdon-Ismail and Atiya, 2004). The formula for calculating Maximum Drawdown is as follows:

$$M = \frac{T-P}{P} \tag{14}$$

where, $M = maximum\ drawdown$, $T = trough\ value$, and $P = peak\ value$.

This calculation helps investors understand the extent of potential losses and the risk of significant declines in their investment portfolios.

Value at Risk (VaR)

Value at Risk (VaR) calculates the potential loss in value of a traded portfolio over a defined period for a given confidence level. It is a widely used risk management tool that quantifies the maximum expected loss with a specified probability. The historical method for calculating VaR involves using the historical return distribution of the portfolio to measure the exact percentile of VaR requested (Duffie and Pan, 1997). The same is defined as

$$Historical\ VaR = -Percentile\ Loss * Portfolio\ Value * \sqrt{T} \tag{15}$$

This method ranks the historical returns from worst to best and selects the return at the desired confidence level as the VaR. For example, at a 95% confidence level, the VaR is the return below which 5% of the historical observations fall. This approach provides a non-parametric estimate of VaR, relying solely on the empirical distribution of past returns without making any assumptions about the underlying return distribution.

Expected Shortfall (ES)

Expected Shortfall (ES), also known as Conditional Value-at-Risk (CVaR), is a risk metric used to measure the average of all potential losses that exceed a specified VaR level. ES provides a more comprehensive assessment of tail risk compared to traditional risk measures like Value-at-Risk (VaR), as it quantifies the magnitude of potential losses in the tail of the distribution, offering a more accurate estimation of extreme downside risk (Artzner et al., 1999).

$$Historical\ Expected\ Shortfall\ (ES) = E(L \mid L > VaR_H) \tag{16}$$

where, L denotes the loss quantiles greater than Historical VaR quantiles.

This metric provides the expected value of losses that occur beyond the VaR threshold, thus capturing the risk of extreme losses more effectively. By focusing on the tail of the loss distribution, ES addresses the limitations of VaR, which only indicates the maximum loss at a certain confidence level without accounting for the severity of losses beyond that threshold.

Omega Ratio

The Omega ratio is a weighted risk-return ratio for a given expected return level that helps identify the chances of winning compared to losing. A higher Omega ratio indicates a greater likelihood of achieving returns above the specified threshold compared to falling below it. This ratio also takes into account the third and fourth moments of the return distribution, i.e., skewness and kurtosis, which enhances its usefulness compared to other risk-return measures (Keating and Shadwick, 2002). In simple terms, Omega ratio is given as:

$$Omega\, ratio = \frac{\sum Winning - Benchmark}{Benchmark - \sum Losing} \tag{17}$$

This formulation provides a comprehensive view of the return distribution, considering both the frequency and magnitude of returns above and below the threshold. By incorporating skewness and kurtosis, the Omega ratio offers a more nuanced perspective on risk and return, making it a valuable tool for performance evaluation and portfolio optimization.

Sortino Ratio

The Sortino ratio, introduced by Sortino and Price Lee (1994), focuses on downside volatility, providing a more targeted evaluation of an asset, portfolio, or strategy's performance during unfavorable market conditions. This ratio is used to assess a portfolio's risk-adjusted returns relative to an investment target by considering only the downside risk, thus offering a clearer picture of performance during adverse market conditions. The Sortino ratio is calculated as follows:

$$Sortino\, ratio \; = \frac{Return_{Portfolio} - MAR}{Downside\ Deviation} \tag{18}$$

By focusing solely on the negative deviations from the target return, the Sortino ratio provides a more precise measure of risk-adjusted performance, particularly in scenarios where downside risk is a primary concern for investors.

Upside Potential Ratio

The Upside Potential Ratio is a measure of the return of an investment asset relative to the minimal acceptable return. This metric helps firms or individuals choose investments that have exhibited relatively strong upside performance per unit of downside risk (Sortino and Van Der Meer, 1991; Sortino et al., 1999). The Upside Potential Ratio is calculated as follows:

$$U = \frac{\sum_{min}^{+\infty}(R_r - R_{min})P_r}{\sqrt{\sum_{-\infty}^{min}(R_r - R_{min})^2 P_r}} \tag{19}$$

where, the returns R_r have been put into increasing order. Also, P_r is the probability of the return R_r and R_{min} which occurs at $r = min$; min is the minimal acceptable return.

This ratio enables investors to identify investments that provide favorable returns above a certain threshold while effectively managing downside risk. By focusing on the potential for positive returns relative to the risk of negative returns, the Upside Potential Ratio offers a nuanced view of investment performance that is particularly valuable for risk-averse investors.

Modified Sharpe Ratio

The Modified Sharpe Ratio is more suited for examining the performance of modern investment vehicles, which exhibit fat-tailed returns and display potential for extreme losses (Chow and Lai, 2015; Xiong and Idzorek, 2018). While the standard Sharpe ratio measures the excess return over the risk-free rate relative to the standard deviation of the portfolio, the Modified Sharpe Ratio uses alternative risk measures such as Value-at-Risk (VaR), Modified Value-at-Risk (MVaR), or Conditional Value-at-Risk (CVaR) in the denominator to focus on downside risk, particularly tail risk. The standard Sharpe ratio is calculated as follows:

$$Modified\,Sharpe\,ratio = \frac{R_p - r_f}{CVaR_p} \tag{20}$$

Where CVaR, also known as Expected Shortfall, measures the average loss beyond the VaR threshold.

By incorporating downside risk measures, the Modified Sharpe Ratio provides a more accurate assessment of the risk-adjusted performance of investment vehicles that are prone to extreme losses. This focus on tail risk makes the Modified Sharpe Ratio particularly useful for evaluating investments in environments characterized by significant uncertainty and potential for large negative returns.

4 Results & Analysis

4.1 Connectedness Matrix Comparison Analysis – Including & Excluding GRBI

The TVP-VAR connectedness decomposition model is utilized to capture the time-varying nature of the underlying dynamic spillover network structure, following the seminal work of Diebold and Yilmaz (2012). Initially, the analysis focuses on the average total connectedness index (TCI) values. To gain further insights into the impact of Covid-19 and the Russia-Ukraine war on the network of asset indices, the total net connectedness and net pairwise connectedness are also examined. Each asset index can function as either a net shock transmitter or a net shock receiver.

Table 1 presents the average results regarding the connectedness within the network for the full sample period. The diagonal metrics represent the impact on their own indices, while the non-diagonal elements indicate the level of transmitting and receiving shocks. The columns correspond to a specific asset index's effect on all other indices, whereas the rows correspond to the contribution of a specific asset index's return to the forecast error variance of all other indices in the network. The combined contribution of the non-diagonal shocks from all other indices to a particular index, row-wise, results in the FROM (FROM OTHERS) column. Conversely, the combined contribution of the non-diagonal shocks from all other indices to a particular index, column-wise, results in the TO (TO OTHERS) row. The difference between the TO and FROM results yields the NET connectedness values. The TCI value is the average of the sum of the FROM/TO column. It is noteworthy that GRBI contributes only 7.65% shock TO the entire system, the lowest among all considered asset indices, followed by BTCI (16.22%) and CAGI (20.67%). In contrast, EQWI contributes a substantial 107.39% shock TO the entire system, the highest among the indices, followed by CXNI (75.84%), OLGI (71.07%), and CXEI (69.62%). The average TCI metric of 53.34% indicates that this network explains 53.34% of the variance in the system of considered asset indices. The average diagonal impact of approximately 46% implies that idiosyncratic effects are lower compared to the effects from other market indices at the aggregate level. At the individual asset level, BTCI, CAGI, and GRBI exhibit the highest self-impact, with 74.58%, 68.88%, and 66.27%, respectively, whereas EQWI, CXNI, OLGI, and CXEI show the lowest self-impact of variance decomposition at 25.29%, 32.30%, 34.08%, and 35.64%, respectively. This suggests that BTCI, CAGI, and GRBI markets' variance is mainly due to internal shocks, while EQWI, CXNI, OLGI, and CXEI sustain significant shock impacts from other markets. On a net basis, GRBI is the largest net receiver of shocks (26.08%), followed by CAGI (10.45%), COMI (9.57%), and BTCI (9.20%). EQWI (32.68%) leads the pack of net shock transmitters, followed by CXNI (8.14%), DOSI (6.43%), CXEI (5.26%), and OLGI (5.15%). These findings confirm the hypothesis of including CXNI and CXEI in the analysis as significant net shock transmitters, along with GRBI, which is the largest net receiver of shocks.

Table 1: Full Period Connectedness Table

	GRBI	DOSI	EQWI	COMI	OLGI	GTBI	CAGI	BTCI	ALEI	VIXI	CXNI	CXEI	FROM
GRBI	66.27	6.00	3.08	1.26	1.26	11.48	1.28	1.05	2.39	1.39	2.43	2.09	33.73
DOSI	0.79	46.33	6.02	4.91	3.48	16.47	3.68	1.44	4.51	2.56	3.31	6.50	53.67
EQWI	0.56	3.76	25.29	3.65	10.61	1.26	1.65	1.96	11.91	14.43	13.99	10.93	74.71
COMI	0.56	5.19	6.78	48.98	15.93	1.11	2.77	1.86	4.53	3.38	3.54	5.38	51.02
OLGI	0.49	2.89	13.91	11.88	34.08	0.95	2.28	1.39	7.54	7.73	8.79	8.07	65.92
GTBI	1.77	21.67	2.96	1.47	2.14	56.56	2.37	0.91	2.76	2.55	2.40	2.44	43.44
CAGI	0.71	4.84	4.02	3.73	3.63	2.72	68.88	1.07	3.30	1.73	2.16	3.21	31.12
BTCI	0.79	2.18	4.26	2.34	2.09	1.36	1.18	74.58	3.14	3.16	2.60	2.34	25.42
ALEI	0.55	4.00	16.71	3.44	8.06	1.70	1.78	2.01	37.29	7.95	8.38	8.12	62.71
VIXI	0.35	2.14	18.79	2.35	7.63	1.42	0.86	1.78	7.11	33.77	15.18	8.63	66.23
CXNI	0.64	2.46	17.00	2.43	8.30	1.09	1.05	1.48	7.00	14.32	32.30	11.92	67.70
CXEI	0.43	4.97	13.85	3.99	7.95	1.05	1.79	1.28	7.38	8.60	13.06	35.64	64.36
TO	7.65	60.10	107.39	41.45	71.07	40.64	20.67	16.22	61.57	67.81	75.84	69.62	640.02
Inc.Own	73.92	106.43	132.68	90.43	105.15	97.20	89.55	90.80	98.86	101.58	108.14	105.26	cTCI/TCI
NET	-26.08	6.43	32.68	-9.57	5.15	-2.80	-10.45	-9.20	-1.14	1.58	8.14	5.26	58.18/53.34
NPT	0.00	5.00	11.00	4.00	7.00	3.00	2.00	1.00	6.00	8.00	10.00	9.00	

Table 2 presents the results excluding GRBI from the analysis to understand its incremental impact on the entire network. The overall average TCI metrics increased marginally from 53.34% to 54.94%, indicating that the addition of GRBI to the network reduces the overall total connectedness index value, thereby lowering the effect of systemic connectedness in the system. This leads to a reduction in TO and FROM values for all asset indices. Notably, the NET values for DOSI change significantly from being a strong shock transmitter (6.43%) to a weak one (0.74%), with similar trends observed for VIXI (from 1.58% to 0.63%), CXNI (from 8.14% to 5.03%), and CXEI (from 5.26% to 3.56%). Conversely, GTBI shows a significant increase in shock reception (from 2.80% to 12.89%). These changes in NET values signify that the addition of GRBI dramatically reduces the net shock received by GTBI and the net shocks transmitted by DOSI, VIXI, CXNI, and CXEI. Thus, adding GRBI to the system acts as a shock absorption mechanism.

Table 2: Full Period Connectedness Table w/o GRBI

	DOSI	EQWI	COMI	OLGI	GTBI	CAGI	BTCI	ALEI	VIXI	CXNI	CXEI	FROM
DOSI	46.22	6.13	4.88	3.59	16.55	3.99	1.46	4.75	2.62	3.29	6.52	53.78
EQWI	3.75	25.45	3.79	10.78	1.28	1.66	2.03	11.84	14.49	13.90	11.03	74.55
COMI	5.04	6.97	48.91	16.14	1.02	2.73	1.88	4.94	3.39	3.46	5.51	51.09
OLGI	2.91	14.02	12.07	34.20	0.97	2.32	1.42	7.53	7.80	8.65	8.11	65.80
GTBI	21.98	3.13	1.43	2.28	57.03	2.68	1.00	3.17	2.49	2.28	2.54	42.97
CAGI	5.18	4.07	3.70	3.68	3.31	68.76	1.12	3.25	1.76	2.06	3.11	31.24
BTCI	2.11	4.42	2.31	2.14	1.38	1.17	75.05	3.23	3.19	2.60	2.40	24.95
ALEI	3.99	16.67	3.71	8.15	1.86	1.77	2.09	37.80	7.82	7.91	8.24	62.20
VIXI	2.18	18.85	2.40	7.75	1.44	0.87	1.81	6.95	33.90	15.21	8.65	66.10
CXNI	2.46	17.13	2.44	8.35	1.06	1.08	1.54	6.63	14.58	32.81	11.92	67.19
CXEI	4.93	13.98	4.15	8.08	1.21	1.76	1.32	7.59	8.61	12.85	35.53	64.47
TO	54.52	105.36	40.89	70.94	30.09	20.04	15.66	59.87	66.74	72.22	68.03	604.35
Inc.Own	100.74	130.81	89.80	105.14	87.11	88.79	90.71	97.67	100.63	105.03	103.56	cTCI/TCI
NET	0.74	30.81	-10.20	5.14	-12.89	-11.21	-9.29	-2.33	0.63	5.03	3.56	60.44/54.94
NPT	4.00	10.00	3.00	6.00	2.00	1.00	0.00	5.00	7.00	9.00	8.00	

4.1.1 Connectedness Matrix Values (FROM, TO, NET) - across Pre-, During-, and Post-Covid

Figure 5 illustrates the connectedness table values (FROM, TO, NET) for all asset indices across three time periods: Pre-Covid, During-Covid, and Post-Covid. This comparison allows for insights into the dynamic time-varying connectedness metrics. Notably, the TO values have increased for all asset indices except CXNI from the Pre-Covid to During-Covid and Post-Covid periods. GRBI continues to contribute the lowest TO shock at the aggregate level, closely followed by CAGI, despite a jump in TO levels between the Pre-Covid and the subsequent periods. This suggests that GRBI and CAGI transmit very low shocks to the system, even during periods of heightened uncertainty, making them suitable candidates for diversification benefits. The FROM values increased for all asset indices from the Pre-Covid to During-Covid and Post-Covid periods, except for COMI, OLGI, and ALEI, which saw a decline. This indicates that commodity and energy-related indices experienced decreased shocks from other asset indices in the Post-Covid period compared to the Pre-Covid period. GRBI experienced the highest increase in shock reception between the sub-sample periods, followed by BTCI and CAGI, suggesting higher susceptibility to shocks from other indices during times of heightened uncertainty. However, given their high self-impact components, these shocks from other indices may be muted compared to their own shocks. Lastly, the NET values clearly highlight GRBI as a net receiver of shocks in all sub-sample periods, with significant spikes during the Covid-19 and Russia-Ukraine war periods, and CAGI as the only other notable net receiver.

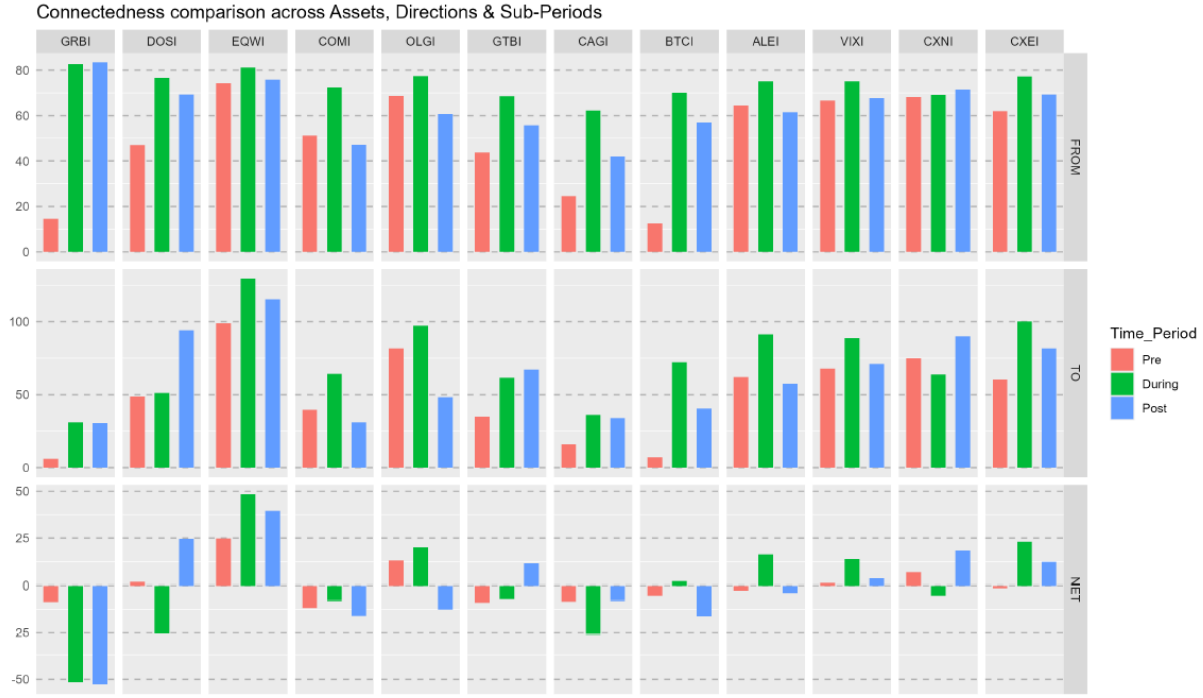


Figure 5: Connectedness comparison Plots

4.1.2 Total Connectedness Analysis

Figure 6 displays the average total connectedness index (TCI) metrics for the entire system over time, illustrating the intertemporal transformation of TCI during periods of stress. High TCI values indicate large spillovers between different asset indices, while low TCI values suggest smaller spillovers. The TCI levels spike during significant events such as the 2015 Chinese stock market turmoil, the Brexit referendum in 2016, the 2018 market crash amid US-China trade tensions, the onset of Covid-19, and the Russia-Ukraine war and its aftermath.

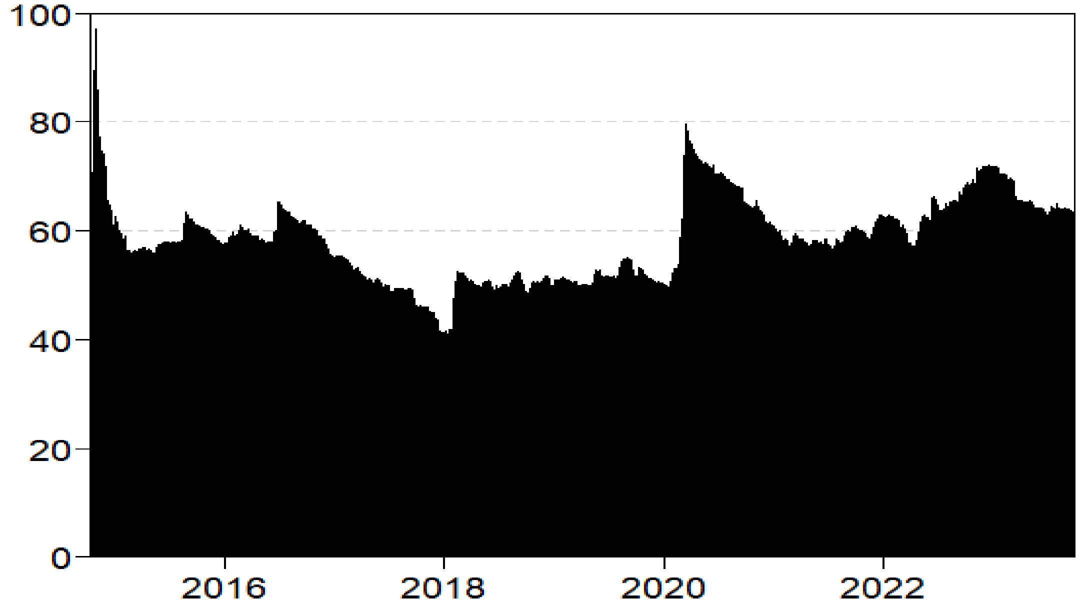


Figure 6: Total Connectedness Index Plot (Full Sample)

4.1.3 Connectedness Network Analysis – Across Pre-, During-, and Post-Covid

Figure 7 shows the change in the connectedness network density across the three sub-sample periods, highlighting the systemic connectedness of the asset indices over time. The nodes represent the asset indices, and the edges signify the net pairwise connectedness values, with edge thickness proportional to the level of connectedness. A threshold level of 0.25 is used to determine significant edges. During the Pre-Covid period, GRBI, BTCI, and CAGI are not part of the network, while EQWI is the most centrally connected index. The During-Covid period exhibits a densely connected network, with almost all asset indices pairwise connected except the GTBI-CAGI pair. The Post-Covid period shows a less dense network but still highlights that all asset indices are interconnected, though at varying levels of pairwise connectedness compared to the Pre-Covid and During-Covid periods. GRBI becomes more involved in the system over time.

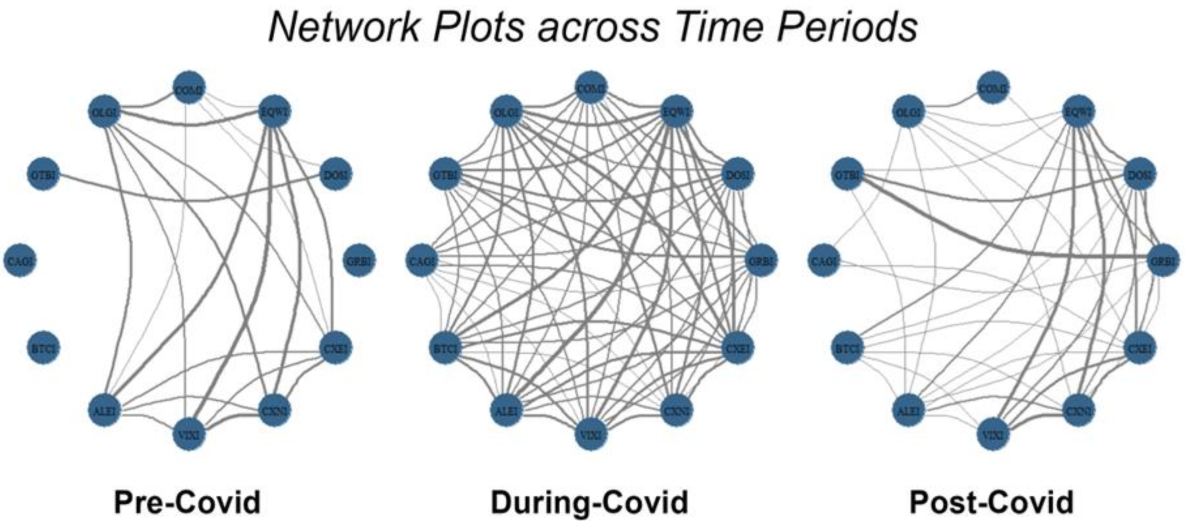


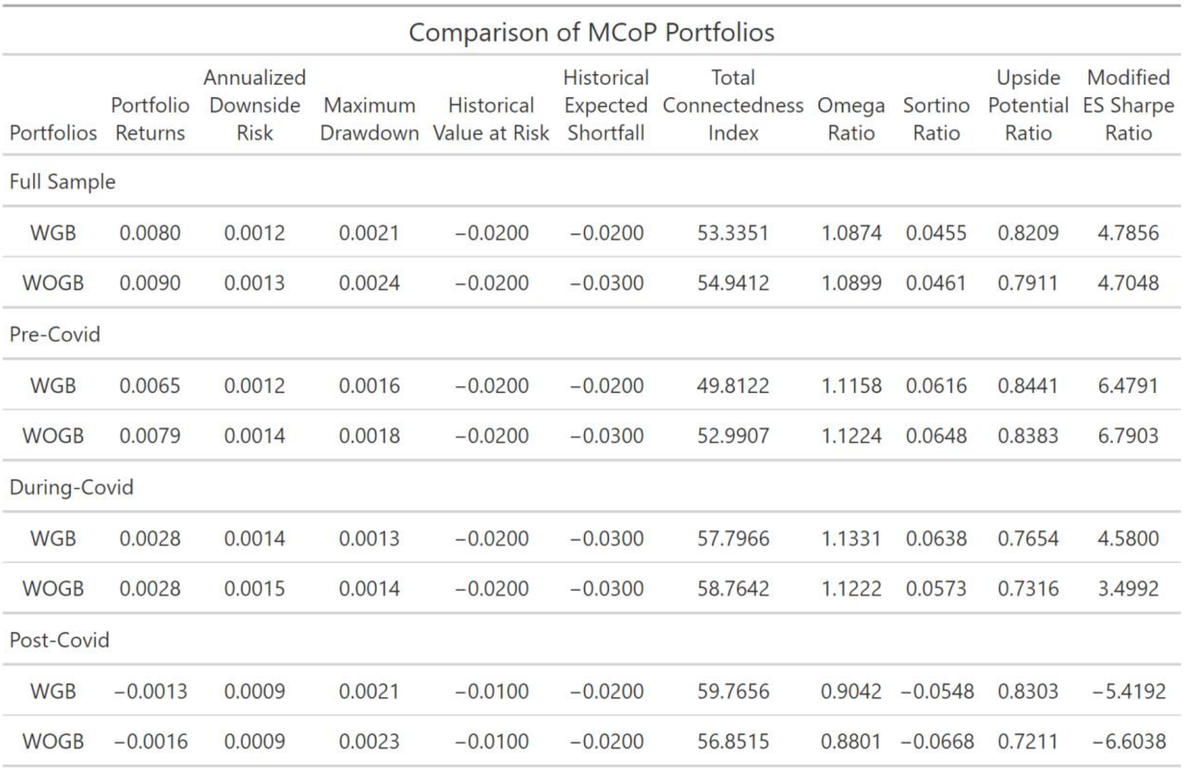
Figure 7: Network Plots across Time Periods

4.2 Portfolio Performance Analysis – with and without GBs

The focus then shifts to studying the impact of adding green bonds (GBs) on portfolio performance using the hedging effectiveness of the minimum connectedness portfolio (MCoP) proposed by Broadstock, Chatziantoniou, and Gabauer (2022). Two MCoPs are compared: the base portfolio without GBs and the delta portfolio with GBs added to the base portfolio. This comparison allows for identifying and gauging the incremental diversification benefit brought by adding GBs to the portfolio.

Table 3 compares the performance of two portfolios— the minimum connectedness portfolio (MCoP) base portfolio (without GRBI) and the delta portfolio (with GRBI)—across various performance measures and different time periods. The analysis reveals that the MCoP base portfolio consistently outperforms the delta portfolio in terms of returns, except during the post-Covid period. Figure 8 further illustrates this by comparing the cumulative returns of both portfolios, reinforcing the consistent better performance of the base portfolio over the delta portfolio in terms of returns.

Table 3: Portfolio Comparison Table Across Time



Comparison of MCoP Portfolios

Portfolios	Portfolio Returns	Annualized Downside Risk	Maximum Drawdown	Historical Value at Risk	Historical Expected Shortfall	Total Connectedness Index	Omega Ratio	Sortino Ratio	Upside Potential Ratio	Modified ES Sharpe Ratio
Full Sample										
WGB	0.0080	0.0012	0.0021	−0.0200	−0.0200	53.3351	1.0874	0.0455	0.8209	4.7856
WOGB	0.0090	0.0013	0.0024	−0.0200	−0.0300	54.9412	1.0899	0.0461	0.7911	4.7048
Pre-Covid										
WGB	0.0065	0.0012	0.0016	−0.0200	−0.0200	49.8122	1.1158	0.0616	0.8441	6.4791
WOGB	0.0079	0.0014	0.0018	−0.0200	−0.0300	52.9907	1.1224	0.0648	0.8383	6.7903
During-Covid										
WGB	0.0028	0.0014	0.0013	−0.0200	−0.0300	57.7966	1.1331	0.0638	0.7654	4.5800
WOGB	0.0028	0.0015	0.0014	−0.0200	−0.0300	58.7642	1.1222	0.0573	0.7316	3.4992
Post-Covid										
WGB	−0.0013	0.0009	0.0021	−0.0100	−0.0200	59.7656	0.9042	−0.0548	0.8303	−5.4192
WOGB	−0.0016	0.0009	0.0023	−0.0100	−0.0200	56.8515	0.8801	−0.0668	0.7211	−6.6038

However, an informed investor understands the importance of considering more than just returns when making investment decisions. Therefore, a deeper analysis of the MCoP portfolios focuses on risk measures to provide a comprehensive assessment. Grootveld and Hallerbach (1999) demonstrate that downside-risk measures are more effective than traditional risk measures. Consequently, this study emphasizes downside risk measures to evaluate the performance of both portfolios more holistically.

Table 3 provides a detailed comparison of various risk measures for both portfolios. The historical value-at-risk (VaR) tail risk measure is identical for both portfolios. However, the delta portfolio (with GRBI) exhibits lower values for annualized downside risk, maximum drawdown, and historical conditional value-at-risk (expected shortfall) compared to the base portfolio (without GRBI). Although the base portfolio delivers higher returns, it also incurs higher downside risks as indicated by these measures. Therefore, evaluating the risk-adjusted return measures is crucial to determine the better-performing portfolio.

The total connectedness index (TCI) for both portfolios is presented in Table 3 and Figure 8. The delta portfolio (with GRBI) shows a lower TCI compared to the base portfolio (without GRBI) during the pre-Covid and during-Covid periods. However, a shift occurs in the TCI levels during the Russia-Ukraine war (post-Covid period). Closer inspection of the TCI comparison charts in Figure 8 reveals that this shift happened in May 2021, within the during-Covid period, likely due to the significant mobilization of Russian military forces near Ukraine and Crimea in March and April 2021, which created an international crisis. This shift indicates that the delta portfolio (with GRBI) was more effective in terms of TCI during the pandemic but less so during the subsequent war crisis.

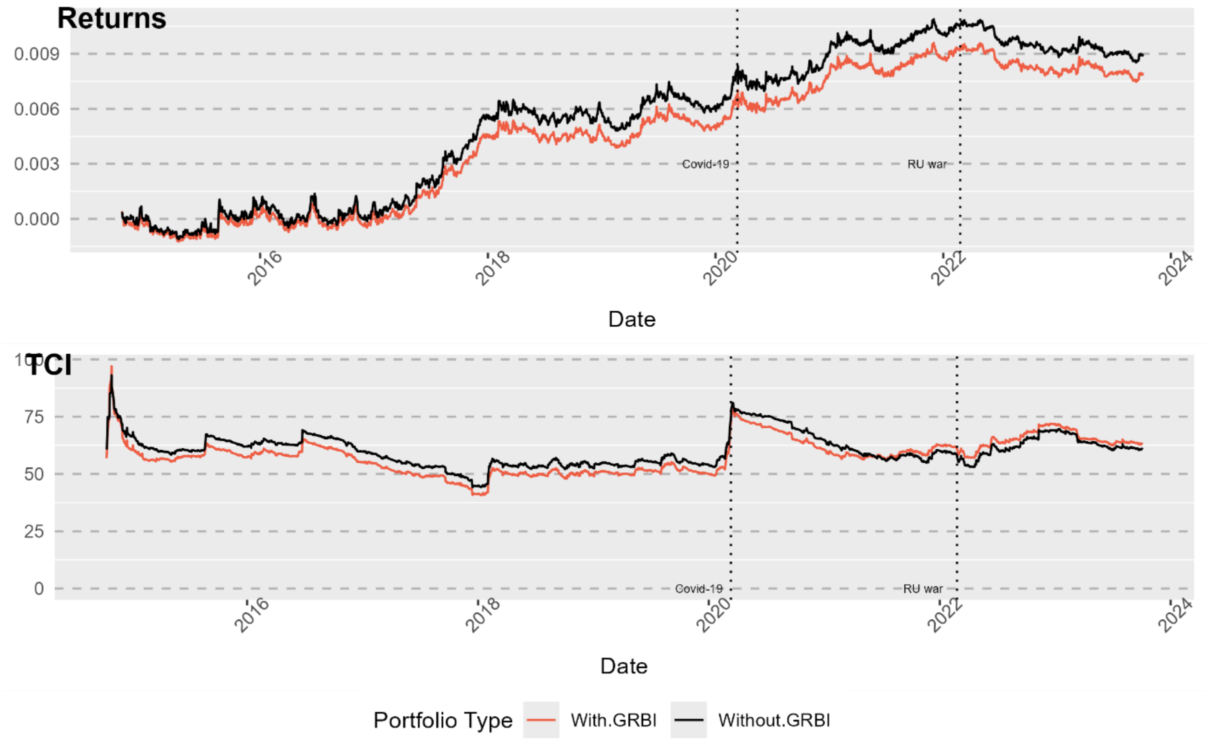


Figure 8: Cumulative Portfolio Returns & Total Connectedness Index Comparison

Figure 9 visually compares the drawdowns of both portfolios, with the base portfolio (without GRBI) represented in black and the delta portfolio (with GRBI) in red. The delta portfolio consistently outperforms the base portfolio in terms of drawdown across the studied period, particularly during the worst scenarios. Recently, the gap in drawdown between the two portfolios has slightly widened, suggesting a potential long-term improvement in the delta portfolio's performance. However, whether this trend represents a short-term fluctuation or a lasting enhancement remains to be seen.

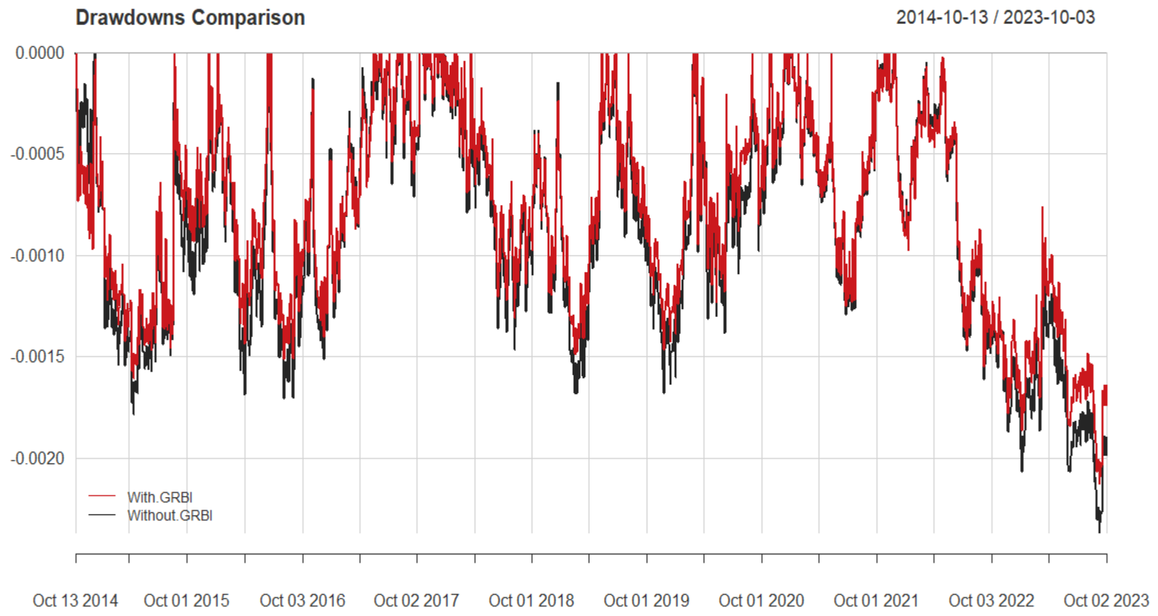


Figure 9: Drawdowns of the Minimum Connected Portfolios

As discussed by Sortino and Van Der Meer (1991), Sortino, Meer, and Plantinga (1999), and Sortino and Satchell (2001), downside-risk-adjusted returns offer a more nuanced view for stakeholders compared to traditional risk-adjusted returns, especially during market turmoil when emotional biases are prevalent. Therefore, this analysis focuses on four downside-risk/tail-risk adjusted return measures: Omega ratio, Upside Potential ratio, Sortino ratio, and Modified Sharpe ratio.

The Omega ratio compares the sum of the gains of the portfolio above a threshold target return to the absolute sum of the losses below that threshold. An Omega ratio greater than one indicates that the portfolio performs better relative to the threshold target return. For the full period and the pre-Covid period, the base portfolio (without GRBI) shows a slightly better performance in terms of the Omega ratio compared to the delta portfolio (with GRBI). This suggests that for a given threshold return, the base portfolio yields more wins than losses, aligning with its better overall returns. However, the delta portfolio's performance improved during the crisis periods (during-Covid and post-Covid).

The Upside Potential ratio, similar to the Omega ratio, considers the sum of gains relative to a threshold target return per unit of downside-risk measure. The delta portfolio (with GRBI) outperforms the base portfolio (without GRBI) across all sub-sample periods. Notably, the delta portfolio's performance advantage increases progressively from the pre-Covid to during-Covid to post-Covid periods.

The Sortino ratio measures annualized excess portfolio returns per unit of annualized downside risk, while the Annualized ES Sharpe ratio measures annualized excess portfolio returns per unit of annualized expected shortfall. Both ratios are true risk-return measures, with the Sortino ratio focusing on downside risk and the Annualized ES Sharpe ratio on tail-risk. The delta portfolio (with GRBI) outperforms the base portfolio (without GRBI) in both measures during the during-Covid and post-Covid periods, though the base portfolio performs better in the pre-Covid period. This indicates that the delta portfolio not only provides better returns relative to downside risk but also optimizes tail-risk during crisis periods, offering significant advantages for investors seeking higher returns while mitigating downside or tail risk during times of stress.

Beyond downside-risk and downside-risk-adjusted return measures, it is essential to consider portfolio performance from a hedging perspective to manage connectedness risks during periods of market uncertainty. Hedging ratios, typically used to measure the hedging performance of futures contracts relative to spot positions, can be extended to portfolio management strategies. Broadstock et al. (2022) and Pham and Do (2022) suggest a trading strategy where a two-asset portfolio involves a long position in one asset and a short position in another. While hedge ratios can express portfolio performance, they do not account for market spillovers.

4.3 Hedging Effectiveness Analysis of Minimum Connectedness Portfolios

Following the approach of Broadstock et al. (2022), and Pham and Do (2022), two minimum connectedness portfolios are constructed: the base portfolio (without GRBI) and the delta portfolio (with GRBI). These portfolios minimize the pairwise spillover connectedness index matrix to calculate optimal weights. The hedging effectiveness, as defined by Ederington (1979), is then evaluated to determine the percentage reduction in the variances of the portfolios for each included asset. This approach helps investors gauge the portfolio's performance from a hedging perspective, providing insights into the effectiveness of adding green bonds to manage connectedness risks. Table 4 depicts the hedging effectiveness comparison of minimum connectedness portfolios. A Hedging Effectiveness (HE) value of zero indicates that the hedge for the particular asset is ineffective, whereas an HE value of one implies a perfect hedge for the asset. The HE values for VIXI, BTCI, and CXNI are high and close to one for both the full-period base portfolio (without GRBI) and the delta portfolio (with GRBI), at approximately 98%, 93%, and 86%, respectively. This suggests that these assets are the most effectively hedged against minimum connectedness criteria within the portfolio.

Conversely, assets with high negative HE values contribute to the portfolio achieving its optimal value (i.e., minimum connectedness) by acting as natural hedges. For the base portfolio (without GRBI), CXEI, DOSI, and GTBI serve as the largest hedge assets, with HE values of -10.41, -10.02, and -9.61, respectively, for the full period. However, in the delta portfolio (with GRBI), the HE values for CXEI, DOSI, and GTBI decrease to -8.26, -7.94, and -7.61, respectively. The introduction of GRBI into the portfolio results in it having the next highest negative HE value of -5.74, highlighting its ability to act as an effective hedge.

Table 4: Hedging Effectiveness comparison of Minimum Connectedness Portfolios

Hedging Effectiveness of MCoP Across Sub-Periods

	MCoP with GRBI								MCoP without GRBI							
	Full Period		Pre Covid		During Covid		Post Covid		Full Period		Pre Covid		During Covid		Post Covid	
Assets	HE	p.val	HE	p.val	HE	p.val	HE	p.val	HE	p.val	HE	p.val	HE	p.val	HE	p.val
GRBI	−5.74	0.00	−10.35	0.00	−6.20	0.00	−1.60	0.00	-	-	-	-	-	-	-	-
DOSI	−7.94	0.00	−9.17	0.00	−8.71	0.00	−4.89	0.00	−10.02	0.00	−12.46	0.00	−14.19	0.00	−3.00	0.00
EQWI	−0.50	0.00	−1.47	0.00	0.17	0.03	−0.11	0.28	−0.85	0.00	−2.26	0.00	−0.30	0.00	0.24	0.00
COMI	−0.51	0.00	−1.11	0.00	−0.38	0.00	0.21	0.02	−0.86	0.00	−1.80	0.00	−1.16	0.00	0.46	0.00
OLGI	0.50	0.00	0.24	0.00	0.73	0.00	0.58	0.00	0.39	0.00	−0.01	0.90	0.57	0.00	0.71	0.00
GTBI	−7.61	0.00	−9.67	0.00	−10.51	0.00	−3.27	0.00	−9.61	0.00	−13.12	0.00	−16.99	0.00	−1.90	0.00
CAGI	0.67	0.00	0.61	0.00	0.73	0.00	0.72	0.00	0.59	0.00	0.49	0.00	0.58	0.00	0.81	0.00
BTCI	0.93	0.00	0.93	0.00	0.93	0.00	0.91	0.00	0.91	0.00	0.91	0.00	0.89	0.00	0.94	0.00
ALEI	0.35	0.00	−0.26	0.00	0.65	0.00	0.60	0.00	0.20	0.00	−0.67	0.00	0.46	0.00	0.73	0.00
VIXI	0.98	0.00	0.98	0.00	0.98	0.00	0.97	0.00	0.97	0.00	0.97	0.00	0.97	0.00	0.98	0.00
CXNI	0.86	0.00	0.81	0.00	0.91	0.00	0.87	0.00	0.83	0.00	0.75	0.00	0.86	0.00	0.91	0.00
CXEI	−8.26	0.00	−12.27	0.00	−4.12	0.00	−8.04	0.00	−10.41	0.00	−16.56	0.00	−7.01	0.00	−5.14	0.00

Figure 10 provides a detailed breakdown of the Hedging Effectiveness (HE) values for GRBI, CXEI, DOSI, and GTBI across the sub-sample periods of Pre-Covid, During-Covid, and Post-Covid, offering insights into how these values have evolved during different crisis periods. The HE values for GRBI and DOSI show a consistent decline across the Pre-Covid, During-Covid, and Post-Covid periods. In the delta portfolio (with GRBI), GTBI's HE values increase during the Pre-Covid and During-Covid periods but decrease in the Post-Covid period. Conversely, CXEI experiences a sharp drop in HE during the During-Covid period, followed by an increase in the Post-Covid period. For the base portfolio (without GRBI), DOSI, GTBI, and CXEI all show a reduction in HE values during the Pre-Covid and During-Covid periods when compared to the delta portfolio (with GRBI). However, this trend reverses in the Post-Covid period, where the HE values increase for these assets in the delta portfolio. The data indicates that during the Post-Covid period, the delta portfolio (with GRBI) exhibits higher hedging effectiveness for all four assets under consideration compared to the base portfolio (without GRBI). This increase in HE values is correlated with the significant rise in the portfolio weights of GRBI in recent times, suggesting that GRBI has become a more effective hedging instrument during periods of heightened uncertainty.

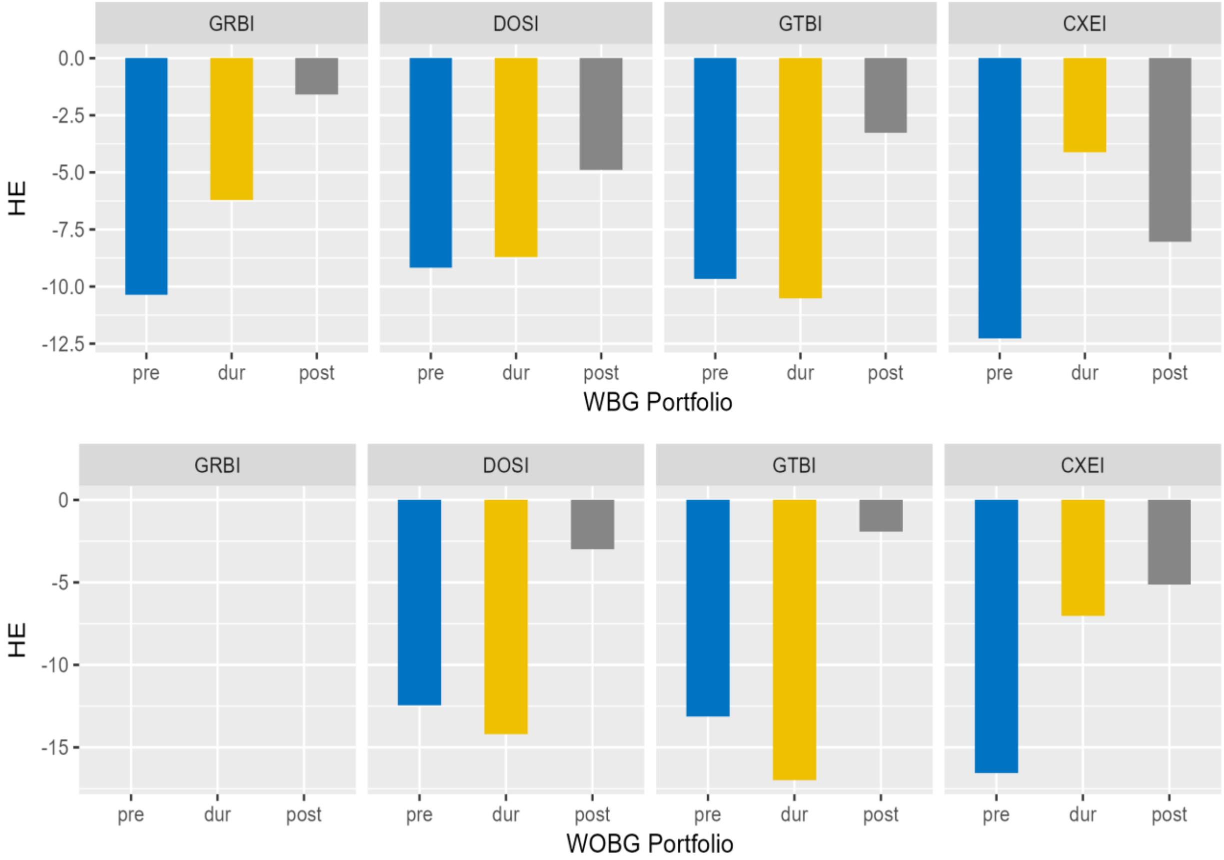


Figure 10: Hedging Effectiveness Plot Comparison

The results of this section offer significant contributions for portfolio managers and traders in the areas of risk management, portfolio diversification, and strategic asset allocation. By utilizing the time-varying parameter vector autoregression (TVP-VAR) model, the research provides insights into dynamic spillover effects and interconnectedness between green bonds (GBs) and other financial assets, enabling proactive risk management. The detailed assessment of downside-risk measures, such as maximum drawdown, annualized downside risk, and conditional value-at-risk, helps mitigate potential losses during market stress. The study highlights GBs as effective hedging instruments, reducing systemic risk and enhancing portfolio resilience. The comparative analysis across different sub-periods offers practical insights for strategic allocation decisions, demonstrating how GBs improve diversification benefits and portfolio performance under varying market conditions. By employing downside-risk adjusted return measures like the Omega ratio, Upside Potential ratio, and Modified Sharpe ratio, the study enables better evaluation of risk-return trade-offs. The analysis of hedging effectiveness shows that GBs act as natural hedges, reducing negative impacts within the portfolio. Traders can leverage these findings to develop sophisticated trading strategies, anticipate market movements, and manage volatility, particularly during periods of heightened uncertainty such as the Covid-19 pandemic and the Russia-Ukraine war. This research equips financial professionals with advanced tools and insights to optimize portfolio performance and navigate the complexities of modern markets.

5 Conclusion

Using the spillover index methodology proposed by Diebold and Yilmaz (2009, 2012, 2014) and the time-varying parameter vector autoregression (TVP-VAR) approach by Antonakakis et al. (2019, 2020), this study analyzes the returns connectedness between the green bond index (GRBI) and various other critical financial indices including stock market (EQWI), bond market (GTBI), commodity market (COMI), oil market (OLGI), currency (DOSI), carbon market (CAGI), cryptocurrency (BTCI), alternative energy (ALEI), volatility index (VIXI), and credit default indices (CXNI and CXEI). The analysis spans the entire period as well as three sub-periods: Pre-Covid, During-Covid, and Post-Covid (coinciding with the Russia-Ukraine war period). It also compares two portfolios: the base portfolio (without GRBI) and the delta portfolio (with GRBI). For the full period, the delta portfolio (with GRBI) shows that EQWI contributes the highest shock to the entire system (107.39%) and receives 74.71% from the system. Conversely, GRBI contributes only 7.65% shock to the system, and BTCI receives only 25.42% from the system. EQWI is identified as the largest net transmitter of shocks (+32.68%), while GRBI is the largest net receiver of shocks (-26.08%). Including both CDS indices (CXNI and CXEI) in the analysis proves crucial, as they are among the largest net shock transmitters, emphasizing their importance in portfolio analysis and hedging strategies. The total connectedness index (TCI) comparison for the full period shows that introducing GRBI to the portfolio reduces the TCI from 54.94 to 53.34, indicating that GRBI acts as a spillover absorber. Adding GRBI dramatically reduces the net receiver shock to GRBI and the net transmitter shock from DOSI, VIXI, CXNI, and CXEI. Across the sub-sample periods, GRBI and CAGI consistently act as net receivers of shocks, while EQWI is a consistent net transmitter. Other assets show varying patterns of net shock reception and transmission. The network plots reveal that the network was sparsely connected during the Pre-Covid period, became densely connected during the During-Covid period, and then less densely connected during the Post-Covid period. This indicates that the uncertainty and disruption caused by the Covid-19 crisis were greater than those caused by the Russia-Ukraine war. Over time, GRBI has become more involved in the network. At the minimum connected portfolio (MCoP) level, the base portfolio (without GRBI) performs better in terms of cumulative returns but performs poorly in terms of downside risk measures such as maximum drawdown, annualized downside risk, and historical expected shortfall. The delta portfolio (with GRBI) shows consistently lower drawdowns during all crisis periods, with the gap in drawdowns widening recently. The delta portfolio also outperforms the base portfolio in two out of three downside-risk adjusted return measures (Upside Potential ratio and Modified Sharpe ratio), though it performs slightly worse in terms of the Omega ratio. The hedging effectiveness (HE) analysis shows that introducing GRBI reduces the largest negative HE values for CXEI, DOSI, and GTBI from -10.41, -10.02, and -9.61 to -8.26, -7.94, and -7.61, respectively, indicating that GRBI acts as a natural hedge within the MCoP. Sub-sample period analysis of HE values reveals that GRBI and DOSI's HE values consistently fall, while GTBI's HE increases in the first two periods but falls in the Post-Covid period. CXEI shows a sharp fall in HE during the During-Covid period, followed by a rise in the Post-Covid period. The HE values for CXEI, DOSI, and GTBI fall from the base portfolio to the delta portfolio in the Pre-Covid and During-Covid periods but rise in the Post-Covid period. From an investor's perspective, the study provides key insights at both the asset and portfolio levels. GRBI, CAGI, COMI, and BTCI are identified as net shock receivers, suggesting that adding these assets to a portfolio can lower the system's total connectedness. BTCI, despite its low correlation with other assets, is highly self-driven, making it a complex choice for diversification. GRBI and CAGI consistently act as net shock receivers across all periods, making them reliable for maintaining minimum connectedness in a portfolio. Investors should consider GRBI, CXEI, DOSI, and GTBI based on hedging effectiveness values. The delta portfolio (with GRBI) performs better in downside-risk adjusted return measures (Sortino ratio and Modified Sharpe ratio) during crisis periods, indicating that it is the superior choice for minimum connected portfolios during times of stress.

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