Mathematical Content Knowledge of Pre-Service Teachers: Implications for Consecutive ITE Programmes

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This study examines the mathematical content knowledge (MCK) of pre-service post-primary mathematics teachers (N=85) on commencing their Professional Master of Education (PME) initial teacher education (ITE) programme. Participants' cognitive and conceptual proficiency with curriculum-aligned mathematical content was evaluated using a validated paper and pencil test, on commencement of their PME studies. Findings indicate that pre-service mathematics teachers exhibit stronger proficiency with Junior-Cycle curriculum-aligned content and weak proficiency with Senior-Cycle concepts, regardless of the curriculum strand. Given the lack of recent research examining MCK relating to pre-service teachers, and the focus of postgraduate ITE programmes on pedagogical aspects of teaching, this research identifies concerns with ITE programmes that may need to be addressed to support pre service mathematics teachers' development and induction into the profession.

Keywords: mathematical content knowledge; pre-service teachers; consecutive ITE

Introduction

The Professional Master of Education (PME), a Level 9 Masters qualification, was established following a review of initial teacher education provision in the Irish context in 2012 (Department of Education and Skills, 2012). Students entering the PME to qualify as post-primary mathematics teachers will have undertaken degree level studies in mathematics or related areas, as outlined by the subject requirements to register with the Teaching Council of Ireland (Teaching Council, 2013). Given the consecutive postgraduate nature of the PME programme i.e. entrants have completed degree studies in their subject area(s), the focus is primarily on the development of pedagogical practices and professional experiences in the classroom. It is fundamentally recognized that ITE teachers' mathematical content knowledge (MCK) does not alter during their teacher qualification programme (Osborne 2013). Accordingly, the purpose of our study was to examine the MCK of pre-service mathematics teachers (N=85) commencing their PME programme[1]. Lowrie and Jorgensen (2016) stress that most significant studies relating to pre-service mathematics teachers' MCK are dated and have been overtaken by studies focused on PCK (pedagogical content knowledge). Given that research has demonstrated the essential function of a teacher's mathematical knowledge base for undertaking key teaching and learning roles such as lesson

[1] This paper is based on Ní Ríordáin, M., Ní Shúilleabháin, A., Prendergast, M. & Johnson, P. (2021). Irish pre-service mathematics teachers' knowledge of curriculum-aligned content.

preparation, facilitating classroom discussion and creating purposeful learning opportunities (e.g. Baumert et al., 2010), it is important that we examine the MCK of pre-service mathematics teachers. Examining MCK can alert us to issues within ITE programmes that could be addressed prior to induction into the teaching profession.

Pre-Service Teachers and Mathematical Knowledge

Mathematical knowledge for teaching (MKT), which includes both subject matter (MCK) and pedagogical (PCK) considerations (Ball, Thames & Phelps, 2008) is essential for mathematics teaching and supporting student learning (Baumert et al., 2010). MKT is utilised for example when using different approaches to explain mathematical concepts to students, interpreting students' answers, using various representations, and selecting examples that help develop students' understanding (Clivaz & Ní Shúilleabháin, 2019). However, despite such obvious importance, research studies have shown evidence of inadequate knowledge of mathematics for teaching amongst teachers (Ma, 1999). Similarly, Lowrie and Jorgensen (2016, p. 205) probe the 'PCK fever' that has emerged in relation to researching teacher knowledge, at the detriment of examining content knowledge. Baumert et al.'s (2010) research demonstrates that limited subject matter knowledge can have detrimental effects on a teachers' PCK and, consequently, negative effects on instructional quality and student progress. Furthermore, they find that these differences in teachers' MCK persist across an entire teaching career.

There are various reasons cited for this in the literature but of concern to us is the role of ITE. Thanheiser et al. (2013) suggest that there are inadequacies in teachers' knowledge of mathematics when they graduate from their ITE programmes and many lack conceptual understanding of the mathematics they will be required to teach (O'Meara, Fitzmaurice & Johnson, 2017). For example, Slattery and Fitzmaurice (2014) carried out a study at an Irish university to measure pre-service post-primary mathematics teachers' conceptual understanding of fraction division. The results showed that participants – who were near the end of their degree programme – had a fragmented understanding of the fraction concept and were unable to explain the invert and multiply rule. They relied on a series of "rules without reason" to answer the questions posed. Thus, when teaching, these pre-service teachers would have to rely on a series of learned procedural steps, as they did not have the conceptual understanding necessary to teach for understanding (O'Meara et al., 2017).

Baumert et al. (2010) suggest that ITE programmes should increase the attention given to teachers' subject matter knowledge and, more specifically, achieve a balance between MCK and PCK. Mathematics teachers need to possess knowledge of the curriculum content at a much deeper level of understanding than their students (Krauss et al., 2008). Affording pre service teachers the opportunity to re-examine post-primary school mathematics content from an advanced perspective may be an important element in preparing them to teach mathematics meaningfully (Artzt et al., 2012). A deeper understanding of mathematical concepts may also enable teachers to access a wider collection of strategies for explaining and illustrating mathematical content to their students (Ma, 1999). This present study looks to examine the MCK of pre-service post-primary mathematics teachers on a broad scale as they commence their ITE programmes and provides evidence of issues in relation to subject matter knowledge that may be addressed during the two-year PME programme. For the purposes of this study MCK is concerned with 'an understanding of mathematics concepts anticipated to be taught' (Norton, 2019, p.530) and, accordingly, pre-service teachers' proficiency with curriculum aligned content (Ní Ríordáin et al., 2017).

Methods

A paper-and-pencil test developed to examine Irish post-primary out-of-field mathematics teachers' knowledge of curriculum-aligned content (see Ní Ríordáin et al., 2017) was utilised in the study. The TEDS-M conceptual framework supported the development of item design for the paper-and-pencil test (Ní Ríordáin et al., 2017; Tatto et al., 2008) and items were developed that closely align with the Irish post-primary mathematics curriculum (Krauss et al., 2008). The original paper-and-pencil test had 24 MCK items, including 10 multiple choice and 14 open-ended items. However, for the purpose of this study a reduced version, 17 MCK items (9 multiple-choice, 8 open-ended), was utilised due to time constraints in collecting the data. Three of the multiple-choice items consisted of two sub-items, therefore participants were asked to complete 20 items in total.

The paper-and-pencil test was administered to participants in their first mathematics pedagogy lecture of the term, on commencement of their PME programme. The test was closed book and no calculators were allowed. All those in attendance at the first lecture consented to participate in the study. Participants (N=85) were recruited from four (of eight) institutions in Ireland offering the PME programme over the course of three intakes (2015 – 2017), allowing for a more considerable sample for data analysis and to make appropriate inferences. A convenience sampling approach was utilised in the study, whereby all four authors had access to the participants through their involvement in the delivery of the PME programme in their participating universities.

The scoring of the MCK items on the paper-and-pencil test involved two different scoring processes. Ní Ríordáin et al. (2021) provide a detailed description of the analysis of the data. The first scoring process involved the calculation of a *cognitive score*, with each item scored based on the correctness of the answer with an overall percentage score returned for each participant. In addition, a cognitive proficiency rate for each individual item was determined by calculating the mean score for each item and then converting to a percentage (multiple-choice items) or the mean score of an item as a percentage of total possible score of that item and then converting to a percentage (open-ended items). The second scoring system employed examined conceptual errors made by participants on items answered. Each item was broken down into key concept(s) necessary for answering the question and linked to the post-primary mathematics curriculum. Based on a participant's solution to a given item, a *conceptual error* score was calculated, and an overall percentage error score returned.

Key Findings

The first stage of the cognitive scoring analysis examined the overall scores for each pre service teacher. This overall cognitive score provides a gauge of a pre-service teacher's performance based on their responses to the test items. The cognitive score was calculated as a percentage out of a possible 42 marks. The first notable finding is the low mean score of only 40.2% with a standard deviation of 18.02. However, there is a range of 88.1% with a maximum scored of 92.86% achieved by one pre-service teacher. The second stage of analysis examained the proficiency rates for each item on the test (Table 1). An item's mean score is divided by its total possible score (1 or 3) in order to calculate the proficiency rate. Only four items (of twenty) had a proficiency rate higher than 50%. It is important to note that all of these four items are linked to Junior Cycle mathematics and that no strengths were recorded at Senior Cycle level. Also included in Table 1 are three items with a proficiency rate between 40%-50% (pass standard). Only one of these items is at the Leaving Certificate level, all others are linked to the Junior Cycle curriculum. Likewise, test items with proficiency rates below 40% are deemed a fail as they fall below minimum standards expected in studies completed by qualified mathematics teachers. Thirteen items out of twenty fell below the required 40% pass rate.

Table 1: Pre-service teacher cognitive proficiency with curriculum-aligned content

Strand & Concepts[2]	**Level: JC/LC**	**Proficiency Rate (%)**	***SD***
A&F: Making predictions about what comes next in a pattern	JC	91.0	27.5
N: Solving problems involving shopping: % discount; Performing calculations with percentages	JC	64.3	34.4
G&T: Properties and equations of a line – slope, x/y intercepts; Relationship between the slopes of parallel lines; Labelling axes w/ appropriate scales	JC	60.6	35.5
G&T: Properties of a Square; Applying Pythagoras's Theorem; Operations with Surds	JC	58.3	34.5
N: Relationships between the number systems	JC	46.3	37.8
S&P: Probability: Finding the probability of equally likely outcomes	JC	46.0	49.7
S&P: Representing Data Graphically: Mean vs. Median	LC	45.0	49.9
A&F: Writing an arithmetic expression for the terms in a sequence	JC	38.7	39.8
S&P: Standard Deviation	LC	35.0	47.8
S&P: Mean vs. Median	LC	29	46.1
S&P: Count the number of ways to select *r* objects from *n* distinct objects	LC (HL)	28.0	45.6

[2] **A:** Algebra; **F:** Functions; **N:** Number; **G:** Geometry; **T:** Trigonometry; **S&P:** Statistics & Probability.

F: Recognize a bijective function and find its inverse LC (HL) 26.3 42.4 **F:** Differentiation LC 25.3 32.5

A: Solve simultaneous equations with two variables and interpret results LC 25.3 39.5 **F:** Graph of the function ��(��) = ��^{3}; Transformations ��(��) + ��; Graphs of inverse functions are reflections over �� = ��. LC 25.3 23.8

S&P: Median LC 20.0 39.5 **S&P:** Interquartile Range LC 19.0 39.5 **F:** Associate derivatives with slopes and tangent lines LC 19.3 39.5 **F:** Relationships between number systems JC 18.0 38.6

A: Solve simultaneous equations with infinite number of solutions LC 15.2 31.0

Overall, the pre-service mathematics teachers demonstrated poor proficiency with the Functions strand. All items had a proficiency rate of below 40%. Although these items relate to LC content, all but one are at the Ordinary level. The case is similar for the Statistics and Probability strand where participants demonstrated poor proficiency in five of the seven items connected to this strand, four of which are at the Ordinary level and relate to representing data graphically, the other item relates to Counting. Pre-service teachers demonstrated, on average, a pass standard proficiency with the remaining two items: one related to JC Probability content and the other LC content identifying data sets in which the Mean and Median were equal. The data relating to the Statistics and Probability content suggests a poor knowledge of concepts overall, particularly items involving Senior Cycle and Higher level.

In general, no distinct pattern emerged from the data in relation to strengths and weaknesses relating to the other strands. For example, Item 13a (finding what comes next in a pattern) with the highest proficiency rate (91%, with a standard deviation of 27.5) involved content from Junior Cycle Algebra. Yet its related item, 13b (writing arithmetic expressions for terms in a sequence), only had a proficiency rate of 38.7%, with a standard deviation of 39.8. The four items with the highest proficiency rates involve three different strands (Algebra, Number and Geometry), but all content is related to Junior Cycle. The items with the weakest proficiency rates involve Number and Algebra also, but at Senior Cycle. Therefore, overall, the data suggests that pre-service teachers demonstrate a stronger proficiency with Junior Cycle curriculum aligned content and poor proficiency with Senior Cycle content, irrespective of the curriculum strand.

An analysis of pre-service mathematics teachers overall conceptual scores establishes a relatively high occurrence rate of errors amongst the 85 participants. Overall, of the questions answered, there was a 45.86% mean occurrence rate for conceptual errors or evidence of incomplete conceptual understanding. The implications of this is that more than two out of every five errors that could be made were made by these pre-service teachers.

Given that there is variation in the response rates for individual items, it is difficult to undertake a comparison of occurrence rates on individual concepts. However, there are several items with a high response rate, with a high conceptual error occurrence rate, that can extend our understanding of the cognitive scores and proficiency rates outlined in the previous paragraphs. For example, items 3-7 relating to Statistics and Probability had a high response rate with a high occurrence rate (50%-78%) for conceptual errors relating to the identified concepts. In general, items around or below the average occurrence rate for conceptual errors and with a high response rate are associated with Junior Cycle content relating to Geometry and Trigonometry, Number and Algebra. It is interesting to note variation in occurrence rates within items with high response rates. For example, Item 10 examines Number Systems. Pre-service teachers demonstrated low occurrence rates for conceptual errors relating to Junior Cycle concepts of Natural and Rational Numbers, but an increase in conceptual errors is evident with higher order concepts such as Prime and Complex Numbers relating to Leaving Certificate content. This is consistent with findings relating to pre-service teachers' cognitive proficiency rates with curricular aligned content. These insights help identify the areas of weakness that may have caused participants to answer an item incorrectly and by profiling pre-service mathematics teachers' knowledge of content related items it can help us address such misconceptions within our teacher education programmes.

Discussion & Conclusion

The findings of this research suggest that while these pre-service teachers may be well equipped to teach Junior Cycle mathematics content, they do not demonstrate the same proficiency with Senior Cycle content. Furthermore, there are strands of the curriculum (e.g. Statistics & Probability and Functions) where pre-service teachers demonstrate poor cognitive and conceptual knowledge. This research aligns with the findings of Ma (1999), and Slattery and Fitzmaurice (2014) that record shortcomings in pre-service post-primary teachers' knowledge of mathematics. Given the importance of teachers' adequate knowledge of content to develop learners' conceptual understanding and to positively impact on learner achievement (Baumert et al., 2010), these findings may have broad implications for consecutive post-primary ITE programmes in Ireland.

The results of this research suggest that, despite the rigorous content requirements of the Teaching Council (2013), pre-service teachers on consecutive programmes may require additional subject matter knowledge prior to their commencement of teaching. In line with recommendations from Baumert et al. (2010), ITE programmes should increase the attention given to pre-service teachers' content knowledge and explicitly align MCK and PCK in PME mathematics pedagogy modules. This is significant as high levels of teacher PCK is not achievable without a high level of subject matter knowledge and both have significant impact on student achievement (Baumert et al., 2010) and consideration needs to be given to LC content. For example, in pedagogy modules, MCK materials should not mirror that of programmes for mathematics majors, but rather explicitly link mathematical content at a deeper level of understanding to that of the school curriculum (Artzt et al., 2012). On the basis of this evidence, any reduction of the current minimum subject specific pedagogy from 5 ECTS credits (Teaching Council, 2013) should be avoided. It may be of interest to repeat this research with additional cohorts of pre-service teachers who have studied the revised post-primary mathematics curriculum as part of their own post-primary education and may therefore be more familiar with the revised curriculum content, particularly that of Statistics and Probability which was not compulsory content prior to 2012.

Given calls for a renewed focus on content knowledge in teacher education (Lowrie & Jorgensen, 2016), further research should be conducted on both pre-service and in-service mathematics teachers' knowledge on a larger and longitudinal scale. Such research would be of particular relevance in informing any future ITE and teacher education policies. In the context of various ITE programmes underway across the country, it may be of particular interest to compare and contrast the MCK of pre-service teachers from consecutive and concurrent programmes. Additional research should be conducted to investigate how best we might support the development of pre-service teachers' MCK over the duration of their PME programme. The strong association between MCK and classroom practices always needs to be taken into consideration and by assessing pre-service teachers' MCK it allows for the development of awareness of the challenges they may face (Norton, 2019). How we utilise this information thereafter needs to be considered in our PME programme delivery.

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