Guiding Young Children's Participation in Early Childhood Mathematics Education: An Ethnographic Study of an Irish Preschool Classroom

A Thesis Submitted for the Degree of Doctor of Philosophy

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DECLARATION

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work, and that I have exercised reasonable care to ensure that the work is original and have conformed to the regulations on the use and declaration of Generative AI, and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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In Ómós do mo Mháthair

(1956 – 2024)

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TABLE OF CONTENTS

DECLARATION ..... **iii**
DECLARATION ON USE OF AI TOOLS ..... **iv**
ACKNOWLEDGEMENTS ..... **vi**
LIST OF ABBREVIATIONS ..... **xvi**
LIST OF TABLES ..... **xviii**
LIST OF FIGURES ..... **xix**
ABSTRACT ..... **xxi**
CHAPTER 1: INTRODUCTION **1**
1.1 Irish Educational Policy Context 2
1.2 The Importance of Early Childhood Education 3
1.3 Play Pedagogy in ECE 4
1.4 Early Childhood Mathematics Education (ECME) 5
1.4.1 The Importance of ECME 7
1.4.2 The Nature of ECME in Preschool Settings 8
1.5 An Overview of the Current Irish ECCE Structure and ECME 9
1.6 Gaps in ECME Research 11
1.7 Personal Interest 12
1.8 Purpose of the Study 13
1.9 Overview of the Thesis 15
CHAPTER 2: Literature Review **16**
2.1 Early Visions of the Child 16
2.1.1 Jan Comenius 17
2.1.2 Johann Pestalozzi 17
2.1.3 Friedrich Froebel 18
2.1.4 Maria Montessori 19
2.1.5 Early Perceptions of the Role of the Adult 20
2.2 Modern Vision of the Young Child as a Mathematician 20
2.2.1 Young Children Being and Becoming Mathematicians 22
2.3 Professional Knowledge for ECME 23
2.3.1 Mathematical Content Knowledge (MCK) 24
2.3.2 The Nature of Mathematics 26

2.3.3 The Big Ideas of Early Childhood Mathematics....27
2.3.4 Knowledge of Mathematical Development....32
2.3.5 Pedagogical Content Knowledge (PCK)....33
2.4 Educator Beliefs....35
2.4.1 The Importance of Educator Beliefs and Attitudes....35
2.4.2 Definitions of Beliefs and Attitudes....36
2.4.3 Attitudes....37
2.4.4 Influences on Beliefs and Attitudes....38
2.4.5 Beliefs about Intentional Teaching....41
2.5 Pedagogical Strategies for Supporting ECME....43
2.5.1 Play as a Vehicle for Learning....43
2.5.2 Mathematically Rich Environments....49
2.5.2.1 Block Play....50
2.5.2.2 Socio-dramatic Play....51
2.5.2.3 Games....52
2.5.2.4 Intentional Teaching....52
2.5.2.5 Circle Time (CT)....54
2.5.2.6 Mathematical Talk and Discussion....55
2.6 Educator Noticing: Definitions and Terminology....56
2.6.1 Professional Educator Noticing: Definitions and Theories....56
2.6.2 Noticing as Equitable Practice....58
2.6.3 Common Frameworks for Educator Noticing....59
2.7 Chapter Summary....64
CHAPTER 3: GOVERNMENT POLICY DEVELOPMENT AND ECE/ECME....66
3.1 ECME in Primary School....67
3.1.1 Curriculum Reform....67
3.2 The Development of the Preschool Sector....70
3.2.1 Child Care Act 1991 & Child Care Regulations (1996; 2006; 2016)....70
3.2.2 Ready to Learn (GoI, 1999a)....72
3.2.3 The National Early Childhood Frameworks: Aistear and Síolta....73
3.2.3.1 Síolta: The National Quality Framework for Early Childhood Education....73
3.2.3.2 Aistear: The Early Childhood Curriculum Framework....74
3.2.4 The Early Childhood Care and Education (ECCE) Scheme....81

3.2.5 Department of Education Early Years Focused Inspections 82
3.3 The National Literacy and Numeracy Strategies 85
3.4 Professional Award Criteria and Guidelines for Undergraduate ECEC Degrees 90
3.5 Nurturing Skills 92
3.6 Chapter Summary 93
CHAPTER 4: THEORETICAL UNDERPINNINGS 96
4.1 Connections to Data Collection Methods 97
4.2 Culture 98
4.2.1 Definitions of Culture 98
4.3 Bronfenbrenner's Bioecological Systems Theory 99
4.3.1 Process 100
4.3.2 Person 101
4.3.2.1 Demand Characteristics 101
4.3.2.2 Resource Characteristics 102
4.3.2.3 Force Characteristics 102
4.3.3 Context 103
4.3.4 Time 105
4.4 Socio-Culturalism and Guided Participation 106
4.4.1 Rogoff's Theory of Guided Participation 106
4.4.1.1 Providing Bridges from Known to New 108
4.4.1.2 Structuring Situations and Transferring Responsibility 108
4.4.1.3 Practising and Playing with Routines and Roles 109
4.5 Chapter Summary 111
CHAPTER 5: METHODOLOGY AND RESEARCH DESIGN 113
5.1 Research Design 113
5.1.1 Philosophical Assumptions 113
5.1.2 Ontological Position 114
5.1.3 Epistemological Position 115
5.1.4 Axiology 116
5.2 Research Approach 118
5.2.1 Conceptual Framework 118
5.3 Strategy of Enquiry – Ethnography 121
5.3.1 Ethnography 121

5.3.1.2 Micro-ethnography....122
5.4 Recruitment Procedures....124
5.5 Data Collection Methods....125
5.5.1 Field Notes....125
5.5.2 Participant Observation....127
5.5.3 Semi-structured Interviews....128
5.5.3.1 Preparation of the Interview Guide....129
5.5.4 Casual Conversations....130
5.5.5 Autophotography....131
5.5.6 Participant Captured Video....132
5.5.7 Reflective Journal....133
5.6 Reflexivity – Positionality....135
5.7 Pilot Study....137
5.8 Data Analysis Procedures....138
5.8.1 Researcher Positionality and the Data Analysis Process....145
5.9 Reliability, Validity and Trustworthiness....146
5.9.1 Establishing Trustworthiness....148
5.10 Ethical Considerations....151
5.10.1 Informed Consent....151
5.10.2 Autonomy....151
5.10.3 Non-maleficence....154
5.10.4 Beneficence....155
5.10.5 Justice....155
5.11 Chapter Summary....156
CHAPTER 6: AN INTRODUCTION TO THE SETTING....157
6.1 Setting the Scene: Staff, Children and Routine (A Verbal Portrait)....158
6.1.1 Staff....159
6.1.1.1 The Manager....159
6.1.1.2 The Lead Educator in the ECCE Room....159
6.1.1.3 The Children....159
6.1.2 The ECCE Daily Routine....161
6.1.2.1 Free Play....162
6.1.2.2 Circle Time (CT)....163

6.1.2.3 Whole Group Time.....167
6.1.2.4 Snack Time.....169
6.1.2.5 Outdoor Play.....169
6.2 Conclusion.....170
CHAPTER 7: CHILDREN'S PARTICIPATION IN ECME.....171
7.1 Number Sense.....172
7.1.1 Number in Free Play.....173
7.1.1.1 Priya and the Counting Game.....173
7.1.1.2 Catch and Count.....175
7.1.1.3 Game of Ladybirds.....177
7.1.1.4 Bingo.....179
7.1.1.5 Number Operations in Free Play.....181
7.1.2 Number during Circle Time (CT).....187
7.1.2.1 Number of the Week/ Counting.....187
7.1.3 Number Activities during WGT.....191
7.2 Measures.....**193**
7.2.1 Measures in Free Play.....193
7.2.1.1 'I need to make it higher.'.....194
7.2.1.2 The Python.....194
7.2.1.3 The Hulk House.....196
7.2.1.4 The Builders Shop.....198
7.2.1.5 The Dress Shop.....199
7.2.2 Measures during CT: Time.....200
7.2.2.1 Days of the Week.....200
7.2.2.2 Months of the Year.....202
7.2.2.3 Seasons of the Year.....204
7.2.3 Measures During Whole Group Time (WGT): Weight and Capacity.....204
7.3 Shape and Space.....208
7.3.1 Shape in Free Play.....208
7.3.1.1 3D Shape in FP.....210
7.3.2 Shape in CT.....213
7.4 Space.....215
7.5 Data Handling.....216

7.5.1 The Garden....216
7.6 Pattern....220
7.7 Chapter Summary....224
CHAPTER 8: CHALLENGES TO SUPPORTING ECME....228
8.1 Structural Constraints....229
8.1.1 Non-Contact time....229
8.1.2 Non-Teaching Related Duties....231
8.1.3 Fluctuating Break Times....232
8.1.4 Missed Opportunities....235
8.1.5 Temporal Impacts on CT....236
8.1.6 Managing Several Children at Once....238
8.2 Siobhán's Perspective on the Challenges in Supporting ECME in the Setting....240
8.2.1 Staffing Issues....240
8.2.2 The Education Gap and Staff Qualifications....240
8.2.3 Staff Confidence with Mathematics....242
8.2.4 Time and Space....243
8.3 Sinéad's Perspectives on Practice....244
8.3.1 Transition to Primary School....244
8.3.2 Sinéad's Perceived Role During Free Play....248
8.3.3 Children's Non-attendance....249
8.4 Chapter Summary....251
CHAPTER 9: DISCUSSION AND CONCLUSION....253
9.1 Overview of the Study....253
9.2 Addressing the Research Questions....254
9.2.1 RQ 1: The Types of Mathematics the Children Participated In....254
9.2.1.1 Number....255
9.2.1.2 Measures....257
9.2.1.3 Shape and Space....260
9.2.1.4 Data Handling....262
9.2.1.5 Pattern....263
9.2.2 RQ 2: Strategies Employed to Support Children's Participation in ECME....265
9.2.2.1 The Influence of Setting Policy....265
9.2.2.2 Professional Development Support....267

9.2.3 Adult Planned Strategies for Supporting Children's Participation in ECME 268
9.2.3.1 Provision of a Mathematically Rich Environment.....268
9.2.3.2 Worksheets.....271
9.2.3.3 Structured Guided Participation.....272
9.2.3.4. Mathematical Talk.....273
9.2.3.5. Noticing as Equitable Practice.....275
9.2.3.6. Intentional Teaching.....276
9.3 RQ3: Challenges Identified in Supporting Children's Participation in ECME 277
9.3.1 Transition to School.....282
9.4 Connections to the Policy Context.....283
9.5 Limitations of the Study.....285
9.6 Reflections on the Research Process.....287
9.7 Contribution to Knowledge.....289
9.8 Recommendations.....291
9.8.1 Recommendations for Policy.....292
9.8.2 Recommendations for Practice.....294
9.8.3 Recommendations for Future Research.....296
9.9 Conclusion.....297
REFERENCES.....301
APPENDICES.....345
Appendix A My Transition Flower.....345
Appendix B Personal Reflection.....346
Appendix C Interview Schedule – Manager.....348
Appendix D Reflective Journal Entry 'The Coat'.....350
Appendix E Data Collection Record.....351
Appendix F Reflective Journal Entry – Adult Support.....352
Appendix G Ethical Approval.....353
Appendix H Sample Codes.....354
Appendix I Example of Coded Data (RC1).....355
Appendix J Plain Language Statement (Parents/Guardians).....356
Appendix K Informed Consent Form (Parents/Guardians).....358
Appendix L Plain Language Statement (Manager / Educator).....359
Appendix M Informed Consent Form (Manager).....362

Appendix N Informed Consent Form (Educators).....364
Appendix O Plain Language Statement (Children) 366
Appendix P Pictorial Assent Form (Children) 372

LIST OF ABBREVIATIONS

BSRLM	British Society for Research into Learning Mathematics
CECDE	Centre for Early Childhood Development and Education
CK	Content Knowledge
CoRE	Competence Requirements in Early Childhood Education and Care
CPOP	Curriculum Planning and Observations Policy
CT	Circle Time
DCYA	Department of Children and Youth Affairs
DE	Department of Education
DECDIY	Department of Children, Disability, Integration and Youth
DES	Department of Education and Skills
DEY	Department of Education and Youth
DLP	Designated Liaison Person
DoE	Department of Education
EAL	English as an Additional Language
EC	Early Childhood
ECE	Early Childhood Educator
ECCE	Early Childhood Care and Education
ECEC	Early Childhood Education and Care
ECERs	Early Childhood Environment Rating Scale
ECME	Early Childhood Mathematics Education
ELC	Early Learning and Care
EMA	Early Mathematical Activities
EYEPU	Early Years Education Policy Unit
EYFS	Early Years Foundation Stage
EYMTLP	Early Years Mathematics Teaching and Learning Policy
FN	Field Note
GoI	Government of Ireland
GP	Guided Participation
HEI	Higher Education Institution
HLE	Home Learning Environment

HSE	Health Service Executive
INT	Interview
INTO	Irish National Teachers Association
ITE	Initial Teacher Education
LE	Lead Educator
MCK	Mathematical Content Knowledge
MKT	Mathematical Knowledge for Teaching
MT	Mathematical Talk
NCCA	National Council for Curriculum and Assessment
NFQ	National Qualifications Framework
NLNDLS	National Literacy, Numeracy and Digital Literacy Strategy
NLNS	National Literacy and Numeracy Strategy
OMEP	Organisation Mondiale pour L'Education Pre-Scolaire
PACEY	Professional Association for Childcare and Early Years
PCK	Pedagogical Content Knowledge
PD	Professional Development
PMC	Primary Mathematics Curriculum
PSC	Primary School Curriculum
PSCF	Primary School Curriculum Framework
QQI`	Quality and Qualifications Ireland
RC	Reflective Conversation
RJ	Reflective Journal
STEAM	Science Technology Engineering, the Arts and Mathematics
STEM	Science Technology Engineering and Mathematics
UN	United Nations
UNCRC	United Nations Convention on the Rights of the Child
UNICEF	United Nations International Children's Emergency Fund

LIST OF TABLES

Table 2.1 Bishop's Six Universal Mathematical Activities……..... 27

Table 2.2 Alignment of Research-based Mathematical Ideas and Irish Curricular Documents……. 28

Table 3.1 Mathematical Concepts and Skills in the Theme of Communicating……... 75

Table 3.2 Mathematical Concepts and Skills in the Theme of Exploring and Thinking…….. 76

Table 3.3 Proposed Aims and Learning Goals for Exploring and Thinking (NCAA, 2003).... 78

Table 3.4 References to Mathematics in the DE Guide to Early Years Inspection…….. 83

Table 4.1 Five Strategies to Support Children in the Guided Participation Process……..….. 110

Table 4.2 Commonalities Across Selected Theoretical Frameworks……...……. 112

Table 5.1 Potential Positive and Negative Participant Perceptions of the Researcher…...…... 135

Table 5.2 Examples of Data Sources Supporting Triangulation……. 141

Table 5.3 How Potential Threats to Reliability and Validity were Addressed…….…..…….. 147

Table 5.4 Actions Taken to Establish Trustworthiness……...…….. 150

Table 6.1 An Overview of the Children Attending the ECCE Class……..…….. 160

Table 6.2 Overview of the Daily ECCE Routine……..…… 161

Table 6.3 Whole Group Time Activities……..……. 168

Table 8.1 Sinéad's Activity During Free Play……..…… 230

LIST OF FIGURES

Figure 2.1 Pedagogical Content Knowledge as the Intersection of Subject Matter, Understanding, Teaching Techniques and Knowledge of Student Development…. 34

Figure 2.2 Continuum of Play-based Learning……... 47

Figure 3.1 Key Competencies as Outlined in the Primary Mathematics Curriculum……. 70

Figure 4.1 Bronfenbrenner's Ecological Systems……... 104

Figure 5.1 Conceptual Framework for Supporting ECME……..... 120

Figure 5.2 Excerpt from the Field Notes Record…… 127

Figure 5.3 Kolb's Experiential Learning Cycle…….. 134

Figure 5.4 Overview of Data Analysis Process…….…... 139

Figure 6.1 Overview of Circle Time Routine…….……. 166

Figure 6.2 Sinéad's Circle Time Board……... 167

Figure 7.1 Example of a Counting Game Card……... 175

Figure 7.2 Catch and Count……..... 177

Figure 7.3 The Game of Ladybirds……..... 179

Figure 7.4 Counting Stick Set……..... 182

Figure 7.5 Numberblocks…… 186

Figure 7.6 Example of a Large Bucket Balance……..… 206

Figure 7.7 Circles and Connectors…….. 210

Figure 7.8 Loose Parts Shelf…….…….. 218

Figure 7.9 Dobble……..... 219

Figure 7.10 Snake Pattern Cards……..... 221

Figure 8.1 Magnetic Building Blocks used by Filip……..... 233

ABSTRACT

Guiding Young Children's Participation in Early Childhood Mathematics Education: An Ethnographic Study of an Irish Preschool Classroom

Córa Gillic

In recent years, policy in Ireland has emphasised the importance of early childhood education, and there has been an increasing focus on the inclusion of mathematics (numeracy) in early childhood settings. This research explored how one Irish preschool setting guided and supported children's engagement and interest in mathematics during their final year of preschool (ages 4-5 years). This detailed study of practice contributes to the current small body of knowledge of preschool mathematics education in Ireland. Adopting a micro-ethnographic approach, qualitative data were generated using a range of data collection methods (participant observation, semi-structured interviews, reflective conversations and autophotography).

Findings indicate a disconnect between pedagogical strategies to support early mathematics highlighted in setting policy documents, such as Math Talk, and those enacted in the preschool classroom. The analysis shows that educator beliefs around non-intervention in children's free play influenced practice. This resulted in inequitable opportunities to explore mathematics, for example, girls' limited opportunities to engage in block play. Beliefs about children and mathematics deemed part of primary school curricula also limited support given at times, for fear of overpreparing children for primary school.

Analysis reveals that the incorporation of mathematics into adult-led activities, such as circle time, was not cognitively challenging for some children at the time of the study. Findings also highlight the impact of children's non-attendance and children with English as an Additional Language, requiring further support.

The study recommends that mandatory initial professional training and professional development focused on pedagogical strategies, such as Math Talk, strategies to support EAL children, and the necessity to engage with a wider variety of mathematical concepts other than number and measures.

CHAPTER 1: INTRODUCTION

"*To develop the whole child, we must develop the mathematical child"* (Clements & Sarama, 2014a, p. 2)

Ireland sits within exciting times in terms of early mathematics education. Over the last decade, the Early Childhood (EC) sector has witnessed an increasing emphasis on Early Childhood Mathematics Education (ECME) across government educational policy. The National Literacy and Numeracy Strategy (NLNS)(DES, 2011) has been reviewed (DES, 2017a) and updated (GoI, 2024a). The accompanying Strategy Implementation Plan (GoI, 2024b) sets targets to support Early Childhood Education and Care (ECEC) settings in providing the youngest citizens with high-quality early mathematical learning experiences. This policy emphasis on ECME offers the context for this study. This study is located within the EC sector. It seeks to explore how ECME is supported at the practice level. The study examines the interplay between different levels and how policy influences practice. This practice element is explored through an ethnographic approach and observations in a preschool classroom offering the Early Childhood Care and Education (ECCE) scheme funded by the Department of Children, Equality, Disability, Integration and Youth (DCEDIY) and which is subject to inspection by the Department of Education and Youth (DEY) and Tusla (The Child and Family Agency).

Providing high-quality preschool practice has become a central tenet of successive Government agendas. The last two decades have witnessed an increased focus on preschool education in Ireland, and, over the last decade, an emphasis on young children's engagement in early mathematics education. Coupled with a parallel agenda to increase engagement with and quality of Science, Technology, Engineering and Mathematics (STEM) education (e.g. DES, 2017b; DES, 2020), Irish early childhood educators (ECEs) find themselves as participants in enacting these new policy developments. This national emphasis reflects an

international trend towards developing a future STEM-literate workforce beginning in children's earliest years through high-quality early childhood education (DES, 2020; Gough, 2015). This chapter begins by outlining current Irish educational policy and ECME. A discussion on the importance of early childhood education and play pedagogy follows. An overview of the current early childhood structure is then provided, along with a discussion on the importance of the nature of ECME in EC. Gaps in current research are then highlighted. My personal interest and purpose of the study are stated. The chapter concludes with an overview of the thesis.

1.1 Irish Educational Policy Context

Following in the footsteps of their international counterparts (Marginson, et al., 2013) and driven by the desire to improve learner outcomes for mathematics (DES, 2011), the Irish Government has increased its focus on mathematics education as part of the broader STEM Education Policy agenda (DES, 2017a; DES, 2020), as well as focusing on mathematics education in its own right, as part of *Ireland's Literacy, Numeracy and Digital Literacy Strategy 2024-2033: Every Learner from Birth to Young Adulthood* (GoI, 2024a) (NLNDLS), and the new Primary School Mathematics Curriculum (DE, 2023a). The NLNDLS identifies early childhood as a key period for development and, consequently, pre-primary and primary settings have been identified as important places for facilitating and developing young children's engagement with mathematics (GoI, 2024a). The NLNDLS further notes that ECEs play a 'pivotal role' in young children's engagement with early mathematics education (p.3). The NLNDLS (GoI, 2024a) also observes that high-quality interactions between children and educators are central to children's mathematical learning experiences. These sentiments are echoed in the *STEM Education: Reporting on Practice in Early Learning and Care, Primary and Post-Primary Contexts* (DES, 2020) report, where it is noted that a highly skilled ECE can enhance young children's learning experiences by asking questions,

engaging children in discussion as well as facilitating children's participation in processes such as comparing, predicting and discovering in a high-quality, enabling environment.

A review of the first iteration of the National Literacy and Numeracy strategy (DES, 2011) in 2017, stated that a more intense focus on mathematics was called for, with an identified need to support early childhood educators in gaining "a deeper understanding of numeracy concepts, the sequence in which children learn early mathematical ideas and identifying and providing materials and activities which further promote learning in this area" (DES, 2017b, p.31). The current strategy reiterates this point and states that 'a renewed focus on numeracy is required' (GoI, 2024a, p.18). The report on STEM Education in Ireland (DES, 2020), an Inspectorate review of STEM practices in educational settings, including EC, supports the need for ECEs to possess a deeper knowledge of ECME. It raised concerns that in one in every five sessions observed (n=29), children's engagement with STEM learning experiences was deemed 'less than satisfactory' (p.13). While this report relates to STEM education rather than mathematics as a stand-alone area of learning, it could be inferred that, in general, early childhood teachers do need support in facilitating early mathematics in practice, as stated by Dunphy (2017). An overview of policy development regarding EC and ECME will be provided in Chapter Three.

1.2 The Importance of Early Childhood Education

Internationally, recognition of the importance of early childhood education is growing (UNICEF, 2019). Early childhood education is critical for development (MacDonald & Carmichael, 2018), to such an extent that early childhood (pre-primary) education has been identified in the United Nations Sustainable Development Goals (UN, 2015), Goal 4 -Quality Education. Target 4.2 aims to ensure that all children have equal access to quality early

development, care and pre-primary education (UNICEF). Mathematics education (numeracy) is addressed in target 4.6, ensuring that all youths and adults are literate and numerate.

The period from birth to five years is acknowledged as one of rapid learning and development. Young children's participation in ECE has proven benefits, including better holistic development during the preschool years, and better outcomes during primary school and beyond (Schweinhart et al., 2005; Temple & Reynolds, 2007; UNICEF, 2019). Irish policy also recognises the importance of this early developmental phase (e.g., *Ready to Learn*, GoI, 1999a; *First 5,* GoI, 2018; GoI, 2024a). This recognition is clear in Irish national early childhood frameworks, *Síolta, the National Quality Framework for Early Childhood Education* (CECDE, 2006) and *Aistear, the Early Childhood Curriculum Framework* (GoI, 2024c). Both frameworks were developed for those working with children across the preschool and primary sectors, specifically for children aged 0-6 years. Shonkoff and Phillips (2000) highlight the importance and role of environments in children's holistic development during this time, noting that what occurs in these environments during this period can positively or negatively affect future development and outcomes.

1.3 Play Pedagogy in ECE

Some consider play to be the defining characteristic of ECE. Both Irish early childhood education frameworks, *Aistear* (GoI, 2024c) and *Síolta* (CECDE, 2006), highlight the importance of employing a play pedagogy in ECE. Children have a recognised right to play (UNCRC, 1989). Play pedagogy is recognised as a key vehicle for young children's learning (Ginsburg, 2006; Pramling Samuelson & Johansson, 2006; Wood, 2014) and consequently is afforded importance internationally across many EC curricular frameworks (e.g. New Zealand's *Te Whāriki*, Australia's *Early Years Learning Framework*, Norway's *Framework Plan for Kindergartens)*. Early childhood theorists such as Froebel, Piaget, Vygotsky, and Bruce advocated for play pedagogy in early education. It is through play that

children learn about their world as they observe, handle, and manipulate objects. Play invites creativity and imagination. A child's social and emotional development develops as they play with others, developing negotiating and collaborative skills. Play offers opportunities to explore, experiment, and investigate (Bruce, 2001; Andrews, 2012; Moyles, 1990). Through play, children build on prior knowledge and experiences. It supports the development of resilience and emotional regulation. In short, play positively contributes to a child's holistic development.

Play pedagogy is fiercely guarded by those working in early childhood education (O'Neill et al., 2022), yet, despite being recognised as a core aspect of children's learning, no agreed definition of play exists (Andrews, 2012). *Aistear* (NCCA, 2009) defined play as 'a way of doing things' (p.53). The nature of play and learning is often contested in early childhood. Some ECEs favour children's engagement in free play, with little adult interaction, while others advocate for a guided play approach, where adults support the play (Weisberg et al., 2013). This debate will be discussed further in Chapter Two (Literature Review).

1.4 Early Childhood Mathematics Education (ECME)

Early Childhood Mathematics Education (ECME) is a traditional component of early childhood education. Early childhood pioneers such as Comenius, Pestalozzi, Froebel and Montessori included aspects of mathematics education in their early education programmes (Saracho & Spodek, 2008). Despite the importance these ECE pioneers placed on ECME, its inclusion in early childhood education has often been questioned and even criticised over time. In 1922, the American psychologist Edward Thorndike questioned the usefulness of engaging young children in arithmetic before Grade 2 (Sarama & Clements, 2009) and suggested replacing the first of the Froebel gifts (six yarn balls) with a toothbrush to emphasise the need for good dental health (Brosterman, 1997). The early years of the 20th

century saw direct instruction in mathematics falling out of favour, to be replaced by a movement that encouraged children to experience mathematics through play, everyday activities, and project work, delaying formal instruction until primary school (Saracho & Spodek, 2008).

The inclusion of mathematics in early childhood education is frequently at odds with the traditional beliefs of ECEs that social, emotional and language development are more important than mathematics, which is often considered too 'academic' for preschool education (Lee & Ginsburg, 2009). Evidence suggests that this traditional view is changing (Benz, 2012; Thiel, 2010). Fuelling this change of perspective is research showing that early mathematical ability is a strong predictor of later academic success (Duncan et al., 2007; Watts et al, 2014) and that young children are competent and capable mathematicians, capable of understanding mathematical concepts (Reikerås et al., 2012) and who enjoy engaging with mathematical ideas (Clements & Sarama, 2021).

Mathematics learning begins at birth (Geary, 1994), with infants able to differentiate between sets with one object and two objects (Clements & Sarama, 2014b). Reikerås et al's (2012) study of toddlers (n = 1003) showed that participants showed some understanding in the mathematical areas of number and counting, geometry and the problem-solving process, showing that very young children can engage with a broad range of complex mathematical ideas. Studies have shown that young children enter preschool with some knowledge of various mathematical concepts (Trawick-Smith et al., 2016) and informal mathematical knowledge gained from home and other non-preschool settings (Ginsburg et al., 2008).

The early childhood research community has recently witnessed a renewed interest in ECME (Elia et al.,2018). Elia et al. (2018) define ECME as involving 'creating learning environments and providing activities by professionals (practitioners and educators) to give young children opportunities and experiences for their mathematical development in child-

care centres, pre-kindergarten, kindergarten and primary schools' (p. 2). This aligns with Irish policy; however, the extent to which this happens in Irish ECE classrooms is unclear.

1.4.1 The Importance of ECME

Mathematics can be described as a 'cultural tool of the world' (Vygotsky, 1978). Being numerate is an important life skill. It enables children to participate in their everyday worlds (DES, 2011). Indeed, research shows that children's early engagement with a broad range of mathematical concepts is beneficial not only to their mathematical development (Aubrey et al., 2006; Duncan et al., 2007; MacDonald & Carmichael, 2018; Sarama & Clements, 2009) but also to their literacy development (Duncan et al., 2007). Elgavi and Hamo (2024) assert that early engagement with mathematical concepts supports the activation of different elements of the brain and the development of neural connections in the brain in the first years of life. Such connections are critical for supporting emergent mathematical understanding.

Sarama and Clements (2009) observe that 'mathematics knowledge on school entry is a stronger predictor than any host of social-emotional skills' in terms of later mathematical achievement (p.6). Research conducted by Watts et al. (2014) indicated that children with strong mathematical abilities in preschool continued to do well in mathematics into adolescence, even after considering the child's characteristics (personal and family background). Interestingly, this study concluded that early mathematical development in the early years of primary school was also associated with achievement in secondary school, thereby showing that consistency across early years setting types was a positive predictor of continued mathematics achievement into the teenage years.

1.4.2 The Nature of ECME in Preschool Settings

The informal nature of ECME in preschool differs from the more formal approach commonly found in primary classrooms (Gasteiger & Benz, 2018). The vision of ECME proposed by Elia et al. (2018) suggests that children should experience ECME through everyday situations (routines, setting the table, dividing and allocating playdough, measuring drinks bottles at snack time) and through play. The learning is situated and rooted in a particular activity as it routinely or spontaneously occurs. It is expected that ECEs notice and respond to children's engagement with and manipulation of materials. They comment on the activity, ask questions, engage in mathematical talk and discussion and model mathematical language as they interact with the child and their activity (Wager, 2014). They mathematise everyday situations for children. The vision of ECME that is often advanced is premised on the idea that learning mathematics in preschool is spontaneous, often unplanned, and naturally occurring throughout the preschool day. The spontaneous nature of ECME relies on the ability of an ECE to identify the mathematics in children's play activity, respond to it, and enhance the learning as it occurs (Dockett & Goff, 2013). For such rich mathematical interactions to occur, ECEs require a high level of mathematical content knowledge (MCK) (Gasteiger & Benz, 2018) and pedagogical content knowledge (PCK) (Lee, 2017). Such knowledge is a requisite component of an ECE's professional knowledge bank (Dockett & Goff, 2013). In preschool, children's engagement with mathematical ideas often occurs during free play. It takes a highly skilled and mathematically knowledgeable ECE to provide a mathematically rich environment, to seed the play-learning space with opportunities that afford the exploration of mathematical ideas and concepts (Gasteiger, 2015). ECEs need to understand the mathematics learning afforded by the materials in the classroom. The required knowledge base of an ECE to engage children in a high-quality ECME is complex and multifaceted.

1.5 An Overview of the Current Irish ECCE Structure and ECME

Chapter Three will provide a more detailed account of the current ECEC structure in Ireland. A brief overview will be provided here to provide a context for this study. A key aspect of this study explored how ECME is supported at the practice level. This component was explored ethnographically in a preschool classroom catering to children before transitioning to primary school. Therefore, educational structures for the 3-5 age group will be discussed here. The Early Childhood Care and Education (ECCE) scheme enables children aged between two years and ten months and five years to attend a free three-hour preschool session five days a week. This universal preschool education, introduced in 2010, is free for all children for two years before transitioning to primary school (Walsh et al., 2024). The most recent EC sector profile report (DCEDIY, 2022) shows that 60038 children between 3 and 6 years old attend a preschool service in Ireland. This figure reflects the number of children potentially engaging in ECME in early childhood settings in Ireland annually.

Under the ECCE scheme, Lead Educators (LEs) must hold a QQI level 6[1] qualification in Early Childhood Education. This is a Further Education qualification. A financial incentive is available to settings to employ graduate LEs. Graduate LEs hold a QQI level 7 (ordinary) or level 8 (honours) undergraduate degree. Content for undergraduate ECEC programmes is outlined in the *Professional Award Criteria and Guidelines for Initial Professional Education (Level 7 and Level 8) Degree Programmes for the Early Learning and Care (ELC) Sector in Ireland* (DES, 2019). The guidelines allude to a preservice ECE's need for 'knowledge of children's different strategies of learning (play-based, social learning, early literacy and numeracy, language acquisition and multilingualism)' (p.16) and be capable of, 'generating an appropriate curriculum that stimulates and promotes positive

[1] QQI Quality and Qualifications Ireland. The awarding body for further education and training, and the majority of higher education institutes.

learning dispositions, emergent literacy, maths and science skills' (p.17). However, these *Knowledge and Practice* statements are vague and do not detail the mathematical or pedagogical content knowledge required to provide a high-quality early childhood mathematics education. Coupled with the statement that 'these criteria and guidelines do not seek to create absolute homogeneity across higher education professional awards in ECEC' (p.15), there is no guarantee that ECEC undergraduates receive the required knowledge base they need to engage children in the broad range of mathematical concepts recommended by researchers (e.g., Clements & Sarama, 2021). This contrasts with the primary school sector, where Initial Teacher Education (ITE) providers undergo accreditation and inspection by the Teaching Council, and mathematics education courses are mandatory. A recent study of ECME content in Irish undergraduate degree courses (Gillic et al., under review, 2026) showed that ECME is generally embedded within consolidated modules, usually within an early STEM education or a Literacy and Numeracy module. Early mathematics education is rarely afforded a module of its own.

Another noteworthy issue is that, at present, over 500 qualifications (both national and international) are deemed suitable for staff working in EC settings (DCEDIY, 2024), resulting in diversity among the ECE population and in languages spoken in ECE settings. The number of foreign national children under the age of 5 in 2022 was 17'799, again contributing to the language diversity within ECE settings (DCDE, 2025). This contrasts with the robust systems in place for educators in the primary and secondary sectors, where the Teaching Council assesses teaching qualifications, and where primary teachers can receive support in teaching children with EAL from the professional development body, Oide.

1.6 Gaps in ECME Research

Despite the apparent advantages of early engagement with mathematics, little is known about preschool mathematics in early childhood classrooms. Internationally, little empirical evidence exists showing how children's early mathematics education is supported in preschool settings (Jegodtka et al., 2022; Wickstrom et al., 2019). Only a few studies (e.g. Trawick-Smith et al., 2016; Lundquist et al., 2023; Wickstrom, 2023) have explored what actually happens in preschool classrooms. In the British Isles, Marks et al. (2019) note that fewer than 1% of articles in the Journal of British Society for Research into Learning Mathematics (BSRLM) reflected research in the English Early Years Foundation Stage (3–5-year-olds). Marks et al. (2019) consider the Early Years sector to be a 'neglected group' that 'represents a sizable proportion of learners and an important aspect of the overall picture in understanding learning and teaching in mathematics education' (p. 42). In the Irish context, only a handful of published studies have explored ECME in preschool settings. Neylon (2014) has explored early mathematics education in 26 Irish preschool settings using the ECERs/E (Sylva, Siraj-Blatchford, and Taggart, 2006) rating scale. Findings from this study indicated that services under Montessori and High/Scope philosophies provided broader mathematical learning experiences than traditional play-based settings. Neylon (2014) concluded that generally, early mathematics education was of poor quality in play-based settings, with settings using the Montessori and High/Scope curricula scoring higher. It must be noted, however, that Neylon's (2014) study was carried out at a time when Síolta (CECDE, 2006), Aistear (NCCA, 2009) and the first iteration of the national literacy and numeracy strategy had only been introduced. It should also be noted that EC staff qualifications were coming on stream or were still in development when the study was conducted. Much has changed in the Irish ECEC landscape over the last decade. Findings from O'Dwyer's (2021) self-study action research on ECME in their setting indicate that the

following are necessary for ECME: a mathematically knowledgeable curriculum/pedagogical leader and ECEs, a well-resourced mathematically enabling environment, and the need to build relationships with parents around ECME. O'Dwyer (2021) recommended that ECE settings produce a mathematics curriculum policy to ensure all staff engage children in ECME. The study presented in this thesis contributes to the Irish and international body of research relating to ECME as it explores how ECME is supported in practice.

1.7 Personal Interest

"*Whenever I have tried to carry out a piece of theoretical work, it has been at the basis of my own experience, always in relation to the processes I saw taking place around me*"

(Foucault cited in Mills, 2003, pp.1-2)

I take the view that young children are capable mathematicians. They are keen observers of the world around them and strive to make sense of that world. I see mathematics as an important cultural tool, embedded in the way we live. Adults enable children to 'join the club' as we interact with them, engaging in interactions such as extended co-constructive dialogue, sustained shared thinking, and facilitating mathematically rich enabling environments, in which young children are free to explore and learn. This view stems from nearly 20 years of experience working with young children from 3-7 years as a Montessori (preschool) and primary teacher (early years), in a variety of educational contexts and through engagement with degree coursework.

My undergraduate research explored the self-reported beliefs about and practices of Irish preschool educators towards mathematics in preschool practice. This interest in preschool mathematics provision continued to Master's level, where I explored my practice concerning the playful teaching and learning of shape and space. Findings from the former

study (Gillic, 2021a) indicated that while preschool educators viewed mathematics as being important and that it had a place in preschool education generally, there was a fear of 'not doing too much' as children should not go to primary school with too much mathematical knowledge, as they will be learning about these concepts in Junior Infants (the first year of primary school). Participants also reported a narrow mathematics focus, concentrating on numeral recognition, counting and basic shape identification.

Findings from my Master's research indicated the usefulness of a *Progression Continua* approach, as evident in the Primary Mathematics Curriculum (PMC) (DE, 2023a). Progression Continua outline sequentially key points in mathematical development in various mathematical topics, and can help educators identify aspects of mathematical activity in young children's free play (Gillic, 2020). Like Foucault, my interest in this topic stems from my experience in the classroom and observations of what was happening around me. While in practice, I felt pressured not to engage children in mathematics, which was deemed part of the Junior Infant curriculum. This pressure came from my perception of the inspectorate and social media fora and filtered down to me through my manager. There has, for me, always been tension as a result of this. My reading and research show young children to be interested in mathematics and who are competent mathematicians. As a Montessori-trained educator, I subscribe to the mantra, 'follow the child'. If a child is interested in something, my instinct is to enable the child to follow this interest until they are satisfied with the knowledge gained. My experiences in the ECCE classroom have led me to explore if other EC educators experience the same tensions between policy, beliefs and practice, and how ECME is experienced by the children in their ECCE classrooms.

1.8 Purpose of the Study

As alluded to above, little empirical Irish research has been conducted exploring how mathematics is supported in Irish preschool classrooms. Given the recent policy focus on

early mathematics education, exploring how young children's participation in mathematics education (3-5 years) is supported in an Irish preschool setting seems pertinent. This study positions young children as having a right to high-quality early mathematics education (Munn, 2005; Cohrssen & Page, 2016) and given the lack of research highlighted above, this ethnographic study of one preschool classroom explores *how young children's participation in ECME is supported in an Irish preschool setting.*

This question can be subdivided into the following sub-questions:

1. What types of mathematics do children participate in?
2. In what ways does the adult guide or support this participation?
3. What are the challenges in supporting children's participation in ECME in this setting?

Research Questions are answered through ethnographic methods of participant observation, reflective conversations (RCs) and semi-structured interviews. The ethnographic methods recorded first-hand the strategies used by an ECE in an ECCE-funded preschool classroom to support ECME. This ethnographic, non-intervention study, the first of its kind in ECME in Ireland, is unique in that children's participation in ECME was observed over the course of the ECCE day, and not just at certain points of the day, for example, free play time or circle time. It offers insights into the beliefs and practices of a setting manager and an individual educator with regard to supporting young children's participation in ECME. It highlights the impact of ECCE structures, routines and administrative procedures on ECME, as well as considering how the children were supported in participating in the different areas or 'Big Ideas' of ECME. The study highlights the need to intentionally support children's engagement with mathematics during free play, as well as to intentionally teach early mathematics to children with English as an additional language (EAL). Implications arising

from the findings of this study include the call for professional development (PD) in ECME for both in-service and pre-service ECEs, addressing the broad range of 'big ideas' and pedagogical strategies to facilitate young children's, particularly girls and those with EAL, participation in ECME. Implications also include the need for further coherence across preschool and primary school curricular frameworks, emphasising the need for cross-sectoral conversations regarding limiting children's engagement with certain mathematical concepts in preschool.

1.9 Overview of the Thesis

This chapter has introduced the reader to the context in which this study is situated. In doing so, it provided a rationale for the study. It stated the research aim and associated questions. The chapter that follows presents the literature review, which highlights key areas central to the study and acts as a rationale for the research questions. Chapter Three will outline the development of ECME in Irish EC policy. Chapter Four presents the theoretical underpinnings that support the analysis of findings. Chapter Five will present details of the ethnographic design, the reason for its selection and associated data collection methods. Chapter Six serves to set the scene, detailing aspects of the setting, participants and daily routine. An analysis of selected examples of children's mathematical activity gathered from my fieldnotes and self-captured video taken by the lead educator in the ECCE room will be detailed in Chapter Seven. Chapter Eight offers insights into the challenges in supporting ECME in this setting. Chapter Nine presents a discussion based on findings from Chapters Five, Seven and Eight, and provides a conclusion of the study along with detailing the implications for future research, policy and practice.

CHAPTER 2: LITERATURE REVIEW

This chapter describes and synthesises relevant empirical literature on ECME to identify key concepts and debates that inform this study. As noted in Chapter One, this study explored how young children are supported to participate in ECME in an Irish preschool setting. This chapter begins by providing an overview of early conceptualisations of the young child as a learner as well as early perceptions of the role of the adult in supporting children's learning. This is followed by section 2.2, which explores my view of the young child as a mathematician and the literature which underpins this. A review of the literature in relation to professional knowledge for ECME, the role of educator beliefs (RQ3), and suggested pedagogical strategies for children's participation in ECME (RQ2), then follows. The chapter concludes by considering the literature surrounding the concept of teacher noticing and its applicability to the field of ECME (RQ2).

2.1 Early Visions of the Child

Early childhood pioneers, Comenius (1592-1960), Pestalozzi (1746-1827), Froebel (1782- 1852), and Montessori (1870-1952) held similar views of the young child, positioning them as active, first-hand, sensorial learners who learn by interacting with and exploring objects in their worlds. Furthermore, they shared a similar vision of pedagogy for early childhood, where play and exploration were central to learning and development (Bruce, 2021; Silber, 1965). For Froebel, the child was competent, agentic and capable of making choices about their learning (Bruce, 2021). This view was shared by Montessori, who viewed the young child as an individual, an observer who experienced sensitive periods to learning. Additionally, she viewed young children as competent mathematicians who were interested in mathematics.

2.1.1 Jan Comenius

Considered a founding father of infant education, Jan Comenius, in his book *School of Infancy,* explored child development and education for children aged from birth to six years (Saracho and Spodek, 2008). In two of his works, *The Great Didactic* and *Pampaedia*, Comenius suggested that Kindergarten programmes should be integrated in nature and comprise the following subjects: language, number, art, music, nature study, moral and religious education (Murphy, 1995). The sole focus on Number in relation to mathematics for Kindergarten children is notable. Comenius' mathematics curriculum for primary education is notably broader, suggesting that children engage in counting with manipulatives and measuring spaces with proficiency (Murphy, 1995). In *Pampaedia*, Comenius further articulated the significance of mathematics in the primary years, portraying it as a foundational discipline that should be accessible to all children. He argued that competencies such as counting, weighing, measuring, and understanding geometric relationships are essential for making sense of, and participating effectively in, social life (Murphy, 1995). This perspective is consistent with contemporary views of mathematics as a cultural tool that enables meaningful participation in culturally situated activities (Nunes & Bryant, 2021; Rogoff, 1990; Vygotsky, 1978).

2.1.2 Johann Pestalozzi

Johann Pestalozzi advocated for young children's engagement with mathematical concepts involving number (counting and fractions) and form (shape) through the use of real-life, everyday objects, e.g. pebbles, fingers, dots, pies for fractions, real-life contexts, and manipulatives/visuals. For the teaching of geometry, referred to by Pestalozzi as '*the alphabet of form'* (Silber, 1965, p.142), children were introduced to a restricted number of basic two-dimensional shapes, including the straight line and the square. Pestalozzi's second stage in learning about form involved cultivating an understanding of the spatial relationships

among shapes through systematic measurement, thereby strengthening the child's capacity for mathematical precision (Silber, 1965). Pestalozzi maintained that this supported the development of children's writing skills, as letters are constructed by combining plane (two-dimensional) shapes.

2.1.3 Friedrich Froebel

Influenced by his strong religious beliefs and his wonder about the beauty of nature, Froebel designed a set of *Gifts for Play* (von Marenholtz-Bülow, 1877/2007), intending to help young children appreciate and understand the beauty of form in nature (Saracho & Spodek, 2008). As noted by Bruce (2021), these gifts comprise 'a core set of spheres, cubes, cylinders and prisms' (p.93) to be explored in an open-ended way. In contrast to Pestalozzi's approach to introducing two-dimensional shapes to children, Froebel began with the three-dimensional cube as the first of his instructional gifts (Silber, 1965). It is not just the materials themselves that lead to the exploration of mathematical ideas; the sequenced process of replacing the materials in their box also facilitates the development of spatial awareness and building a sense of part-whole relationships (Bruce, 2021). The gifts offer children an opportunity to explore 'sorting, positioning, spatial awareness, size, shape and dimension' as well as to engage in processes such as creativity and problem solving (Rose, 2022, p.45). Saracho and Spodek (2008) suggest that although the intention behind a child's exploration of the gifts was not necessarily to learn mathematics, children would have learned the mathematical ideas of shape and form inadvertently as they manipulated the deeply mathematical materials in a structured, sequenced, yet sensorial way (Balfanz, 1999) 'almost by osmosis' (Bruce, 2021, p.96). The concept of children sensorially learning through interacting with objects/materials remains a central tenet of EC practice today, where educators perceive themselves as facilitators of rich learning environments, enabling

children's exploration of objects, supporting their learning and understanding of the attributes of the objects under consideration in a sensorial way.

2.1.4 Maria Montessori

The Montessori curriculum for three-to-six-year-old children comprises five curriculum areas: Practical Life, Sensorial Exercises, Language, Mathematics, and Culture Activities (including science, history, and geography) (Montessori, 1964). Although a separate mathematics curriculum exists, mathematics is integrated into the broader curriculum. For example, in practical life, as children transfer objects from one bowl to another, they informally engage with capacity, and the ideas of measure (more/less/full, empty). The sensorial exercises offer opportunities to engage with shape (form), comparisons, similarity, difference, matching, ordering, and geometry. Similarly to Froebel, children are introduced to three-dimensional solids before two-dimensional plane shapes. Like Pestalozzi before her, Montessori believed that sensorial exploration of two-dimensional shapes prepared children for writing. The skill of classifying is developed in some of the cultural exercises (such as living/non-living, animals with hooves/those without).

Through the use of specifically designed materials, which gradually increase in difficulty (Milinković & Bogavac, 2011), the Montessori mathematics curriculum introduces young children to mathematics in a hands-on, concrete way, leading to a more abstract experience towards the upper end of the three-to-six phase (Basargekar & Lillard, 2021). The Montessori mathematics curriculum chiefly focuses on numeration and counting. Numerals are introduced via sandpaper numerals, and counting to ten is experienced through engagement with number rods, bead stair, and the spindle box introduces children to the concept of zero and the empty set.

2.1.5 Early Perceptions of the Role of the Adult

Comenius advocated a responsive pedagogy for young children in which the adult's role was to observe the child closely, identify readiness for new learning, follow the child's emerging interests, and use these observations to construct a sequenced curriculum (Silber, 1965). Froebel similarly endorsed a child-centred approach, emphasising high-quality adult–child interactions as a means of supporting learning. He viewed the adult as responsible for creating richly resourced environments that promote exploration, for designing an integrated curriculum, and for serving as the primary mediator between the child and learning (Bruce, 2021). Froebelian philosophy, therefore, proposes that adults offer guidance within children's play. Aligned with Froebel's ideas, Montessori also conceived of the adult as a curator of a "prepared environment," characterised by order and by the careful placement of didactic materials, each assigned to its own designated space in the classroom. The 'directress' bore responsibility for maintaining this environment and, like Froebel, adopted the role of an observer who intervenes only when necessary. As Montessori stated, 'it is imperative that a teacher understand and appreciate his position as observer' (Montessori, 1915/1997, p. 51).

This section has considered the philosophies and pedagogical ideas of some of the EC pioneers in relation to ECME. Their work continues to influence the field, as will be indicated in my discussions on mathematical content for ECME (Section 2.3.2) and on educator beliefs (Section 2.4).

2.2 Modern Vision of the Young Child as a Mathematician

Young children have a right to an education and the right to engage in mathematical activity and education from birth (Munn, 2005; Cohrssen & Page, 2016). Dockett and Goff (2013) assert that young children possess a right to engage in mathematics education, and to have their mathematical activity noticed, valued and responded to. In this study, the young

child is positioned as being a competent and capable mathematician, experienced in the everyday mathematics of their environment, and who draws upon their cultural 'funds of knowledge' (Moll et al., 1992; Williams et al., 2016), to make sense of their cultural worlds through play (Papandreou & Tsiouli, 2020; Worthington & van Oers, 2016). Moll et al. (1992) define funds of knowledge as 'historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual functioning and well-being' (p.133). Emphasising and responding to children's funds of knowledge facilitates a strengths-based approach which recognises all a child brings to a learning situation. This seminal theory continues to shape EC research (e.g., Fenech et al., 2025; Ruscoe et al., 2025; Worthington, 2017). Chesworth (2016) acknowledges the usefulness of this theory in supporting educators in noticing children's knowledge, competencies, and interests gained from the home learning environment (HLE), and to use this knowledge to support children's participation in cultural practices, such as mathematics.

This study views young children as active agents in their learning, constructing knowledge through their explorations of their environments in conjunction with the people in them. This view aligns with the social-constructivist approach to education, where cognitive development is facilitated through a child's interactions within their social and cultural environments (Smidt, 2009). Through these interactions and explorations of cultural tools (such as mathematics), children make sense of their world through enculturation, a process in which their engagement with mathematics is "culturally guided" (van Oers, 2010, p.34). [See section 4.2 for further discussion on culture]. A responsive pedagogy engages children in interactions with their environment and cultural tools through adult support and guidance, facilitating young children's participation in mathematical activity within the early childhood classroom.

2.2.1 Young Children Being and Becoming Mathematicians

The conceptualisation of young children 'being' and 'becoming' is not new. Its roots can be felt in Froebelian philosophy, where education and life are seen as 'a process of becoming' (Bruce, 2021, p.4). Froebel believed that children were living in the moment, experiencing life as it happened, and learning in the moment. They are not preparing for life; they are experiencing and learning through everyday life experiences, and interacting with their environments, in the here and now. People, therefore, experience 'becoming' as a continuous, life-long process (Bruce, 2021).

The post-modern paradigm of the *new sociology of the child* positions young children as active learners, who are competent, capable and knowledgeable, and who are experts in their own lives (Dahlberg et al., 1999). This theory frames young children as 'beings' rather than 'becomings'. Moss (2001) suggests that when society focuses on preparing the child for the future (the becoming), the 'being' can often be overlooked. Aligning with Froebelian philosophy, the new sociology of the child encourages educators to focus on the 'being' of the child, responding to their needs and interests in the present, rather than preparing them for the future. This theory permeates Aistear (GoI, 2024c), where educators are encouraged to observe and listen to children, and to provide an emergent curriculum in response to their observed interests and ideas (Hayes, 2013).

The narrow view of 'becoming' as preparation for life is often associated with s*choolification*. Schoolification reflects a phenomenon observed in preschools, where the focus lies on skills perceived as being academic (mathematics, reading and writing) in preparation for the transition to primary school (The Professional Association for Childcare and Early Years (PACEY), 2013). PACEY (2013) asserts that, where schoolification occurs, it occurs at the expense of developing personal, non-academic skills, such as social skills, independence, curiosity and child agency.

Positioning young children as rights bearers, with a right to ECME (Cohrssen & Page, 2016), this section proposed a vision of the young child as a capable and competent mathematician, actively being and becoming a mathematician as they interact with their worlds. The next section reviews the literature on the types of knowledge required by ECEs, including the range of mathematical ideas suggested by researchers, to engage young children in ECME as mathematicians.

2.3 Professional Knowledge for ECME

The nature of mathematical knowledge needed for teaching has been investigated and deliberated for decades (e.g., Ball et al., 2008; Copr-Gencturk et al., 2019). Exactly how it should be conceptualised is regularly debated and contested, particularly within the domain of ECME (e.g., Gasteiger & Benz, 2018; Lee, 2017). Shulman's (1986) foundational work on the categories of teacher knowledge proposed the categories of Content Knowledge (CK), Pedagogical Content Knowledge (PCK) and Curricular Content Knowledge (CCK). Ball et al. (2008) further subdivided these categories to include sub-categories such as horizon content knowledge, knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. While Ball et al. (2008) is widely cited and has strongly influenced the conceptualisation of Mathematical Content Knowledge (MCK), the proposed subdivisions remain subjects of ongoing exploration and debate (e.g., Copur-Gencturk et al., 2019; Copur-Gencturk & Tolar, 2022). Given the contested nature of their suggested subdivisions, this review will utilise Shulman's categories of CK, 'the necessary subject matter content knowledge for teaching (Shulman, 1986, p.8) and PCK, the ability to effectively convey (teach) mathematical content to students in a way that is comprehensible to them (Lee, 2010). Curricular knowledge will be considered in Chapter Three, where Aistear (GoI, 2024c) will be discussed.

Professional competence for ECME is complex and multifaceted, requiring situation-specific skills and types of knowledge. Within the ECME research space, some researchers conceptualise educator competence differently, emphasising aspects such as knowledge of early mathematical development (Clements & Sarama, 2021; Priestley, 2022) and reflection (Dockett & Perry, 2013). Dockett and Goff (2013) suggest that reflection is being able to reflect both *in the moment* and retrospectively on children's learning experiences. The possession of positive beliefs about the relevance and appropriateness of including ECME in ECE is also a prerequisite of professional competence in ECME (Gasteiger & Benz, 2018). Together, these elements of professional competence in ECME support a responsive pedagogy that enables educators to observe, interpret, respond to, and enrich children's mathematical activities.

2.3.1 Mathematical Content Knowledge (MCK)

Acknowledged as a core component of professional knowledge for teaching subject matter across all educational levels, content knowledge is defined by Shulman (1986) as "the necessary subject matter content knowledge for teaching" (p.8). Ball et al. (2008) assert that subject content knowledge is a 'foundational' aspect of a teacher's competency (p.402). Without it, they cannot support children in learning mathematical concepts. Ball et al. (2008) contend that MCK is not simply a deep knowledge of mathematics. In teaching terms, it embodies the idea of understanding how mathematics might be presented in various classroom contexts, as college mathematics differs significantly from the mathematics taught in primary, secondary, or early childhood classrooms.

Linder and Simpson's (2018) and MacDonald and Murphy's (2019) systematic reviews of research in ECME identified educator content knowledge as one of the dominant themes in the literature. This explicit knowledge of mathematics content (Gasteiger & Benz, 2018) is required of preschool teachers, not only to recognise mathematics in children's play

(Dunekacke et al., 2015; Oppermann et al., 2016), but also to interpret and enhance it (Lee, 2017). Rowland et al. (2009) contend that Mathematical Knowledge for Teaching (MKT) forms an integral part of a teacher's toolkit when needing to respond 'in the moment' to unexpected ideas or responses that arise in everyday classroom situations, or, as in the preschool context, in children's play. Dockett and Goff (2013) concur with this view, but emphasise that educators must also draw on such knowledge to engage in sustained reflection on children's observed mathematical thinking. This reflective process, they argue, is essential for designing activities that effectively extend and enhance children's mathematical learning. Both skills (observation and reflection) are necessary to provide a holistic picture of a child's mathematical activity and development, and are sharpened by an educator's level of mathematical content knowledge. These ideas will be explored further in section 2.6.

Gasteiger and Benz (2018) contend that preschool educators do not require the same general mathematical content knowledge as their peers in primary school, but a type of '*horizon content knowledge'* (Ball et al., 2008) of the foundational concepts of mathematics. Ball et al. (2008) define horizon content knowledge as 'an awareness of how mathematical topics are related for mathematics included in the curriculum' (p.403). Ball et al. (2008) suggest that all teachers, at any level of the educational system, who possess this type of knowledge are capable of seeing the bigger picture; they understand where content connects to further learning as children advance through the grades.

The next section begins by considering the nature of mathematics, before examining the mathematical concepts (content) that young children should engage with. The *Big Ideas of Early Childhood Mathematics* discussed below indicate the wide range of mathematical content ECEs need to support young children's mathematical learning (RQ1).

2.3.2 The Nature of Mathematics

Ernest (2016) asserts that the nature of mathematics can be understood from two differing perspectives. One perspective views mathematics from a traditional, positivist stance, in which mathematics is static in nature (Dossey, 1992), reflecting an agreed body of definite facts and procedures, and where these remain 'certain, cumulative and untouched by social interests' (Ernest, 2016, p.17) An alternative perspective views mathematics as a dynamic entity, one which constantly changes and evolves through creativity, experimentation and application (Dossey, 1992). Ernest (2016) posits that a fallibilist, socio-cultural view of mathematics perceives the nature of mathematics as being 'fundamentally social, with cultural limitations to its claims of certainty, universality and absoluteness, and is much subject to conceptual revolutions as in science' (p.17). Hersh (1997, as cited by Dooley et al., 2014) positions mathematics as a socially constructed phenomenon; its characteristics and nature are determined by human endeavour, creativity, and the individual culture in which it evolves (Hersh, 1997, as cited by Dooley et al., 2014). Mathematics is a culturally devised tool that enables us to engage with and interpret our world (Vygotsky, 1978). Mathematics education is a social process shaped by socially and politically determined content, societal values and individual learner attitudes towards the subject (Bishop, 1988). The mathematical knowledge deemed important for a particular society is determined by policymakers in response to desired societal outcomes (Bishop, 1988). Bishop (1988) described mathematics as a 'pan-cultural phenomenon'; it exists in all cultures to support our engagement with our environments (p. 19) and identified six universal mathematical activities common across all cultures (p. 24): Counting, Locating, Measuring, Designing, Playing, and Explaining. Table 2.1 explains each of these activities.

Table 2.1

Bishop's Six Universal Mathematical Activities (1988, pp. 182-183)

Activity	Description
Counting	The use of a systematic way to compare and order discrete phenomena. It may involve tallying, or using objects or string to record, or special number words or names
Locating	Exploring one's spatial environment, and symbolising that environment with models, diagrams, drawings, words or other means.
Measuring	Quantifying qualities for the purposes of comparison and ordering, using objects or tokens as measuring devices with associated units or 'measure-words'.
Designing	Creating a shape or design for an object or for any part of one's spatial environment. It may involve making the object, as a 'mental template', or symbolising it in some conventionalised way
Playing	Devising and engaging in games and pastimes, with more or less formalised rules that all players must abide by.
Explaining	Finding ways to account for the existence of phenomena, be they religious, animistic or scientific.

It is worth noting that Bishop (1988) identified playing as a social learning process and 'a crucial activity for mathematical development' (p.48), acknowledging the mathematical nature of many games across cultures. Bishop (1988), in line with Vygotsky (1978), suggested that the imaginary nature of play lends itself to the development of abstract thinking as well as to the application of rules and structure. Section 2.5.1 explores play as a pedagogical tool for early childhood education.

2.3.3 The Big Ideas of Early Childhood Mathematics

Research focusing on relevant and suitable mathematical content for EC often refers to the '*Big Ideas of Early Mathematics'* (Dooley et al., 2014). Sarama and Clements (2009) propose that 'big ideas' should meet the following three benchmarks: they must be mathematically coherent, build on children's informal mathematical knowledge, and lay foundations for future mathematical development and understanding. However, currently, there is no commonly agreed set of mathematical ideas for early childhood mathematics education across international preschool curricula (Thiel & Jenssen, 2020), and there is some

variation in the expectations of what content should be explored within preschool contexts, and whether this is specified in curricula or not.

In the US, Sarama and Clements (2009) suggest 12 big ideas of mathematics that young children should engage with. Dooley et al.'s (2014) commissioned report to inform primary curricular reform noted that the big ideas suggested by Sarama & Clements (2009) lack a focus on mathematical processes such as communicating, argumentation, justifying, generalising, representing, problem-solving, reasoning and connecting. However, Sarama and Clements (2009) argue that skills and processes are inherent in their big ideas or mathematical learning goals. Explicit connections between mathematical content (Big Ideas) and mathematical processes are necessary in EC curricular frameworks if ECEs are to support and guide young mathematical learners effectively.

Table 2.2

Alignment of Research-based Mathematical Ideas and Irish Curricular Documents

Research			**Irish Policy / Curricular Frameworks**	
Perry & Dockett (2008) The Powerful Ideas of Mathematics (Australia)	**Sarama & Clements (2009)** Big Ideas of Mathematics (USA)	**Early Math Collaborative (Brownell et al, 2014)** Big Ideas of Early Mathematics (USA)	**Aistear The Early Childhood Curriculum Framework** (NCCA, 2009)	**Department of Education (2023a)** Primary Mathematics Curriculum (Ireland)
	Counting	Counting	Counting skills	Numeration and Counting
Number sense	Quantity, number, subitising	Number Sense	Numerals (symbols)	Number patterns, sequences & relationships

Data	Comparing, ordering, and estimating	Sets (sorting) Data Analysis	Comparing, ordering, sorting, and matching	Data
Mental computations	Early addition/subtraction strategies	Number Operations		Operations
	Composition of number, place value			Place value, fractions
Spatial & Geometric Thinking	Spatial Thinking	Spatial Relationships	Space, Place	Spatial Awareness & Location
	Composition & decomposition of shapes	Shape: developing definitions	Shape	Shape, transformation
Probability	Length, Area, Volume, Angle	Measurement	Measures: Length, Weight, Height, Capacity, Money, Time	Measuring Time Money
	Pattern & Structure	Pattern	Pattern	Patterns, Structures, Rules
			Mathematical Language	
Mathematical Processes				

Mathematization Connections Argumentation	Reasoning Problem-Solving Classification Seriation		Problem-solving Predicting Analysing Questioning Justifying	Understanding & connecting Communicating Reasoning Applying & Problem-Solving

In Table 2.2. I have sought to demonstrate the alignment between research-based mathematical ideas for ECME and Irish curricular documents. Table 2.2 provides an overview of the general mathematical topics that are included in selected well-regarded ECME research and current Irish policy. The 'big ideas' are provided in chronological order of their appearance in the literature and policy, and not due to any one proposed set of ideas being deemed more important than another. Australian researchers, Perry and Dockett (2008), proposed seven powerful mathematical ideas and processes that young children should have access to: "mathematization, connections, number sense, argumentation and mental computation, algebraic reasoning, spatial and geometric reasoning, as well as data and probability sense" (p.16). The Early Math Collaborative (US) recommends the following mathematical areas for young children: sets, number sense, counting, number operations, patterns, measurement, data analysis, spatial relationships, and shapes (Brownell et al., 2014). Inherent within each of these areas is a set of 26 'big ideas', aiming to break down each area into easily understandable skills and concepts related to an area, e.g. Sets

- Attributes can be used to sort collections into sets
- The same collection can be sorted in different ways
- Sets can be compared and ordered (p.13).

When mathematical concepts are broken down in this way, it can support early childhood educators in better understanding the mathematical content, skills and processes associated with mathematical concepts (Brownell et al., 2014). From the set of mathematical ideas proposed above, some commonalities are emerging across the sets of 'Big Ideas' and within Aistear (NCCA, 2009; GoI, 2024).

Despite some variations in the mathematical concepts to be included in ECME, the following common concepts are evident across the literature: number, counting, shape, spatial thinking, operations, measures, data and pattern. These are inherent in Bishop's (1988) Six Universal Mathematical Activities outlined in section 2.3.2. Probability is included only in the Australian literature. Its absence from other suggestions for the *Big Ideas of ECME* could stem from a belief that young children are incapable of engaging in probabilistic thinking (English, 2023). This may change over time as more studies focus on this area of mathematics with young children (e.g. Kingston, 2024). However, a knowledge of mathematical ideas and concepts alone is insufficient to support ECME in preschool classrooms; a knowledge and understanding of early mathematical development is also key (Clements & Sarama, 2021; Priestley, 2022).

This literature review shows broad alignment between research and curriculum understandings of early childhood mathematics. It is for this reason that I deliberately chose to focus on these particular topics in my analysis (see Chapter Four). Findings relating to the mathematics experienced by the children (as reported in Chapter 7) reflect the Big Ideas of Early Math (Brownell et al., 2014): number sense (including counting and number operations), measurement, data analysis, shape, spatial relationships, and pattern. As presented in Table 2.2, these mathematical areas are common across the research literature and are reflected in Irish Curricular frameworks such as PMC (DE, 2023) and in Aistear (GoI, 2024c). It should also be noted that the mathematical content and processes for ECE

suggested by researchers and included in national curricular documents reflect Bishop's (1988) six universal mathematical activities of counting, locating, measuring, designing, playing and explaining.

2.3.4 Knowledge of Mathematical Development

Connected to MCK and PCK is a knowledge of mathematical development. An ECE not only needs to possess deep knowledge of the *Big Ideas of Early Mathematics* (e.g. Brownell et al., 2014), the ways to support children in recognising and learning mathematics in play and everyday activities (Brunner, 2022), a knowledge of individual children (Ball et al., 2008) but they also need an understanding of the sequence of development of mathematical ideas (Priestley, 2022). Other researchers (Anders & Rossbach, 2015; Aubrey, 1997; Lee & Ginsburg, 2009; Brown, 2005) assert that educators' knowledge of mathematical development also underpins the ability to see the mathematics in children's play scenarios, and consequently extend the learning therein. Despite the growing emphasis on mathematics education in early childhood, many early childhood education professional development programmes do not include content on children's mathematical development or how to convey (teach) mathematical ideas to young children (Gasteiger & Benz, 2018; Gillic et al.,2026, under review). Many early childhood educators are therefore unaware of how mathematical thinking develops in young children (Priestley, 2022), which makes it difficult for them to recognise and assess each child's stage of mathematical development. Notably, the 2017 (DES, 2017a) review of the 2011 *NLNS* (DES, 2011) states that ECEs need to 'gain a deeper understanding of …the sequence in which children learn early mathematical ideas" (p.31), signifying government recognition of the need to develop ECE knowledge of ECME.

2.3.5 Pedagogical Content Knowledge (PCK)

Pedagogical content knowledge (PCK) is acknowledged in the literature as a key component of effective teaching and learning (Lee, 2010), a requisite skill for recognising mathematics in everyday situations (Gasteiger & Benz, 2018; Lee, 2017; McCray & Chen, 2012), and for supporting and enhancing mathematics observed in children's play (Lee, 2017; McCray & Chen, 2012). PCK has been defined as:

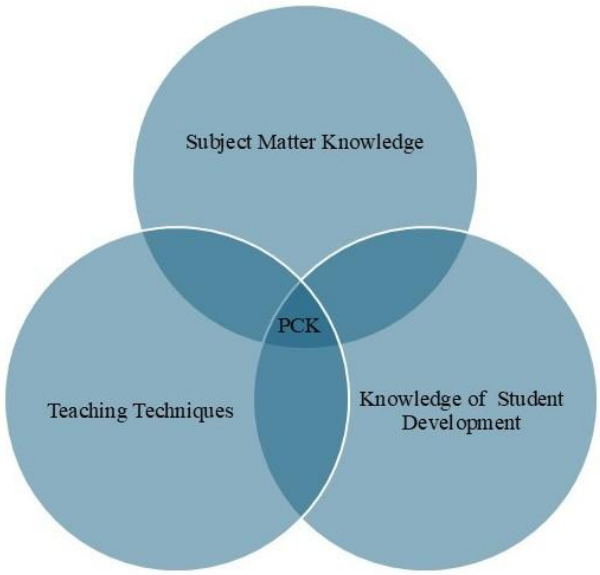
> The most useful ways of representing and formulating the subject that make it comprehensible to others…pedagogical content knowledge also includes an understanding of what makes learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons (Schulman, 1986, p.9)

While an established general definition for PCK exists, much debate remains about what constitutes PCK for ECME. Bruner (2022) suggests that the discourse is underscored by competing schools of thought. The first is driven by developmental psychology (e.g. Sarama & Clements, 2009) and the second is underpinned by the socio-pedagogical tradition of preschool as an institution with 'an educational mission' (p.221). Opperman et al. (2016) further assert that these two fields of thinking contribute to two different interpretations of PCK for ECME. They suggest that the former, developmental psychology, interpretation follows the 'school readiness' tradition, where a teacher-led, direct instructional approach is applied. The second interpretation of PCK is more applicable to EC settings. Following a child-led, play-based approach, McCray and Chen (2012) developed an adapted view of Schulman's (1986) definition of PCK for ECME, which they define as an educator's ability to help children 'recognise, name, and experiment with the mathematics in their classroom environment' (p.292). McCray and Chen (2012) see PCK for ECME as comprising three interrelated types of knowledge: knowledge of mathematical content, knowledge of appropriate teaching strategies, and knowledge of student development. McCray and Chen (2012) assert that ECEs should possess knowledge of all three components to explore

mathematical concepts effectively with young children. This view of PCK, as more than teaching techniques and arising from subject matter knowledge and knowledge of student development, acknowledges the interplay between Shulman's categories.

Figure 2.1

Pedagogical Content Knowledge as the intersection of subject matter understanding, teaching techniques and knowledge of student development (McCray & Chen, 2012, p.295)



Lee (2017) offers another definition. Also using a tripartite framework, Lee (2017) frames PCK for ECME as comprising three competencies: an awareness of opportunities for mathematical learning, an ability to draw upon knowledge of ECME concepts to interpret mathematics in children's play, and an ability to enhance mathematical thinking at a deep level. Within this definition, the need for a strong level of MCK underpins an educator's ability to notice and respond to mathematics observed in a child's activity, and to support mathematical thinking to a higher level. Also embedded within this definition is the

competency of situational awareness or being open to noticing the mathematics in play and other everyday situations (Gasteiger & Benz, 2018).

This section has reviewed the literature on mathematical content and pedagogical content knowledge. Some researchers (e.g. Blömeke et al., 2015; Dunekacke et al., 2015) consider the interconnected affective traits of beliefs and attitudes towards mathematics, early mathematics education and educator self-efficacy in ECME as critical elements of PCK as they play a role in determining if, and how ECME is enacted in early childhood classrooms. The section that follows explores these affective components of PCK.

2.4 Educator Beliefs

Alongside subject-matter knowledge and pedagogical content knowledge, educators' beliefs are recognised as a key element of professional competence (Dunekacke et al., 2015) and as an aspect of structural quality in preschool practice (Roßbach, 1993, as cited in Thiel, 2010). To date, the majority of research on teacher beliefs centres on primary and secondary educators. Despite the increasing emphasis on ECME and the identification of beliefs as an important component of educator competence, there is a lack of research generally exploring the beliefs of preschool educators towards mathematics in preschool (Benz, 2012; Sumpter, 2019). This section considers components of the affective domain, including beliefs about and attitudes towards mathematics and mathematics education, educator self-efficacy in ECME (Liljedahl & Oesterle, 2020).

2.4.1 The Importance of Educator Beliefs and Attitudes

The literature on early childhood mathematics education explores educator beliefs about self-efficacy (Blömeke et al., 2017; Chen et al., 2014; Takunyaci & Takunyaci, 2014; Brown, 2005), about the importance of mathematics (Thiel, 2010; Sumpter, 2019), what constitutes mathematics in early childhood (Lee & Ginsburg, 2007), how children learn

mathematics (Platas, 2015), and what mathematics (if any) young children should learn and associated pedagogies (Chen et al., 2014; Lee & Ginsburg, 2009). Beliefs may also impact an educator's ability to notice, interpret mathematical concepts in children's play (Benz, 2012; Oppermann et al., 2016). An educator's views on early childhood teaching, learning, and children's conceptual systems are closely interconnected. Beliefs shape whether learning occurs through free exploratory play, educator-directed activities, or a blend of teaching approaches (Lee & Ginsburg, 2009). As Dunphy et al. (2014) note, 'a teacher's conceptualisation of mathematics and what it is *to do* mathematics have strong influences on pedagogic practice' (p. 33). The following sections begin by defining beliefs and attitudes, then examine how beliefs, attitudes, and self-efficacy interact to influence children's experiences of ECME in the preschool classroom.

2.4.2 Definitions of Beliefs and Attitudes

There is no agreed-upon definition for the term 'beliefs' within the research literature (Benz, 2012). Perspectives of educators about the teaching and learning of mathematics in preschool education have been explored through the lenses of attitudes (Benz, 2012), beliefs (Chen et al, 2014; Lee & Ginsburg, 2007; MacDonald, 2020; Platas, 2015; Youmans et al., 2018), views (Hunting et al, 2012), emotional directions (Sumpter, 2019) and perceptions (Knaus, 2017). Philipp (2007) posits that beliefs are 'lenses through which one looks when interpreting the world' (p. 258), and that our beliefs influence how we interact with the world. In educational terms, Kagan (1992) defines educator beliefs as 'tacit, often unconsciously held assumptions about students, classrooms, and the academic material to be taught' (p.65). Fives and Beuhl (2012) identify teacher beliefs as an 'integrated system of judgements that relate to teachers' classroom work" (p. 114). Beliefs about education, its purpose and nature are informed by personal experiences with the educational system and with individual subjects, like mathematics (Johnston & Bull, 2021). Such beliefs may, to some extent, be

influenced by Government policy (Fosse & Lossius, 2016) and cultural values. Liljedahl and Oesterle (2020) note that the literature characterises beliefs as being static in nature and that once established, they are hard to change. Beliefs, therefore, are complex constructs, informed and underpinned by factors such as personal experiences, education and engagement with policy. This mesh of experience and education frames how teachers think about education, how children learn, what content is appropriate, classroom interactions and how content should be explored with children.

van Oers (2002) uses the term *mathematical epistemology* to encapsulate pedagogical beliefs held by teachers towards mathematics - what is a teacher's understanding of mathematics? What are their views on mathematics? How do they think it should be taught? What should not be taught? This understanding develops through personal, professional, and academic experiences with mathematics in school, college, and ongoing professional development. The research literature (e.g. Fosse & Lossius, 2016) also suggests that preschool educators' views towards mathematics can be influenced by government policy in terms of curricular documentation.

2.4.3 Attitudes

Stemming from beliefs and defined as 'manners of acting, thinking or feeling, that show one's disposition or feeling' (Philipp, 2007, p. 259), educator attitudes may be positive or negative in nature and although generally considered static and unchangeable (Liljedahl & Oesterle, 2020), can be quicker to change than beliefs (Philipp, 2007). Educators' attitudes towards mathematics directly bear on the inclusion of mathematics in daily work with young children (Anders & Rossbach, 2015). Fleer (2010) notes how educators' attitudes towards young children's learning/acquiring concepts heavily influence the type, quantity, quality, and potential for conceptual development in early childhood classrooms. Research suggests that pedagogical beliefs—ideas about teaching and learning—are shaped by many factors.

Personal experiences, education, and cultural expectations all play important roles in forming these beliefs.

2.4.4 Influences on Beliefs and Attitudes

The literature offers reasons for differing beliefs: differences in the educational level of the educator, differences in teaching experiences, and differences in cultural background (Lee & Ginsburg, 2007). Bruner (1996) suggests that educators' beliefs are often subject to examination by those who subscribe to the main cultural beliefs of a society or a profession. In Irish preschool practice, Aistear (GoI, 2024c) advocates a democratic, socio-cultural, co-constructive and playful approach to early childhood pedagogy, where premature engagement with formal academic skills is to be avoided (DES, 2011). This often conflicts with the Montessori preschool curriculum, which teaches mathematics and literacy skills directly to young children (Gilpin, 2020).

Little Irish research has been carried out at the individual setting level on the prevailing beliefs of individual settings towards mathematics in ECE. A general study of 10 Irish Montessori educators conducted by Gilpin (2020) showed that increasingly, Montessori educators adopt a curricular approach in line with government conditions to avail of government funding to administer the ECCE scheme. Settings must conform to the pedagogy of play, inherent in Aistear, and practice is inspected against the tenets of Aistear (DE, 2025a; Gilpin, 2020). Although some participants felt compelled to change their practice, mixing Montessori materials with general play equipment in light of inspector comments, their belief in the Montessori method remained fixed. These static beliefs align with the accepted understanding that beliefs, once established, are difficult to change (Liljedahl & Oesterle, 2020).

In the case of attitudes, some studies show that attitudes may not be as static as once believed. A mixed-methods study of 157 preschool educators by Sumpter (2019) demonstrated that attitudes towards the inclusion of mathematics in preschool can be positively influenced and can change with the introduction of new curricula and accompanying professional development. Johnston and Bull's (2021) study of 55 Australian early childhood educators found that initial negative attitudes toward mathematics shifted over time as participants engaged with their own children's schoolwork and worked with young children in preschool settings, providing a new perspective on how mathematics can be approached and enjoyed.

Research studies demonstrate that many early childhood educators hold positive beliefs towards mathematics in the preschool years (e.g., Thiel, 2010; Benz, 2012; Hunting et al., 2012; Gillic, 2021; Johnson & Bull, 2021). Hunting et al. (2012) explored the beliefs of 64 Australian preschool educators, through structured interviews, about how young children engage with and think about mathematics. The majority of participants believed that young children were capable of engaging in mathematical activity and thinking. 88% of participants noted that mathematical activity and exploration begin earlier than three years of age. MacDonald's (2020) national survey of 506 Australian preschool educators demonstrated that the 195 respondents who worked with children under three believed that mathematics is a natural feature of very young children's lives, and that their role was to enhance children's mathematical thinking and understanding in response to observed interests and abilities. These beliefs are encouraging; however, it must be noted that mathematics has been a feature of early childhood curricula in Australia for a decade and that both pre- (e.g. Cohrssen & Tayler, 2016) and in-service continuing professional development in mathematics for ECEs has been available in Australia for some time now (e.g. Perry & Dockett, 2013). A more recent study by Johnston and Bull (2021) exploring the perspectives of Australian ECEs

working with 0–3-year children indicated that although the majority (75%) of survey participants shared the view that ECME for this age group was extremely important, the qualitative component of the survey suggested that, for the majority of participants, ECME was not as important as social and emotional development. Such findings show the complexity of gauging beliefs and attitudes of ECEs towards mathematics using a survey method of data collection, and indeed the complexity of exploring educator beliefs more broadly.

In the US context, Lee and Ginsburg's (2009) study, conducted through interviews, reports nine common misconceptions of early childhood educators;

- young children are not ready for mathematics education; mathematics is for some bright kids with mathematics genes;
- simple numbers and shapes are enough; language and literacy are more important than mathematics;
- teachers should provide an enriched physical environment, step back, and let the children play;
- mathematics should not be taught as a stand-alone subject matter;
- assessment in mathematics is irrelevant when it comes to young children;
- children learn mathematics only by interacting with concrete objects, and computers are inappropriate for the teaching and learning of mathematics (p.38).

The study concluded that mathematics education should be an essential part of early childhood pre-service teacher programmes. Educators need up-to-date research showing that young children can engage with a broader range of mathematical concepts than once

believed, as well as an understanding of how guided play fosters mathematical language and concept development. Australian studies indicate that when educators grasp what early childhood mathematics involves and how to support it, they are more likely to develop positive beliefs and attitudes toward mathematics in early learning.

A review conducted by MacDonald and Murphy (2019) showed that there is a lack of studies that focus on early childhood educators' beliefs about young children's capabilities in mathematics. My small-scale Master's study (Gillic, 2021a) showed that eight Irish early years educators believed that children were capable of engaging with a wide range of mathematical concepts; however, the educators themselves focused on numbers, counting and shape. Studies by Browne and Wong (2016) and Knaus (2017) suggest that, generally, ECEs underestimate children's mathematical capacities (MacDonald, 2020). This feeling was expressed by one of the participants in my previous study; however, the educator expressed a reluctance to follow the child's mathematical interest to avoid overlap with mathematical content in the primary curriculum (Gillic, 2021a).

2.4.5 Beliefs about Intentional Teaching

Pre-service preschool teacher education has previously focused on supporting young children's social and emotional development (Ginsburg, 2016). Some pre-service programs advised that intentionally teaching mathematics in preschool classrooms was unsuitable, unnecessary and potentially harmful to child development (Gifford, 2003; McCray & Chen, 2011). The dispute concerning the benefits of free play over direct instruction for child development remains a key debate in EC research literature (Skene et al., 2022), with Lee and Ginsburg (2009) stating that these beliefs have held a long-lasting effect on early childhood pedagogy, where mathematics is often linked with direct instruction and the use of didactic equipment, strategies that are seemingly in opposition to the mantra of free play and a play pedagogy. Pollitt et al. (2020) maintain that these beliefs continue to influence the

incorporation of mathematics in preschool curricula. Pollitt et al (2020) see the continuation of these outdated beliefs as problematic. McCray and Chen (2011) concur with this statement and suggest that this belief in and commitment to Developmentally Appropriate Practice (DAP), coupled with a belief about what mathematics education is, facilitates the assumption that it is 'developmentally inappropriate' to engage young children with mathematics (p. 256).

A marked preference for promoting social and emotional development in early childhood practice is common in the USA (Chen et al., 2014; Hollingsworth & Winter, 2013; Lee & Ginsburg, 2009; Ginsburg et al., 2008), in Australia (Johston & Bull, 2021) and in the UK (Costa et al., 2023). However, a study of 114 preschool educators across Tennessee (US) by Denny (2009) showed that practitioner attitudes towards the inclusion of more academic subjects, such as mathematics and literacy, were changing. A more recent US study by Platas (2015) noted that preschool educators view early childhood mathematics education (ECME) as an important part of a preschool curriculum. European-based research, too, shows positive practitioner attitudes towards mathematics and the inclusion of mathematics in early childhood curricula (Benz, 2012; Björklund & Barendregt, 2016; Gillic, 2021a; Sumpter, 2019; Thiel, 2010).

Some researchers (e.g. Copley & Padron, 1998; Ginsburg et al., 2008) suggest that negative experiences with mathematics can cause educators to dislike the subject or feel incompetent in mathematics, potentially hindering the inclusion of mathematics in curricula. Oppermann et al. (2016) specifically highlight practitioners' 'math self-concept' (p.176), a practitioner's perception of their ability in mathematics. Oppermann et al. (2016) explain that this belief plays a critical role in practitioners' ability, willingness and motivation to identify children's mathematical ideas in play situations. There is still a need for early childhood educators to increase their sensitivity to mathematics in play-based situations (Anders &

Rossbach, 2015). The ability to analyse a play situation and identify its opportunities for mathematics education relies on a combination of content knowledge, knowledge of student development, and a belief that ECME is important and worth noticing and enhancing. A key component of PCK is knowing how to communicate mathematical ideas in relevant and meaningful ways to young children in play and everyday situations (Gasteiger & Benz, 2018). The next section of this review considers strategies to facilitate this in the early childhood classroom. The section begins with play, the cornerstone of early childhood pedagogy (O'Neill et al., 2022).

2.5 Pedagogical Strategies for Supporting ECME

Research (e.g., Dooley et al., 2014; Purpura et al., 2011; Trawick-Smith et al., 2016) recommends that a range of interactional strategies (e.g. mathematical talk and discussion, direct instruction, co-playing) should be employed when responding to, reflecting on and supporting young children's developing awareness of mathematics in their everyday environments (MacDonald, 2018). This section reviews the literature on the common strategies used to support ECME in preschool settings, beginning with a focus on play as a strategy for early learning.

2.5.1 Play as a Vehicle for Learning

Bishop (1988) noted that play is a well-documented cultural activity that supports engagement with mathematics and included it as one of his six universal mathematical activities. Play is a key vehicle for children's learning and a cornerstone of early childhood practice (Ginsburg, 2006; Pramling Samuelsson & Björklund, 2023; Wood, 2014). Consequently, it offers rich opportunities for mathematical learning (Gasteiger 2015; van Oers, 2010). Play pedagogy aligns with a socio-cultural view of early childhood education, as children have opportunities to gain knowledge from interacting with a more knowledgeable

other (Rogoff, 1990; Weisberg et al., 2013), either a fellow child or an adult acting as a co-player (Wood, 2014), during a guided activity. Mathematical exploration through play has been a traditional focus within the pedagogies of EC pioneers such as Montessori (Standing, 1957) and Froebel (Bruce, 2021). Within play lie other contexts for learning, such as extended talk and discussion, rich imaginative experiences and imitation (Dunphy, 2012; Rogoff, 1990). Each of these contexts has the potential to support early learning of mathematics. Play enables children to make sense of their environments and accommodates the child as an active learner (Kernan, 2007). Play provides opportunities to try out new ideas, to test new theories, and to take risks in learning (Bruce, 2001). Moyles (1990) observes that play offers children a means to develop problem-solving skills and to promote critical, creative and divergent thinking.

Play facilitates mathematical learning. Research suggests that mathematical concepts are encountered frequently during children's free play (Seo & Ginsburg, 2004). Seo and Ginsburg's (2004) study showed that 79 out of 90 child participants engaged in at least one mathematical activity during their play. Mathematical activities observed included those associated with many of the big ideas discussed in section 2.3.2: classifying, magnitude, enumeration, pattern and spatial relations. An observational study by van Oers (1996) showed that mathematical concepts were so frequent in observations of children's play that an educator could not seize all the 'teachable moments' inherent in the play. Nevertheless, mathematical concepts can go unobserved if the teacher is unwilling or lacks the mathematical content knowledge to identify them. van Oers and Poland (2012) note that to be educationally beneficial, educators must engage in children's play, challenge children's thinking by introducing problems and provide feedback on the play.

Sproule et al. (2021) observe that ECEs are more secure when promoting social and emotional development through play, and find promoting more disciplinary learning, such as

mathematics, through play more difficult. This difficulty, Sproule et al. (2021) assert, leads ECEs to adopt a more formal approach to teaching these topics. Fisher et al. (2010) agree, indicating that many practitioners believe that play and academic learning are fundamentally incompatible, such that they must either engage in instructional activities to ensure intellectual gains or allow children to play freely for their holistic development.

The research literature suggests that the strategy of *guided play*, where educators gently and gradually scaffold children's learning in play towards a specified learning goal, may be an effective method of intentionally engaging children in mathematical activity (Skene et al., 2022). Drawing on Vygotskian thinking that learning occurs when supported by a more knowledgeable other, guided play offers child-led learning opportunities supported by adults (Pyle & Danniels, 2016; Weisberg et al., 2016). Guided play still adopts a child-led approach, while the adult uses open-ended questions, active listening and co-constructive discussion to guide the child towards the mathematical goal (Weisberg et al., 2016). According to Weisberg et al. (2016), guided play can take two forms: educators curate the classroom to facilitate children's achievement of a learning goal through free exploration of this prepared environment, *or* educators observe children at play and support children's learning through questions, comments and modelling of relevant language to achieve a learning goal. This type of play exemplifies an 'enhanced discovery approach' (Hassinger-Dass et al., 2017), in which educators immediately respond to support children's exploration, ideas, and learning.

Current perspectives on play-based learning represent a shift from a dichotomous conceptualisation of play as a child-led/directed activity versus an adult-led, intentional teaching position. Instead, current thinking asserts that a high-quality early childhood education programme weaves fluidly between child-led and adult-led playful activities across the EC day, depending on the learning goal or outcome of an activity (Pyle & Danniels,

2017). Zosh et al. (2018) assert that this conceptualisation of play can be positioned as a spectrum or continuum. Such a conceptualisation, Zosh et al. (2018) contend, enables us to acknowledge and understand the many types and functions of play.

Pyle and Danniels (2017) proposed a *Continuum of Play-based Learning* to support educators in navigating the tensions that occur between perceptions of the adult as a facilitator of a rich learning environment and as a non-participant in children's play and intentional teaching. They argue that a singular focus on free play may limit academic progress, while excessive direct instruction can stifle developmental benefits. Zosh et al. (2018) agree, stating that a continuum of play facilitates a balance of child and adult-led learning throughout the preschool day. Pyle and Danniels (2017) five-point continuum framework supports the recognition of a broad spectrum of play types, including both free (child-led) and guided (adult-supported) play. Pyle and Danniels (2017) assert that such a framework can support educators in moving away from the dichotomous model of play versus work. Pyle and Danniel's (2017) continuum is outlined below:

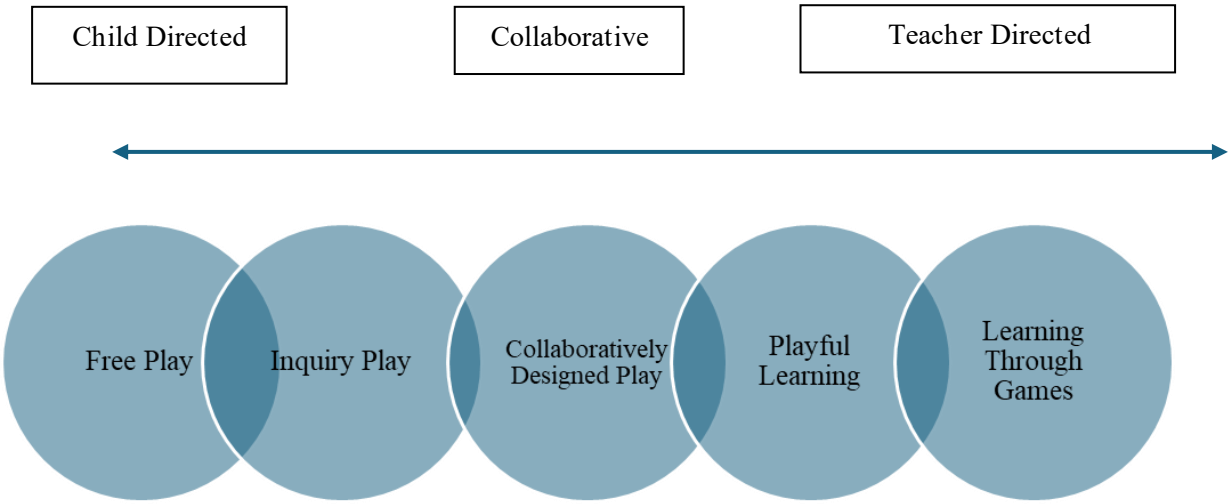
1. **Free Play:** Entirely child-directed and voluntary, with no adult interference in the narrative or resources. This is characterised by high levels of child agency, child-directed goals, and intrinsic motivation.
2. **Inquiry Play:** Child-initiated play where the teacher "steps in" to extend the activity by integrating related curricular learning outcomes.
3. **Collaborative Play:** A shared locus of control where teachers and students co-design the play context (e.g., a veterinary clinic), but children direct the narrative within that environment. Guided play is an example of this type of play.

4. **Playful Learning:** Prescribed, teacher-directed activities designed to be engaging, such as a "flower shop" where children must complete specific order forms to practice math and literacy.

5. **Learning Through Games:** The most intentional level, where teachers use games with set rules (e.g., Bingo) to teach discrete skills such as number sense.

Figure 2.2

Continuum of Play-based Learning (Pyle & Danniels, 2017, p. 282).



Pyle & Danniels' (2017) continuum of play-based learning positions play (in all formats) as a sophisticated pedagogical tool, where adult or child leadership varies in intensity and intent, across the preschool day. It must be noted that no single point on the continuum is superior to another; rather, a high-quality program moves fluidly across the spectrum based on the learning objective. When conceptualised in this way, we 'move away from a binary integration of perspectives and practices, with different types of play perceived as complementary rather than incompatible" (Pyle & Danniels, 2017, p. 311).

Zosh et al. (2018) also propose a spectrum or continuum of play with some similarities to that proposed by Pyle and Danniels (2017). In both proposals, the continuum is defined by the distribution of agency, with a balance of both adult and child-led activity. It is this characteristic, Zosh et al. (2018) argue, that is a sign of a high-quality early childhood learning environment, which leads to deep conceptual learning. Zosh et al. (2018) further assert that a spectrum of playful activity supports the educational aspects of play as learning is meaningful, engaging, and socially interactive.

Wood (2022) asserts that play continuum frameworks 'remain a problematic construct depending on whose purposes are privileged – those of the children or those of the educators (p.19). Wood (2022) further observes that the work of Fleer (2018) offers a way to challenge the dichotomous narrative between adult-led and child-initiated activities. Fleer (2018) argues that a play continuum enables the integration of everyday mathematical and scientific concepts to be explored and intentionally taught in a fun and accessible way. Fleer's (2018) *Conceptual Playworlds,* a programmatic model of early playful teaching and learning, enables educators to introduce early STEM learning within an adult-led playful context. In line with Vygotskian principles, the *Conceptual Playworlds* approach values imagination and creativity as the chief characteristics of play. In this collaborative learning method, learning occurs in a social context, typically with a more knowledgeable peer, in this case, the teacher. Conceptual Playworlds are characterised by five pedagogical processes: selecting a story for the Playworld, collaboratively designing a conceptual Playworld space based on the story, entering and exiting the conceptual Playworld space, planning the inquiry and planning of teacher interactions to build conceptual learning when in role (Li & Disney, 2023). Both Fleer's (2018) and Li and Disney's (2023) research demonstrates that intentional adult involvement in a type of collective play strengthens the learning rather than inhibiting it. Through this model, children's awareness and knowledge of their cultural context are

enhanced. (Fleer, 2018). Furthermore, Li and Disney's (2023) research showed that a Conceptual Playworld approach supported the development of early mathematical problem-solving skills when the collaborative imaginary learning scenario proved meaningful to the children.

Sproule et al. (2017) propose the concept of 'playfulness' or the 'playful teacher' as an effective pedagogical tool to positively engage children in academic learning. They further assert that the characteristics of a playful teacher include: 'use of voice and facial expressions, use of gestures and whole body behaviours, use of props and popular culture and the use of humour and play' (p. 234). This approach, they believe, may help to bridge the perceived divide between learning through free play and intentional adult-led teaching.

Pramling et al. (2019) propose the term 'play-responsive teaching' as a way to describe educators' deep participation in children's play and how they respond to the concepts noticed in the play. Fleer (2019) describes this responsive process where educators tune in to the ideas in the play and intentionally introduce new ideas or language to develop the play in line with the children's ideas, thereby facilitating a child-led approach. However, as Dockett and Goff (2013) note, "this potential for mathematical learning in play is only realised if the maths in play is noticed, explored and talked about" (p.772).

2.5.2 Mathematically Rich Environments

Just as an engaging adult fosters mathematical thinking and learning, a mathematically rich physical learning environment is also a crucial contributing factor in facilitating early childhood mathematics learning (Clements & Sarama, 2014a). Indeed, a well-resourced learning environment creates positive learning outcomes (Varol & Farran, 2006). Montague-Smith et al. (2018) note that high-quality mathematical environments are those that enable children to experience mathematical concepts in a way that makes sense to

them. Play resources, familiar to young children, such as construction toys, jigsaw puzzles, tangram puzzles, as well as creative activities such as paper folding, all contribute to mathematical development and learning (Hawes et al., 2015). Wager (2013) describes how early childhood educators 'seed' the environment with 'mathematical tools' that enable children to engage with mathematics in a natural, meaningful way (p.166). This, in turn, enables the educator to respond to the mathematics that occurs as children play with these tools.

2.5.2.1 Block Play

'Block play is defined as any time a child manipulates proportional wooden blocks, and uses actions, and/or language to represent realistic or imaginary experiences' (Cohen & Uhry, 2011, p.1). Stemming from Froebelian Kindergarten (Bruce, 2021), block or construction play maintains its position as a core activity in EC settings. Playing with blocks and other construction materials enables children to mentally design objects, thereby engaging in Bishop's (1988) activity of designing. In her seminal project, Gura (1992) asserted the important role of peers in the development of construction ability, with children engaged in collaborative block play developing skills at a higher rate, thereby enhancing social development (Rogers, 2006). This echoes a socio-cultural stance to learning (Rogoff, 1990; Vygotsky, 1978). Adult input is critical to support the development of relevant vocabulary (Cohen & Emmons, 2016; Ferrera et al., 2013; Schmitt et al., 2023; Trawick-Smith et al., 2017), and the use of open-ended questioning, problem-posing and challenge setting have proved effective strategies in facilitating children's deeper thinking and problem-solving abilities (Cohen & Emmons, 2016; Trawick-Smith et al., 2017). In terms of ECME, block play affords opportunities to explore patterns, 3D shapes, measures and spatial relationships (Brownell et al., 2014). This makes a case for guided block play (Weisberg et al., 2013) and the wide range of mathematical topics that can be addressed. Trawick-Smith et

al.'s (2017) study concluded that introducing small-world play items, such as cars or mini-figures, did not enhance children's block play

Hawes et al. (2015) identify block building as a key activity to support spatial relationships, develop spatial reasoning and visualisation skills. Spatial reasoning has been afforded importance in supporting the development of STEM skills and mathematical reasoning more generally (Hawes et al., 2015; Verdine et al., 2017), with early spatial skills being a predictor of later mathematics success (Rittle-Johnson et al., 2019). As Clements and Sarama (2021) note, young girls do not engage in block play as often as boys, thereby limiting their opportunities to develop spatial reasoning skills as a result. Despite the importance of early spatial skill development in the literature, spatial reasoning remains a space where educators lack confidence (Bates et al., 2023), where variations in instructional quality occur (Schmitt et al., 2023), and where Irish students fall behind their peers internationally (McHugh et al., 2024). Verdine et al. (2014) contend this is an area that remains under-researched in early childhood.

2.5.2.2 Socio-dramatic Play

Afforded high importance by Vygotsky (1978), socio-dramatic, pretend, or make-believe play facilitates collaborative learning (Bodrova & Leong, 2024) and opportunities for imitating social routines, actions and language (Vygotsky, 1978; Rogoff, 1990). Findings from Worthington and van Oers' (2016) study exploring children's mathematics in socio-dramatic play show that children draw on their cultural funds of knowledge (Moll et al. 1992) when engaged in this type of play. The imagined scenario (recreated from observed cultural practices) provides a space rich in mathematical dialogue (Worthington & van Oers, 2016). Findings from this English study indicated that number and counting were the most frequently observed, followed by money, time, and measuring. It was also found that

participating teachers rarely intervened in children's play; however, they mediated maths learning via other means, such as providing a rich mathematical learning environment.

2.5.2.3 Games

Balladares et al.'s (2023) review of the literature found that games with rules, such as board games, encourage social skill development. Games facilitate mathematical learning (Bishop, 1988). Board games are a playful way to develop early number skills (Moomaw, 2017), and are ideal for small groups (Ramani & Eason, 2015). Games involving dot die and spinners develop numeral recognition and subitising skills, as well as the quantification of small numbers. Board games support understanding of the five principles of counting: one-to-one correspondence, stable order, cardinality, order-irrelevance and abstraction (Gelman & Gallistel, 1978). A study by Cohrssen and Niklas (2018) provided preschool teachers with games and prompts for open-ended questions. Findings indicated long-lasting positive gains in children's number sense development. This was particularly the case for children with EAL. Pyle and Danniels' (2017) findings indicated that, despite the advantages of games for both cognitive and social development, they are a rare feature of EC pedagogical practice in Canada. There is little to no evidence of the prevalence or nature of their use in preschool education in Ireland.

2.5.2.4 Intentional Teaching

Despite the reservations some ECEs hold about intentional teaching, Aistear (GoI, 2024c) advocates for intentionality in early learning. Intentional teaching has been defined as 'responsive adult intervention with a purpose, i.e. intentional pedagogical actions to support the achievement of a learning goal' (Epstein, 2007, cited in Grieshaber et al., 2021). Aistear (GoI, 2024c) advocates for this approach, defining intentional pedagogies as:

> "The purposeful actions of educators to facilitate, support and extend babies, toddlers and young children's learning experiences, needs, interests, ideas, skills and strengths" (p.32).

As discussed in section 2.4.5, the role of intentional teaching or intentional pedagogy in ECE remains a much-contested concept (Björklund et al., 2018; Edwards, 2017; Grieshaber et al., 2021; Walsh et al., 2019). Grieshaber et al.'s (2021) scoping review of 101 studies on intentional teaching in early childhood education highlighted the complexity of examining educators' understanding of what it means to teach intentionally in EC settings. This is especially relevant to the adult's role in children's self-chosen play, a topic that remains debated in early childhood circles (Skene et al., 2022). McInnes (2019) notes that ECEs are often reluctant to become involved in children's free play. Current research advocates for the introduction of intentional mathematics teaching, but not as a substitute for play-based pedagogy. It is proposed that intentional teaching be introduced through the provision of playful, meaningful, relevant, and active early mathematical experiences such as real-world problem-solving, reasoning, explaining and justifying, based on active and hands-on experiences (Clements & Sarama, 2021). MacDonald (2018) notes that both indirect and direct teaching strategies are required to support young children's mathematical development. Guided play (discussed above) can be seen as a playful way to engage young children in the intentional teaching of mathematics.

Research suggests that the intentional teaching of mathematics in early childhood contexts is more beneficial than the traditional 'laissez-faire' play-only approach to early childhood education (Clements & Sarama, 2021), and Björklund (2014) conceptualises intentional teaching as a 'powerful teaching strategy' (p.391). Despite the research evidence supporting the necessity for intentional teaching in EC contexts, this topic remains under-researched (Grieshaber et al., 2021).

Explicit intentional teaching is considered a necessary strategy for supporting children with EAL to participate in mathematical activity (Doabler et al., 2016). Doabler et al.'s study found that using a sequenced instructional curriculum, based on hands-on active learning, with a focus on target mathematical vocabulary, was successful in narrowing the achievement gap between EALs and their English-speaking peers. Clements and Sarama (2021) note that EAL students benefit from direct instruction in mathematics in their first language, building knowledge and understanding in their home language before transference to English. However, further research is needed to determine the most effective ways to support young children with EAL in mathematics learning (Baker et al., 2018).

2.5.2.5 Circle Time (CT)

Bustamante et al. (2018) argue that early childhood educators often struggle to schedule dedicated time for the intentional teaching of specific concepts, whether mathematical or otherwise. They identify CT as a part of the daily preschool routine that facilitates intentional teaching (Bustamante et al., 2018) and children's participation in discussion (Zhang & Quinn, 2018). This daily dedicated whole-group time, often lasting up to 20 minutes, potentially offers a perfect vehicle for teaching specific concepts to young children. However, findings from Bustamante et al.'s (2018) US study, showed that teacher talk during this time was of low quality, with few open-ended questions being asked, with teacher talk chiefly characterised by comments or statements, with few, if any, of these relating to mathematical skills. Children's talk was characterised by answering teacher questions or repeating what had been said by the teacher. Few opportunities were afforded for children to offer their opinions on topics being discussed. Bustamante et al. (2018) concluded that, in general, CT does not provide rich learning opportunities or opportunities for children to talk, share ideas or think critically. Lundquist et al.'s (2023) study of mathematics education in a Swedish preschool found that all children participated in the same activity

during CT, offering little to no challenge for more advanced learners. Lundquist et al. (2023) suggested that the focus was on supporting the less able children rather than on differentiating learning activities to cater for all children. The article did not address how this affected the more advanced children. A final observation of CT is the inclusion of the daily calendar discussions. Both Bustamante et al. (2018) and Zhang and Quinn's (2018) studies found that such discussions were a typical feature of CT. The inclusion of these discussions is contested in the literature. The concept of time is one that young children find difficult to understand (Clements & Sarama, 2021). Beneke et al. (2008) contend that young children struggle with concepts associated with time, such as past, present and future. Bustamante et al. (2018) highlight the paucity of research in relation to CT content and pedagogical actions that occur within it.

2.5.2.6 Mathematical Talk and Discussion

Mathematical talk equates to Bishop's (1988) mathematical activity of explaining. Research increasingly shows that engaging young children in intentional mathematical discussions is highly effective and a major indicator of early mathematical understanding and future mathematical success is mathematical language ability (Purpura et al., 2011). Klibanoff et al.'s (2006) seminal study showed extensive variation in the amount of time early childhood teachers spent engaging children in conversation about mathematical concepts. This, in turn, affected the levels of mathematical language the children used themselves. The study highlighted the important need to engage children in mathematical language and ask them to vocalise their mathematical thinking. A small-scale study found that using mathematical storybooks during shared reading sessions prompted more mathematical talk from educators, especially when they were given instructional guidelines (Hojnoski et al., 2016). Indeed, the use of dialogic reading techniques was found to be successful in increasing not only mathematically specific language but also the mathematical

knowledge in young children (Purpura et al., 2017). Research conducted by Cohrssen et al. (2014) demonstrated that teachers pausing during conversations with children allowed children to think about, formulate, and articulate their mathematical thinking during play-based activities. Such mathematical interactions enable children to reconstruct mathematical understandings and to reflect on these understandings as new knowledge is constructed (Cheeseman, 2015). These mathematical discourses between educators and children encourage children to mathematise their world (Dunphy et al. 2014).

2.6 Educator Noticing: Definitions and Terminology

Aistear (GoI, 2024c) refers to the practice of noticing, interpreting and enhancing learning in children's play and everyday activity. This section begins by considering the varying definitions of the concept of professional noticing, followed by an overview of the various frameworks designed to help educators notice children's mathematics in discrete mathematics lessons and in young children's free play. Firstly, noticing is framed within the work of Goodwin (1994) on *Professional Vision* before considering the concept of professional noticing in educational contexts.

2.6.1 Professional Educator Noticing: Definitions and Theories

In his seminal paper, Goodwin (1994) describes professional vision as "discursive practices used by members of a profession to shape events" and that "this shaping process creates objects of knowledge that become the insignia of a profession's craft" (p. 606). In other words, professions collectively develop tools and ways to enable them to notice salient aspects of their work. Goodwin (1994) described professional vision as consisting of three practices, providing a basic framework for noticing in professional contexts: coding, highlighting and producing and articulating material representations of the noticed work. Coding involves interpreting what has been noticed in terms of professional knowledge.

Highlighting identifies what is important about the observation, and producing or articulating material representations can be interpreted as documenting or sharing findings with others.

To date, educator noticing research has largely focused on the teaching and learning of mathematics in primary, secondary and tertiary (educator training programmes) education, with little focus on preschool education (Parks & Wager, 2015). Within the literature, noticing has been conceptualised in different ways, including intentional noticing (Mason, 2002), adaptive teaching / responsive teaching (Gibson & Ross, 2016), and equitable noticing (Louie, 2018).

The noticing required of educators differs from everyday noticing (Louie, 2018). Noticing effectively in a dynamic educational environment is a complex and challenging skill (Ball, 2011). Effective noticing of mathematics in ECE contexts relies on educators drawing on various types of knowledge: mathematical content knowledge, knowledge of mathematical development and how to communicate mathematics in relevant and meaningful ways to young children (Priestley, 2022).

Specifically writing about noticing in mathematics education, Mason (2011) discusses the necessity for mathematics educators to engage in 'intentional noticing' (p. 36), i.e. developing the skill of purposively tuning in to children's mathematical activity. Intentional noticing is a component of the *discipline of noticing* (Mason, 2002), where educators need to respond in new ways to observe aspects of children's mathematics rather than relying on old habits when responding. Mason (2002) explains that the *discipline of noticing* comprises two techniques: pre-paring to notice in the moment and post-paring, reflecting on what has been noticed in the past to inform the response or action taken to what is being noticed now. This conceptualisation supports the discipline of noticing, advocated as a means of reflective practice. Building on the work of Schön (1983), such a conceptualisation marries well with

EC pedagogical practice, where reflection in and on practice to support development is a core component of the daily work of educators (Hayes, 2013).

Although not within the sphere of mathematics education, Gibson and Ross (2016), citing Sawer (2004), refer to the noticing process as one of 'adaptive teaching', where children, through their engagement and responses to instruction, influence classroom activity so that, consequently, teachers must adapt their guidance or teaching methods. ECEs need to be "decision-makers, who think on their feet to adjust instruction or their responses to circumstances appropriately" (Gibson & Ross, 2016, p.181). Often, these 'micro-adaptations', these in-the-moment responses to noticed aspects of children's learning, and these spontaneous assessments can offer opportunities for future learning. Professional noticing in education can then be viewed as a component of responsive pedagogy.

2.6.2 Noticing as Equitable Practice

There is a growing trend in mathematical research to establish effective methods that facilitate equitable mathematical education (Wager, 2014). Hand (2012) contends that equitable mathematics teacher noticing can be defined as, "attending to socio-political processes that privilege some forms of mathematical activity over others and position some students as mathematically capable and others as incapable" (p.242). Wager (2014) suggests that inequitable noticing of children's participation in mathematical activity and knowledge of mathematical ideas can negatively impact children's developing identities as learners and doers of mathematics, as it is the adult who determines what is mathematical activity and what is not. Adults decide which aspects of mathematics should be documented (or not). They decide what mathematics activities children should engage in and when to stop providing more challenging concepts/activities (Gillic, 2021a). Adults decide who is mathematically capable and who is not, who is worthy of attention and who is not, thereby affecting individual learner identities (Wager, 2014).

In early childhood classrooms, educators can, and do, ignore children's mathematical ideas (Gillic, 2021b; Lee & Ginsburg, 2009), thereby not giving children a chance to purposefully experience mathematics during their earliest years. This links directly to the concept of young children bearing a right to mathematics education (Cohrssen & Page, 2016) and highlights the issue of adult power in the classroom. Louie (2018) highlights that noticing is subjective; it is not an unbiased act. A clear thread in the research is that individual educators view classroom activity differently, and that these differences can be influenced by their individual experiences of mathematics education (Anders & Rossbach, 2015), educational philosophies (McCray & Chen, 2011), cultural backgrounds (Hand, 2012) and that these distinct types of experiences can influence how and what they choose to notice in the classroom (Jacobs et al., 2010). In the EC classroom, as noted elsewhere in this review EC educators can take the view that their role is to facilitate a rich learning environment and to allow the children to learn through their interactions with objects in the environment (Lee & Ginsburg, 2009; McInnes, 2019) rather than taking a more proactive role in the teaching and learning process through asking open-ended questions, engaging in math talk (Klibanhoff et al., 2006; Cheeseman, 2015), and engaging in intentional teaching of mathematics (Clements & Sarama, 2021). In the laissez-faire approach (Clements & Sarama, 2021) alluded to above, it is the teacher's beliefs and personal educational philosophies, informed by policy, curriculum or education, that determine whose mathematical activities are noted and enhanced, and more importantly, if the mathematical activity is noticed and attended to at all (Vogt et al., 2020).

2.6.3 Common Frameworks for Educator Noticing

A review of available frameworks for educator noticing mathematics (Amador et al., 2021) showed that the two most commonly used frameworks are those developed by van Es & Sherin (2002) and Jacobs et al. (2010). Similar to Goodwin (1994), Van Es and Sherin

(2002) proposed a three-part skills framework for analysing educators' noticing skills in mathematics lessons. The first skill is to 'identify what is important in a teaching situation. Educators need to decide what is worth attending to during a lesson' (p. 573). Second is to 'make connections between specific events and broader principles of teaching and learning' (p. 574), that is, connecting what is noticed to issues of teaching and learning, such as learning processes, equity, funds of knowledge, etc. The third skill is to 'use what one knows about the context to reason about a situation' (p. 575). Noticing skills develop within the specific classroom context where lessons are taught. When analysing a teaching and learning situation, educators draw on their knowledge of the subject being taught, their knowledge of how students think about the content being covered, and their knowledge of the context in which they work to determine what is noticed.

Building on the work of van Es and Sherin (2008), Jacobs et al. (2010) also propose three interrelated skills for educator noticing children's mathematical thinking: attending to children's strategies, interpreting children's understandings and deciding how to respond based on children's understandings (p.172). If educators are to successfully build on children's mathematical thinking, then they need to acquire the skills of attending to and interpreting children's strategies and then draw on a range of pedagogical strategies to be able to respond appropriately. Jacobs et al. (2010) contend that all of the above skills are employed simultaneously by the educator before they actually respond to the child.

Both of these frameworks were developed to help pre-service (van Es & Sherin, 2002) and in-service (Jacobs et al., 2010) teachers notice aspects of student mathematical thinking in primary (kindergarten to grade 4) and secondary contexts. However, Lee (2017) contends that traditional frameworks developed for educator noticing in primary/secondary contexts do not apply to play-based pedagogies, which are inherent in preschool settings.

Lee (2017) adapts the framework proposed by Jacobs et al. (2010). Similar to Jacobs et al. (2010), Lee's (2017) framework also comprises three components –

"*Noticing* mathematical situations in which children engage,

Interpreting the nature of children's math activity and

Enhancing children's mathematical thinking and understanding." (p.233)

Although the keywords have changed, the basic skills or processes remain the same as those proposed by Jacobs et al. (2010). Therefore, one could argue that the skills put forward by Jacobs et al (2010): Attending, Interpreting and Responding are essentially the same skills suggested by Lee (2017): Noticing, Interpreting and Enhancing. This is particularly the case if one views responding in terms of enhancement by engaging in mathematical talk or co-playing, where educators scaffold children's mathematical thinking. Therefore, the framework for noticing children's mathematical thinking developed by Jacobs et al. (2010) could be applied to preschool contexts and playful pedagogies, where the skills of observation, attunement and responding are key features of early childhood pedagogical practice (Dockett & Goff, 2013).

Lee's (2017) study explored the ability of 30 Korean preschool educators to notice mathematics in pre-prepared hypothetical scenarios (see McCray & Chen, 2012). Participants read the scenarios and were then asked a series of questions in relation to them. The scenarios covered several early mathematical concepts, e.g., numbers, shapes, spatial relations, measurement, pattern. Participants' levels of PCK were measured on three components of PCK: noticing, interpreting and enhancing mathematics in play scenarios. Findings showed that participants with Master's degrees in ECE showed higher levels of PCK than those with BA degrees, while participants who had 5+ years of experience also had higher levels of PCK than those with fewer than 5 years of experience. Findings also showed that the most

frequently identified mathematical concepts by participants were classification (90%), number sense (86.7%), measurement (86.7%) with pattern (63.3%), shape (46.7%), operations (46.7%), and spatial relationships (13.3%), scoring lower. These scores were interpreted by Lee (2017) as demonstrating that participants had adequate knowledge in the classification, number sense and measurement. Findings also indicated that although participants with lower levels of PCK were capable of noticing the mathematics in the scenarios, they were not necessarily capable of interpreting the scenarios mathematically. However, participants who could interpret the scenario mathematically were capable of suggesting ways to enhance mathematical thinking. Lee's (2017) findings echo those of Jacob et al. (2010), where a combination of factors, such as higher levels of experience and professional development, led to higher levels of noticing ability.

Within the concept of a responsive pedagogy, Dockett and Goff (2013) offer a two-fold model for noticing mathematics within early childhood contexts:

(a) Recognising: where adults pay attention to mathematics, both in planned and spontaneous play experiences. Within this lies the concept of an educator's mathematical content knowledge. Dockett and Goff (2013) state that in order to recognise the mathematics within the play, educators must have a sound understanding of foundational mathematics (p.773). In conjunction with mathematical knowledge, educators also need to be *willing* to notice the mathematics in the play; they need to be open to noticing mathematical ideas along with other areas of child development. Carr and Lee (2012) note that often what we notice relies on what we 'expect' to see, i.e., what we have been trained to see, e.g. developmental stage, abilities, interests and so on. Goodwin (1994) speaks of *discursive practices* (p.606); these discourses, commonly accepted within a profession, guide members of that profession to think in certain ways, helping the

profession to distinguish itself from another. As MacNaughton (2005) observes, developmental discourses, commonly found in EC textbooks and policy documents, influence the thinking of EC educators, their daily practices and decision-making processes, and can be found in the observations that EC educators document (p. 16).

(b) Responding to the mathematical content identified within a play scenario: Dockett and Goff (2013) identify three roles of the adult in responding to mathematics in children's play, which, if used effectively, can be used to both 'affirm and stretch' children's understandings (p. 773).

- role of provocateur
- sustained shared thinking
- co-construction

Finally, MacDonald (2018) provide a three-component framework for educator noticing of young children's everyday mathematical explorations: Notice, Explore and Talk. MacDonald (2018) asserts that 'noticing' is an essential component of everyday practice in early childhood settings, which allows educators to get to know individual children, what they know about mathematics, their understanding of mathematical ideas and how they apply this knowledge in their play and their daily lives (Marcus et al., 2016).

There is potential to make links between the frameworks for noticing mathematics discussed above and aspects of Goodwin's (1994) conceptualisation of professional vision. Observation, interpretation, and responding/enhancing have become part of the 'insignia' (Goodwin, 1994, p.606) of ECE professional practice. As noted elsewhere in this chapter, an ECE's beliefs about the importance of ECME and the importance of noticing and enhancing mathematical aspects of children's play are key factors in *coding, highlighting* and *interpreting* the mathematics in a child's play. Opportunities for *representation* arise from

physically documenting the observation, interpretation and plans for enhancement. However, as also noted in this chapter, a tendency remains to focus on social, emotional and language development over mathematical development in practice.

2.7 Chapter Summary

This chapter discussed the mathematical content knowledge, both subject content knowledge and horizon content knowledge, required for ECEs for ECME. The *Big Ideas of Mathematics* for ECME were also reviewed. This chapter explored the literature concerning the influence of educator beliefs towards mathematics in ECE, on self-efficacy and towards the intentional teaching of mathematics in ECE. Coupled with mathematical subject and pedagogical content knowledge, beliefs are central to facilitating a responsive pedagogy and mathematics in ECE (Anders & Rossbach, 2015; Perry & Dockett, 2013). Furthermore, Dockett and Goff (2013) also suggest that preschool educators' noticing mathematics in children's play requires critical reflection on educators' values and expectations of young children's learning. Educators need to look beyond their expectations and understanding of what children are capable of and reflect upon what they deem important for young children to learn and experience.

This chapter provided definitions for professional vision or educator noticing of mathematical activity. It has also described various frameworks for educators' noticing of children's engagement with mathematics. In essence, noticing involves the skills of observation, interpreting the meaning of the observation and responding to or enhancing the mathematics observed. Responding may occur 'in the moment' or later, giving the educator time to reflect on the child's learning and to develop further opportunities to develop the mathematical concepts being explored by the children (Mason, 2002). Noticing is shaped by educator MCK, PCK beliefs and the tools of the ECE trade, such as the skills embodied

within an observation cycle. Chapter Three presents a contextual overview of ECME and Irish educational policy development.

CHAPTER 3: GOVERNMENT POLICY DEVELOPMENT AND ECE/ECME

Chapter Two presented an overview of key findings from the literature about how best to support high-quality ECME. This chapter offers another lens. Situated within the *macrosystem* (Bronfenbrenner, 1979), it examines governmental policies which set the contexts and constraints that directly impact practice in ECE classrooms (RQ3). An analysis of government policy over time is necessary to understand the systemic factors that drive social and educational practice. It is these policies, informed by societal views, culture and ideologies that influence the culture and practice of associated microsystems (Hayes et al., 2017; Rosa & Tudge, 2013). Therefore, to understand policy is to understand culture and practices enacted at the local level. As noted in Chapter Two, Bishop (1988) asserted that what is understood as mathematics education is underpinned by socially and politically determined content as well as current social values. This is important for this thesis, which seeks to understand how young children's participation in ECME is supported in an Irish preschool setting. Mathematics education is a social process shaped by socially and politically determined content, societal values and individual learner attitudes towards the subject (Bishop, 1988

It is not within the remit of this thesis to analyse every Irish government policy concerning the development of EC and ECME; rather, I focus on key policy developments and initiatives over the last twenty-five years. This chapter also examines the mathematics embedded in Aistear and the PMC (DE, 2023a), aligning with the key ideas of ECME presented in Chapter Two. The chapter begins with an overview of ECME in the early years of primary school. An overview of ECEC policy development in the preschool sector then follows.

3.1 ECME in Primary School

This section outlines the inclusion of ECME in the primary school system, as it was in the context of the primary school that, until recently, the majority of young children (4–6-year-olds) experienced a more formal introduction to mathematics education. The curriculum introduced at the inception of the state in 1922 was to last for nearly half a decade, with few amendments. This chapter charts developments from 1971, when a revised child-led, inquiry-based approach to primary education was introduced, to the more recent revision of the Primary Mathematics Curriculum (DE, 2023a).

3.1.1 Curriculum Reform

A 1967 review of primary education informed the revised curriculum in 1971 (INTO, 1995), replacing its prescriptive, subject-based predecessor, which had been in operation for nearly half a century at that point (Murphy, 2006). The new, research-based 1971 curriculum advocated a play-based approach to teaching and learning in the infant classes (the first two years of primary school). It championed inquiry-based, discovery learning, active discussion, and the first-hand exploration of concrete materials. Teaching and learning for young children were to be based on relevant and meaningful experiences. In short, pedagogical approaches specified in the 1971 curriculum, which were holistic, developmentally appropriate, activity-based, and child-centred, were the expectation (Murphy, 2006). Although discrete subjects were included in the curriculum, they were, wherever possible, to be taught in an integrated manner (DoE, 1971; Murphy, 2006).

Regarding mathematics, the 1971 curriculum emphasised that young children should discover mathematical concepts and relationships through interaction with concrete materials within the learning environment (DoE, 1971). Mathematics education was structured around free play and continuous provision of materials, such as sand, water and natural materials

(shells, wooden blocks, etc.). It is interesting to note that activities such as threading, sequencing and jigsaws were offered to develop fine motor skills, hand-eye coordination, and left-right orientation (INTO, 1995, p.33). Mathematical affordances of these materials, such as pattern, sorting, and spatial awareness, were not alluded to.

The primary school curriculum (PSC) was revised and updated in 1999. Reiterating the importance of a child-centred, developmentally appropriate and inquiry-based curriculum as stated in the 1971 curriculum, this new document called on educators to celebrate "the uniqueness of each child" and "ensure the development of the child's full potential" (GoI, 1999b, p. 8). Just like its predecessor, this curriculum advocated for a playful approach to teaching and learning in infant classes without premature formality (GoI, 1999b). Integrated learning for the infant classes was recommended (Murphy, 2006).

The 1999 primary mathematics curriculum (PMC) defined mathematics as "the science of magnitude, number, shape, space, and their relationships and also a universal language based on symbols and diagrams' (p.2) and proposed that mathematics offered children a way to understand their environments and to participate in society. Although the strands (number, algebra, shape and space, measures and data) were presented separately, they were to be communicated to children as interrelated entities. The number strand contained a section entitled 'Early Mathematical Activities' (EMA) (GoI, 1999c) which comprised four strand units: classifying, matching, comparing and ordering. These strand units would be included in Aistear (NCCA, 2009) later. Mathematics teaching and learning were to take place through a strategy of playful guided discovery, following a constructivist model of teaching and learning.

More recently, the PSC has been reviewed and transfigured. This revised curriculum framework has at its core a focus on the development of seven key competences which apply

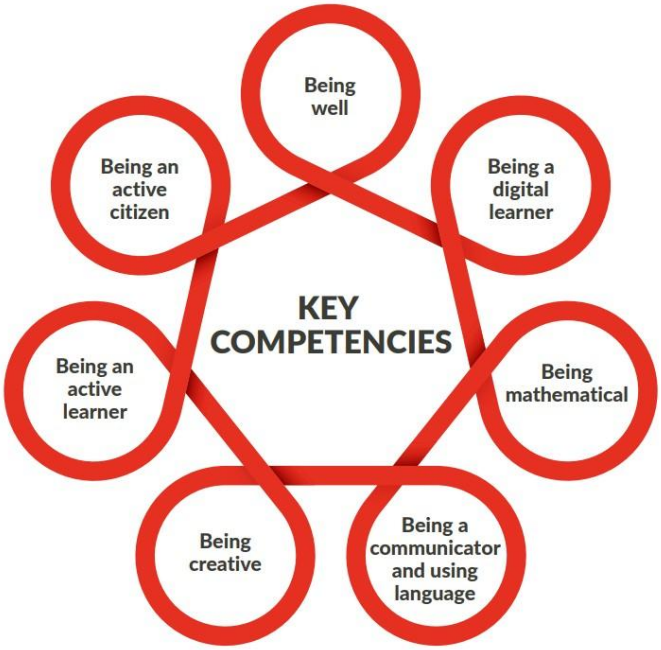
to each of the curriculum areas. One of these competences is *being mathematical* (see Figure 3.1). The PMC (DE, 2023a) defines *being mathematical as:*

- Framing real-world information and situations in mathematical terms
- Engaging in mathematical activities that involve enjoyment, effort, risk-taking, critical thinking and reflection (p.63)

Forming part of a broader primary STEM curriculum (DEY, 2025), while remaining a subject in its own right, the current primary mathematics curriculum (DE, 2023a) was developed based on two major research reports, commissioned by the NCCA (Dooley et al., 2014: Dunphy et al., 2014) and several ancillary reports (e.g. Dooley, 2019; Nic Mhuirí, 2020). This revised curriculum maintains the child-led, inquiry-based and playful approach to teaching and learning advocated by its predecessor. Situating the child as capable of engaging with mathematics, the curriculum positions mathematics as an enabler. It enables children to engage with their social and cultural worlds, supporting the idea that mathematics should be seen as worthwhile - not only for participating in society, but also for its intrinsic beauty as a creative endeavour (p. 67). In terms of mathematical content for the infant classes, much remained the same as in the 1999 version, except for the removal of the EMA strand. Aspects of this strand (classifying, matching, comparing and ordering) are now incorporated into the other mathematics strands.

Figure 3.1

Key Competencies as Outlined in the Primary Curriculum Framework (PCF) (DE, 2023b).



3.2 The Development of the Preschool Sector

In comparison to the long-established primary school sector, the preschool sector developed in a rushed, ad hoc manner. This section considers the development of the preschool sector from the 1990s, a period of economic growth and the resulting demand for childcare to support working families, to the present day. The rushed, ad hoc nature of the sector's establishment resulted in a fragmented system with many government departments responsible for overseeing sector development and administration (Walsh, 2016). The implications of this process can still be felt today.

3.2.1 Child Care Act 1991 & Child Care Regulations (1996; 2006; 2016)

The Childcare Act (1991), part VII, was Ireland's first regulatory framework for the early childhood care and education sector (Egan & Garrity, 2022). The Act granted responsibility to the Department of Health to monitor the welfare, safety and development of

young children in early childhood services, with inspectorate roles allocated to health professionals rather than those working in ECEC. This could be attributed to the lack of graduate professionals working in ECEC contexts when the early childhood sector was emerging as a professional entity, or, as Egan and Garrity (2022) posit, an inspection process centred on health and safety, rather than from an education and care perspective. These regulations outline minimum requirements that settings are required to meet, and penalties are imposed if requirements are not met. The 1996 regulations were revised in 2006, with the introduction of *Regulation Five*. Regulation Five placed an onus on inspectors to assess how the 'learning, development and well-being' of young children were catered for in EC settings. These revised regulations did not state the need to employ qualified staff in EC services; rather, 'a sufficient number of suitable and competent adults' were required to work directly with the children at all times (Regulation 8.1). (*Child Care (preschool-services)(No.2) Regulations, 2006*). Inspections took place under the newly established Health Service Executive (HSE), before transferring to TUSLA (Child and Family Agency) in 2014 (Egan & Garrity, 2022). Egan and Garrity (2022) suggest that the retention of the health service as an inspectorate cemented the health and safety model of the inspection process.

The 2016 revision of the Childcare Regulations introduced minimum qualification requirements for the sector, with a minimum level 5 qualification in ECEC – the equivalent of one year of study, being recommended for the work involved. As detailed later in this chapter, education-focused inspections, overseen by the Department of Education and Skills, were introduced for the sector in 2015 (Duignan & McDonnell, 2022), thereby subjecting EC settings to two inspection processes. Hayes (2022) notes that both inspection processes were intended to supplement each other; however, in reality, the dual inspection process contributes to the 'structural split between care and education' (p.49).

3.2.2 Ready to Learn (GoI, 1999a)

Drawing on the United Nations Convention on the Rights of the Child (UNCRC) and the young child's right to a high-quality education, *Ready to Learn* (GoI, 1999a), was the first Irish educationally focused policy paper to recognise early childhood as the period from birth to six years. Published at a time of curricular reform within the primary education sector, this white paper noted that high-quality preschool experiences contribute to the development of necessary learning dispositions that support young children's successful transition to more formal schooling. A chief focus was on supporting early education for children from disadvantaged areas and those with additional educational needs. To this end, the paper outlined the need to develop both a universal system of preschool education, combined with a highly-qualified and skilled EC workforce, as a means to support the educational needs of all children. A universal curriculum (methodology and content), quality guidelines and a specific EC inspection system were also to be developed. The paper defined readiness to learn in the following way:

> 'Readiness to learn relates to the fact that children who begin schooling with solid foundations in place will be better placed to develop to their potential. It involves many aspects, including health, social and emotional development, language use, and general knowledge' (p.15)

It is noteworthy that, although the definition promotes a holistic approach to early childhood education, it makes no specific mention of mathematical or numeracy learning and development. This is surprising considering that the document concerns education from birth to six years, including children attending primary school. The term numeracy occurs five times within the document. However, the focus is on parental capacity and confidence in numeracy to help support their child's learning, as well as noting the inclusion of numeracy in international EC curricula rather than its inclusion in preschool education in Ireland. The

following sections detail actions arising from objectives outlined in *Ready to Learn*: universal quality and curriculum frameworks, universal access to early education and the inspection process.

3.2.3 The National Early Childhood Frameworks: Aistear and Síolta

Policies, such as quality and curricular frameworks, conceived and produced at government level, contribute to societal thinking in a given social arena. Referred to as the pillars of practice (Hayes, 2013) for early childhood education and care and central to early years practice and pedagogy in Ireland, *Síolta, the National Quality Framework for Early Childhood Education* (CECDE, 2006), and *Aistear, the Early Childhood Curriculum Framework* (NCCA, 2009) will be explored here. Initially, these key documents were introduced to ensure consistency of quality practice and pedagogy in early childhood settings across the 0-6 age range.

3.2.3.1 Síolta: The National Quality Framework for Early Childhood Education

Published in 2006, Síolta: The National Quality Framework for Early Childhood Education aimed to bridge the gap between care and education, which characterised the Irish early childhood landscape at the time (Dunphy & MhicMathúna, 2022). The strategy was to develop a quality framework for all those working with children from birth to six, including the first two years of primary school. Síolta positions the child as a rights-bearer, who has a right to a high-quality early education. It advocates for a responsive and play-based pedagogical approach to early learning. Síolta (CECDE, 2006) offers a framework for self-reflection on practice based on 16 standards and accompanying components of quality practice.

3.2.3.2 Aistear: The Early Childhood Curriculum Framework

Closely aligned to Síolta (CECDE, 2006) and stemming from Ready to Learn (Walsh et al., 2024), Aistear, the Early Childhood Curriculum Framework, was introduced by the NCCA in 2009. Similar to Síolta (CECDE, 2006), Aistear (NCCA, 2009) adopts a socio-cultural perspective on ECE, advocating that learning arises from social interactions. Like Síolta, Aistear positioned young children as rights-bearers and citizens of the world, central to their own communities. Young children are viewed within Aistear (NCCA, 2009) as unique individuals who are active learners, who learn by interacting with the people and objects in the world around them, echoing the vision of early childhood pioneers (see Section 3.1). They learn holistically, exploring the world through their senses, and actively engaging with people and things through play. Aistear (NCCA, 2009) asks adults to celebrate each child's uniqueness and individual interests, strengths, and abilities. Aistear (NCCA, 2009) perceives adults as guides or facilitators of early learning, devising learning experiences that are relevant and meaningful in nature. The adult is a provider of rich, enabling learning environments where young children are stimulated, challenged, and empowered. The updated version (GoI, 2024c) maintains this vision while also highlighting the role of the educator as an agentic one where pedagogical actions such as reflecting, noticing, nurturing and responding play a key role in facilitating early learning experiences, through a slow-relational pedagogy. In terms of numeracy, Aistear (NCCA, 2009) simply defined numeracy as, 'Numeracy involves developing an understanding of numbers and mathematical concepts' (NCCA, 2009, p.56). The updated version provides a more detailed view of numeracy:

> Emergent numeracy: Babies, toddlers and young children develop a growing understanding of mathematical concepts in the world around them. Numeracy in early childhood involves the exploration and development of mathematical skills, concepts, knowledge and understanding. Through play and hands-on experiences, babies, toddlers and young children see and interact with concepts such as number, shape and space, size, volume, etc. as they become curious about and build an awareness of their functions and purpose (GoI, 2024c, p. 31).

Note that the term numeracy is used rather than mathematics in Aistear (NCCA, 2009; GoI, 2024c), aligning with adopted terminology in policy documents (e.g. *Ready to Learn*, GoI, 1999a; DES, 2011). This distinction between the terms mathematics and numeracy is common in EC discourses, where the term numeracy is often preferred as it is perceived to align with the favoured socio-cultural perspective, which advocates for discovering, collaboration, problem-solving, and children's application of emerging mathematical knowledge and skills in their everyday lives (O'Neill & Gillic, 2024). In Aistear (NCCA, 2009), learning goals for mathematics were indicated in the two themes of *Communicating* and *Exploring and Thinking*, as detailed in Table 3.1 below:

Table 3.1

Mathematical Concepts and Skills in the Theme of Communicating (NCCA, 2009)

Communicating	
Aim	Learning Goals
Aim 3 Children will broaden their understanding of the world by making sense of experiences through language	In partnership with the adult, children will: Use language to interpret experiences, to solve problems, and clarify thinking, ideas, and feelings Build awareness of the variety of symbols (pictures, print, numbers) used to communicate, and understand that these can read by others Develop counting skills, and a growing understanding of the meaning and use of numbers and mathematical language in an enjoyable and meaningful way

Table 3.2

Mathematical Concepts and Skills in the Theme of Exploring and Thinking (NCCA, 2009)

Exploring and Thinking	
Aim	Learning Goals
Aim 1 Children will learn about and make sense of the world around them	Develop a sense of time, shape, space, and place. Come to understand concepts such as matching, comparing, ordering, sorting, size, weight, height, length, capacity and money in an enjoyable and meaningful way.
Aim 2 Children will develop and use skills and strategies for observing, questioning, investigating, understanding, negotiating, and problem-solving, seeing themselves as explorers and thinkers	Recognise patterns and make connections and associations between new learning and what they already know. Demonstrate their ability to reason, negotiate and think logically. Collaborate with others to share interests and to solve problems confidently. Use their creativity and imagination to think of new ways to solve problems.
Aim 3 Children will explore ways to represent ideas, feelings, thoughts, objects, and actions through symbols.	Make marks and use drawing, painting, and model-making to record objects, events and ideas. Become familiar with and associate symbols (pictures, numbers, letters, and words) with the things they represent. Build awareness of the variety of symbols (pictures, print and numbers) used to communicate, and use these in an enjoyable and meaningful way, leading to early reading and writing. Use letters, words, sentences, numbers, signs, pictures, colour and shapes to give and record information, to describe and make sense of their own and others' experiences.
Aim 4 Children will have positive attitudes towards learning and develop dispositions like curiosity, playfulness, perseverance, confidence, resourcefulness, and risk-taking.	Develop higher-order thinking skills such as problem-solving, predicting, analysing, questioning, and justifying.

As can be seen from Tables 3.1 and 3.2 above, mathematics features explicitly in the themes of *Communicating* and *Exploring and Thinking*. The remaining two themes, *Well-being* and *Identity and Belonging,* contain learning goals that refer to the development of learning dispositions necessary for mathematics: curiosity, persistence, perseverance, resilience, self-confidence, determination, creativity, and skills such as collaboration, negotiation, risk-taking, reflexivity, motivation, and self-assessment (NCCA, 2009). The 2009 iteration of Aistear asked educators to engage young children in a broad range of mathematical concepts, skills, and dispositions, which were mirrored in the 1999 Primary School Mathematics Curriculum (GoI, 1999c), affording consistency in approach and content from preschool to the first two years of primary education. This broad focus is emphasised in a recent literature review to support the updating of Aistear (French & McKenna, 2022). However, a draft of the proposed updated version of Aistear (NCCA, 2023) did not state the wide variety of mathematical concepts as in the 2009 edition. In fact, the document muted rather than highlighted mathematics education in ECEC settings. Table 3.3 shows these redrafted proposals for the learning goals for *Exploring and Thinking*.

Table 3.3

Proposed Aims and Learning Goals for Exploring and Thinking (NCCA, 2023)

Exploring and Thinking Through nurturing relationships within a supportive environment, babies, toddlers, and young children will:	
Aim 1 Babies, toddlers and young children will learn about and make sense of the world around them	Have opportunities to explore their world through integrated artistic, linguistic, geographical, historical, scientific, technological, engineering and mathematical learning experiences through play
Aim 3 Babies, toddlers and young children will have opportunities to connect with nature and spend meaningful time in the outdoors	Develop sensory awareness of colours, shapes, smells, sounds, and movement; challenging themselves in adventurous and active play.
Aim 4 Babies, toddlers and young children will have positive attitudes towards learning and develop dispositions like independence, curiosity, playfulness, perseverance, resilience, confidence, resourcefulness and risk-taking	Demonstrate growing confidence in being able to do things for themselves, seeing themselves as agentic learners open to new ideas and experiences. Develop higher-order thinking skills such as problem-solving, predicting, analysing, questioning and justifying through play and playful explorations. Know that their voice is heard and responded to in sharing their ideas, thoughts and questions on their learning journey.

It is clear from Table 3.3 that it was proposed that specific mathematical content be removed from the national early childhood curriculum framework. While the mathematical processes (problem-solving, analysing, predicting, questioning, and justifying), as well as learning dispositions (seeking and welcoming challenge, curiosity/wonder), were clearly indicated, specific mathematical concepts were not. Hayes (2013) asserts that 'practice aimed at encouraging the development of learning dispositions and meta-cognitive skills cannot be content-free' (p.53), suggesting that learning environments must provide challenging content and associated language. Hedges and Cullen (2005), cited in Hayes (2013, p.53), argue for subject content knowledge as one of their proposed four pillars of early childhood practice. A

curriculum framework should outline the content that students should engage with and act as a scaffold to support educators in planning their curriculum. The proposed version of Aistear (NCCA, 2023) could have been interpreted as loosening the emphasis on early mathematics. Given that Aistear (NCCA, 2009) was a source of curriculum content, this de-emphasis on mathematics was worrying on many levels. This position did not align with the tenets of national policy (e.g. DES, 2011, 2017a; DES, 2017b) and national research (Dooley et al., 2014), which clearly emphasised the need for young children to engage in a broad range of mathematical activities. It may also have inadvertently contributed to the belief that early mathematics is not important in the pre-school years, and on a classroom level, educators with no previous training in early mathematics education would not have had a framework to guide them in their work. However, after further consultation with the wider EC sector, the updated version of Aistear (GoI, 2024c) included a specific reference to mathematics. Again, presented in list form, children:

> 'through nurturing relationships within a supportive environment, babies, toddlers and young children will: 'come to understand concepts such as matching, comparing, ordering, sorting, estimating, counting, spatial awareness, size, weight, height, length, capacity and money through súgradh (play) and playful experiences in an enjoyable and meaningful way.'(p.29).

In 2009 specific reference was made, under the theme of Communicating, to the use of mathematical language, with time and pattern being included in one of the lists of mathematical concepts presented under the theme of Exploring and Thinking. The lack of reference to mathematical language or engaging children in Math Talk in the revised version is concerning, given the importance afforded to it, both in the research literature (e.g. Klibanoff et al., 2006; Purpura et al., 2011) and in the recently updated PMC (DE, 2023a). The current version includes a reference to a supportive learning environment and is reflected in the theme of Communicating. Here, the focus is on the written communication of

mathematics, awareness of symbols and exposure to a 'print-rich environment' (p.27). Shape is presented separately, under Aim 3, where children develop a sensory awareness of shapes. There is no reference to learning the names of shapes, to comparing them, or observing their properties. This could suggest to ECEs that such engagement in mathematical talk is unnecessary and should remain within the realm of primary school.

Pattern was not listed alongside other mathematical concepts in the 2009 version of Aistear, but was mentioned separately. It was described as the ability to 'recognise patterns and make connections and associations between new learning and what they already know' (p. 44). This wording does not afford 'pattern' a strong mathematical foundation and may have led educators to see it as about making connections across different areas of knowledge rather than about mathematics. In the revised Aistear (GoI, 2024c), pattern is alluded to under the theme of Communicating and referred to as 'exploring sound, pattern, rhythm, rhyme and repetition' (p.27), suggesting a linguistic rather than a mathematical interpretation of the term. Gripton (2022, 2023) contends that the positioning of pattern within early childhood curricula often lacks prominence, despite the importance afforded to it in the research literature (e.g. Brownell et al., 2014; Rittle-Johnson et al., 2019) in terms of future mathematical understanding and development. This is in contrast to its inclusion from Junior Infants to sixth class in the PMC (DE, 2023a).

Number Operations are excluded from the list of mathematical concepts for ECE in Aistear (GoI, 2024). This is strange given their inclusion in the big ideas for ECME proposed by Perry and Dockett (2008), Sarama and Clements (2009) and those by Brownell et al. (2014). They are included in the PMC (DE, 2023a) for Stage 1. This section has presented a discussion on the development of Aistear and the inclusion and exclusion of mathematical concepts within it. The next section considers the introduction of the ECCE scheme, which provides free preschool education under the guidance of Aistear and Síolta.

3.2.4 The Early Childhood Care and Education (ECCE) Scheme

Government commitment to Aistear/ Síolta, and to ECE more broadly, was evident through the introduction in 2010 of the Early Childhood Care and Education (ECCE) scheme. This initiative provided, for the first time, a universal right for children 3-4 years old to access preschool education for the year before school entry (Hayes, 2022). Children were afforded fifteen hours of preschool education and attended an ECCE class for five days and three hours each day for 38 weeks of the preschool year, aligning with the primary school year. The scheme proved incredibly successful with high uptake, with 94% of eligible 3-4-year-olds in ECCE classes in 2013. The scheme was extended for two years in 2016, when children aged 3 could attend for two years before transitioning to primary school. In 2018, the scheme was extended again, and children from 2 years and 8 months could attend an ECCE class (Walsh et al., 2024). Government figures show that in 2019, 806359 children took part in the scheme, rising to over one million in 2022. Government figures show that in 2022, 17799 foreign national children were present in the country. While no figure for how many of these children attended ECE settings, it could be suggested that many of these children do attend a preschool setting, thereby increasing the number of children with EAL in EC settings. One outcome of the scheme is a rise in the age of young children attending their first year of primary school (the Junior Infant class), with the majority of children now starting primary school at five (Walsh et al., 2024). To avail of funding for the ECCE scheme, as set out in their contracts, early childhood settings must use both Síolta and Aistear to guide their practice (French, 2013). As a result, children attending an ECCE class would potentially be exposed to mathematical concepts in an educational setting under trained educators. As noted elsewhere in this section, the qualification required in 2010 was a level five award in ECEC. This requirement was raised to a level 6 qualification in 2016. Settings that employ a level six lead educator receive €69 per week per child. To encourage settings to employ ECEC

graduates as lead educators, a higher capitation grant of €80.25 per child per week (gov.ie, 2023) is currently offered as a measure to improve quality in ECCE rooms.

3.2.5 Department of Education Early Years Focused Inspections

Introduced in 2016, the Department of Education Early Years Focused inspections originally served those settings which offered the ECCE scheme (2 years 8 months, and five years). The inspection process was recently revised, and now inspections are conducted in settings caring for children from birth to six (DE, 2025). Settings are notified of the intended inspection. The inspection team, guided by the *Guide to Early Years Inspection* (DE, 2025), assesses the quality of learning, learning interactions, and learning environments as well as the appropriateness of the activities provided for young children. Evaluation of practice is guided by four principles: 'The child at the centre of practice, the child as an active learner, building strong connections to support children's learning, and laying the foundations for children's learning' (DE, 2022, p.5). Underpinned by both Aistear (GoI, 2024c) and Síolta (CECDE, 2006), the inspection guide considers practice in relation to the centrality of play as a pedagogy for learning, the use of a strengths-based approach to planning for individual children, holistic development, children's well-being and the adult role in facilitating high-quality learning experiences. Four areas of practice are examined:

- The quality of the context to support children's learning and development
- The quality of the processes to support children's learning and development
- The quality of children's experiences and achievements, and
- The quality of management and leadership for learning (DE, 2022, p. 10)

Appendix 1 of the Inspection Guide details the Signposts for Practice to be considered by members of the inspectorate.

Table 3.4 below shows the signposts for early mathematics education, which are located under Area 2 – Quality of Processes to support children's learning and development:

Table 3.4

References to Mathematics in A Guide to Early Years Education Inspection (DE, 2025)

Learning Outcome	Signposts for Practice
10. Emergent language, literacy, and numeracy skills are fostered	Early years educators model appropriate language, including mathematical language, and encourage expanded use of vocabulary through the use of open-ended questioning and language enrichment during interactions Mathematical thinking and learning is promoted through the use of open-ended resources and games, linked to the everyday lives of children Children have opportunities to engage with activities, both indoors and outdoors, that build early positive dispositions towards science, technology, engineering, the arts and mathematics (STEAM) through hands-on, active experience
15. Children communicate their experiences, thoughts, ideas and feelings with others in a variety of ways	Children demonstrate an awareness and emergent understanding of the meaning and uses of symbols, pictures, print and numbers as a means of communication Children have a growing understanding of the meaning and use of mathematical language

The table above shows that ECEs are expected to use mathematical language with children and to provide early learning experiences that help them understand mathematical representations (such as symbols and numbers) through active, hands-on, everyday routines. Positive dispositions towards mathematics are to be fostered. Much of the language is consistent with that in the current and previous iterations of Aistear.

It is intended that the inspection process support the role and work of the pedagogical leader, through reflective, professional conversation between themselves and the inspector. A key part of the inspector's role is to assist staff in reflective self-evaluation of practice. This

aspect of the inspector role is detailed in the Interim Review of the NLNS (DES, 2017b), which states,

> "The Department of Education and Skills' Early Years Inspectorate will encourage the practitioners to develop their self-evaluation processes within the setting to assess current levels of numeracy provision and plan further learning opportunities" (p.32).

The interim review (DES, 2017a) also notes that

> "The team also provides guidance and advice to practitioners to ensure that children have daily exposure to key early numeracy concepts, experiences and materials. These concepts include exploration by children of number words and symbols, shapes, counting patterns, spatial awareness, measurement and data analysis all experienced through the medium of play rather than didactic practices" (p.28).

It is interesting to note the range of mathematical ideas presented in the statement above. This is inconsistent with the mathematical ideas suggested in Aistear (GoI, 2024c), the LNDLS (GoI, 2024a) and in the *Guide to Early Years Inspections* (DE, 2025). One could ask, as the Guide is freely available to setting staff online, is there a possibility that educators may think that the basic ideas outlined in the Guide are representative of what is expected in terms of early mathematics education? Nine early years inspectors were appointed at the time of the introduction of the education-focused inspections, with the number rising to 19 in total in 2020 (Duignan & McDonnell, 2022). To be appointed, inspectors are required to possess a degree in ECEC as well as five years of leadership experience. In addition to the EC graduate inspectors, two members of the EY inspectorate team have a background in early childhood education at the primary level (Duignan & McDonnell, 2022). Supports for setting staff include the provision of verbal feedback post-inspection as well as a written report. This report is sent to the setting for approval. Recommendations for improvement are made, and settings are given a certain time frame to make the required changes. A follow-through inspection is then carried out to note the implementation of changes. Settings have the right to appeal any judgment they do not agree with. The report is posted on the Department website and is publicly available.

3.3 The National Literacy and Numeracy Strategies

Acknowledging the importance of numeracy skill development from early childhood to adulthood, *Literacy and Numeracy for Learning and Life, The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020* (DES, 2011) set 'ambitious targets' to improve the quality of teaching and learning of mathematics across all educational sectors (p.5). Recognising the importance of the early childhood phase (birth to six years) as a critical period for growth, learning and development, and the important role that ECEC educators play in this development, a key objective of the strategy was to develop a highly skilled workforce in delivering the aims and targets outlined. To this end, the strategy recommended that all ECEC courses should include both mathematical content and pedagogical content knowledge. Further objectives stated the need to provide detailed guidance for ECEC educators (e.g. handbooks, online courses) in best practices in the teaching and assessment of early numeracy, as well as cross-sectoral professional development (PD) for preschool and primary school educators. To date, PD materials (videos) are available on the Aistear Síolta website. However, unlike their counterparts in primary education, where PD is provided by the State agency Oide, no State-sponsored handbook or formal PD course in ECME has been made available for early childhood educators. The strategy also noted that early numeracy education should avoid premature formality, and take place in everyday, routine activities through a broad, holistic and developmentally appropriate educational programme (DES, 2011). Aligning with research (Ginsburg, 2006; Pramling Samuelson & Johansson, 2006), the strategy stated that play pedagogy is central to learning and development in early childhood education. It also stated the need to guarantee that young children's early mathematical experiences were guided by Aistear (NCCA, 2009).

A mid-way review of the 2011-2020 strategy was conducted in 2017. This review noted a stronger focus on literacy than numeracy for the first half of the strategy's duration and that 'a stronger focus on numeracy is now warranted' (DES, 2017, p.7). A key action of the plan for the remaining years of the strategy was to strengthen children's transition from EC settings to primary school. In 2018, the *Mo Scéal* templates were developed by the NCCA to support this transition. Completed with parents, these templates support early childhood educators to share the 'story' of a child's learning journey, detailing aspects of learning and development across the four themes of Aistear (GoI, 2024). Two versions of the Mo Scéal template exist. Template one is generic and provides a space for educators to detail a child's learning journey under each of the Aistear themes. Template two also supports reporting on children's learning and development in line with the Aistear themes. Each theme includes a set of statements (e.g., "The child has good fine motor skills"), and educators indicate the child's progress using a Likert-type scale (always, most of the time, sometimes, rarely). There are no references to mathematics (e.g. counting) in the comments for Communicating, despite counting and the use of symbols being included in this theme in Aistear (NCCA, 2009). Rather, this is included under the theme of Exploring and Thinking, along with the following comments: 'child uses mathematical ideas in everyday interactions, child enjoys using numbers, child is developing a sense of shape and space, and child enjoys solving problems with others' (NCCA, n.d.). The use of these templates is not mandatory, and other transition documents exist. *My Transition Flower* (Appendix A) is another example. This template is freely available from the Health Service Executive (HSE) website. Again, the completed template is shared with parents and ideally forwarded to the child's intended primary school. The template presents a series of skill-based comments for educators to highlight once achieved by the child. The math-related comments presented under *Thinking Skills* include:

*Knows some shapes, e.g. circle, triangle, rectangle

* Understands matching, sorting, ordering

* Understands some numbers using concrete objects

* Counts to 5

* Recognises most colours

* Likes doing jigsaws (HSE, n.d.).

There is also a space for educators to add further information about the child's learning and development. Again, there is no obligation to use this or any template to share aspects of a child's preschool learning journey with a primary school at present.

Pillar two of the Interim Review of the DES (2011) strategy focused on 'improving teachers' and ECEC practitioners' professional practice' (p.23). Developments in this area included the online publication of the Aistear Síolta Practice Guide (NCCA, 2015), the continued implementation of Aistear (NCCA, 2009) and the introduction of the DES Early Years Education Focused Inspections. The key role of the Inspectorate in supporting early childhood settings to engage children in playful mathematical learning experiences, rather than didactic approaches, was identified (DES, 2017). The inspectorate would also aid educators in understanding the 'sequence in which children learn early mathematical ideas' and 'gain a deeper understanding of numeracy concepts' (p.31). A question could be asked if members of the inspectorate team understand these sequences. Although inspectors hold ECEC qualifications, they may not have studied ECME in their courses. As mentioned earlier, not all degree programmes include ECME modules, and there are few professional development opportunities in this area for the sector. The interim review proposed that the inspectorate support settings in completing a self-review with due attention to literacy and numeracy development in the setting. This is a heavy workload for a team of only 19 early childhood inspectors and 4022 settings contracted to deliver the ECCE scheme (Pobal, 2022).

In relation to workforce development, a new target of upskilling educators with a level five qualification in ECEC to at least a level six qualification, set out in the Interim Review (DES, 2017a) was now on the agenda. There was also a focus on increasing the mathematical content in ECEC degree programmes. An additional action under Pillar Three of the Interim Review (DES, 2017a) focused on developing leadership for literacy and numeracy within early childhood settings. This was to be achieved through engaging with PD through the Aistear/Síolta initiative and reflective practice on numeracy provision in line with the themes, aims and learning goals of Aistear (NCCA, 2009).

2023 saw the publication of *Towards a New Literacy, Numeracy and Digital Literacy Strategy: A Review of the Literature* (Kennedy et al., 2023). This collection of systematic reviews of systematic literature reviews included a specific review focused on mathematics in ECE (O'Neill, Gillic & Kingston, 2023). This ECME-specific review found strong research evidence that play is an effective way for young children to engage with and explore mathematical ideas and concepts, particularly when combined with intentional teaching of problem-solving through real, first-hand experiences. The review highlighted the role of the adult as a critical component of high-quality ECME, foregrounding the necessity for knowledge of the sequence of mathematical development, mathematical content and pedagogical content. Therefore, mathematical content should be a feature of all ECEC training programmes. (pre-service and in-service). PD should include the use of pedagogical strategies such as picture book discussions, maths talk, enabling environments and mathematics for all children within the birth to six age range. Some of the above findings and recommendations are featured in the original and interim strategies; however, this recent literature review strongly connects the targets and actions outlined in the strategy to current research and supports in developing a strong rationale for numeracy/mathematics in early childhood settings.

The revised Literacy, Numeracy and Digital Learning strategy was introduced in 2024. It is noteworthy that it was jointly presented by both the Department of Education (DE) and the Department for Children, Equality, Disability, Integration, and Youth (DCEDIY), where the previous iteration was presented by the Department of Education and Skills alone. This joint presentation demonstrated that numeracy learning and development from birth onwards was a focus of government/ inter-departmental policy. Early mathematics (numeracy) was firmly on the educational policy agenda. Not only did the strategy emphasise the significance of numeracy, but it also highlighted the importance of math talk and discussion as a key pedagogical strategy to support young learners in participating in the mathematical processes of problem-solving, reasoning, connection making and critical thinking. Learning dispositions, such as those listed in Aistear (GoI, 2024c) of perseverance, concentration, and creativity, were also deemed important to foster in a child's early years. The strategy lists mathematical concepts young children should encounter in pre-primary education:

> Collecting, sorting, sequencing, ordering, and identifying and creating patterns.
>
> Developing skills such as counting, doing calculations (adding, subtracting, multiplying, and dividing), measuring (weight, length, height, capacity, volume, money, time), estimating, and predicting.
>
> Recognising and using symbols and engaging in mathematical mark making (GoI, 2024a, p.31)

Note the inclusion of 'doing calculations', 'creating patterns', and 'time', none of which are included in the list of mathematical concepts that preschool children should engage with in Aistear (GoI, 2024c). It is important to highlight that the numeracy elements of the strategy originated from the research of mathematics educational researchers, whereas Aistear had a broader scope of development. Despite both documents being published during the same year, there is little evidence to suggest inter-departmental communication, which could

potentially confuse the EC community as to which mathematical concepts should be engaged with in early childhood settings.

On a positive note, plans are in place to support ECEs in their work supporting children's participation in numeracy/mathematics. The accompanying strategy implementation plan (GoI, 2024b) proposed the establishment of Communities of Professional Practice as well as the development of professional learning programmes for ECEs in STEM. The implementation plan outlines intentions to develop guidelines to support lecturers on undergraduate ECE programmes in relation to mathematical content, pedagogy and assessment for student ECEs. This task was allocated solely to the DCEDIY, suggesting that primary teachers lie firmly within the remit of the Department of Education and Youth (DEY), and are supported and inspected by branches of this department, while ECE is overseen by the DEY inspectorate but supported by the DCEDIY. An observation is that the implementation plan outlines plans to support teachers in primary schools in their subject knowledge in the areas of shape, space and data, an area of mathematics where children are underperforming (McHugh et al., 2024). This professional learning opportunity is not extended to the ECE community. The implementation plan will be revised in 2028. Time will tell if the intended plans are enacted.

3.4 Professional Award Criteria and Guidelines for Undergraduate ECEC Degrees

In 2018, a draft version of the Professional Award Criteria and Guidelines for undergraduate degree-level programs, devised by the Early Years Education Policy Unit (EYEPU), was made available to the HEI sector for consultation. The draft guidelines were also informed by a survey of ECEC educators, seeking their perspectives on their training and perceptions of the knowledge and skills required to work in early childhood education and care settings (DES, 2016).

Following the consultation period, an evaluation report (Fillis, 2018) was published. The report concluded that there was a consensus among providers of undergraduate degree courses that the professional competencies presented in the CoRE report (Urban et al., 2011) applied to the Irish ECEC context, and consistency across all programmes was desirable, with room for individual institutions to decide on content and/or method of delivery. Only one of the respondents (OMEP Ireland[2]) called for a change of wording in relation to mathematics. They suggested that

> 'generating an appropriate curriculum that stimulates emergent literacy, maths and science skills' be changed to, 'generating an appropriate curriculum that promotes positive dispositions such as creativity, curiosity and perseverance' (Fillis, 2018, p.7).

OMEP suggested the overt reference to specific subject labels to more generic learning dispositions. The suggested removal of literacy, maths, and science is strange, given that two of these subject areas, literacy and mathematics, are explicitly noted within Aistear, and scientific skills of observation, questioning, reasoning, etc, are embedded within the learning goals for Exploring and Thinking. Perhaps, as it is understood that the content of Aistear (NCCA, 2009) forms a chief component of early childhood degree programmes generally, the overt inclusion of 'literacy, maths and science skills' was deemed unnecessary.

The final version of the guidelines was published in 2019 by the DES. The guidelines are a tool for providers of ECEC degree programmes to 'design and deliver a course…that develops the necessary values, knowledge(s) and practices in their graduates to enable them to practice as ELC professionals' (DES, 2019, p.9). Again, it was emphasised that the guidelines were not intended to produce homogeneity or to standardise delivery and content across courses offered, but rather 'outline essential components in the professional formation

[2] OMEP Organisation Mondiale pour L'Education Pre-Scolaire [World Organisation for Early Childhood Education]

of ELC professionals' (p.11). A review of the Knowledge, Practices and Values in the finalised document, shows that the Knowledge Competence '*knowledge of different strategies of learning (play-based, social learning, early literacy, and numeracy…*' (DES, 2019, p.16) was retained, and the associated practice competence had a slight change in wording (in response to the OMEP suggestion noted above), to '*generating an appropriate curriculum that stimulates and promotes positive dispositions, emergent literacy, maths and science skills'* (DES, 2019,p.17).

One could argue that although the majority of respondents welcomed the idea of consistency across courses with the affordance of autonomy in programme content and delivery, leading to some heterogeneity, this has led to some institutions cherry-picking content they felt was more important than others. This is particularly evident in relation to mathematics/numeracy. Gillic et al.'s study (under review, 2026) confirms this. Findings show that while the majority of ECEC undergraduate courses do include some mathematics/numeracy education content in their programmes, others do not. Only two higher education institutions offer a stand-alone module for early mathematics education; the remaining courses on offer combine mathematics education with literacy education, or are located within a dedicated early STEM education module. Therefore, not all early childhood education graduates have the depth and breadth of knowledge required for early childhood mathematics education as recommended by the DES (2017b) and in the NLNDLS (DES, 2024a).

3.5 Nurturing Skills

The current workforce development plan *Nurturing Skills: The Workforce Plan for Early Learning and Care and School-aged Childcare 2022 – 2028,* was published by the DECDYI in 2021. This version of the workforce development plan clearly defines the roles and activities of persons working within an ECEC setting. The title *Early Years Educator* is

given to every qualified member of staff, irrespective of qualification. *Lead Educator* is given to a staff member who 'leads practice with a group of children' (p. 19). Furthermore, the document states that the term Lead Educator implies a pedagogical leadership role, not only leading and guiding children but also other adults who work within the room alongside them. In terms of mathematics, both Early Years Educators and Lead Educators must be capable of executing appropriate pedagogical strategies to support emergent numeracy and STEAM, while managers are expected to ensure the development and implementation of pedagogical strategies to support children's emergent numeracy and STEAM skills.

3.6 Chapter Summary

This chapter has presented an outline of the development of early childhood education and care policy in the Republic of Ireland since the 1990s. It described an ECE landscape that is continually evolving and shaped by continuous research and political leanings. However, some aspects of early childhood practice have remained stable and consistent across policy and curricular documents. Those, such as active, hands-on learning with real objects, the child as central to the learning process, high-quality learning environments that facilitate and encourage supportive, reciprocal interactions, and the need for a play-based pedagogy, are reflected in Síolta (CECDE, 2006), Aistear (NCCA, 2009; GoI, 2024c), and the DE early years inspection guide (DE, 2025). All of the above are central too, for high-quality ECME, nonetheless, mathematics has at times fallen victim to the prevailing thinking of certain eras, as exemplified by the proposed omission of mathematical concepts in Aistear (NCCA, 2023), to the opposition of the inclusion of numeracy or mathematics in the professional award guidelines for EC undergraduate programmes by OMEP preferring the inclusion of learning dispositions instead.

This chapter highlighted the gap in ECME training for those undertaking courses in EC and for those who deliver ECME on undergraduate courses. Time will tell if the promised

PD for ECEs in numeracy (GoI, 2024b) will occur. The content of such PD will also be interesting, with differences in Aistear and the Literacy, Numeracy and Digital Literacy Strategy (GoI, 2024a) in relation to mathematical concepts to be encountered in early childhood settings. As the words of Urban et al. (2017, p.5) remind us,

> "Central to a competent system is that shared orientations are not only required of practitioners 'on the ground' but of all professionals and institutions that together constitute the early childhood system: early childhood settings, training and professional preparation, research, regulation and governance, inspection and evaluation."

In other words, all government departments and agencies should work collaboratively to determine the core mathematical concepts to be encountered in early childhood settings and to ensure consistency in all documents used by key stakeholders: ECEs, setting managers, DE inspectors, lecturers and students.

This chapter provided a comprehensive history of Irish government policy and curriculum development regarding early childhood mathematics education. It tracked the Infant (Primary School) curriculum development from 1971 and the development of the preschool sector, highlighting a consistent shift towards play-based, inquiry-led learning for young children (birth to six years). This emphasis on play pedagogy and its many interpretations (see Chapter 2) is an important factor for this study, where adult perceptions of their role in play and in supporting mathematical learning through play determine the level of support given during free play sessions.

Key policy documents (e.g. NLNDLS, DE, 2024) and early education frameworks like Aistear, Síolta and the Early Years Focused Inspection Guide (DE, 2025) were examined. Problematic inconsistencies were noted in how mathematical concepts and numeracy are defined and prioritised for this age group across different government departments and policy documents, potentially causing confusion among ECEs and members of the inspectorate as to

what mathematical concepts young children should experience in their preschool education. This policy context undoubtedly influences preschool practice, and it is an important rationale for my focus on RQ 1. *What types of mathematics do the children participate in?* This policy context is also highly relevant to how my findings should be interpreted and understood. This will be further discussed in Chapter 9, Section 9.4.

Furthermore, the chapter identified a significant gap in professional training, as not all early childhood degrees or inspection processes offer sufficient focus on mathematical knowledge or pedagogy. This is a critical finding of this review. As noted in Chapter 2, the research literature states that to facilitate high-quality early mathematics education, ECEs must possess knowledge of mathematical concepts and how to communicate them to children in age-appropriate, playful, and meaningful ways.

Ultimately, the chapter argued that a competent educational system requires better alignment between national strategies, educator qualifications, and classroom practice to support children's early development. Chapter Four outlines the theories that underpin the analysis of this study.

CHAPTER 4: THEORETICAL UNDERPINNINGS

Lerman (2002) asserts that 'an object of research on mathematics teaching and learning can be seen as a particular moment in the zoom of a lens' (p.87), focusing on complex classroom interactional processes, by considering the broader cultural issues (pulling back) as well as the pedagogical processes in the classroom (zooming in). Furthermore, Lerman (2001) contends that researchers of mathematics education should consider how the situation under study is situated in 'the wider macro situation and the micro situations…to make the links between structure and agency, and between culture, history and power and students' learning mathematics' (p.90). Adopting Lerman's (2001) approach of 'pulling back' and 'zooming in' (p.87), this study aimed to explore how young children were guided in engaging with early mathematical concepts in their ECCE room (zooming in), as well as considering the broader societal values and government educational policy development that influence practice in Irish ECEC settings, as described in Chapter Three.

Imenda (2014) contends that a theoretical framework 'evolves from a review of the literature and or/from the data collected' (p.193). It guides the researcher in the data collection and analysis process. Notwithstanding the valuable contributions to the field informed by historical (e.g. Froebel, Montessori) and sociological theories (e.g. Dahlberg et al., 1999; Moss, 2001), from the early stages of my study, I identified *Bronfenbrenner's Bioecological Systems Theory* (1979) as an important means of conceptualising ECME. Findings from the pilot study (Section 5.7) suggested that a critical lens may be productive. I explored using the work of Foucault (Foucault, 1977) and Gore (Gore, 1995) to analyse the collected data; however, in this exploratory analysis, the specific detail of the mathematics was lost, and it did not allow me to fully address the research questions.

Ultimately, I selected two theories from developmental psychology to underpin my work as this is most appropriate to investigating the learning of mathematics, and aligns with respected work in the field (e.g. Clements & Sarama, 2021; Perry & Dockett, 2018). Firstly, to facilitate 'zooming out', I retained Bronfenbrenner's (1979) *Bioecological Systems Theory* to analyse the wider (macro) issues that affect how children are supported to engage in ECME (see Chapter Three). Secondly, I chose Rogoff's (1990) *Theory of Guided Participation* (GP) to analyse how children are supported in learning cultural practices, in this case, the cultural practice of mathematics, in a particular ECCE classroom ('zooming in'). This chapter begins by considering how each of these theories aligns with my study, before exploring differing definitions of the concept of culture, a core concept in both theories. These discussions are followed by presenting each theory in turn, identifying the chief concepts within each and their relationship to the topic under study.

4.1 Connections to Data Collection Methods

Ethnography is increasingly acknowledged as a useful method of exploring the lived experiences of those working in and attending the microsystem that is an ECEC setting (Albon & Huf, 2021; Köngas & Määttä, 2023), and complements Lerman's (2002) conceptualisation of mathematics research as the capture of practice in a moment in time. Bronfenbrenner (1979) stated that 'development never takes place in a vacuum, it is always embedded and expressed through behaviour in a particular environment' (p.29). This statement supports the choice of an ethnographic approach to data collection and analysis, with the method of participant observation lending itself to observing the impacts of institutional structures on interactions and children's participation in aspects of daily routine in the microsystem (Fields & Kafai, 2009; Tudge & Hogan, 2005). Ethnographic studies (e.g., Campbell & Speldewinde, 2022) support the application of Bronfenbrenner's bioecological systems theory when examining early STEM education. An exploration of the

wider social and policy environments (macrosystem) was provided in Chapter Three, with further analysis of the macrosystem provided in Chapters Eight and Nine.

Rogoff's (1990) theory of guided participation provides a valuable framework for examining how children are supported in their mathematical exploration and learning. According to Rogoff, learning occurs through active engagement in shared activities with more experienced others, where guidance is embedded in social and cultural interactions (Rogoff, 1990; 2003). This perspective highlights that children's mathematical understanding develops through observation, guidance from a more knowledgeable other, as well as through participation in collaborative activities.

4.2 Culture

A shared foundational concept within the theories of Bronfenbrenner and Rogoff is that of culture, the ways in which cultural norms, values, and practices support or constrain a person's participation in everyday life events. Socio-cultural theory emphasises the role of prevailing culture in the interaction between individuals and learning and development, where knowledge acquisition is scaffolded through the actions of a more knowledgeable other. Using the term *cultural historical*, Fleer (2010) alludes to the role of culture, historically developed, and its impact on the ways people act within society.

> 'In a sociocultural theory of thinking and knowledge generation, the present social context bears the birthmarks of historically created ways of being and doing that shape what people pay attention to and how they participate in activities, mediate events and learning, and communicate with each other' (p. 70).

4.2.1 Definitions of Culture

No agreed-upon definition of culture exists. It remains a contested term. Bruner (1996) attests that 'culture is man-made…and learning and thinking are always situated in a cultural setting'(p.4), perceiving culture as 'all the ways groups have worked out passing on

the things that matter to them: the behaviour patterns, norms, language, the arts and crafts, the rituals and beliefs, values and principles and structures and institutions' (Smidt, 2011, p.10). Bryman (2016) agrees, noting that 'cultures and subcultures can be viewed as repositories of widely shared values and customs into which people are socialised so that they can function as good citizens or as full participants' (p.29). It follows then that the EC setting and the individual classroom cultures are mediated by a complex mix of socially accepted norms and practices, government policy, personal histories, experiences and beliefs about young children, mathematics and pedagogy. However, Bryman (2016) further notes the potential of culture as a mechanism of constraint, limiting participation in activities as a result of assimilation of societal norms, beliefs and values, where cultural values determine how we act. This is reflected in Bronfenbrenner's view of culture and its influence on human actions, where human beings are not only producers of culture, but also products of culture, where the actions and thinking of an individual reflect the culturally accepted norms and ways of society at a given point in time (Bronfenbrenner, 1989, as cited in Navarro & Tudge, 2023).

4.3 Bronfenbrenner's Bioecological Systems Theory

Bronfenbrenner's *Bioecological Theory of Human Development* (Bronfenbrenner & Morris, 2006) posits that the developing human organism evolves through its interactions with symbols, objects and people within their immediate environments or microsystems. In later iterations of the theory, Bronfenbrenner placed great emphasis on the importance of these interactions. These interactions or *proximal processes* are critical to early human development and are characterised by their reciprocal nature, increasing complexity and regular occurrence over time (Bronfenbrenner & Morris, 2006). They are the means by which young children make sense of their environments (Hayes et al., 2017). Bronfenbrenner posited that these proximal processes or interactions are influenced by person characteristics, contexts and time (Bronfenbrenner & Morris, 2006). Together, these four interrelated

elements: process, person, context and time (PPCT) form the development model (Tudge et al., 2009). This study explored the various factors that affected an educator's support of mathematics in a preschool environment. Educator practices are the products of personal characteristics, context and time (Bronfenbrenner, 2000). The influence of how these elements influence mathematical development in preschool is illustrated in Figure 5.1. Hayes et al. (2017) emphasise the importance of the proximal processes or interactions on the role of the adult within early childhood settings. This point is central for this study, as the research questions consider the types of adult-child interactions and environmental factors that facilitate and support early participation in mathematics. Hayes et al. (2017) highlight the importance of examining the diverse ways in which adults' reciprocal responses to children's interactions contribute to each child's individual development. Hayes et al. (2017) also note that adults must engage in 'reflective observation' so that they can 'understand the characteristics of the child and the environment', so that quality learning can occur (p.10). For this study, proximal processes are defined as those interactions which occur between educators and children. An explanation of each of these components follows.

4.3.1 Process

Identified by Bronfenbrenner (Bronfenbrenner & Morris, 2006) as the chief mechanism for development, proximal processes have been conceptualised as interactions with the environment and the objects, symbols and people in it. Involvement in these processes (activities and interactions) facilitates a person's ability to create meaning, to make sense of their environment, and their role within it (Tudge et al., 2009). Such interactions are reciprocal in nature. They involve interactions between child and educator, child and peer or child and objects in their environment (Hayes et al., 2017). These reciprocal interactions grow in complexity over time, and they must be consistent in nature to ensure efficacy (Bronfenbrenner & Morris, 2006). As Tudge et al. (2009) observe, these processes or

interactions are critical in supporting a young child's development, and how they understand the interconnected concepts in their environments. From an early mathematics perspective, consistent and reciprocal interactions play a crucial role in shaping young children's understanding of foundational mathematical concepts, the development of related skills, and their ability to recognise the relevance of mathematics in everyday contexts (Gasteiger & Benz, 2018). However, as noted by Bronfenbrenner and Morris (2006),

> "The power of such processes to influence development is presumed, and shown to vary substantially as a function of the characteristics of the developing person, of the immediate and more remote environmental contexts and the time periods, in which the proximal processes take place" (p. 795).

Tudge et al. (2009) state that any research study claiming to use the bioecological theory of human development as a theoretical frame should address all four components of the PPCT model, with the proximal processes being an integral part of the study. Each of the remaining three components and connections to this study is now presented.

4.3.2 Person

A person brings to any situation a set of personal characteristics. These may be biological or experiential, or both. These characteristics are further broken down into demand, resource and force characteristics (Bronfenbrenner and Morris, 2006).

4.3.2.1 Demand Characteristics

These characteristics include age, gender, temperament or appearance (Perry & Dockett, 2018). These characteristics influence how an individual engages with people and objects within their environment, as well as how others respond and interact with that individual. In terms of supporting early mathematical development, support may be strong or weak depending on the gender of the educator, as there is some evidence in the literature to suggest that female educators suffer from 'maths anxiety' (Lake & Kelly, 2014) and poor levels of self-efficacy when supporting early mathematical development (Chen et al, 2014).

Internationally, much research has been conducted to explore both levels of mathematics anxiety (e.g. Lake & Kelly, 2014) and beliefs towards mathematics in preschool (e.g. Anders and Rossbach, 2015; Benz, 2012).

4.3.2.2 Resource Characteristics

These characteristics include mental, emotional and material resources. They can encompass past experiences, skill possession and level of education (Bronfenbrenner & Morris, 2006). Previous experiences shape the way early childhood educators engage with mathematics in the classroom. For example, an educator who had negative prior experiences with mathematics during their school years may choose not to participate in ECME in the classroom. Educators may choose to support or not to support mathematics according to the importance they accord to mathematics, based on their own experiences of and engagement with the subject during preservice courses, or due to their belief that the adult role is to provide a mathematically-rich learning environment for children to explore, and to support only when approached by the children (McInnes, 2019). [For a discussion as to how beliefs are formed and influence practice, see section 2.4.]

4.3.2.3 Force Characteristics

These characteristics are related to dispositions, curiosity, persistence, motivation, etc. (Bronfenbrenner &Morris, 2006). These can reflect an individual's willingness to interact with or to respond to the needs of others. In this study, force characteristics could influence how educators respond to young children's mathematical activity. Educators may demonstrate comparatively less motivation to address mathematics than domains such as early literacy or social-emotional development (Costa et al., 2023; Johnston & Bull, 2021).

4.3.3 Context

In early iterations of the bioecological theory for development, Bronfenbrenner placed major importance on the interaction between the five systems or contexts that influence a child's development: macrosystem, exosystem, mesosystem, microsystem (Bronfenbrenner 1979) and later, the chronosystem (Bronfenbrenner & Morris, 2006). Earlier versions of the theory positioned the developing child as centrally located within this nested set of interrelated systems, with the child's development being influenced by the interactions between each of the systems. The macrosystem is characterised by societal culture, government policies and ideologies and broader political systems (Rosa & Tudge, 2013). The exosystem is a system where the developing human is not directly located, but is subject to its influence nonetheless. For example, educational polices devised at government level may determine a child's time and ways of learning in preschool. Similarly, at setting level, the structural or procedural decisions a manager makes may impact adult behaviours or practices in the preschool room (Hayes et al., 2017). The mesosystem has been described as a 'system of microsystems' (Rosa & Tudge, 2013), where interactions and processes occur across microsystems.

Microsystems represent the child's immediate environments, where they spend the majority of their time (Tudge et al., 2009) and engage in face-to-face interactions with others (Rosa and Tudge, 2013), for example, home, preschool, and school. Microsystems are characterised by interpersonal interactions and interactions with objects and symbols in the environment over time. They are sites of proximal processes (Bronfenbrenner & Morris, 2006). This study positions the preschool setting as an identified microsystem.

Figure 4.1

Bronfenbrenner's Ecological Systems



While the microsystem of the preschool is the focus system of this study, it would be prudent to consider the influence of the other systems on a child's early mathematical development. An educator's willingness to engage in mathematical interactions may be influenced by broader societal views, such as the nature of early mathematics and its place in preschool practice. Such views can be formed by current curricular documents, which advocate a certain pedagogical approach and where, although mathematical activity is referred to, it is not positioned as being important. The lack of mathematics in pre-service courses may also contribute to early mathematics not being viewed as something that occurs in the preschool context. Examples, such as these, illustrate the influence of larger issues controlled by external bodies in the exo and macro systems. These have been discussed in Chapter Three.

4.3.4 Time

Referred to in early iterations of the theory as the chronosystem (Tudge et al., 2009), time constitutes a core element of bioecological developmental theory. This element can be further broken down into micro, meso and macro time. Microtime describes what is actually happening during the interaction or activity, mesotime considers how often the proximal processes occur in the environment and the level of consistency of this over time, and macrotime, also referred to as the *chronosystem* (Tudge et al., 2009) or *historical time* (Rosa and Tudge, 2013). Bronfenbrenner and Morris (2006) define macro-time as:

> 'The changing expectations and events in larger society, both within and across generations, as they affect and are affected by, processes and outcomes of human development over the life course (p.796).

It is worth noting that resource and force characteristics can change over time. For example, an educator's capacity to identify, respond to, and intentionally extend young children's mathematical learning may evolve positively over time through participation in PD initiatives (Blömeke et al., 2017) or the adoption of a new curriculum (Sumpter, 2017). Changes in belief and attitudes towards mathematics education for young children over time through engagement with other people, research, and continuing education opportunities are reflective of the dynamic quality of the PPCT model.

This study adopts the premise that both mathematical knowledge and positive dispositions towards mathematics and ECME are socially constructed through interactions with peers and educators in everyday preschool activities. These reciprocal, supportive interactions between child and educator facilitate mathematical development in young children. How people interact with their environments, and others in it, is dependent on the personal characteristics of the person themselves, the environment and the time in which the interactions take place. As this study explores how educators support mathematical

development, a focus on interactions, these proximal processes in the microsystem of the preschool room, it can be concluded that Bronfenbrenner's *Bioecological Model of Human Development* provides a suitable theoretical underpinning for this particular study, with its focus on how mathematical development is supported within preschool settings.

4.4 Socio-Culturalism and Guided Participation

In line with socio-cultural theory, this study conceptualises young children as cultural beings who live and participate in particular cultural times and activities, and who contribute to the construction of a particular history. Rogoff (1990) emphasises that cognitive development occurs through social interactions, where children are active learners and the interactions with adults or more knowledgeable peers are critically important for learning (Dunphy, 2012). In terms of mathematics, a child may spontaneously recite the number names during play; however, it is the adult's reaction or response that can transform this spontaneous concept of the child into a scientific one, and one which is recognised as mathematical by the prevailing culture and its participants (van Oers, 2010). Such interactions facilitate young children's gradual acquisition of culturally accepted concepts and processes in a process known as enculturation (Bishop, 1988).

4.4.1 Rogoff's Theory of Guided Participation

In conjunction with Bronfenbrenner's theory outlined above, Rogoff's (1990) theory of *guided participation* (GP) provides a framework for the selected research methods and analysis of guiding children's participation in mathematics. Rogoff (1990) conceptualises culture as 'the organised and common practices of particular communities in which children live' (p. 110). Rogoff (2003) interprets Vygotsky's theory as the learning and promotion of academic skills, such as mathematics, predominantly through instructional interactions. Rogoff's (1990) theory of *GP* takes a broader view of children's learning and their

enculturation into the cultural norms of the society in which they grow and develop. In addition to instructional interactions, GP focuses on the side-by-side or distal arrangements in which children participate in the values, skills, and practices of their communities without intentional instruction or even necessarily being together at the same time. (Rogoff, 2003, p.284)

GP conceptualises children as active learners and participants in the learning process, who learn through observation, imitation, and conversing with others to gain knowledge about their world. Rogoff (1990, 2003) envisions the young child as an *apprentice*, learning to live in a specific cultural context, learning how to become a member of that culture under the guidance of a more knowledgeable other. Observation is a key skill in social referencing. That is, through observation of the actions of others, children learn not only how things are done, but also what they are allowed or not allowed to do (Rogoff, 1990, 2003). Rogoff (1995) suggests the term *Participatory Appropriation* to describe the active role children play in their learning. Rogoff (1995) defines participatory appropriation as 'the personal process by which, through engagement in an activity, individuals change and handle a later situation in ways prepared by their own participation in the previous situation' (p.142). That is, children draw on knowledge gained from previous experience to solve a current problem.

GP considers the role of parents and other more knowledgeable others in the learning process and the strategies they use to support learning and development. This includes the language used in the guiding process, the questions adults ask, the comments they make, and their thinking aloud. It includes the physical tools or resources in the learning environment, and how children interact with these. Moreover, GP considers the role of activities that are not specifically designed to teach, but which result in enhancing learning and development (Rogoff, 1990). It includes various forms of participation in culturally guided activities

through the use of particular tools and involvement with cultural institutions. The key processes of GP are discussed below.

4.4.1.1 Providing Bridges from Known to New

This process acknowledges the key role communication, both verbal and nonverbal, plays in supporting new understandings and thinking, enabling children to make connections between what is already known and new information (Rogoff, 1990). Embedded within this are the skills of social referencing (mentioned above), joint attention (the coordinated focus of two people on a shared object or event) and intersubjectivity. Intersubjectivity can be defined as the shared understanding of an object or event, providing a stimulus for mutual communication (Rogoff, 1990). Smidt (2009) observes that 'where a teacher or other adult interacts with the child without trying to really know what the child is doing, the interaction lacks intersubjectivity and will not result in learning' (p.137).

4.4.1.2 Structuring Situations and Transferring Responsibility

In GP, children and caregivers engage in joint activity, mutually structuring the learning situations in which children are involved. This structuring is crafted by the adult, as they determine children's access to opportunities where children can observe cultural skills and practices and participate in them. These opportunities include participation in everyday routine activities, play and conversation (Rogoff, 2003). GP also includes activities where adults do not allow children to participate in, or limit their participation in, e.g., adults choose to do chores while children sleep, limiting children's participation in these as a result. Adults often constrain opportunities to explore – perhaps out of safety concerns. Such constraints are part of the participatory and guided nature of development. GP includes efforts by social partners and by the children themselves to avoid some kinds of learning; for example, free play may enable children to choose their own activities and to avoid math activities. Conversely, adults specifically design activities for children to participate in, e.g., the design,

resources and routine activities of a preschool room. In summary, GP is a theory of learning rather than a pedagogy. Learning occurs spontaneously through observation of another's activity, through collaboration in play, as well as through adult-designed activities.

4.4.1.3 Practising and Playing with Routines and Roles

Rogoff (2003) asserts that children's engagement in routines and play allows them to become familiar with local traditions and practices, and that through their participation in such activities, children alter these traditions over time. Rogoff (2003) identifies role or socio-dramatic play as an opportunity, where children can practice observed everyday cultural activity, in a space that is not bounded by time or adult conceptualisations of objects. It provides a context to 'work out the scripts of everyday life, to (they) emulate adult/community activity/ roles they observe' and to 'experiment with social rules' (p.298). Rogoff (2003) proposes five strategies adults may employ to engage children in the guided participation process. These are summarised in Table 4.1 below

Table 4.1

Five Strategies to Support Children in the Guided Participation Process (Rogoff, 2003)

Strategy	Description
Responsive Assistance	Adult responsiveness to support learning is characterised by waiting and being ready to help children in a manner where the child is afforded agency in directing the pace and direction of the activity. It involves responding in the moment to a child's need for support, rather than adult planning of instruction.
Initiative in observation and pitching in	Children are afforded opportunities to observe activities to learn how to participate in them. Observation is a skill that requires a child to actively attend to a situation being observed. Once the skills or tasks of the activity observed are noted, the child may be enabled to join in with the tasks at hand, and with practice will be able to do the task independently. Rogoff (2003) suggests that even very young children can learn to participate in activities this way.
Observing with time-sharing of attention	A more demanding subskill of observation is that of monitoring several events at one time. It requires great skill to attend to many interesting activities at once; attention needs a broader focus. This is particularly the case when a child learns through observation without guidance from an adult. The development of a heightened sense of alertness to many events at once is crucial to learning how different things are done by different people.
Intent participation in apprenticeship	Children's observation of everyday activities serves as an apprenticeship in thinking and learning. Learning results, not only from observation, but also from participating in the activity under the guidance of a more experienced person, an adult or peer.
Learning through listening in	Equally important to the learning process as observation. Children's listening to adult conversations is key to language development, and is a key learning strategy in some cultures.

4.5 Chapter Summary

This chapter has described the theoretical framework for this study. It described the chief concepts and theories underpinning this study. These concepts and theories contribute to the conceptual framework (outlined in Chapter Five, section 5.2.1) for the study. Bronfenbrenner's (1979/2006) Bioecological Systems Theory supports the analysis of the wider societal and governmental influences on ECE practice (Lerman's (2001) 'zooming out'), while Rogoff's (1990) theory of guided participation supports the analysis of classroom practices and interactions used to guide children in participating in mathematics (Lerman's (2001) 'zooming in'). Table 4.2 illustrates the commonalities across the theoretical frameworks. Chapter Five introduces the reader to the selected methodology and research design for this study.

Table 4.2

Commonalities Across Selected Theoretical Frameworks

Overarching Theoretical Approach: Socio-cultural theory Common Foundations: Culture and Interactions		
Element	Bronfenbrenner's Bioecological Systems Theory (theory of development)	Rogoff's Guided Participation Theory (theory of learning)
Process	Proximal Processes – recurring, reciprocal, meaningful interactions within the environment with people, objects and symbols that support or drive development	*Guided Participation* – children learn through a process of *Apprenticeship* - through joint activity and interaction with more experienced others through observation, collaboration and shared activities. Intersubjectivity, a shared understanding of the activity, is a key process in GP.
Person	The individual's characteristics influence how they interact with people and objects in their environments	*Participatory Appropriation:* The child is an active learner, whose developing skills, interests and cultural understanding shape how they participate in shared activities. The child learns from previous experiences.
Context	The environment is organised into five nested systems, and development is influenced by the interactions between the systems	Learning takes place within social and cultural contexts (e.g. institutions such as preschool settings)
Time	Development emerges through sustained interactions with people in their environments in the moment (microtime), regularity of interaction (mesotime) or activity and over periods of time (macrotime/chronosystem)	Culture defines how participation occurs, what skills are valued, and how guidance is structured. Development occurs in immediate interaction and continues across a person's lifespan (historical time).

CHAPTER 5: METHODOLOGY AND RESEARCH DESIGN

Chapter One stated the purpose of this study: to explore how young children's participation is supported in an Irish preschool setting. This research observed and analysed the types of mathematics the children engaged in, the strategies practised by the adults to support ECME, and the challenges they faced in supporting ECME. My reasons for exploring this topic and the research questions posed were also outlined in Chapter One.

This chapter details the methodology for this ethnographic research study. The chapter begins by presenting the research design and providing a rationale for the chosen design. The research approach and strategy of enquiry are outlined. The chosen data-collection methods to answer the research questions are described. An overview of the pilot study follows. The selected framework for analysis is documented, and examples of its application are supplied. The reliability, validity and trustworthiness of the study are then discussed. Ethical considerations relating to the study are then reviewed.

5.1 Research Design

5.1.1 Philosophical Assumptions

Cohen et al. (2018) assert that "planning research is not an arbitrary matter" (p.177). Choosing a research approach and adopting a particular design requires careful consideration. Creswell (2013) notes that the research process must move seamlessly from "philosophical assumptions, to interpretive lens, and to the procedures involved in studying social or human problems" (p.44). The research process requires the researcher to consider how they view reality (ontology), how they view knowledge, and how it is acquired (epistemology), as well as how personal values can affect or influence the interpretation of the research situation (axiology). Each of these influences the research question, design, approach and methods

used in the study (Bryman, 2016). This section details this study's ontological, epistemological and axiological underpinnings.

5.1.2 Ontological Position

According to Palaiologou et al. (2016), ontology asks, 'What is there to be known? What is the nature of social phenomena?' (p.22). This study focused on how an early childhood educator supported participation in early mathematics learning in an early childhood setting. This research adopts a constructionist worldview (paradigm). This ontological position considers social phenomena as continuous, as they are constantly executed and re-executed by people or "social actors" (Bryman, 2016, p.29). Research conducted from a constructionist perspective aims to tell the story of what is happening with respect to a social phenomenon in a specific context (Braun & Clarke, 2022). Research underpinned by a constructionist lens views findings as representing a "specific version of social reality" (p.29). Phenomena are socially constructed in nature, and the categories generated by these social actors (participants) as they interact with their worlds enable them to make sense of phenomena encountered. Therefore, meanings and understandings of the world are generated through interacting with the world. In this study, I, as a researcher, interacted with participants through conversation and participant observation. I observed strategies and interactions used to support early mathematics education; strategies that early childhood educators execute and re-execute daily, thereby recording the nature of activity in the preschool classroom as it occurred.

My position as a researcher is that participants guide children's participation in early mathematics education in their own ways; they have their own ways of responding to and enhancing mathematical activity. Participants have their reasons for supporting mathematics in the ways that they do. These differences in levels and types of support are natural and are influenced by participants' prior experiences, education and to some extent, curricular

expectations and their interpretations of these. Support strategies may also be influenced by management, setting policy and procedures.

5.1.3 Epistemological Position

Epistemology, the philosophy of knowledge, is our understanding of how we have learned our view of reality. In other words, we ask, "How do we know what we know?" This study is underpinned by an interpretivist view of knowledge construction, where knowledge is gained through interacting with people and their environment, and where researchers aim to "grasp the subjective meaning of social action" (Bryman, 2016, p.26) and try to *understand* rather than *explain* human actions in an empathetic way. This interpretivist position assumes that reality is subjective and socially constructed. Interpretivist philosophies seek to 'understand individual human action either in terms of their daily interactions and common-sense ideas or in the context of the wider culture' (O'Reilly, 2009, Interpretivism). Within this paradigm, research participants are understood to hold diverse views, opinions, beliefs, and experiences of a given phenomenon. These perspectives may vary significantly across individuals. Consequently, interpretivism is concerned with generating a nuanced and in-depth understanding of an issue as it is experienced and articulated by participants (Braun & Clarke, 2022). O'Reilly (2009) asserts that this understanding is easily accomplished through researcher participation in participant cultural activities and contexts, supporting researcher empathy development, therefore, a better understanding of the issue and context under study. My interpretive stance led me to adopt an ethnographic approach to the study. This is discussed further in Section 5.3.

5.1.4 Axiology

A central tenet of qualitative research is that of the researcher as a data collection instrument (Marshall & Rossman, 2016). Researchers bring prior experiences, knowledge and values to their research study. Closely linked to a researcher's positionality, qualitative researchers should acknowledge the value-laden quality of their research study. Such values should be explicitly stated, and their influence on devising the research question, data collection, data analysis and interpretation should be reflexively considered (Cresswell & Poth, 2018). It is crucial that any analysis or interpretation of the phenomenon explored reflects the lived experiences of participants rather than those of the researcher (Dibley, 2011). Hammersley (2018) notes that a researcher's observation of a situation will produce an account of what occurs in a research setting (a person's actions, responses, interactions, etc.); however, findings are 'more likely to be valid and correctly interpreted' (p.8) when participant perspectives on the situation are included in the data set. Hammersley (2018) suggests this is due to a researcher being unaware of the nature/context of the setting in which participants carry out their work. To gain a true picture of the situation, a researcher should seek the thoughts and knowledge of participants, as they may perceive the reality of the situation differently from the researcher. Being a former member of the social world that is ECEC, I do share some of the perspectives and experiences of participants; however, I did not work in the same setting, with the same people, policies and outlooks as my participants, so I cannot truly interpret their situation entirely from my own experience. Therefore, in line with the constructionist, ontological, and interpretivist epistemological positions, my axiological position is that I seek to understand the situation with the support of my participants.

In conducting this study, I documented what I observed and sought to interpret it. Participants could have felt that they needed to focus on the mathematics to ensure I captured

it. My interpretation of their work will be published and therefore publicly available, and may be perceived as fact by other stakeholders in ECE. My decisions regarding what is deemed relevant to document and analyse, and what is deemed unnecessary to report, may impact the relationships between the owner-manager and participating staff. To mitigate this, my biases, subjectivities and motivations for carrying out this research were regularly checked by keeping a reflective diary and through the PhD supervision process. Drafts of chapters Six, Seven and Eight were sent to one of the participants for review and comment.

Braun and Clarke (2022) note the value of reflecting on the researcher's philosophical assumptions, educational background, skills and life experiences and how these influence the research study. To articulate my values, I considered my relationship with mathematics over time. This reflection revealed how interactions and contexts had a negative influence on my personal experiences with mathematics, and contexts and interactions positively influenced the researcher's current relationship with mathematics education. A copy of this reflection can be found in Appendix B. From this reflection, the following values emerged:

- Mathematics is a cultural tool that enables a person to participate in their world.
- Mathematics is a component of holistic child development.
- Mathematics development is supported by knowledgeable, responsive adults who provide a mathematically rich environment (context) and are tuned into individual children's personal characteristics and respond accordingly.
- Knowledge is constructed through interaction with objects, symbols and people, and individual people experience phenomena in individual ways depending on their contexts and personal characteristics.

Engaging in this exercise not only allowed me to articulate my values towards early mathematics education but also to understand the formation of these values over time. My

positionality, and how personal values and biases were checked, will be discussed in detail in a later section of this chapter.

5.2 Research Approach

This study employed a qualitative approach, as it sought to explore how early childhood educators support young children's mathematical learning in preschool settings. An ethnographic design was used, incorporating data collection methods such as informal conversations and semi-structured interviews, alongside ethnographic content analysis of national and local policy documents. According to Denzin and Lincoln (2000), qualitative research can be defined as: "…situated activity, that locates the observer in the world. This means that qualitative researchers study things in their natural settings, attending to make sense of or to interpret phenomena in terms of the meanings people bring to them" (p.3).

The above definition demonstrates that qualitative research focuses on the social world, a type of 'social enquiry' (Greig et al., 2013, p.172) and researchers can best understand social phenomena in the places where they occur and from the perspectives of those who experience them. Conducted in natural settings as opposed to clinical sites (Hammersley, 2013), qualitative enquiry seeks to explore the activity that occurs as people go about their routine daily activities. In this study, participants were observed and interviewed in their preschool rooms (their natural setting) so that their actions and interactions with young children to support early engagement with mathematics could be studied.

5.2.1 Conceptual Framework

Crawford (2020) states the importance of constructing a conceptual framework for any research study. Conceptual frameworks provide direction for a study, show how individual aspects of the study align, and, most importantly, demonstrate how the 'study design and methodology meet rigorous research standards' (p.36). Maxwell (2013) asserts

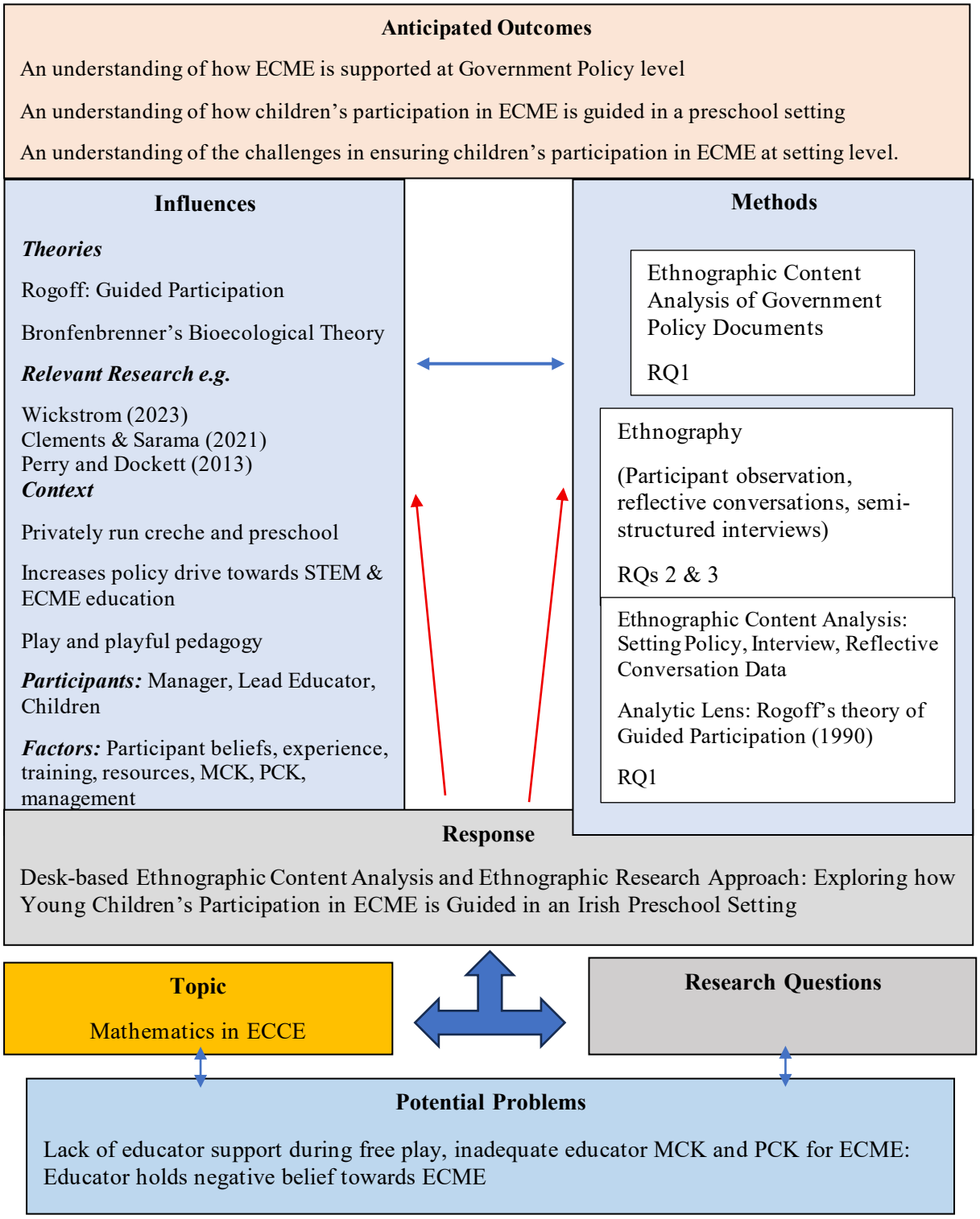
that a conceptual framework should be constructed by the researcher, and include the following: philosophical and methodological paradigms, research questions, prior theories and research. Theories chosen to support data analysis were described in Chapter Four. The template for this conceptual framework was adapted from the work of Gurhy (2017). See Figure 5.1 (p.120) for the conceptual framework for this study.

Aligning with the philosophical assumptions outlined above, an ethnographic approach was chosen for this study. Previous studies (e.g. Anders & Rossbach, 2015; Chen et al., 2014) explored ECEs' beliefs towards mathematics in preschool using quantitative survey methods. While such a methodology can serve well when researching large sample populations (Cohen et al., 2018), it was deemed incompatible with the aim of this study to observe how participation in mathematics is supported in an ECEC setting, due to the spontaneous and embedded nature of mathematics in ECEC settings. (Hammersley, 2013). Also, survey studies report on self-reported descriptions of practice. These can be problematic as one has to trust that respondents are reporting accurately on practice. Therefore, one cannot be sure of the reliability of self-reported practices (Cohen et al., 2018).

Questionnaire surveys could have been administered to individual participants within the setting; however, I felt that this approach was at odds with my constructionist worldview of reality and interpretive epistemological stance and the research aim of exploring how participation in mathematics is supported in a preschool setting.

Figure 5.1

Conceptual Framework for Supporting ECME. Adapted from Guhry, 2017.



5.3 Strategy of Enquiry – Ethnography

Chapter Four outlined the connections to the selected theories and research strategy, including ethnography and associated data collection methods (Section 4.1). From an ontological perspective (section 5.1.2), ethnography supports the exploration of practices that occur in a specific context, in this case, an ECCE setting, in relation to ECME. Ethnographic data collection methods such as participant observation, conversations and interviews support the epistemological stance that knowledge is constructed with people in their natural environment (section 5.1.3).

As noted in section (5.2.1) above, alternative strategies (e.g. surveys) were considered to explore how children's participation in ECME was supported in this setting and were dismissed due to their incompatibility with the rich data set I wanted to procure. I wished to observe and document first-hand how ECME was supported in an ECCE classroom.

5.3.1 Ethnography

Ethnography is classified as a qualitative approach suited to educational research (Flewitt & Ang, 2020) and is characterised by participant observation. Indeed, the term participant observation is often used inter alia with ethnography. Bryman (2016) notes that this use of the term participant observation to represent ethnography serves to undermine the complexity of the approach as features of ethnographic research include: long periods of observing and collecting data on-site, high levels of interaction with participants, occurring in natural settings, documenting what happens in settings, and providing a holistic view of the phenomenon being explored (Hammersley, 2018). However, a generally accepted definition of ethnography is lacking in the research literature (Flewitt & Ang, 2020). Hammersley (2018) contends that there is much variation as to the interpretation and the definition of ethnography, and when positioned as a research approach, a consensus as to its definition

may never be reached. The lack of an agreed definition has been attributed to the approaches' origins in anthropology and various refinements over time (Flewitt and Ang, 2020; Hammersley, 2018). Bell (2009) cites Brewer's (2000) definition of ethnography, as "the study of people in naturally occurring settings or 'fields' by methods of data collection which capture their social meanings and ordinary activities, involving the researcher participating directly in the setting" (p.6). Holloway (2010) offers this definition, ethnography as, "the description and interpretation of a culture or social group" (p. 76). Fields and Kafai (2009) go further and describe ethnography as the study of participant feelings, values, beliefs, relationships, interactions and power dynamics that occur within cultures and subcultures. In the early childhood education research arena, ethnography can offer "rich and deep insights into how children make sense of the world" and why adults who support them act in the ways they do (Flewitt & Ang, 2020, p.123). In this study, the aim of understanding how children's participation in mathematics is guided will be met by studying how early childhood educators in one early childhood setting work and noting how they guide young children's mathematical learning. The study also examines the culture surrounding early mathematics education in the setting. Culture, a contested and debated term (see Chapter 4), can be defined as the values, beliefs, and actions of a particular group (Roper & Shapira, 2000). When social scientists research the culture of a group, they seek to explore participants' beliefs, knowledge and actions, to understand why they act in the ways they do.

5.3.1.2 Micro-ethnography

Traditionally, ethnographical research was expected to take up to 12 months in the field (Bryman, 2016; Fusch et al., 2017). However, perceptions of how long a researcher needs to stay in the field are changing (Rashid et al., 2015). Micro, mini or focused ethnography (Fusch et al., 2017; Knoblauch, 2005) is a type of ethnography that is increasingly used, as traditional extended periods in the research field become less common

(Fusch et al., 2017). Focused ethnography is characterised by shorter research periods in the field. Technical data-collection methods may be used to address the shorter data-collection period (Knoblauch, 2005). Time spent in the field can vary from as little as a week to a full year for a micro or focused ethnography. In designing this study, I considered whether the data should be collected one day a week over a year or whether a more intense, focused period of data collection would be more beneficial. Beyond the practical constraints of my role in the university, many factors were considered during the decision-making process, including establishing, developing and continuing relationships with participants, becoming 'part of the team', participant comfort with another adult in the room, the ECCE calendar and so on. When reflecting on factors such as establishing, developing and continuing relationships with participants, I felt that it would be easier to develop and maintain these over a continuous four-week period, rather than an intermittent period of one day a week. Also, being a participant observer over a continuous period enabled me to observe participants responding to and enhancing young children's mathematics and interests over consecutive days rather than losing potentially rich data during days not on site, when such pedagogical interactions occurred over shorter periods. Such interactions could have gone unreported by participants during weekly visits.

The use of communication methods, such as casual conversations and interviews, is also a feature of this approach. Whereas, in traditional ethnography, researchers may enter the field without a specified research question, micro-ethnographers enter the field with predetermined, focused research questions (Roper & Shapira, 2000). Micro- or focused ethnographical studies aim to comprehend participants' values, cultural standards and roles in their organisations or societies (Fusch et al. 2017). This marries well with the aims, research questions and theoretical framework of this study as it considers how ECEs support mathematics within one early childhood setting.

This study adopted a focused or micro-ethnological approach where I was situated within a preschool room for a short period (Bryman, 2016). In this case, 2.5 hours per day, five days a week, for four weeks, with specific research questions to be answered (Roper & Shapira, 2000). During these four weeks, I witnessed first-hand the pedagogical actions and interactions that support young children's engagement with mathematical concepts using traditional ethnographic data-collection techniques of participant observation, interviews, conversations and field notes (Mukherji & Albon, 2015). As a former preschool and primary educator, with twenty years of experience and extensive knowledge of policy and practice, the shorter period was perceived as a strength, easing initial entry into the field (Bryman, 2016). This practical knowledge enabled me to become a 'team player' quickly and a group member more readily. This idea will be further considered in a later section on reflexivity.

5.4 Recruitment Procedures

This study employed a purposive recruitment strategy. Creswell (2007) notes that with purposive sampling, the researcher actively looks for participants who possess a depth of knowledge about the situation being explored. Managers from both the pilot and the main setting were known to the researcher for their interest in and knowledge of early mathematics education and early STEM education more broadly. This interest was deemed important for the study as it was hoped that observed practices would show how early mathematics education is enacted in settings where the manager is an advocate for young children's engagement with a broad range of mathematical concepts. Participating settings receive funding from the DCEDIY to provide the Early Childhood Care and Education (ECCE) scheme, where children, from two years and ten months onwards, receive two years of preschool education, underpinned by the tenets of Aistear (GoI, 2024c) until they start primary school.

5.5 Data Collection Methods

This section outlines the data collection methods used to answer the research question, 'How is mathematics supported in preschool settings?' In line with the ethnographic tradition of seeking to gain a rich, thick and comprehensive understanding of the situation under exploration, a range of data collection methods were selected for use in the study. The data set consisted of an interview (INT) with the setting manager, Siobhán, two reflective conversations (RC) with the lead educator, Sinéad (pseudonym), participant collected photographs and videos, as well as daily field notes (FN) arising from participant observation. Two of the setting's policies were also collected for analysis.

5.5.1 Field Notes

Field notes are written accounts of observations made in the field and are a key component of ethnographic research and participant observation (Tenzek, 2018). They enable researchers to write a detailed account of what has been observed. Field notes can be descriptive, analytical or reflective. Bogdan and Biklen (2003) note that field notes contribute rich data and "can be an important supplement to other data collection methods" (p.111). In this study, field notes supplemented data collected from reflective conversations and semi-structured interviews. Denscombe (2010) recommends that field notes be written up as soon as possible after a period of observation, facilitating researcher reflection on the experience, documenting initial thoughts and analyses (Bryman, 2016), and that time for this activity should be built into the research. Bogden and Biklen (2003) suggest that this reflective scrutiny of researcher actions and reactions can help address researcher bias. In this study, I observed during the preschool's morning session, keeping a handwritten account. Field notes were expanded and written up in a digital format each afternoon post-session, while the events of the day were still fresh in my mind.

Tenzek (2018) notes that field notes should include details about the environment/setting under study as well as interactions between participants. Such information enables the researcher to identify patterns in the data and, in doing so, consider what is happening in the context being studied. This information contributes to the rich description required of qualitative research and ethnographic studies. Lareau (2021) suggests the following criteria be included in field notes: date, time, place of observation, details of what happens while the researcher is on-site, sensorial impressions, conversation summaries, as well as questions arising from the data (after reflection/analysis). Analytical notes were made as field notes were captured during the data collection phase, during the daily write-up, as well as during the analysis phase of the study. Figure 5.2 provides an example of a daily write-up of my field notes. It should be noted that both analytic and reflexive notes were continuously reviewed and expanded. Writing reflexive memos supported me in the continuous questioning of my positionality as a researcher (Braun & Clarke, 2022). For me, this was particularly important as someone with intimate knowledge of working in the EC sector; I felt I needed to reflect on how my personal experiences and beliefs shape the research. This will be discussed further in section 5.6.

Figure 5.2

Excerpt from the Field Notes Record

Project: Preschool Children's Participation in Mathematics		
Site: Little One's Creche (ECCE Room)		
Date: 8/5/23		
Observation Notes	Analytical Notes	Analytical Memoes
Sinéad asks what shape the group are saying goodbye to "rectangle", shout the children and say hello to "circle" Chandra gets up from her seat and finds a plastic ring – "This is a circle", she says. Other children follow her lead, leave their seats and find other items in the room that are circular in shape. After affirming all the circles, Sinéad says, "Everyone, put your things back, 5,4,3,2,1". When all children are back on their chairs, Sinéad says, "everyone get out your magic wands and we'll draw a circle". She models drawing a circle, saying "go around" as she draws a circle in the air. The children copy her.	The children can readily identify a rectangle and a circle. This is not the first time they have been introduced to one during CT. Environment affords opportunities to identify circles, plenty of examples to choose from. Chandra's agency to change the CT routine is clear here, supported by Sinéad. This was a missed opportunity to compare examples of circles children brought back. Scope to discuss size, material, thickness, etc. Sinéad affirms but does not engage in math talk, and carries on with her usual routine of air drawing the shape.	My training drew attention to the importance of discussing different attributes of shapes, noticing similarities and differences. My Montessori geometric cabinet enabled this. I love the way the children followed Chandra's lead here. A well-resourced environment facilitates exploration of and engagement with shape; a philosophy of 'follow the child' is evident here, although limited to action of the children and not discussion. Lack of time? Commitment to the CT routine? Or a combination of both?

5.5.2 Participant Observation

Both ethnography and participant observation require researchers to immerse themselves in the site(s) of research for extended periods. Becker and Greer (1960) offer this definition of participant observation:

> "the method in which the observer participates in the daily life of the people under study, either openly in the role of researcher or covertly in some disguised role,

observing things that happen, listening to what is said, and questioning people, over a length of time" (p.28).

In this study, I opted for overt participatory observation and became 'another body in the room', helping with daily activities. In this way, I hoped to 'blend in' with the daily life of the setting, gaining trust from the adult participants and the children in their care. Cohen et al. (2018) note that the participant observation technique is useful for researching small groups and that when the observer has been with the group for a while, the 'observer effect' can be reduced. The observer effect, also known as the '*Hawthorne Effect',* refers to the idea that participant behaviour may change in response to being observed (Sadek & Sadek, 1996). However, Roberts-Holmes (2014) suggests that continuous periods in research settings can help to offset this effect as participants relax and feel more at ease with the researcher's presence. As noted in an earlier section, my background as an ECE may have helped me to gain the trust of participants more readily; however, it would have been naïve of me to assume this to be the case. In practice, this appeared to be the case as the lead educator and I were able to chat about practice, our experiences of working in a Montessori environment and the educational system in general. Such conversations were documented in the same way as the field note example presented in Figure 5.2 above.

5.5.3 Semi-structured Interviews

Semi-structured interviews are a flexible method of gaining insight into participants' views on a topic (Greig et al., 2013) and allow participants to expand on issues that are significant and meaningful to them (Bryman, 2016). Interviews enable participants to consider answers carefully (Lareau, 2021). Semi-structured interviews align with my interpretivist position, as they enable researchers to gain detailed knowledge of the views and beliefs held by participants on a particular topic. Interviews were audio-taped using my iPhone, uploaded to a password-protected Google Drive and subsequently deleted from the phone. Interview transcripts were produced manually by me.

5.5.3.1 Preparation of the Interview Guide

Lareau (2021) contends that interview guides are "a set of open-ended questions that help you (researchers) reveal what you want to learn in the study" (p.74). This structure acts as a guide to ensure that similar key topics pertinent to the research are explored, that is, those necessary to answer the research questions, while still providing flexibility to pursue the topics or issues participants deem important. Brinkmann and Kvale (2018) propose that qualitative interviews contain nine different types of questions, including lead-ins, probing and open-ended questions. This study used a selection of successful questions used during my undergraduate study exploring the beliefs of early childhood educators towards mathematics in early childhood, as well as questions based on the work of Chen et al. (2014) and Hunting et al. (2012), who used survey statements to explore preschool educators' beliefs and self-reported practices about mathematics in early childhood contexts. Questions such as, *How should young children experience mathematics in preschool?* and *What challenges did you/do you currently face in terms of providing a mathematics curriculum in this setting? How can these be addressed?* aided in exploring the beliefs of the manager towards ECME in the setting and potential difficulties she perceived staff may experience in enabling children's participation in ECME.

Interview questions were grouped according to the concepts being explored. Lareau (2021) suggests asking general lead-in questions first to relax the interviewee. These questions should be followed by more specific topic-related questions, with more sensitive questions being asked towards the end of the interview when participants are fully relaxed (Lareau, 2021). Lareau (2021) maintains that a guide of twelve main questions is sufficient for an interview lasting between 90 and 120 minutes.

For this study, it was anticipated that interviews would take no longer than an hour, so a reduced set of questions was prepared. In reality, interviews lasted 25 minutes for the pilot

manager interview (see Section 5.7) and 35 minutes for the main study interview. It should be noted that both manager participants were time-poor; therefore, interview times were constrained by temporal and structural elements. Initially, the main study interview was conducted in the preschool room, but transferred to the manager's office when the children returned from the garden. This could have interrupted the flow of the conversation, but happily, it did not. However, due to the location of the manager's office close to the main entrance/exit area of the setting, a minute of the conversation was inaudible due to children preparing to leave the setting for home. Interviews with the manager in the main setting occurred during the first week of the observation period. This time frame worked well, as I had time to become familiar with the setting and staff before the interviews. It also enabled me to comment on and question observed practice during the interview. The interview schedule for this study is located in Appendix C.

5.5.4 Casual Conversations

As I acted as a participant observer during my time in the setting, and in keeping with an ethnographic approach, I anticipated engaging in natural, casual conversations with participants during their working day. These informal conversations enabled me to probe participants as to the nature of their interactions with young children, helping me to understand why they interacted in the way they did. For example, during a conversation before the children arrived (FN) 3), the Lead Educator (LE), Sinéad, mentioned that the setting had lost their full-time cook and that they were looking for someone to fill this position. The position had been unfilled for some time. This added pressure on the managers, as they had to take on extra duties, such as cooking up to ninety meals a day. Thereby, taking away time spent monitoring in the classrooms. Such conversations offered information that could be used to glean further information during the semi-structured interview with the manager and aided analyses of interview content, providing another layer of context to the

information shared. In this particular instance, the information above enabled me to understand the pressure the managers were under and explained the varying times for their coverage of Sinéad's break. Casual conversations were recorded within my field notes.

5.5.5 Autophotography

Compatible with the ethnographic approach (Cohen et al., 2018), photographs capture events and objects in real-time. This study adopts autophotography as a data collection method. Autophotography asks participants to take images of aspects of their environment, a way of capturing a participant's view of their environment and offers researchers an opportunity to discuss the photographs with participants, thereby providing participant and researcher interpretation of events (Glaw et al., 2017). The purpose of autophotography in this study is to explore how participants view their preschool rooms from a mathematical point of view. Throughout the observation period, the LE was asked to capture images of materials/ equipment that support mathematical development. The LE was asked to provide an oral narrative explaining the mathematical content of the image. These conversations took place at the end of weeks two (RC1) and three (RC2), during the children's 'outside time', as the owner-manager was free to supervise the children, leaving the LE free to talk with me. The LE and I engaged in a discussion as to how the mathematics present in the images might be supported and developed. This collaborative data-collection technique can serve to empower participants in the research process and help to build a positive relationship between the researcher and participants as meanings are shared and explored (Torre & Murphy, 2015). It should be noted that these reflective conversations formed part of the data set rather than the photographs themselves. Such practice aligns well with the constructionist paradigm adopted in this research, where meanings are constructed between people, in this case between researcher and participant. Capturing interactions as they occur, either via photograph or video, is standard practice in preschool rooms, so it was not envisaged that

requesting photo capture of instances where mathematics is supported would place an extra burden on participants. Conversations between a researcher and participants about the photographs can help to minimise researcher bias. In this instance, participants share their interpretations and meanings they attach to the photographs they have taken with the researcher.

5.5.6 Participant Captured Video

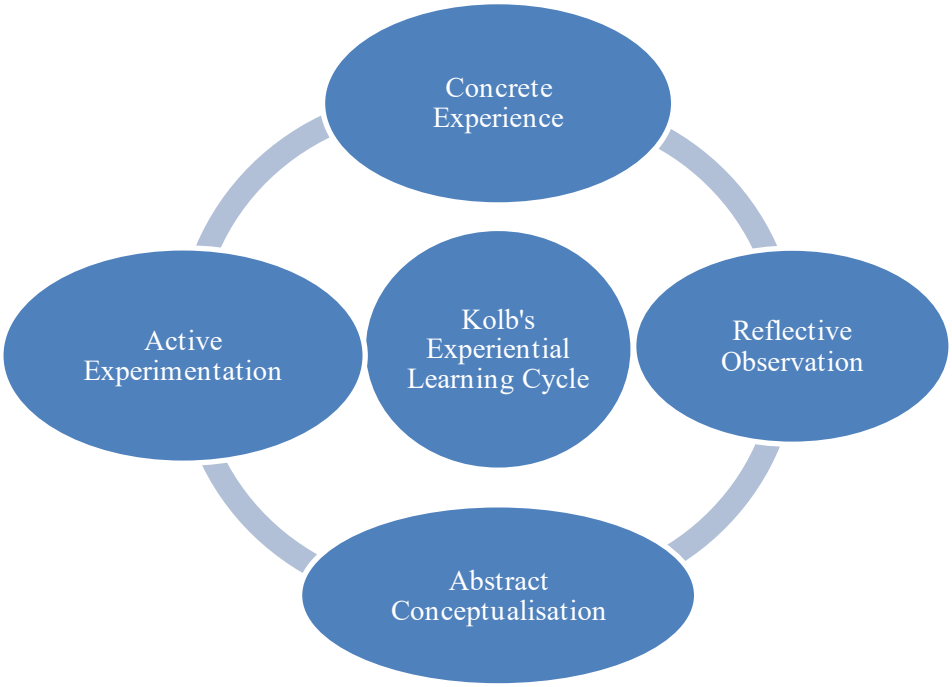
The benefits of using video in educational research are many. Video captures 'naturally occurring' events (Jewitt, 2012) and multi-modal behaviours and classroom interactions in real time (Cohen et al., 2018). Video has the added advantage of facilitating numerous reviews, and enabling the capture of non-verbal data (Cohen et al., 2018), catching details an audio or written observation may miss (Flewitt, 2006). Cohen et al. (2018) note that video data can be prone to researcher bias and subjectivity, as typically, the researcher decides what is important to record and what is not. In this study, videos of children engaging in mathematical activity were taken by the LE, thereby offering an insight into what the LE deemed to be a mathematical activity, allocating the power of what to record to her. Researchers such as Jewitt (2012) and Robson (2014) note that video footage often needs to be accompanied by a conversation or interview to interpret the reasons for the reactions/interactions captured by the video. Therefore, as with the LE-captured photographs, the videos were reviewed by me and the LE. The LE captured two videos over the 4 weeks. I had hoped that the LE would be able to capture more; however, the busyness of the preschool room did not facilitate this. Again, the conversation was documented digitally by iPhone and supplemented by field notes. As with the photographs above, data was generated from the reflective conversations about video content.

5.5.7 Reflective Journal

A researcher's reflective journal can act as a 'repository' for recording the researcher's ideas and reflective analysis of events observed in the field. The journal can provide a space for questioning assumptions and values as well as a space for interpretation or 'meaning-making' (Braun and Clarke, 2022, p.19). As a former early childhood educator, I have engaged in the process of reflective practice both informally as an educator and more formally as a student of early childhood studies. The model for reflection most familiar to me is that proposed by Kolb (1984). Kolb's reflective model, also referred to as an *Experiential Learning Cycle,* comprises four distinct stages: concrete experience, reflective observation, abstract conceptualisation, and active experimentation. Kolb's model is a holistic approach to reflection. It encourages reflection using the senses, feelings and intellect as well as drawing and reflecting on past experiences to make sense of and gain a deeper understanding of a current situation (Kolb, 1984). By engaging in these stages of personal reflection, I developed a framework for a reflective journal (RJ) and used it to analyse personal interpretations of phenomena observed in the field.

Figure 5.3

Kolb's Experiential Learning Cycle (1984).



I wrote in my reflective journal often, daily during the first week, as I began to get to know Sinéad, the children and the routine. These first few entries centre around the children and me. I developed a rapport quickly and easily with Sinéad as we had much in common. However, it took time to gain the trust of particular children. Appendix D is an example of a reflection on an encounter with a particular child. I was too eager to assist him in putting on his coat, and he was not impressed. The reflection reveals my feelings after the incident. Although this interaction may seem harmless enough, it showed that I had reverted too quickly into the role of an ECE, and I had crossed a boundary. I did not know this child well enough to think they would welcome my assistance. Indeed, my intended assistance was perceived as an interference. I was a visitor in the children's space. I needed to learn the classroom rules. Children are encouraged to do things for themselves. Lesson learned! This example illustrated how keeping a reflective journal provided space for me to consider my

role as researcher in this study (Braun & Clarke, 2022). An overview of the data collection record is located in Appendix E.

5.6 Reflexivity – Positionality

A researcher's identity and experiences can shape how they interpret research data and how they interact with and are viewed by participants (Flewitt & Ang, 2020). In light of this, Braun and Clarke (2022) assert that researchers should consider their positionality within the research as an important requisite of any study. This process, referred to as *reflexivity*, is a process of questioning our attitudes, thinking, personal values, habits and assumptions, while also seeking to comprehend our role in research with our participants (Bolton, 2010). My positionality has been described in Chapter 1; however, a consideration of the potential influence of my positionality within this study and how this may be perceived by participants, and the likely influence that this might have had on my acceptance into the field, and the quality of data collected is warranted here. Table 5.1 below details the possible perceptions of me by participants. A discussion follows concerning these possible perceptions and, in the case of negative perceptions, how these may be mitigated.

Table 5.1

Potential Positive and Negative Perceptions of the Researcher from Participants

Positive Perceptions	**Negative Perceptions**
Mentor/Expert	Know it all
Researcher of good practice	Judge
An asset to developing practice	A hindrance – obstructing daily practices, getting in the way
Part of the EY community – one of them, someone who understands the sector	Lucky, privileged – have left the sector, Teaching on B.Ed. rather than on BECE

It is important to state that these possible perceptions of the researcher were self-generated as part of the reflexive process, and that I understood that perceptions could change as relationships were forged between myself and the participants during the fieldwork period. Although I have considerable experience working in early childhood settings as a Montessori teacher and have carried out research in the area of preschool mathematics, I do not see myself as an 'expert', and particularly not as an expert in how children participate in mathematics in a particular setting.

I met with participants ahead of the data collection process. This informal information session offered an opportunity to introduce myself and the research outlining my position as someone with a keen interest in how mathematics is supported in preschools. It was important for me that a shared understanding of what the study aimed to do and how this would be achieved was agreed upon. For me, as a researcher, participants must view the process as non-threatening and as a means to highlight how mathematics is supported in preschool. This meeting took place on the 26th of April 2023. It was hoped that this introductory session would ease the process of the researcher 'settling in' to the setting and being accepted as a group member. I intended to work as part of the setting, helping out as necessary, e.g. handing out food at snack time, mopping up spills, etc. However, I recognise that although I could be seen as an 'insider', who shares a particular background and experiences with participants, I could also be considered an 'outsider' as I no longer work in the preschool sector (Braun and Clarke, 2022). I believe that my experience of being an ECE did help me to quickly become part of Sinéad's room, and after the first week, she was comfortable in sharing her thoughts about practice and policy with me, as noted in this reflective diary entry.

> "I feel Sinéad is more comfortable with me in the room. I feel less awkward in this setting than I did in the pilot…I feel comfortable helping out here. Sinéad is the only adult in the room with no one to assist her. Maybe her acceptance of my presence is

> because she appreciates the help and knows that I understand what she speaks about" (RJ3).

This feeling of being comfortable and of Sinéad being appreciative of the help was validated and acknowledged by Sinéad in the later stages of the data collection phase as she stated during a casual conversation, "it was lovely to have someone else in the room, someone qualified, and who knows what they are doing" (FN 15). While this may be interpreted to speak to my perceived academic expertise, I interpreted this comment as Sinéad recognising that I had experience working in a preschool setting and, therefore, I was able to 'fit in' to support Sinéad in her work with the children.

I am aware that reflexivity is a continuous process. Therefore, the reflective diary remained a constant feature of my research process from data collection to data analysis, and as I moved toward submission and publication. A reflection on Sinéad's comment above can be found in Appendix F.

5.7 Pilot Study

Bryman (2016) advocates for a 'pilot study' to be carried out to test the efficacy of research instruments, to evaluate whether or not they capture what they are intended to capture. Pilot studies also allow researchers to assess the order in which data collection takes place, for example, interviews before or after observation. In terms of an interview schedule, piloting can offer an opportunity not only to gauge the quality, phrasing and pitch of questions. The process can provide practice in the interview process and develop personal confidence. The pilot study was carried out in March 2023, with ethical approval (Appendix G) obtained from Dublin City University in January 2023.

The pilot study was conducted in Rainbows Community Childcare, a community-run ECEC setting. The manager was also the lead educator in the preschool room in which the pilot study was conducted over one week. I attended the setting for two and a half hours per

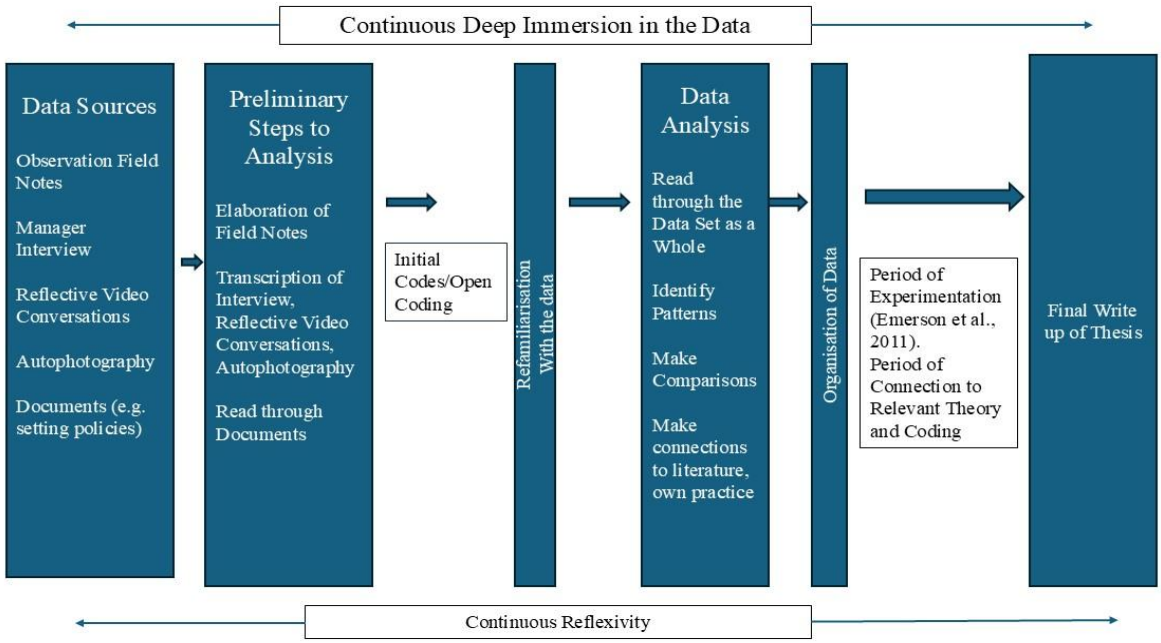
day over five days. A preliminary visit took place the week before the start of the pilot, to allow familiarisation between staff, children and me as researcher, and to enable staff to ask any questions they had about the study. The interview schedule for the manager was deemed fit for purpose. Due to technical difficulties with the class iPad, staff were unable to take videos of the children's play and mathematical activity. The ECE, Maureen (pseudonym), took photographs of her key children engaged in play with mathematical aspects. Reflective conversations with Maureen discussed these aspects and showed that Maureen was capable of interpreting the play mathematically. The pilot study confirmed the suitability of an ethnographic approach for this study and that the selected data collection procedures were effective in collecting data that would answer the research questions.

5.8 Data Analysis Procedures

The micro-ethnography method, adopted for this study, focused on studying aspects of classroom practices used to support/guide young children's participation in ECME. A dual process of ethnographic writing and *Ethnographic Content Analysis* (ECA)(Altheide & Schneider, 2013) was used to analyse the dataset. The data set was extensive, comprising many pages of detailed field notes, analytical and reflexive memos, interview transcripts, and two setting policy documents. Van Maanen (2011) asserts that 'the qualitative data analysis and writing process is anything but predictable and linear' (p.153). This has been my experience. I have engaged in the analytic process continually since the beginning of data collection (Lareau, 2021; van Maanen, 2011) and have continued to do so iteratively throughout the study. Figure 5.4 provides an overview of the data analysis process.

Figure 5.4

Overview of the Data Analysis Process



Analysis was an iterative process beginning with recording and reflecting on field notes. Field notes were made and extended from the preliminary visit to the setting. Daily field notes were written up, elaborated and reflected on as soon as possible after capture. As the field notes were elaborated, my initial thoughts on the data were recorded, and tentative codes were assigned. These were reviewed and reflected on each day as the data collection grew, as I sought to interpret what I saw and heard, to produce a 'cultural portrait' of practice in relation to children's participation in early mathematics learning in the setting (Van Maanen, 2011). Connections (similarities and differences) to content across the data set were also recorded in these elaborated field notes, along with further questions to explore in subsequent days.

Analysis of the interview data began by listening to each audio file several times, initially to the audio only, and subsequently in tandem with consideration of field notes made

at the time of the interviews. A process of transcription followed. The resulting manuscripts were read many times while listening to the recordings, amending transcription errors as they were identified. This process also facilitated familiarisation with the data set. Again, initial thoughts and codes were jotted into the margin of transcriptions. This open coding (Emerson et al., 2011) included my initial comments and interpretations, fuelled by my own experiences as a Montessori teacher, reading of relevant literature, and the context of Irish ECE policy and practice. I remained aware of my professional and personal biases throughout the analysis phase and actively took measures to minimise their impact on the coding process (see Section 5.6) (Emerson et al., 2011). Following this, comparisons were made across content in the interviews and in the field notes transcriptions, reviewing the dataset as a whole and documenting thoughts, making connections to relevant literature and Irish ECE policy continuously.

As each data source was collected, the contents were compared and contrasted to help answer each research question in turn. Each data source was analysed against previously collected data, relevant literature and government policy documents, noticing similarities, differences, and looking for common elements relating to the research questions in turn, such as mathematical concepts experienced across the daily routine, pedagogical strategies, and challenges to providing early childhood mathematics education in the setting. This was an iterative process; I constantly went back and forth between data sets. This constant analysis, comparing and contrasting data sources, served as data triangulation, a method acknowledged to strengthen a research study, supporting the validity and consistency of findings across the data set (Cohen et al., 2018). Table 5.2 illustrates this triangulation process. In addition to analysing data sources, reflective and analytical memos were kept continuously and checked for emerging biases in data interpretation and analysis.

Table 5.2

Examples of Data Sources Supporting Triangulation

Date of Collection	Data Source	**Provided Triangulated Evidence in Relation to:**
26/4/23 3/5/23-25/5/23	Field Notes: Preliminary visit, Daily visits to the ECCE classroom	Sourcing and review of classroom resources. Number (Addition) in FP Counting to bigger numbers (John) Collaborative game playing with EAL (Bingo) Staff recruitment and retention issues Limitations on number learning
4/5/23	Interview with Siobhán (25 minutes)	Staff recruitment & retention issues Following the child's lead / no limitations on number learning
8/5/23	Reflective Conversation 1 (RC1) with Sinéad Autophotography (29 minutes 5 seconds)	Use of collaborative game playing to support EAL Knowledge of how resources can support ECME Non-consideration of using PDR during CT to share children's math activity during FP
9/5/23	Curriculum Planning & Observations Policy (Early Years Mathematics Teaching & Learning Policy)	PDR is mentioned as a strategy in the CPOP to support oral language development and math talk
22/5/23	Reflective Conversation 2(RC2) with Sinéad Reflections on self-captured video episodes Autophotography (35 minutes)	Sourcing & review of resources Video: John counting to 'bigger numbers' Knowledge of how resources can support ECME

Once I had read through the entire dataset and ideas for themes generated, I took some time away from the data and analysis process. That is not to say that my mind was not completely on the data. I continued reading on my thesis topic and took notes of ideas as they came to me, as my mind subconsciously considered the data and the story it was trying to tell. Emerson et al. (2011) assert that this period of non-engagement with the data can help to develop a sense of objectivity as researchers distance themselves from the data collection site and participants.

Analysis of the interview and reflective conversations on autophotography and participant self-captured video data began by listening to each audio file several times, initially to the audio only, and subsequently in tandem with field notes made at the time of the interviews. A process of transcription followed. Again, initial thoughts and codes were jotted into the margin of transcriptions. This open coding (Emerson et al., 2011) included my initial analytical and reflexive comments and interpretations, fuelled by my own experiences as a Montessori teacher, reading of relevant literature, and the context of Irish ECE policy and practice. I remained aware of my professional and personal biases throughout the analysis phase and actively took measures to minimise their impact on the coding process (see Section 5.6) (Emerson et al., 2011). As this process progressed, comparisons were made across the entire data set. The dataset as a whole was reviewed, and thoughts and insights were documented, and connections to relevant literature and Irish ECE policy were consistently made to facilitate the rigour and validity of the study through this triangulation process.

After this period of non-engagement, I engaged in a period of re-familiarisation, re-reading the entire data set and accompanying analytic and reflexive memos. This time away proved fruitful as new ideas and connections to policy, literature, theory, and practice became clearer. I began to write to analyse, constantly moving between rereading the data set and

writing (Emerson et al., 2011), identifying connections and patterns across each part of the data set. A process of active analytic writing ensued as I endeavoured to organise data content into coherent pieces of text. Firstly, I organised the data by segments of the daily routine, which was followed by organisation of the data by mathematical topic observed (RQ1)(See section 2.3.3 for a justification of the mathematical topics selected), and finally by interactional strategy (RQ2). Data analysis to identify challenges associated with supporting ECME in this setting (RQ3) then took place. The data analysis and write-up were part of a highly iterative process, writing to understand the data, reading, highlighting and ordering the details (Emerson et al., 2011). As I progressed through this analytical and organisational phase, referred to by van Maanen (2011) as a period of 'considerable thought and experimentation', topics relevant to the research questions were determined as I interpreted the data. (p. 153). I drafted a structure to answer the research questions, as well as providing a 'thick description' (Geertz, 1973) of the situation under study.

Ghodsee (2016) asserts that theory shapes the selected research approaches and data collection techniques. Furthermore, she notes that while some theories appear relevant at the beginning of a study, at times, others become relevant during the analysis process. In this study, both processes occurred. Bronfenbrenner's *Bioecological Systems Theory* (Bronfenbrenner & Morris, 2006) was identified early in the study as a means to understand the pedagogical interactions observed in the ECCE classroom and the influence of systemic processes on practice. As I engaged in the 'write-up' process, it was apparent that another theoretical lens was needed to underpin my findings, particularly those relating to children's informal learning of mathematics and how this learning was facilitated and guided in the setting. As presented in Chapter Four, I chose Rogoff's (1990) theory of *Guided Participation* to analyse these aspects of practice and coded relevant data as described below.

Interview /Reflective video conversation data were analysed using ethnographic content analysis (Altheide & Schneider, 2013). Content analysis was applied to setting documents (setting policy documents, transition documents), as well as to interview and reflective conversation data to identify potential evidence for the concepts associated with Guided Participation (Rogoff, 1990), and to note aspects of mathematical activity documented within these documents. Due to its permanent form, content analysis can facilitate review, reanalysis and replication (Cohen et al., 2018). Initial codes, derived from Rogoff's (1990; 2003) work on guided participation, were used to guide the analysis process. While drawing up a schedule of codes can be considered an aspect of quantitative analysis, these codes were used as a guide and were constantly revised and added to. Altheide (1987) notes that within analyses of ethnographic data, 'categories and "variables" initially guide the study, others are allowed and expected to emerge throughout the study' (p. 68). I remained open to new codes or themes as they arose from the data collected. Altheide and Schneider (2013) claim that this form of content analysis is iterative. The researcher constantly re-reads, re-codes and re-categorises themes as they engage and re-engage with documents under analysis, comparing and contrasting information within. Connections to practices and comments were recorded in observational field notes. These jottings or 'asides' (Emerson et al., 2011, p.80) were written in the margins, analysing and documenting 'tentative hunches' as I went along (Emerson et al., 2011, p.81). A sample of codes produced from the analysis process can be found in Appendix H. An example of coded data is located in Appendix I. Artefacts produced by participants, such as setting policies/procedures, were also analysed using a content analysis method. The data analysis period extended over two years, with a focused time frame of nine months.

5.8.1 Researcher Positionality and the Data Analysis Process

My positionality was presented in Chapter One, and also considered in relation to participants in Section 5.6. However, a further consideration of how my positionality may have impacted the data analysis process is warranted here. As noted in Section 5.6, a researcher's personal experiences, beliefs, values and assumptions can not only influence study design, data collection procedures and relationships with participants, but also influence the way in which the data is analysed and interpreted (Braun & Clarke, 2022). This stems from the view of the researcher as an instrument for analysis (Nowell et al., 2017). As with the consideration of my relationship with participants (5.6), I used my reflexive journal to examine the potential influence of personal interests, beliefs, and values on my data analysis and the creation of knowledge. Nowell et al. (2017) consider this critical reflective journaling process as a quality strategy for qualitative research. Reflexive questions asked included: who will benefit from this study? What are the goals of this study? With my professional and research interests in early mathematics and my belief that early mathematics education is important, I adopted an advocacy position. I wished to show the mathematics that preschool children engage with and how their educator supported this engagement. This position guided my data analysis as I looked for evidence of mathematical concepts in children's play and observed how the environment and the strategies employed by their educator facilitated this engagement. This position influenced the coding of my data (looking for specific mathematical concepts, pedagogical strategies). Reflexive journaling enabled me to consider how I was coding and whose perspective I was coding from, questioning personal beliefs and assumptions, asking myself, *why did I code that information in that way,* perhaps drawing on literature or a policy document, or *why did that statement affect me that way?,* drawing on a personal experience (Braun & Clarke, 2022). Answering these questions enabled me to interrogate personal biases, and as data analysis was an iterative process, new

questions came to light, questions I needed to ask participants to help me make sense of the situation. Sometimes, my professional ECE background and experience helped me to understand the story the data was trying to tell, and at other times, I needed the experience of participants to clarify points for meaningful interpretation. Reflections on the researcher/participant relationship as experienced in this study are further considered in Chapter Nine.

5.9 Reliability, Validity and Trustworthiness

Reliability, in qualitative research, can be defined as "a fit between what researchers record as data and what occurs in the natural setting being researched" (Cohen et al., 2007, p. 149). Within micro-ethnography research design, researcher bias and subjectivity are widespread (Fusch et al. 2017). Researcher bias has been explored elsewhere in this chapter; however, the concept of triangulation in limiting researcher bias is considered here. Denzin (2012) suggests that methodological triangulation, that is, the use of many data collection methods, can be effective in limiting researcher subjectivity and bias within ethnographic research. In this case study, several data collection methods were used: semi-structured interviews, casual conversations, participant video review, content analysis of setting documentation, photographic documentation and field notes. The use of a wide range of data collection tools provided a detailed, descriptive record of how early mathematics education is supported in an early childhood setting. The potentially varied and rich data set contributes to the reliability of the research findings (Cohen et al., 2018).

Using a multi-data collection instrument approach facilitates triangulation, which establishes validity in research. Cohen et al. (2007) define triangulation as an attempt to "…explain more fully, the richness and complexity of human behaviour by studying it from more than one standpoint" (p.254). Cohen et al. (2018) propose that *methodological triangulation*—the use of multiple data collection methods—provides a means of rigorously

examining qualitative data. The various data collection methods have been detailed elsewhere in this chapter.

Bryman (2016) notes how high internal validity may be achieved in qualitative research when "there is a good match between the researcher's observations and the theoretical ideas they develop" (p.273). In this study, conclusions were drawn from data analysis from various sources, as well as from the literature review concerning mathematics education in early childhood contexts. Discussions with university-appointed supervisors concerning the above also contribute to conclusions drawn from the data.

It was intended that this study would be high in ecological validity. Ecological validity is concerned with the naturalness of the research approach and a study's ability to capture the everyday occurrences, opinions and knowledge of participants (Bryman, 2016). In this study, I located myself in an early childhood setting as a participant observer. I took part in the daily activities of the preschool room during each morning preschool session. It was anticipated that some note-taking/jotting would occur during the observation period, with field notes being written up in more detail post-session. It was envisaged that the children in the setting would be familiar with the note-taking process of the observation method, as it forms part of everyday practice in the setting as part of the assessment/documentation process. It was also envisaged that this familiarity with adult note-taking would not detract from children's natural play activity. And this was indeed the case, as children, for the most part, did not pay attention to my note-taking activity. Table 5.3 below outlines how potential threats to reliability and validity were addressed:

Table 5.3

How Potential Threats to Reliability and Validity were Addressed

Potential Threat	How This Was Addressed.
The researcher, in exploring the present, may be unaware of important antecedent events.	The researcher was present in the setting for 5 morning sessions for 4 weeks. Conversations with educators before each observation period elicited information regarding events during the previous day's afternoon session. This information was recorded in the field notes.
Informants may be unrepresentative of the sample in the study.	This study aimed to explore how mathematics education is supported in preschool education more broadly. It was accepted that findings from the ethnographic component may not be generalizable to the wider population.
The presence of the observer might bring about different behaviours.	It was hoped that with the researcher being present over 4 weeks, participants would become less aware of the researcher's presence and revert to normal practices – minimising the Hawthorne Effect over time. However, it must be stated that one cannot ensure that the researcher's presence will not alter participant behaviour generally. Participants may engage in more mathematical activity than is usual as they are aware of the nature of the study.
The researcher may become too attached to be objective.	Data collected and associated field notes will be shared with and reviewed by university supervisors for aspects of researcher bias. The researcher kept a reflexive journal and constantly reviewed for evidence of bias. Piloting and review of data collection methods highlighted any potential for bias within instruments, as well as the researcher's interpretation of the data generated.

5.9.1 Establishing Trustworthiness

Ahmed (2024) asserts that trustworthiness is central to establishing the credibility and reliability of findings in qualitative research. Lincoln and Guba's (1985) framework supports qualitative researchers to establish trustworthiness in their work. Key elements of this framework include credibility, transferability, dependability and confirmability (Ahmed, 2024). Credibility can be achieved through extended involvement with participants in the field and continuous observation of participants in their natural settings. Credibility may also

be addressed through member checking, where interview transcripts and/or findings are shared with participants to check for accuracy (Nowell et al., 2017). See section 9.5 for a reflection on this process. Triangulation, the collection and analysis of several data types, also supports the credibility of a study (Ahmed, 2024; Nowell et al., 2017). Transferability can be inferred through the inclusion of *thick descriptions* (Geertz, 1978) that depict experiences which may be applicable to the situation of others. Such detailed descriptions can help to determine the appropriateness of transferability of methods and findings from one context to another (Lincoln & Guba, 1985). Dependability was facilitated through rigorous documentation of the theoretical and methodological choices made throughout the study, the rationale for decisions made and the creation of an audit trail (Ahmed, 2024; Nowell et al., 2017). Confirmability requires the researcher to acknowledge that complete objectivity cannot be achieved, and that processes are in place to consider the influence of personal experience and values in the interpretation of findings (Bryman, 2016). I sought to address the confirmability of this study through the processes of member checking and the keeping of a reflexive journal (Ahmed, 2024), as well as the keeping of raw data and field notes to support the verification of findings, *referential adequacy* (Lincoln & Guba, 1985; Nowell et al., 2017). Table 5.4 below shows actions taken to establish the trustworthiness of this study.

Table 5.4

Actions Taken to Establish Trustworthiness (Lincoln & Guba, 1985)

Actions	Description	Enables
Prolonged engagement (Micro-ethnography)	Four weeks spent in the ECCE room, observation, reflective conversations with the EC lead, exploring their interpretation of ECME in their classroom	Credibility
Persistent observation (Micro-ethnography)	Participant observation, consistent engagement with the data set, enabling identification and assessment of important factors, and investigation in sufficient detail	Credibility
Triangulation	Data collection and analysis interpretation of multiple sources (manager, EC lead, observation, setting and government policy), methods (participant observation, reflective conversations, interview, policy analysis)	Credibility
Member checking	Findings and conclusions shared with the setting manager to ensure accurate representation of interview data and setting procedures and practices	Credibility
Thick Description (an artefact of micro-ethnography)	Description of procedures, context (Chapters 7 & 8), and participants (Chapter 6) in sufficient detail to permit judgment by others of the similarity to other EC settings	Transferability
Audit trail	Record keeping: documenting methodological and theoretical choices made, data reduction process, field notes, reflexive notes, interview schedule development and pilot process	Dependability Confirmability
Reflexive journal	Maintained throughout the research process. This included my personal insights and understandings of the research process, interrogation of personal biases and their influence on data interpretation	Credibility Transferability Dependability Confirmability
Referential adequacy	Keeping raw data for future analysis and interpretation, for verification of initial findings and conclusions	Credibility

5.10 Ethical Considerations

Ethical approval for this ethnographic study was granted by the Research Ethics Committee at Dublin City University in January 2023. I am committed to maintaining ethical standards when conducting research in early childhood settings. Beauchamp and Childress (2008) present the following five ethical principles: informed consent, autonomy, non-maleficence, beneficence, and justice.

5.10.1 Informed Consent

Written permission to carry out the study was sought from gatekeepers: setting managers, head teachers, boards of management, and parents. Gatekeepers were provided with a plain language statement detailing the particulars of the study (Appendix J). Written permission was obtained from the parents/guardians after they had read the leaflet via an Informed Consent Form (Appendix K). The plain language statement was jargon-free and easily understood by participants. Confidentiality and anonymity of participants were maintained by attributing aliases to both adult participants and children during observation transcription, field notes, data analysis and the write-up of the study. The names and location of the settings were changed and generalised, e.g. an early childhood setting in County Dublin (Lareau, 2021). Video footage captured by participants was only viewed by the participants and me, and the setting retained the footage.

5.10.2 Autonomy

Permission to conduct the research in the EC setting was sought from the setting manager. Adult participants (manager and ECEs) were provided with a Plain Language Statement (Appendix L) and an informed consent form (Appendices M & N). I visited the settings before the commencement of the project, to answer any questions adult participants or children may have and to clarify the researcher's role within the setting during the

fieldwork period (Mereweather, 2018). All participants (adults and children) were reminded throughout the research period that they could withdraw from the study at any time.

I was aware that I was entering into the children's learning environment and that it was necessary to obtain assent from the children to carry out observations and take field notes in that environment. The children were informed about the study through child-friendly language during the pre-study visit using a researcher-produced picture book (Appendix O). I presented myself to the children in role as 'Córa the Maths Detective' with a puppet as an assistant detective. Adopting a researcher-in-role approach, I conducted a maths hunt around the room with the puppet and magnifying glass. Children in the pilot setting spontaneously joined in this hunt, getting out of their seats and finding objects they perceived as mathematical (PFN 1).

Written / pictorial assent was sought from the children (Appendix P). Reflections recorded in my Reflective Journal from my initial visit in the pilot setting note that '*the gaining of assent was not as easy as I had expected. It was so time-consuming… and some verbal re-wording of the statements was necessary, particularly for the youngest children'* (RJP1). My informed assent form had been designed for older children (4–5-year-olds). I was placed in the second year of ECCE, and therefore, I was not expecting a child as young as two and a half to be present in the room. It was apparent that this method of informed assent was not appropriate for such a young child. After consultation with the LE, I decided to assess the child's assent to be observed through her body language (Informed consent had been obtained from the child's parents). Based on my previous experience of conducting research with preschool-aged children, it was anticipated that children's participation would be fluid as they leave an observed activity for reasons of their own choosing. As well as obtaining non-verbal assent, ongoing oral assent was also sought during the study. Children were reminded that they were not obliged to participate, and were free to leave an observation

or ask me not to record them if they did not wish to be observed. I was also mindful of children's body language, as children may not verbally state that they do not wish to participate but indicate their wish to withdraw in other ways (Dockett et al, 2012). An example of this occurred in the main setting with Chandra. Chandra had been absent from the setting on my preliminary day. Therefore, she had not been introduced to me and my project. As I tried to gain informed assent from her, she continuously turned her back to me as I approached. I took this as an indication that she did not wish to chat with me. This continued for 3 days. The following week she became less wary of me and approached me with a smile, and some pretend food from the home corner. I did not immediately ask her to fill in the informed assent form. I waited another day until I felt she was truly comfortable with my being there, and she had time to observe me and my note-taking. As with adult participants, children were continuously reminded that they were free not only to opt out on any given day but also to withdraw from the study at any time.

I was aware of potential 'power inequalities' that could arise during the data collection process (Lareau, 2021, p.99). Lareau (2021) asserts that power dynamics exist during the interview process as interviewers need to guide the interview to obtain the required data. However, as Lareau (2021) notes, interviewees may continue with interviews because they feel they have to, despite being assured that they can withdraw from an interview at any time. Lareau (2021) suggests being mindful of participants' body language, as it may suggest unease in the participants. If participant unease is suspected, a check-in is required. A termination of the interview is possible. In this study, during the reflective conversation conducted with the ECE in the pilot setting, the participant was visibly nervous with hands clasped and eyes down (FN P3). The participant was reassured that this was a conversation rather than an interview. This situation was not new to me, as noted in my reflective diary, '*I have experienced this nervousness before when conducting interviews for my BA research,*

even with very experienced educators. The most nervous in that study were those with level 6 qualifications' (FN P3). Such discomfort and unease should be on the radar of a researcher and acted upon if suspected, either by checking in with the participant or simply turning the recording device off. We agreed not to audio-record the conversation; the participant was happy for me to make notes, she relaxed immediately and shared photographs of her key children engaged in mathematical activity.

A further example of my observation of and mindful respect for participants' body language occurred during Sinéad's preparation and planning time, typically between 8.45 and 9.20 am. I had identified this as a time to follow up with Sinéad with some questions or observations from the previous days, or from our reflective video conversations. However, some mornings, Sinéad appeared busy; her head was down. She was concentrating on getting her work done. She spoke little. I took this as a sign that she did not wish to be disturbed, and I kept my questions for another time.

5.10.3 Non-maleficence

A philosophy of 'do no harm' was adopted. All adult participants and children had the right to withdraw from the study at any time. According to Greig et al. (2013), researchers must be cognisant of what it actually means in terms of researching with both adults and children. In this study, I was committed to ensuring that both adult participants and children did not feel compelled to participate and that each person understood that it was acceptable not to participate. This was repeatedly stated to both adult participants and children using appropriate language.

I have completed *Children First* training, a course in child protection. I understand my ethical obligation to commit to ensuring the well-being of each child in the study, and

made myself aware of the *Designated Liaison Person* (DLP) in each setting and committed to report concerns relating to child protection to the DLP should they arise.

5.10.4 Beneficence

The principle of beneficence alludes to the positive facets of a research study (Flewitt & Ang, 2020) and researchers acting, 'for the positive benefits of others' (Greig et al., 2013, p.247). Flewitt and Ang (2020) assert that researchers should design their study to ensure faithful, true, reliable and verifiable findings, to contribute high-quality research to the field. This action, Flewitt and Ang (2020) suggest, not only demonstrates respect for the academy but also towards the participants themselves. The researcher should ask themselves, 'What are the benefits of taking part in this study for participants? What are the benefits of the research to society? This study explored how ECME is supported in an early childhood setting and intended to share findings with participants and the wider research community. Adult participants may have found participating in the study enhanced their practice as they engaged and conversed with me in relation to mathematics in early childhood educational settings. It was not envisaged that any harm would come to either the adult participants or the children in the settings as a result of participating in this study. It is intended that the study will make a valid contribution to existing research on mathematics education in early childhood and that findings will be disseminated through academic journals and relevant conferences.

5.10.5 Justice

I, as a researcher, am committed to conducting a fair and equitable research study, ensuring that all participants and children have equal access to participation and that their opinions concerning the study are valued and respected. Participants and children were

afforded time to consider the information provided to them and to ask questions about their involvement in it.

5.11 Chapter Summary

This chapter provided a rationale for the qualitative research approach employed in this study. The ontological, epistemological, and axiological assumptions have been detailed. The chapter described the conceptual framework underpinning the study, and a description of the methods for data collection and analysis was provided. The chapter considered the reliability, validity and trustworthiness of the study and provided a synopsis of how potential pitfalls in these areas could be avoided. Consideration was also given to the ethical implications of the study. Chapter Six presents a thick description of the setting, its people, and routines.

CHAPTER 6: AN INTRODUCTION TO THE SETTING

'children have their own preschool arithmetic, which only myopic psychologists could ignore' (Vygotsky, 1978, p.84)

An ethnography aims to tell a story (Van Mannen, 2011). In this instance, I tell the story of how an ECE setting supports ECME in a preschool classroom at a particular point in time. I have chosen to follow Vygotsky's (1978) advice in the quote above in that, through my observations, I deliberately set out to observe and document the mathematics young children experience in their preschool classroom, and how these mathematical explorations were supported by the adult in the room. I aimed to make visible the mathematics that may often be visible in the EC classroom but which remain invisible to the outside world, privy only to those working in this EC setting. As discussed in Chapter Four, EC classrooms provide children with opportunities to observe mathematical cultural practices in action (Rogoff, 1990), and may or may not feature intentional instruction (Pollitt et al., 2020). The analytical framework draws on Rogoff's highly influential concept of *Guided Participation* (1990; 2003). As noted in Chapter Four, Rogoff's theory considers how adults support children's participation in learning about their world, in learning how to participate in the cultural practices that characterise everyday life, in this case, mathematics. It considers the role of adults and more able others in children's learning. It considers how learning experiences are structured, even those where the adult is not directly involved in the learning (Rogoff, 2003). In this study, I observed the learning environment, its resources and people, how they come together to support, direct or constrain children's participation in and learning of mathematics in the preschool classroom. This study aimed to contribute to the knowledge gap noted in Chapter One, which is that very little is known about children's participation in early

mathematics education in Irish preschool settings. To this end, findings presented in the following chapters answer the following sub-questions:

1. What types of mathematics do children participate in?
2. In what ways does the adult guide or support this participation?
3. What are the challenges in supporting children's participation in ECME in this setting?

The data set was analysed in the first instance to identify aspects of mathematics children engaged with during the ECCE daily routine (Chapter Six), followed by a second analysis identifying the strategies used by the adult to support children's engagement with mathematical concepts (Chapter Seven). Chapter Eight provides findings in relation to the challenges in supporting children's participation in ECME in this setting. As described in Chapter Five (Methodology), data were gathered using several strategies aligned with an ethnographic research approach. This chapter sets the scene, providing a narrative context of staff, children and the daily ECCE routine, i.e. describing the elements of the microsystem that is the ECE setting and the ECCE classroom.

6.1 Setting the Scene: Staff, Children and Routine (A Verbal Portrait)

In this section, the reader is introduced to the participants. The daily routine is summarised to provide a context for the study. With any story, the characters are central to the events that take place within the narrative. With this in mind, the subsequent paragraphs introduce the reader to the staff and children in the setting. Staff who were directly involved in the study are described here. This is then followed by a narrative description of the setting and the daily routine of the ECCE class.

6.1.1 Staff

6.1.1.1 The Manager

Siobhán[3] has owned and managed this EC setting for the past 20 years. She is qualified to master's level in ECE and has a keen interest in ECME, one which she hopes to promote in her setting. In her capacity as curriculum leader, Siobhán wrote an Early Years Mathematics Education Policy, implemented training in ECME for staff and reviewed and sourced resources that support children's participation in ECME (INT1). However, due to external interests, Siobhán delegated much of this aspect of the managerial role to the deputy managers (INT1), but still maintains the position of lead manager. Siobhán tries to be present when the setting opens in the morning to discuss any matters arising and to be on-site as often as possible.

6.1.1.2 The Lead Educator in the ECCE Room

Sinéad is a graduate educator with a level 8 degree in ECE and three years' experience in the early childhood sector. Before working in this setting, she worked in a Montessori-based ECCE classroom, although she was not specifically trained in the Montessori method of education. Sinéad has a keen interest in sport and enjoys travelling. She has an engaging personality and is easy to talk to. Sinéad is the only educator in the room.

6.1.1.3 The Children

Table 6.1 provides an overview of the 11 children who attended the setting during the study. Originally, the class comprised six girls and five boys; however, two female children left the setting suddenly at the end of the first week to return to their home country for an extended period of time, leaving nine children on the roll. Of these nine, eight were regular attenders, with one child, Audra, attending only two or three days per week. These days

[3] The names of the owner/manager, lead educator and children have been changed to protect the identity of participants.

varied each week. As alluded to in Table 6.1, the majority of the children spoke English as an additional language (EAL). Their level of fluency was quite high. Audra spoke an Eastern European language at home, and her English was not as fluent as the other children in the group. The others had all been born in Ireland and had attended the preschool for two years before the study. Audra was born outside of Ireland and had only joined the group this academic year. Both of Seán's parents were Irish, and Seán was the only child who spoke English alone. Based on my observations, Table 6.1 also gives an insight into the personalities of the children. Two children, Chandra and John, were adept at gaining adult attention. Both children had strong personalities. John, in particular, was at his happiest when working with an adult and often sought attention from either Sinéad or me.

Table 6.1

An Overview of the Children Attending the ECCE Class

Name	Age	EAL	Description
Chandra	4	✓	Vivacious, big personality, often demanded adult attention, confident, infectious laugh
Filip	4	✓	Relatively quiet, confident, happy to chat on a 1:1 basis, mostly played with the other boys, but sometimes preferred to play alone
Seán	4		Sensitive, quiet, observant, obedient, preferred to play with others
Priya	4	✓	Extremely quiet, easily upset, often observed playing alone,
Arjaat	4 (turned 5 during the study)	✓	Loved to chat, friendly, observant, preferred to play with others
John	4	✓	Preferred to play alone or with an adult, interested in numbers, number operations and

			constructing 3D shapes. Often sought adult attention or company. Loved Numberblocks
Aparna	4	✓	Rarely attended. Very quiet.
Adomas	4	✓	Sensitive, preferred to play with the other boys, construction was his preferred activity, easily upset
Audra	4	✓	Very quiet, rarely spoke, rarely interacted with other children, once her two friends left.
Lucy	4	✓	These two children left suddenly after week 1. Up until then played chiefly with Audra and often in their home language
Daisy	4	✓	

6.1.2 The ECCE Daily Routine

The ECCE session lasted for three hours, from 9:30 to 12:30. Children arrived from 9.20 and were greeted by a member of staff at the main door before entering their ECCE room. Here they were greeted either by Sinéad (if not on door duty) or another member of staff. Children hung their coats independently and placed their bags on a shelf close to the door. Table 6.2 provides an overview of the daily routine.

Table 6.2

Overview of the Daily ECCE Routine

Time	Activity Type	Characterised by:
9.30 -10.25	Free Play	Children are free to choose their own activity, and adults support as necessary
10.25 – 10.30	Tidy up & set up for Circle Time (CT)	Adult and children tidy up, set out chairs for CT, children sit on mat
10.30 -11.00	Circle Time Chiefly held indoors, some days outdoors	Children allocated chair, CT adult-directed set of learning activities, chiefly focused on passage of time, weather,

		number, counting & letters of the alphabet Aspects of weekly topic were also discussed
11.00 – 11.20	Whole Group Activity Time	Adult-led activity, activities based on weekly topic: Creative, Exploratory play, Fridays: number worksheets
11.20-11.40	Snack Time	Children's bags given out by a child, children eat snack from home at tables, chat. Little adult interaction.
11.40 – 12.10* **Sometimes longer**	Outdoor Time	Free play in the outdoor garden
12.10- 12.25	Story time/video time (if not outside)	Sinéad reads a story to the group
12.25	Getting ready to go home	Children put on coats, fetch bags and sit on chairs – lined up in pairs by the door
12.30	Session Finishes / Children go home	

6.1.2.1 Free Play

The first hour of the day was *Free Play Time*. This period of the ECCE day was characterised by children's demonstration of individual or collective agency to select a resource/toy they wished to play with. The choice or power to either seek or reject adult support lay entirely with the children during this time. They were free to play alone or with others and could choose where in the room they wished to play. Generally, Sinéad refrained from initiating interactions with the children at this time unless approached by the children to do so. As she herself commented, "there are times when the room goes so quiet, no one comes near me, and I'm like someone ask me something, they get so involved in their play"

(FN 1). This philosophy is embedded in the setting's Curriculum Planning and Observations Policy (CPOP), which states that the learning environment should be arranged 'for children to do things for themselves with a minimum of adult intervention' (p.4). As presented in Chapter Two, this conceptualisation of the adult as facilitator of a rich learning environment and the notion of adult restraint in intervening in children's play for fear of interfering is a central tenet in early childhood discourse (McInnes, 2019), and in EC practice. Indeed, Sinéad spoke about advice given to her on practicum: "It's amazing what you see when you just watch…when I was on placement, my supervisor told me to just sit and watch and to think about what I was seeing when children play" (FN1). Free play time then in this setting is a space for adults to observe, to notice and interpret what is seen, and interact when invited to.

Over the fieldwork period, my reflective notes document my surprise at the lack of interaction initiated by Sinéad during this time (RJ1; RJ3). However, this is not to say that Sinéad did not interact with the children at all. Numerous instances were observed and documented in which Sinéad, when called upon by the children, actively engaged with them either individually or in small groups. Despite Sinéad's limited interaction with the children during FP, much rich mathematics occurred. This will be discussed in Chapter Seven. This contrasts with the more adult-directed *Circle Time* (CT), which followed free play and lasted twenty to thirty minutes.

6.1.2.2 Circle Time (CT)

This section begins with a narrative overview of a typical CT session as recorded in my field notes. It identifies the aspects of mathematics covered during CT and the interactional strategies used by Sinéad to engage the children with them. Field notes document that CT followed the same routine structure each day. As noted in Table 6.2 above, CT was scheduled from 10.30-11.00. Occasionally, the start time was delayed due to Sinéad

being relieved for her break by an assistant manager. On these days, the assistant manager guided the children through the setting-up process but did not initiate CT, and therefore, CT started later than planned and was often rushed in nature as a result (FN1; FN5; FN7; FN9). To begin, Sinéad and the children set out eleven chairs of varying colours in a row in front of the large car mat on the floor, in front of some low wooden shelves. There was only one green chair, which was reserved for the best 'tidy upper'. Sinéad's adult-sized chair was positioned on the mat facing the row of child-sized chairs. The linear furniture layout facilitated the adult-dominated nature of this part of the routine. Sinéad could see all of the children.

The CT display board (Figure 6.2) was located on the wall behind this chair. Positioned to the right of the chair was a wicker basket containing laminated pedagogical resources, including two-dimensional geometric shapes, labels for the days of the week and months of the year, and pictorial representations of various weather conditions. Adjacent to the basket was a portable whiteboard, which functioned as a display space for visual materials aligned with the weekly thematic focus. For example, during the first week, the board featured imagery associated with the theme *Under the Sea*. This theme lasted for the first two weeks, and the theme of *Science* was explored in weeks three and four.

Once the chairs had been arranged, the children sat on the mat and were assigned a chair to sit on by Sinéad, who, after identifying the 'best tidy upper' and asking them to sit on the green chair, says, 'Seán, can you sit on a blue chair? Chandra, can you sit on a yellow chair?' (FN 2). CT then began with the 'Hello Song', which is sung by the whole group. The same song was sung every day during the observation period

Let's say hello to Audra, let's say hello to Audra, let's say hello to Audra, we're glad you're here today.

This song was repeated eleven times, with each child being individually greeted through the song. This was followed by a 'Listening Song' to aid children in settling down and focusing on CT content.

Ooha ooha ears are listening. Ooha ooha eyes are watching. Ooha ooha feet are quietening, now we're ready to learn

Once the group had settled, Sinéad began CT by asking some questions. The questions followed the same sequence every day. The conversation started by discussing the day of the week, with Sinéad asking, "Who can tell me what day it is today?" The children offered their ideas and agreed upon a day, and the appropriate label is added to the display. Discussions were followed by singing the Days of the Week song, sung to the tune of the Addams Family theme tune (Mizzy, 1964). It was sung with great gusto every day. The children liked singing loudly.

This discussion was followed by the question, "Who can tell me what month we are in?" The children offered their answers. Again, a song was used to help children learn the months of the year and the sequence they appear in. As a day, month and season were agreed upon, Sinéad took the appropriate label from the basket and added it to the CT board (see Figure 6.2 below).

The conversation then moved to the weather with Sinéad asking, "What is the weather like today? Look outside the window and tell me." The children do as asked and shout out adjectives such as sunny, dry, and not windy. They chose an appropriate picture from the collection in the wicker basket to represent the weather; this was added to the CT display. Once the discussions regarding the weather were concluded, Sinéad moved the conversation on to the shape of the week. Again, Sinéad kept several laminated 2D shapes (heart, square, rectangle, circle, triangle, star) in her basket (FN 2). She asked the children to wave goodbye to the shape they had looked at the previous week and to welcome the shape for the current

week. Over the observation period, the children revisited rectangles, circles, stars and squares. Sinéad decided which shape would be looked at each week. It should be noted that Sinéad only intended to focus on 2D shapes and informed me, "I haven't introduced them (3D shapes) during circle time…just in passing if they've made a 3D shape in their play" (FN 16). A discussion on the number of the week followed. Sinéad's focus here was on recognising a numeral and how to write it. Sinéad typically modelled to the children how to write the number in the air using their index finger as a 'magic wand. There was also a focus on counting that number on fingers, head taps, claps, and knee taps. This part of CT concluded with a look at the selected letter of the week. The second part of CT centred around a theme of the week. Sinéad had some pictures related to the theme on an A1 whiteboard, and they discussed each picture in turn.

Figure 6.1

Overview of Circle Time Routine (10.30 – 11.00 approximately)

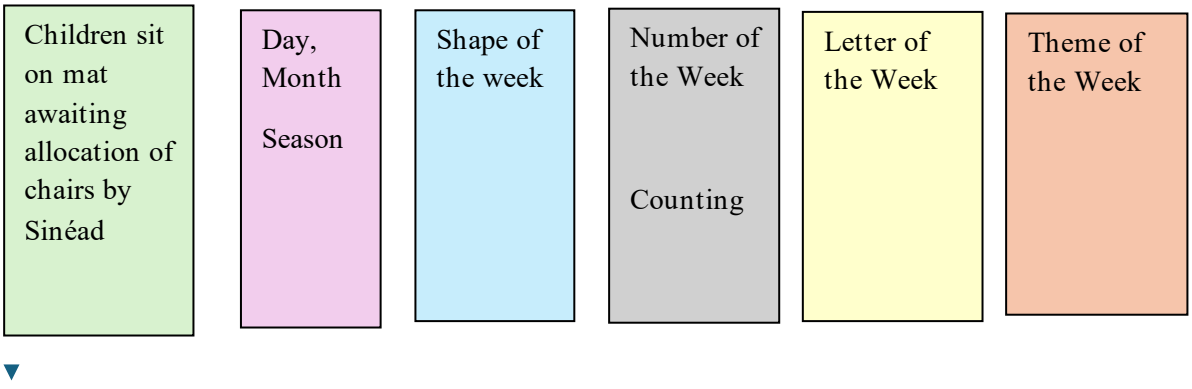


Figure 6.2

Sinéad's Circle Time Board



6.1.2.3 Whole Group Time

The end of CT, characterised by Sinéad introducing language or ideas associated with the weekly theme, seamlessly morphed into whole group time, by Sinéad asking the children to bring their chairs back to the tables and sit there awaiting further instructions. From Monday to Thursday, whole group time (WGT) activities reflected some aspect of the weekly theme and took the form of either a sensory-based or creative/art activity. The weekly theme was determined by Sinéad as a result of her observations of children's interests, reflecting the emergent aspect of her curriculum. A thematic approach to teaching is common in early childhood settings and offers a structure to support learning in ways that are meaningful and relevant to children (Min et al., 2012). Fridays were different, however. Here, children were given photocopied worksheets reflective of the number of the week learned during CT. The majority of the WGT activities took place at the tables in the ECCE room. There was one exception to this. During the final week, Sinéad took the children outside where the large bucket scales were located (Figure 7.6) to explore weighing objects. This is described in

detail in Chapter Seven. Table 6.3 provides an overview of the activities that took place during WGT.

Table 6.3

Whole Group Time Activities

Date	Activity	Theme
3/5/23	Painting: pre-cut crabs, orange/red paint provided	Under the Sea
4/5/23	Sensory Play: Sand – plastic sea creatures to hide; Water – plastic sea creatures to wash	Under the Sea
5/5/23	Worksheet – Number 5 (writing numerals and representing quantity)	Number of the Week
8/5/23	Glueing & Sticking – making an octopus, children given pre-cut template and 4 strips of paper to glue on as tentacles	Under The Sea
9/5/23	Music & Movement: video with song: focus is on spatial & positional language (moving over, under, around sea creatures in different ways – jump over the jellyfish, swim around the shark)	Under the Sea
10/5/23	Glueing & Sticking – making starfish, children given precut template and 5 stickers to add as the starfish's 5 eyes and some Cheerios to add as the starfish's suckers	Under the Sea
11/5/23	Playdough: Making Sea Creatures Additional materials: fusilli pasta & matchsticks	Under the Sea
12/5/23	Worksheet – Number 6 (writing numerals and representing quantities)	Number of the Week
16/5/23	Floating & Sinking (Forces): children given a worksheet to complete, children to predict if items will float or sink and record predictions and results	Science
17/5/23	Magnetism (Forces): exploring different types of magnets and the materials they attract	Science
18/5/23	Materials: Making lava lamps	Science
19/5/23	Worksheet – Number 7 (writing numerals and representing quantity)	Number of the Week
22/5/23	Materials: properties of materials – mixing oil, water and pepper	Science
23/5/23	Weight/measures	Science

24/5/23	Colour Mixing (Painting)	Science
25/5/23	Chemistry – reactions: adding baking soda to lemon slices	Science

6.1.2.4 Snack Time

Every day, one child was selected to hand out the lunch bags to the other children who were seated in self-selected spaces at the tables. The children were free to chat amongst themselves. Sinéad typically used this time to complete some administration work on the tablet, often sending photographs she had taken of the children at play that morning to their parents via an App. Children who had finished their lunch, replaced their bag on the allocated shelf and were free to go to the Reading Area, choose a book and wait for the others to finish. At 11.40 (approximately), Sinéad and the children arranged the chairs in pairs next to the door, sat and waited for Sinéad to call their name to get their coat and put it on for Outside Time. On days that were deemed dry enough, children simply lined up at the door and skipped this step.

6.1.2.5 Outdoor Play

This period of Free Play lasted 30 minutes. Children were free to choose their activity. Sinéad's role during this time was chiefly supervisory in nature, and the children's safety was the overarching concern during this time. While I recognise that it is possible to encounter mathematical ideas in free play outdoors, I decided to observe outdoor play on one occasion and noted that no specific mathematical interactions took place between Sinéad and the children during this time. In fact, Sinéad only interacted when there was a safety concern or there was a conflict to resolve. On this basis, I decided not to report on this part of the ECCE routine in terms of ECME. Once concluded, the children went home at 12.30.

6.2 Conclusion

This section provided the reader with an overview of the setting; the staff, children and daily routine based on my observational fieldnotes, interviews and conversations. This narrative account established the contextual foundation for the subsequent findings and analyses, situating the reader within the organisational and pedagogical structures of the daily routine and the strategies employed to support ECME in this setting. During free play, the children freely explored their learning environment, where all resources were accessible to them. Here, they drove the learning agenda, while Sinéad supported when requested, rather than engaging in direct or intentional teaching. This is in contrast to the more teacher[4]-led approach adopted during Circle Time (CT), where Sinéad controlled the focus and opportunities for learning.

[4] The term 'teacher' is used here as this is how Siobhán referred to her staff.

CHAPTER 7: CHILDREN'S PARTICIPATION IN ECME

Children display a universal love of mathematics, which is, par excellence, the science of precision, order, and intelligence (Montessori)

This chapter presents findings that answer research questions one and two relating to the events, actions and interactions that occurred within the microsystem of the ECCE classroom, and answers the following research questions;

1. What types of mathematics do children participate in?

2. In what ways does the adult guide or support this participation?

In line with Montessori's observation above, findings show that the children liked working with mathematical ideas and concepts, and in some cases, there was evidence of their joyful participation in mathematics. Aistear (GoI, 2024c) recommends that children engage in learning experiences that foster a foundational understanding of "concepts such as matching, comparing, ordering, sorting, estimating, counting, spatial awareness, size, weight, height, length, capacity and money through *súgradh* (play) and playful experiences in an enjoyable and meaningful way" (p.29). Unlike the Irish Primary Mathematics Curriculum (DE, 2023a), Aistear (GoI, 2024c) does not place each of these mathematical concepts into categories or strands. For ease of understanding findings, I have chosen to present findings under the categories of the *Big Ideas of Early Math* (Brownell et al., 2014): number sense (including counting and number operations), measurement, data analysis, shape, spatial relationships and pattern.

Emerson et al. (2011) note that the selection of fieldnotes (FN) for inclusion in ethnographic texts is 'not a simple matter of picking the most interesting examples' of a phenomenon under study (p.207). FN may be chosen to highlight key aspects of behaviour

under study, to share typical routine situations and to help the reader to get a sense of the setting, characters, behaviours and actions (Emerson et al., 2011). I selected these FN excerpts to provide the reader with a thick description (Geertz, 1973) of the learning environment and resources used to support children's participation in ECME, and to provide evidence of the pedagogical routines and strategies that support or hinder early mathematical learning (Rogoff, 2003). Selected FN excerpts illustrate the range of mathematical concepts that the children engaged with, helping to provide evidence for answering research question one. I aimed to include examples from several different children. The FN excerpts included in this section reflect children's own, self-chosen participation in mathematical activity during Free Play (FP) as well as their more structured participation in adult-led segments of the daily routine (Rogoff, 2003), such as Circle Time (CT) (Bustamante et al., 2018; Lundquist et al., 2023; Zhang & Quinn, 2018) and Whole Group Time (WGT), therefore helping to answer research question two. I will present these examples of classroom activity with some commentary on the levels of mathematical understanding that the children displayed. Beginning with the most frequently observed aspect of mathematics, number, the chapter discusses children's participation in Number, Measures, Shape, Spatial Relations, Data and Pattern. Adult strategies used to guide participation in these activities are also presented. Further discussion of broader issues will be included in Chapter Nine.

7.1 Number Sense

Similar to previous studies (Gillic, 2021a; Worthington & van Oers, 2016), of all of the big ideas of mathematics, children's participation with Number was the most frequently observed over the study period. This section discusses how children participated in activities involving Number during the study. Counting dominated, with some findings indicating individual children's interest in sets and number operations. The findings indicate that children engaged with aspects of number in multiple ways.

7.1.1 Number in Free Play

Children's participation in number-related activities during free play chiefly occurred through interaction with games (Bishop, 1988; Ramani & Eason, 2015). There was an abundance of freely available number games in the ECCE room, some commercial and some teacher-made. Sinéad stated that some games, for example, Pattern Snakes and the Counting game, had been downloaded from a commercial teaching resource site and prepared for one child, John, as a response to observations of his interest in counting. Sometimes the children engaged in a self-selected activity by themselves, and at other times they sought input from Sinéad. The selected FN excerpts that follow illustrate how individual children explored Number through the differing activities available to them. They show the children actively engaging in number activities. They exemplify children's interest in number, their knowledge, competency, mastery and positive disposition towards mathematics. The first field note excerpt shows Priya's engagement in a self-selected counting activity.

7.1.1.1 Priya and the Counting Game.

In FN6, I observed Priya, a four-year-old girl, sitting by herself with some number and quantity cards on the table in front of her. Later, I discussed this with Sinead in RC1. Sinéad identified this game as being 'really good' for developing children's counting skills. She continued, 'they just love this activity… I keep adding more cards to the box, and they love completing them all" (RC1). In this excerpt, Priya selected the activity from a range of similar activities on the shelf and sat next to Sinéad, who was working with another child. The cards have a random array of objects to be counted (Figure 7.1). The child must place a counter on the matching numeral, choosing from three numerals at the bottom of the card. There were 20 cards in total

> For the activity, Priya needed to count the objects on the card and choose the correct numeral (from a choice of three) to place a counter on – four flowers, place the

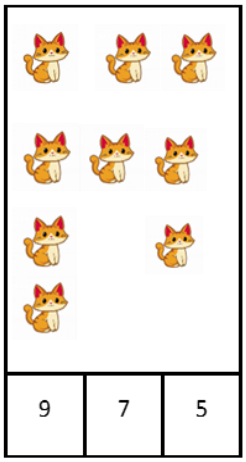
> counter on four. The maximum number of objects on a card was ten. As I watched, Priya already had ten cards spread out in a line in front of her with the counters on the correct numeral and was reaching into the box to take another card. She placed this card directly on top of the first line. She looked at the card, placed the counter on nine, then went back and counted the cats, placing one finger on each one as she counted (silently). She proceeded in this manner until all 20 cards were completed. She did not talk or try to catch an adult's attention for the duration. Her face showed intense concentration, wrinkled brow and intent gaze (FN1)

Based on this observation alone, one could say that Priya is a competent counter to ten. Using the one-to-one principle (Gelman & Gallistel, 1978), she counts accurately and reliably, checks her work, and works methodically. Priya's placing of the counter on the number nine before counting the set suggests that she had experience with this activity (Rogoff, 1990). She recognised how nine was represented on the card, but checked to see that she was correct. Priya must have been shown how to do this type of activity before, either directly with Sinéad or by observing a more knowledgeable peer (Rogoff, 1990). She knew exactly what to do. The relatively small number of children (11) in a small space perhaps enabling children opportunities to observe others and to learn from these observations (Rogoff, 2003) as Sinéad noted, "my room is so small that they all see it anyway, they're seeing it and they'd normally come over and say, oh that's so cool, look what you've done" (RC1). Priya was deeply engaged in the activity. She was afforded the freedom, time and space to become so involved. Such deep engagement supports the development of learning dispositions such as determination and persistence (GoI, 2024c). She was sitting at a table next to another child and Sinéad. Priya briefly looked at what the other child was doing before setting out the twenty cards in front of her. The items on these cards were not arranged in a typical representation, encouraging the practice of the order-irrelevance principle, as well as the skill of subitising. During this 20-minute observation, Priya was so focused on her activity. Again, her face was full of concentration. After completing all 20 cards, she sat up and scanned her work, smiled and said, "Yes!" Sinéad also tried to observe Priya's activity. However, her attempt to observe Priya was interrupted by the competing demands of

Chandra, who was sitting next to her. This highlights the practical challenges of sustaining individualised attention within EC classrooms. From an ECME perspective, such moments are significant, as they reveal how easily opportunities for mathematical scaffolding can be missed when practitioners are required to prioritise immediate needs over focused observation. This point will be revisited in Chapter Eight.

Figure 7.1

Example of a Counting Game card



As noted above, Priya's completed activity brought her joy. The following excerpts demonstrate how games facilitate joyful participation and serve as a motivational catalyst (Rogoff, 1990) for early childhood engagement with fundamental numeracy, including numeral recognition and arithmetic operations

7.1.1.2 Catch and Count

In Catch and Count, players compete to collect the most fish while navigating a 'shark' penalty that resets progress. The following excerpt is analysed to demonstrate how this game fosters social learning environments and motivates student engagement with numerical operations.

> Sinéad was invited to play 'Catch and Count' by Priya. Sinéad asked Priya to get a dice as the spinner for the game was missing. Priya chose a numeral dice and said, "if we roll a six, we can go again". [The game covers numbers one to five]. They needed to find a bubble card with the number of bubbles rolled by the dice. Priya rolled a three, looked at the cards, chose one and counted "one, two, three", putting a finger on each bubble as she did so. Then she took another card, turned it over to show a puffer fish. John was watching the game from the other table. Priya rolled a four, scanned the cards again, selected one and counted "one, two, three, four, four bubbles" and turned it over. It was another single puffer fish (she laughed) "Only one", she said, laughing. Five minutes later, all the children had gathered around this table watching the game. Priya drew the shark card – all her cards were taken away. There was a collective sigh in the group, and Filip said, "Oh no, it's the shark…all your cards are gone. Priya laughed. (FN 4).

It is noteworthy how one simple game can captivate and gain children's collective attention. The anticipation of who would get the shark card was palpable, and the collective empathy for Priya when she drew the shark card was evident. Priya demonstrated her ability to understand the rules of the game and to count up to four bubbles. She recognised numerals on the dice. She understood cardinality she knew that she had four bubbles at the end of her count. The numeral spinner for the game was missing; this is why Priya was asked to fetch a die. Figure 7.2 shows that the spinner represented numbers with numerals rather than dots. Perhaps this is why Priya chose a numeral dice rather than a dot dice. The game not only supported Priya's engagement with mathematics but also her personal resilience and general social development in a way that is fun and enjoyable (Balladares et al., 2023). This is evident by Priya's laughter when she continuously drew the single puffer fish, and especially when she eventually drew the shark card. Sinéad's lack of participation is interesting. Although she had been invited to play with Priya, Priya played mostly by herself. Perhaps Priya sought Sinéad's participation as emotional support rather than a co-player (Rogoff, 1990). Priya knew how to play the game, so she did not need advice on how to play it. She took control of the game, continually rolling the dice without offering a turn to Sinéad, who waited patiently and did not interfere with Priya's agenda. Perhaps, knowing she had a

captive audience in the onlooking children, Priya kept playing, looking for the reaction and camaraderie of the other children. Sinéad was not needed.

Figure 7.2

Catch and Count



Picture: orchardtoys.com

I selected the following FN excerpt to show John's agency in directing his activity. Similarly to that above, it demonstrates Sinéad's sensitivity to children's self-directed activity and flexibility of approach when playing with him.

7.1.1.3 Game of Ladybirds

The following excerpt illustrates John's agency in determining his own rules for the game. It shows a flexible educator who sensitively responds to John's request for her to play with him. In this long field note excerpt, John approached Sinéad (who was working with Chandra) to play this game with him. Sinéad joined him on the mat. They did not play the game according to the official rules. No die was rolled. The objective of their version of the game was to match quantities.

> Sinéad "we're dividing these leaves up – these leaves have no ladybirds on them and these ones have some – she made two piles of cards on the mat. Sinéad: "now, we turn this leaf over (a leaf with nothing on it) and we need to find its match…can you pick a leaf from the other pile and turn it over?" "How many dots can you see?" "Two", said John (without counting). "OK, can you find a leaf with two ladybirds?" asked Sinéad.

> John did so, quickly and arranged both leaves, one on top of the other, with the ladybird side showing. Sinéad: Pick one leaf, how many dots?
>
> John answered 'one' and then found the correct corresponding leaf, and again placed one card on top of the other and placed the pair next to the first pair on the mat. Sinéad left John to go to Chandra to see how she is getting on and then came and sat by John again on the mat, looked at the pairs made by John and said, "Do you want them on top of each other or beside each other?". John pointed to his piles, indicating that he'd like them on top of each other. "Ok, we can do it like that, we can put them together like that", Sinéad responds, and John matched another pair and put them next to the other pairs. He counted his piles: "one, two, three"
>
> John turned the leaves in his matched pairs to show the dots. He now has three pairs matched and lined up. They are one, three, and five. He matched four and Sinéad asked, (looking at his line) "one, three, five" "where will the four go?". John moved the piles and placed four between three and five. He looked at the big piles of leaf cards again and matched another pair, "I've found more threes", and added them to the three pile. He then matched a pair of sixes and added it after the five in his pile. Sinéad affirmed his actions with a "good job". John then found a pair of two and added them between the one and three. He then counted from right to left, placing one finger on each pile as he counted "one, two, three, four, five, six". John then put a five-dot card and a two ladybird card together and put them side by side on the mat. Sinéad asked, in a playful tone, "Are they the same?" John laughed and replied, "No, I was trying to trick you" (FN 3).

In this excerpt, John and Sinéad make sets, sets of quantities. John can perceptually subitise small amounts to five. He can match numerical patterns, noticing similarities and differences between the quantities quickly and easily. Sinéad motivates and prompts his interest in the game with her questions, "Can you find a leaf with two ladybirds?" "Where will the four go?" She uses praise to further motivate his participation (Rogoff, 2003). For John, it seems, the goal is to match the pairs as quickly as possible. Sinéad practices responsive assistance (Rogoff, 2003) as she offers choice within the game, asking him how he would like to arrange his piles, respecting his way of doing things. John ordered the piles in relation to their increasing quantity, despite the quantities being represented in dot/ladybird form only. His perceptual subitising (Clements & Sarama, 2021) skills are on show. John understands the stable order principle; he understands that four goes between three and five and that six comes after five. He exercises the one-to-one correspondence principle as he

places his finger on each pile as he counts. Cardinality is understood. Finally, Sinéad's playful tone at the end of the excerpt serves to motivate John to participate further (Rogoff, 2003). His attempt to 'trick' Sinéad is evidence that he finds joy in his mathematical activity and her participation in it. This excerpt shows a warm, positive relationship between John and Sinéad. Sinéad adopts a sensitive, responsive approach (Rogoff, 2003) as she follows his lead throughout the activity. It demonstrates John's understanding of the stable order principle (Gelman & Gallistel, 1978), his ability to perceptually subitise (Clements & Sarama, 2021) and to match small quantities.

Figure 7.3

The Game of Ladybirds



Image: CleverKids.ie

7.1.1.4 Bingo

The focus of this game is on number recognition and matching numerals from zero to twenty. In her conversation on self-captured photographs of resources (RC1), Sinéad commented on a photo she had taken of this as a mathematical resource and took a positive view of this game: "I kinda would do that the odd time on a Friday if I didn't have the worksheet…I think it's a great game". Sinéad also felt that this game supported informal

mathematics learning in a fun and engaging way, “they don’t know it's…I know it’s a maths activity in my head, they just don’t understand that, they think they’re just playing a game”. However, Sinéad stated that two children who had prolonged absences struggled with numeral recognition and that she had to adapt the game to support their participation in this group activity, “they struggle more with that one (Bingo) cos they couldn’t, if I said the number they didn’t know which one it was, so I’d have to show them, and then they were able to match it up…I was kinda targeting both the people who had the knowledge and people who were just recognising the numbers” (RC1). The following FN excerpt was selected to exemplify Sinéad’s strategies in supporting every child’s participation in this game, tailoring her support for different learners.

> Priya and Filip brought a number bingo game to the table where John, Seán and Sinéad were sitting. Filip commented, “It’s hard to play this game”. Sinéad responded by saying, “Can you not remember how to play? We’ve done it before”. She then reminded the two children how to play. She moved her chair and positioned it between Priya and Filip. “I will call out the numbers for you, and you take a board each”. Audra expressed an interest in playing, so a space was made for her at the table. Sinéad begins the game, “our first number is…12…a one and a two”. Filip: “I have a 12” and placed a red counter on the 12 on his board. Sinéad continued, “Our next number is…19”. Filip quickly jumped in “one and a nine”. Sinéad “Well done, Filip, a one and a nine is 19”. Filip looked down at his board and quickly scanned those of Priya and Audra and observed, “We don’t have any 19”. Seán wanted to join in the Bingo game. Sinéad gave him a board and called out “11” and Filip shouted “one, one”. Seven was next, and Seán shouted out excitedly, “I have a seven”. Sinéad called out 15, and Filip says “five and a one”. Sinéad replied, “Very good, but it’s the other way round, one and a five”. “OH”, said Filip (laughing) “, a one and a five”. The game carried on like this for another five minutes, with Filip winning the round. He was beaming and was beyond delighted with himself! The children remained very interested in playing this game and wanted to play it again (FN 16).

In the FN excerpt above, Sinéad supported the two children by reminding them how to play the game and suggesting that she would help them play. As other children joined, they were given a board. Any interested child could play. She enabled those children who recognised the numeral and those who needed help with this. She called out the number and then described it, “12…one and two”. Filip, who had demonstrated a keen interest in numbers

past ten over the observation period, was quick to locate 12 and identify that no participant had 19: “We don’t have any 19”. When Sinéad called out 15, Filip tried to imitate her (Rogoff, 1990) by saying, “five and one”, to which Sinéad praised (Rogoff, 1990) him for his efforts before informing him that it’s one and five. The excerpt concludes with Filip winning the game, and his beaming face communicates joy, reflecting his enjoyment of the game. Both Filip and Audra spoke English as an additional language, and Sinéad’s participation in the game provided support in terms of guidance and modelling of mathematical language (Cohrssen & Niklas, 2019). As noted in Table 6.1, Audra’s friends had left the setting, and she frequently played alone. Her participation in the game was heartening, and Sinead’s tailoring of instruction supported her in accessing the group activity. This further demonstrates Sinéad’s ability to engage in adaptive teaching (Gibson & Ross, 2016), in response to her noticing of children’s differing abilities in recognising numbers to 20. This activity also served as a mode of apprenticeship or ‘intent participation’ (Rogoff, 1990, p. 317) with the children observing and listening to Sinéad and each other as numbers were called out and described. Sinéad’s alteration of her instructional approach exemplifies Rogoff’s (2003) concept of Responsive Assistance as she called out the numbers and described them for those children who were unfamiliar with the teen numbers.

7.1.1.5 Number Operations in Free Play

The selected excerpt below exemplifies Sinéad’s engagement in the discipline of noticing (Mason, 2002) as she noticed John’s particular interest in number operations and reflected on potential activities to support his further engagement with it. Her planned response, an activity to support John in his fascination with the structural aspects of number, is detailed. The excerpt reflects John’s ability to engage with small sets, to combine sets together to make one set, and his joy in being able to do so.

Sinéad and John were working with a box containing four sets of coloured rods, and small wooden tiles with numerals and symbols for addition, subtraction and equal to on them. Sinéad asked John, "Take three red and five blue" rods. John did this and Sinéad placed the plus sign between the sets, and asked, "How many do you have altogether?". John worked from left to right, placing one finger on each rod as he counted "one, two, three," continued "four, five, six, seven, eight" and stated, "there's eight". "Well done", praised Sinéad, "now can you get eight yellow and seven blue?" John counted out each set and placed the eight yellow rods to one side of the plus sign and the seven blue to the other. Again, he was asked how many he had altogether. And again, working from left to right, John counted each rod and said "15". Sinéad asked him, "Can you make 15?" John took a one and a five tile from the box and placed them side by side on the table. It reads 51. Sinéad commented, "That makes 51. If we turn them around, we can make 15".

Sinéad followed this with, "Now we can do take-away. Can you take four red rods from the box?" John did this and placed them on the table. He was then asked to take another two red rods, and these were placed to the right of the minus sign that Sinéad had placed on the table. Sinéad said to John, "Now what is four take away two?" "Two!" shouted John (nearly jumping from his chair with excitement). "Great job", replies Sinéad, "can you get two blue rods from the box?" John did so and placed them to the right of the equals sign, which Sinéad had placed on the table. John was then asked to place three green rods on the table, then the minus sign, then two green rods and was asked, "What is three take away two?" "One", shouts John and takes a yellow rod from the box and places it to the right of the equals sign. Sinéad is then relieved for her break (FN 5**).**

Figure 7.4

Counting Stick Set as Used by Sinéad and John



Image source: ebay.co.uk

This is an example of an extended structured guided participation (Rogoff, 1990).

Such interactions are characterised by their length and sustained joint activity. Sinéad

structured the activity through instructional interactions; she shaped John's contribution through actions and thinking, initially placing the plus sign between the sets, then guiding him to place each set on either side of the sign for the second operation (Rogoff, 1990), a process necessary to engage in the cultural mathematical practice of addition. Her repeated question, *How many do you have altogether?* indicated to John that he needed to combine his sets to answer the questions. Praise was given as he completed the task. Sinéad structured John's participation in the activity in a staged manner; she scaffolded his developing understanding of how to physically lay out the components of the operation, in this way becoming an apprentice (Rogoff, 1990) in learning how to carry out the calculation. For both addition and subtraction operations, she began with small quantities, increasing the quantities slightly for subsequent calculations, each operation a little more challenging than the previous one. The challenge was a motivator; it helped to maintain John's interest and participation in the activity (Rogoff, 2003). He liked this activity; he wanted to succeed and master this skill. Mastery brought joy. I wondered how far the challenge would have been extended had Sinéad not been relieved for her break. FNs show that as soon as Sinéad left the room, John lost interest in the activity and found something else to do. Perhaps, for him, Sinéad's involvement in the activity was what made it so joyful.

On a previous day (FN 1), Sinéad identified John as a child who was ready for such mathematical activity and showed me the resource (Figure 7.4) that she intended to use to support him. She intentionally planned to guide his developing interest and knowledge using a specific piece of equipment designed to support John's developing understanding of number operations (Rogoff, 1990). The above excerpt exemplifies a responsive pedagogical approach as well as Sinéad's agency to shape and tailor her curriculum to cater to the needs of individual children. This pedagogical action is echoed in Siobhán's belief that "if the child

is interested, that's the whole ethos of early years as far as I can see, you go with the child, you meet them where they're at" (Int. 1).

A week later, Sinéad captured a similar event with John on video (RC2). Triangulation across the data sources found consistent evidence that Sinéad viewed good practice as observing and enhancing (Lee, 2017). Again, as with the vignette above, they are working with the counting sticks. Some sticks are on the table in front of John, who appears focused and listening to Sinéad.

> Sinéad: That's three red and four yellow. What do we get? If we add them together?
>
> John proceeds to count each of the sticks, one at a time, in the correct sequence from one to seven. He puts a middle finger on each stick as he counts. "7" he said loudly, with a smile. Sinéad responds with a 'good job', and asks, "Can you find number 7? Where's the number 7?"
>
> John looks in the compartment in the box, on top of the 4 compartments for the sticks. The wooden tiles for the numerals and mathematical symbols are in this compartment. He takes out the number 7 tile and says "7", smiling. He looks at Sinéad. Sinéad continues, "Ok, can you put those back (pointing to the sticks)? We'll do a new number". John puts the sticks back into the correct compartment – yellow with the yellow, red with the red.
>
> John shouts, "I'll get 5 red and 5 yellow?" To which Sinéad responds, "Then you already know the answer. We'll try 6 green and..." Before Sinéad can finish, John protests, "No, I want to do 10". Sinéad changes tack and says, "I'm going to do 10, but I'm going to do it a different way". John smiles. Sinéad continues, "We'll go more than 10, 6 green and 5 blue. John takes 5 green sticks and lays them on the table. He does not count out loud, goes back to the box, takes another green stick and places it with the set of 5. He then takes out 5 blue sticks and lays them on the table next to the set of green sticks, but with a gap between the sets.
>
> Sinéad says, "6 green plus 5 blue. How many do you have altogether?" Use your finger to count them all. He counts all the sticks, shakes his head and says nothing. Sinéad encourages him to try one more time. John has started to lose interest. Sinéad steps in, holds his finger as she counts out loud, "1,2,3,4,5,6,7,8.." "11" shouts John before Sinéad has finished counting. Sinéad asks, "Can we make number 11 with the number tiles?" " I have to make 11?" He takes two 'one' tiles from the compartment and says, "two ones, two ones," and puts them side by side on the table. Sinéad comments, "We put it to the side, don't we, and that makes it 11" (RC2 on video).

The video vignette captured by Sinéad and discussed in RC2 above reflects Sinéad's noticing, interpreting, responding and enhancing (Lee, 2017) John's interest in numbers and

addition. To do this, Sinéad drew on her knowledge of John, her knowledge of mathematics and the knowledge of resources she could use to develop this interest further (Priestly, 2022). She purposively tuned into John's mathematical activity (Mason, 2011). During RC2 on this video, Sinéad offered this explanation for her choosing to record this activity:

> "I kinda chose this video because, with this child, he's kind of advancing onto the next stage of his maths skills and to try and keep him like entertained and kind of involved in the numbers because he's a lot more ahead than the other children".

Sinéad's knowledge of John's ability was revealed as she tried to dissuade him from making 10, and tried to make a number higher than 10, to challenge him. However, John was not happy. Sinéad resorted to trickery, saying she would do 10 another way before choosing two quantities that make 11. John acquiesced; however, he indicated his unhappiness by not speaking, then shouting the total before the count was finished as Sinéad took his finger to count the sticks. This episode demonstrated Sinéad's wanting to extend John's addition/counting skills, and she did so despite John not being a willing participant in the activity. John tried to exercise some agency in the activity, but Sinéad's learning goal won through. She engaged John in an apprenticeship (Rogoff, 1990) in the cultural practice of counting and addition (Bishop, 1988), with the equipment serving as a bridging device between current and new knowledge (Rogoff, 1990). Sinéad facilitated a structured learning activity, giving some direct assistance, while John was given some responsibility in a joint problem-solving process (Rogoff, 1990). Sinéad's guidance in this learning activity was an intentional response to her noticing of John's mathematical knowledge (Mason, 2011)

Interestingly, when asked how else she might further John's number sense, during RC2, Sinéad replied, "I think by doing the worksheets, that's the next level, like the one that I have printed off for him specifically… I don't know…the numbers beside it, was it the owls or something, and he had to match the number to the actual objects and kinda do that". This response further demonstrated her intentional noticing of John's interest (Mason, 2011) and

her strategy to intentionally enhance both his interest in and learning of counting and addition.

Number operations, such as addition and subtraction, are a feature of the Primary School Infant Mathematics Curriculum (DE, 2023a) and are not listed as a mathematical concept that preschool children should experience in Aistear (GoI, 2024c). Despite this non-inclusion of number operations in Aistear (GoI, 2024c), Sinéad encouraged John's engagement in this activity. She facilitated John's participation in his emerging interest in number operations, one which was fuelled by his funds of knowledge (Chesworth, 2016; Moll et al., 1992; Worthington, 2017) in the area, and which stemmed from watching the TV programme Numberblocks at home (FN 14). Numberblocks supports young children's understanding of the structural aspect of numbers. Sinéad supported him in being a mathematician. She recognised that such activity brings John joy and planned an interaction to support this. The episode shows the importance of a well-resourced environment, having appropriate equipment to hand, as well as the importance of a willing educator, equipped with the skills to notice, interpret and respond to individual mathematical interests (Lee, 2017).

Figure 7.5

Numberblocks (John's Favourite TV Show)



Image source: cogstoysandgames.ie

7.1.2 Number during Circle Time (CT)

The literature (e.g. Bustamente et al., 2018; Zhang & Quinn, 2018) positions CT as a potential space for the intentional teaching of specific concepts. As noted in Chapter Six, the adult-directed Circle Time (CT) facilitated the children's developing numeral recognition and counting skills, with the chosen number of the week being displayed on Sinéad's CT board. In RC 1 Sinéad shared her reasons for including a *Number of the* Week in her CT routine, "I tried to implement it this year a lot earlier on, I started it in September and decided to do a number of the week and a letter of the week because a lot of my kids didn't recognise them last year so I wanted to change that for this year". This quote reflects Sinéad's reflections on her teaching the previous year and her intentions to make changes for this cohort of children.

7.1.2.1 Number of the Week/ Counting

The number of the week was introduced and taught the same way every day over the data collection period. At the beginning of each week, the children waved goodbye to one number and welcomed another. For example, during the first week of the study, the children waved goodbye to four and welcomed number five (FN 1). Once the children had welcomed the new number each week, they were asked to show that many fingers, give that many claps/head/nose taps, before air-writing that number with Sinéad. Sinéad modelled the formation of the numeral in conjunction with verbal prompts. For instance, with number five, she said, "to the side, down and curve" (FN 1). During the first week, this routine was followed for the first two days. On Friday, Sinéad held CT in the outdoor classroom. Again, the children were asked what the number of the week was, then Sinéad asked them to find the number five in the outdoor classroom. The children looked at one of three blackboards; this one had numbers zero to nine, and the letters of the alphabet on it. All of the children found the number five on this board. This was followed by clapping and air writing exercises. The

routine outlined above was followed for numbers six, seven and eight over the remainder of the observation period.

During the second week, when the children were looking at the number six, there were some days when the children seemed restless, and there was a lot of shouting. Their response to Sinéad's "What is our number this week?" was a deafening "six". In the midst of this unrest, Sinéad tried to continue with her usual CT routine by asking the children to give six claps and count to six as they did so. Adomas counted very quickly (and loudly!) to ten and was about to go on when Sinéad asked him to slow down to the pace of the others and explained that he might confuse those who are still learning to count to six. She asked him to count to whichever number he liked – but he had to whisper. She then asked the other children to count six nose taps. Adomas appeared to move his head up and down as if counting to himself. I could not hear his voice – the other children were so loud. When the other children had finished counting to six, Sinéad asked Adomas how far he got to. "Sixteen". To me, the children's behaviour was a sign that they were bored with this routine and were ready for a more challenging learning experience.

Observations in the FNs show other instances during CT where some children wanted to count higher. For example, FN 1 notes that when looking at number five, Sinéad asked the group to give five claps. Arjaat began counting at 100 and continued to 110. Sinéad's response was to ask him to "slow down and count with the other children". She did not acknowledge Arjaat's interest or ability to start the count from a number other than one or to count in higher numbers. The response focused on not doing what was asked. Later, when the group were asked to count to eight, Arjaat shouted, "I can do 100". This was greeted with a laugh by Sinéad. No further comment was made (FN 15).

This was the third time over the academic year the group had covered numbers one to ten during CT. During RC1, when I commented on individual children's interest in higher numbers and I inquired about this repetition, Sinéad stated that, for her, the main priority with Number was to focus on recognising numbers 1-10, with a different number to be focused on each week. In RC 2 she gave her reasons for not going further than 10, "I'm almost a bit too afraid to go too far ahead cos then, I know that when I was in my Montessori a lot of them (children) were too far ahead…they knew all the stuff for junior infants level that I was afraid they were going to be bored". As part of her practicum, Sinéad had spent some time in a Junior Infant classroom and had some knowledge of the 1999 Primary School curriculum (FN 1). This suggests that Sinéad acted out of care for the children; she did not want them to be bored and 'act out' (RC2) in the Junior Infant class as a result of knowing too much. For Rogoff (1990), guided participation includes actions by well-intentioned adults to limit children's participation in certain learning activities, where adults control what is learned and how this occurs. In this ECCE routine, CT content was selected by Sinéad; it was she who decided what was to be learned and how. This was based on her own prior experience, and the 'not doing too much' was done out of concern for children's future experience and behaviour in primary school. Sinéad's belief that she cannot do 'too much' with the children seems at odds with her actions with John, noted above (7.1.1.5). With John, this belief appears to be suspended, as she facilitated his participation in a number activity required in the Infant curriculum, but not stated in Aistear. Perhaps this situation arose out of John's obvious interest in numbers, and Sinéad's interpretation that he was 'ready for the next step', and because she knew him well, she understood that he would push for a bigger challenge. She had to respond to this interest.

The FN below shows that John was not the only child interested in number operations. For example, during one CT session, Seán demonstrated some structural[5] understanding of the number four. The field note excerpt below details the incident when Sinéad asked the children to show four fingers:

> "Show me four fingers". Sinéad scanned the group and congratulated them for all showing 4 fingers. Seán displayed two fingers on each hand and said, "Two and two is four". Sinéad acknowledged this with a smile but made no further comment. Adomas copied his peer and stated, "two and two makes four," but Sinéad had moved on (checking her watch) (FN 1)

In the FN excerpt above, Sinéad noticed Seán's contribution. All of the children, bar Seán, represented four on one hand. He was the only child to represent four on two hands. He had done something different, and although this was acknowledged by Sinéad, albeit non-verbally, she did not engage in a conversation about how four could be represented on hands. Rogoff (1990) contends that young children are adept at picking up adult behavioural cues. Perhaps Seán understood Sinéad's non-verbal acknowledgement as a sign that he should not pursue his observation further. His observations and comments would have provided an ideal opportunity for Sinéad to support Seán in developing his further understanding of the structure of four. Following up on this would also have supported Adomas, who was clearly interested in Seán's action. This was a clear example of a missed teachable moment, where Sinéad could have taken the time to explore Seán's observation further. However, Sinéad had checked her watch, and it was time to move on. The setting's *Curriculum Planning and Observations Policy* (CPOP) advocates for the use of teachable moments as a key strategy for responsive teaching and learning in the setting. The policy defines teachable moments as, 'This is a time at which a child is likely to be particularly disposed to learn something or particularly responsive to being taught or made aware of something'. The FN excerpt above

[5] Structural aspects of number: Understanding how numbers can be composed or decomposed into component parts

suggests that Seán, and indeed, Adomas, were interested in the different ways in which the number four was represented. However, Sinéad's time checking and need to keep to the schedule did not facilitate this at this time.

7.1.3 Number Activities during WGT

As noted in Chapter Six, the WGT sessions on Fridays were distinguished from other WGT sessions by their specific focus on Number, with a particular focus on numeral recognition, writing numerals and representing quantities by drawing. Sinéad photocopied worksheets from number-focused workbooks provided by Siobhán (RC 2). I first encountered the use of number-focused worksheets in the ECCE room during week one. After the CT session, which had been held outside, the children hung up their coats and sat at a table. No instructions had been given; this was an accepted and anticipated part of their Friday routine, and they knew what to do. This activity had become part and parcel of the classroom culture (Rogoff, 1990). They were each given a worksheet on the number five by Sinéad. As Seán handed out the pencils, one to each child, Sinéad explained that they had to trace over the number five (presented in dotted form) and then draw five candles on the large cake underneath, before being allowed to colour their cake. The children were encouraged to write their name on their work independently and could refer to a picture of themselves with the printed version of their name if they needed support with this (these pictures were tacked onto the wall underneath the CT board). There was silence in the room as the children concentrated on this activity. The majority of children could trace the number, and most drew the correct number of candles. Once completed, the children could then put this work into their bags to take home. Sinéad remarked that she had intended to make a number book with these weekly worksheets to send home at the end of the year, but the children were so proud of them that they wanted to bring them home every week, she recalled that one child, Seán,

said “my mom will be so proud of me and my numbers” (FN 3). Seán’s comment suggests that in his household, knowing your numbers in preschool was a valued skill to have.

Worksheets provided in subsequent weeks followed a similar format in that the children were required to trace or free write a number and then represent that quantity in pictorial format. There was some flexibility in the recording of the quantity. In week two, the children were asked by Sinéad to draw six stars (FN 8). However, if they felt they could not draw stars, they could draw circles. Sometimes Sinéad gave instructions, for example, on the third week, when referring to the number seven, she instructed the children “to draw over the dotted number 7s” and then “draw seven lollipops” (FN 12). While Arjaat handed out the pencils, Sinéad continued addressing the group, asking, “Do you draw ten lollies?” “No”, chorused the children. “Do you draw 100 lollies?” “No”, replied the children. Sinéad concluded by saying “only seven”. Sinéad also instructed the children to write two number sevens – one on either side of the jar. This was a free writing exercise, as there were no dots on either side of the jar to scaffold the writing of number seven.

On two occasions, the worksheets had dot patterns representing numbers one to ten across the top of the worksheet (FN 8; FN 12). In week 2, John recognised the dot pattern for 10 and pointed this out to Sinéad before asking her if he could draw ten circles. Sinéad permitted him to do this, nodding her assent. On the same day, just as this part of the day was coming to an end, Sinéad noticed that Priya had drawn four circles, and Sinéad asked her how many more she needed to make six (the focus number that week). Priya answered Sinéad by saying “one more”. Sinéad responded by saying, “OK, try adding one more, count and see,” and turned to Arjaat, who called out to her, “I have eight sixes,” and he proceeded to count them for her. He had indeed written eight sixes. Unfortunately, there was no chance to follow Priya’s progress following her conversation, as Sinéad, immediately after interacting with Arjaat, asked the children to put their sheets into their bags as it was time for Snack.

In a research space where little is known about the use of worksheets in Irish preschool practice, this study contributes to this research knowledge gap. Sinéad uses number worksheets to consolidate children's knowledge of counting and writing numerals to ten, fostered during CT. Rogoff (2003) suggests that caregivers provide activities that resemble those experienced in primary school, preparing them to participate in primary school. Sinéad did not state that this was her intent with using the worksheets, however. For her, the focus was simply for the children to practice what they had learned that week. The children were keen to share this work with their parents, and when the children decided to bring them home rather than have Sinéad collect them and make a number book, Sinéad went with the children as she understood how important it was for the children to make their efforts visible at home and to gain praise and recognition for this outside of the setting.

7.2 Measures

This next section considers how children participated in activities involving measures. The FN excerpts that follow include examples of children participating in activities that support their growing understanding of comparing, weight, height, length, capacity and money. As with the data provided for Number, the selected excerpts reflect child-led and adult-led activity. The first three excerpts, documented during free play, illustrate the social collaborative nature of play and learning (Rogoff, 1990), as the boys engage in discussion about how best to construct their 'spider house'.

7.2.1 Measures in Free Play

Measures was the second most frequently observed big idea of mathematics over the fieldwork period. Observations show that the boys, in particular, were especially interested in exploring this aspect of mathematics, using different resources to compare length and height.

7.2.1.1 'I need to make it higher.'

The field note excerpt below shows Arjaat's interest in measure, particularly height and how it becomes a meaningful and relevant part of his play agenda, and that of his friends.

> Using 2x2 unit blocks and the longer blocks, the boys (Arjaat, Seán, Adomas and Filip) built their 'spider house' five levels high and measured themselves against it. Arjaat stood next to the structure and compared his height to it. "It's nearly as big as me…I need to make it higher," he exclaimed. His friends had the necessary blocks to hand, and they built it higher. Arjaat stood next to it again, and this time it was taller than him. He was very excited, "We made it bigger than me". At this point, I stood beside the structure to steady it. "We can make it as big as you", Filip said. They set about their task, asked me to stand close to the structure every time they added a level, and compared my height to it. Eventually, I needed to add to the structure as it became too high for them to do so (I am 5'8), so they handed me the blocks to place. When the structure was as tall as me, they shouted, "Yes, we did it, it's as big as you". (FN 1).

This excerpt shows Arjaat's interest in measuring, specifically in measuring himself and in comparing his height to collaboratively made objects. He understands that to make the comparison, he needs to stand beside their creation. He made a direct comparison (Brownell et al., 2014). As I stood beside their construction, saying nothing (intending only to steady it), the boys saw my position next to their work as a challenge. I inadvertently restructured their activity plan and became a participant in the activity (Rogoff, 2003). The children directed the activity by wanting to make it taller than me. Again, their understanding that I needed to stand close to their construction to make a comparison was evident. Eventually, I had to take a more active role in the activity, as they were no longer able to construct it. After five minutes, the structure was as tall as me, and the boys decided when this was the case. They made the comparison. Their joy in completing the task was apparent.

7.2.1.2 The Python

The field note excerpt below describes a cooperative endeavour between Adomas, Arjaat and Filip. They have used snap cubes to build a snake, a python. However, they encounter a problem as they try to make their snake longer.

> I sat at the table where Adomas, Arjaat and Filip had made a snake with the snap cubes. It was a Python and it measured the length of the table. The boys wished to make it longer, and Adomas began to add four cubes to the snake. It broke as it did not have the table to support it. Adomas looked annoyed, stood back from the snake, then went to the unit block shelf and took three standard unit blocks, stood them upright, side by side, on a chair and aligned the snake over the top of the blocks. He was delighted with himself as the snake was no longer breaking and falling to the ground. However, Arjaat pointed out that the same thing was happening on the other end of the snake as he added more cubes. The more he tried to fix it, the more it broke. Arjaat went to the block area and brought back three blocks identical to those Adomas chose and set them up on the chair. Both boys were happy with their accomplishment. (FN 4)

The above excerpt demonstrates the boys' collective interest in length, particularly in making things longer. Adomas's problem-solving skills are interesting. He identified the problem, stood back and considered and tried out a solution. His solution of using the unit blocks is innovative. Having a culture where children can select and use any resource they wish to during free play time facilitates this creative thinking and problem-solving. Previous experience with stacking the unit blocks to measure height in previous play episodes (FN 1) may have helped to support his thinking and problem-solving ability here. Furthermore, his strategy was observed by Arjaat, who copied his actions to support his end of the snake. The field note excerpt exemplifies Rogoff's (2003) concept of guided participation, illustrating how children's mathematical learning can be fostered through joint activity in which observation and imitation serve as key mechanisms for engaging with and constructing numerical understanding in early childhood contexts.

In general, the boys dominated construction play. In fact, no observations of the girls playing with these blocks were documented during the fieldwork period. This finding is in line with the observations of Clements and Sarama (2021). The unit blocks, in particular, were commanded by Seán, Filip, Adomas, and Arjaat, with some days spent reconstructing their creations, refining and amending the design as they did so. On occasions, they enhanced

their constructions by using other materials such as doll's house furniture. The following observational fieldnote excerpt shows their continued interest in measuring.

7.2.1.3 The Hulk House

The boys created a long, rectangular base, approximately half a metre in length with the unit blocks. Sinéad, who was standing at the sink, asked, "What are you making?" "A hulk bed", replied Arjaat, "we are making something bigger than you" (laughing). Sinéad (jokingly) "You can't make something bigger than me".

> A toy chair and a TV were placed three-quarters the way of the length of the base. Seán had begun to build up the walls, placing a row of blocks on the perimeter of the structure. Arjaat placed a table and two chairs behind the chair and the TV. He placed the chairs at opposite sides of the table. Adomas asked Arjaat to "put two more chairs here" and pointed to the empty spaces at either side of the table. Seán: "I know, we can have different rooms", he said, excitedly. [They had no dividing walls, but the areas were being delineated – kitchen items such as a fridge, cooker and sink were lined up against the walls at one of the ends, and a bed was placed next to the walls at the other end. They are now making a 'Hulk house'.
>
> Seán began placing his second round of blocks on the perimeter wall. He noticed that the fridge and the cooker hood were still higher than the current level of the wall and said, "We can't put the roof on", shaking his head. Arjaat replied: "We can make the walls bigger and then put the roof on. Seán put two long triangles together to make a rectangular shape and added these to the wall, and continued building.
>
> Seán added another layer of blocks to the wall. He stopped, looked at the part he had built, then looked at it and looked at the cooker hood. "Look, guys, it's almost there". He continued building. It needed to be high enough to put the roof on without breaking the fridge or cooker hood. (FN 13)

The excerpt above shows the boys engaged in collaborative sustained activity, and Arjaat's interest in measure is clearly evident. He was keen to make it 'bigger' than Sinéad. Sinéad's playful comment, 'you can't make it bigger than me,' may have served as a motivator for the boys to continue their activity (Rogoff, 2003). The use of other resources (furniture) contributed to the making of the real and meaningful context of a house, with the addition of 'rooms', through the delineation of space. The furniture was arranged in accordance with culturally accepted conventions; for example, kitchen items such as the

cooker and refrigerator were positioned along one wall, while bedroom furniture was located in a separate area of the space, reflecting processes of classification. Again, the reference to real-life situations (Rogoff, 2003) is clear with Adomas commenting on the lack of chairs at the table, stating, "we need two more chairs", asking for chairs to be placed at each side of the square table. Seán made comparisons between the height of the walls and the furniture inside, as he noted, "We can't put the roof on". The walls were too low, a problem solved by Arjaat's solution of making the walls 'bigger'. Seán solved the problem of not having the required rectangular piece by putting two triangular blocks together, combining shapes to make a new one. The activity also reflects the boys' collective interest in Marvel superheroes. Sinéad's participation was minimal, observing and responding only when asked to do so. Had Sinéad taken a more active role in the activity, she could have modelled the related vocabulary, taller rather than bigger, as used by the boys, or invited some of the girls to participate.

The interest in measure came to the fore again when Arjaat called for Sinéad's attention:

> Arjaat to Sinéad, "Look what we made, look how long it is". Sinéad to Arjaat: "Lie down beside it, is it bigger than you?". Seán opted to lie beside it as Arjaat was reluctant to do so. "I'm longer, I'm longer", he cries excitedly. Finally, after approximately forty minutes, the walls were deemed high enough, and the house got its roof. They covered three-quarters of the structure with long blocks, leaving a gap. Sinéad commented, "I like the roof, but it's not finished". Seán informed her (quite seriously) that this was a glass roof and it was finished. (FN 13)

Length was not the only aspect of Measure that the children encountered in their activity. Children's engagement with money and the concept of exchange was facilitated by the provision of a toy till and plastic coins, and notes located in the home corner. This provision created opportunities for children to enact familiar social practices (Rogoff, 2003), such as shopping or selling, while simultaneously exploring mathematical ideas related to

counting, value, and transactional exchange (Worthington & van Oers, 2016). Field notes document two occasions where children participated in socio-dramatic play scenarios involving money. Both were shops, stereotypically gendered, a builder's shop and a dress shop.

7.2.1.4 The Builders Shop

In the excerpt that follows, the boys used the home corner as a builders' shop. They were imitating the 'script of everyday life' (Rogoff, 2003, p. 298) in that, not only were they engaged in the task of measuring, they were also considering the role of money in a shop scenario.

> Filip and Seán have moved to the home corner and are playing with a box of tools. They are in their 'builders' shop', using a rectangular piece of plastic to 'measure' "Let's measure this shelf", said Filip and placed this rectangular piece against the frame of the 'shop'. They moved to the toy till on a nearby shelf. Seán took out some coins from the till (I can't see how many he has as they are in his fist). "I have four euros, I have four euros' he shouted and waved his fist in the air. Filip returned to his 'measuring "a hundred thousand minutes, it takes so long" (to fix the builder's shop). (FN 1)

Within this excerpt, three aspects of measure are evident. In the first instance, in the 'builder's shop', Filip engages in the everyday practice of measuring a shelf, using a rectangular piece of plastic as his measuring tool. It can be assumed he has observed this activity elsewhere in his everyday experience and emulates that here (Rogoff, 2003). Filip's declaration that 100,000 minutes is so long may indicate his understanding that a hundred thousand, a large number and therefore if the job takes a hundred thousand minutes, then it is taking a long time to complete. It is unclear whether the move towards the till was part of the shop play. However, Seán counted his coins and declared he had four euros, demonstrating his understanding of the concept of coins representing a form of currency. It is worth noting that none of the girls showed an interest in playing this game with the boys.

7.2.1.5 The Dress Shop

Using the home corner as a shop was also observed among the girls. In this instance, the girls, Chandra and Aparna, chose a dress shop for their play scenario.

> Chandra and Aparna are in the home corner hanging up dresses. Chandra moved to the couch in the reading area, which is next to the home corner. She had lined up coins she had taken from the till in the home corner. "I have eight monies", she exclaimed. She went back to the till and took more coins. She lined these up underneath the first row of coins. She counted nine, placing one finger on each coin as she counted. I could clearly see the extra coin when both sets were lined up underneath one another. She was very excited and fetched a purse to put her coins into. She returned to Aparna in the home corner, which they have decided is a dress shop. Chandra to Aparna, "I want this one", as she points to a black Flamenco-style dress with red spots. Aparna hands over the dress, and Chandra says, 'Bye. ' I ask her how much she paid for the dress. She put her hand to her mouth and said, "Oh, I forgot", and headed back to Aparna to pay for the dress. One coin is offered to Aparna, who accepted it – no other conversation followed (FN 7)

This excerpt demonstrates that Chandra can count accurately to nine. She understood that she should line up the coins to facilitate an accurate count and had applied the one-to-one principle when counting, placing one finger on each coin as she counted. Cardinality of the count was also understood. Drawing on her funds of knowledge (Moll et al., 1992), Chandra knew that money was typically kept in a purse and had a sense that money was used in exchange for goods or services as she went to Aparna's 'dress shop'. The fact that no money had been exchanged for the chosen dress and that Aparna did not ask for money is interesting. I would have expected her to ask Chandra for some. When Chandra returned to Aparna and offered her a single coin, Aparna accepted the exchange, no questions asked. Had I not asked Chandra how much she paid for the dress, no exchange would have taken place. This could be explained by their observations of the increasingly common cultural practice of adults using digital payments on a card/phone rather than using cash/coins. The play naturally evolved to a shop-based scenario from hanging dresses to Chandra taking money from the toy till, to 'buying' the dress. In both of the shop scenarios described above, the home corner

allowed the children to change its use from a home corner to a shop. It was equipped with a till, notes, coins, and dresses for the children to use in their role-play scenarios, to design, emulate and rehearse real-life scripts (Rogoff, 2003). The children showed an awareness of money, Seán, in his use of the term 'euros' and Chandra in her knowledge that money is used to buy things.

7.2.2 Measures during CT: Time

The following FN excerpts were selected, as they illustrate the pedagogical strategies used by Sinéad to teach temporal aspects of measure. As noted in Chapter Six, Sinéad had the names of days of the week and the months of the year positioned in order to the side of her CT display board, thus representing for the children the sequence of these aspects of time.

7.2.2.1 Days of the Week

The CT segment that focused on time typically began with Sinéad asking questions such as, "Who can tell me what day it is today?" and the children offered a response. As noted in Chapter Six, Sinéad used the Days of the Week song to support children's understanding of the days of the week.

> Sinéad: "What day is it today?" "Friday", offers one child. "No", replied Sinéad, "tomorrow is Friday…*sing the song in your head*…yesterday was Wednesday". The children started guessing, shouting out random days. Anika's head was nodding, as if to a beat. It appears that she could be singing the song in her head, and after some time, she shouted, "Thursday". "Yes, Anika, it is Thursday", said Sinéad and began to sing the Days of the Week song with the children joining in. A few minutes later, Sinéad asks, "Now, who can tell me what month it is?". The children begin to shout out days of the week (FN 2).

The FN excerpt above shows that despite regular exposure to hearing the days of the week, the children struggled to identify the day, suggesting that findings are in line with recognised assumptions (e.g. Beneke et al. 2008; Clements & Sarama, 2021) that time is a hard mathematical concept for children to master. Haylock and Cockburn (2017) assert that

consistent modelling of the language of time in early childhood is beneficial. The song, together with Sinéad's prompts (tomorrow is Friday, yesterday was Wednesday), acted as a scaffold for Anika to determine which day of the week it was. A further example of using the song as a scaffold occurred several days later:

> …when asked what day of the week it was, Arjaat shouted "Wednesday" to which Sinéad replied, "It is Wednesday, well done Arjaat. I could see you singing the song in your head" and on the following day (FN 7), Sinéad asked "what day comes after Wednesday? Sing the song in your head." "Thursday", shouted the children, to which Sinéad replied, "What day will it be tomorrow? Sing the song in your head again". "Friday", shouted the majority of the children. "Let's sing our song".

Sinéad often coupled the singing of the song with verbal prompts, such as those mentioned above. For example, after singing the song, Sinéad asked the group what day it was. The majority of the children shouted out "Wednesday"; however, Anika answered "Monday", so Sinéad used temporal word prompts to scaffold her thinking. Sinéad said, "Yesterday was Tuesday, what comes after Tuesday?" to which Anika replied, "Wednesday". This was followed by the group singing the song together in unison (FN 6). It is important to note that the framing of the questions did not change over the course of the study; the same questions were asked in the same order every day, even when Sinéad was pressed for time. The selected FN excerpts show that some children were unsure of which day it was when asked, so it could be assumed that this strategy was not working.

However, Sinéad's use of prompting, for example, "yesterday was…" proved more useful in terms of getting children to identify the day. Referencing the song in these prompts was also useful in supporting individual children to sequence days in their minds. The month and season were easier to remember, as although cyclical in nature, these did not change every day. Rogoff (1990) observes that adults often use questions or prompts to support children's thinking and memory. Here, Sinéad connects the remembering of the sequence of

the days of the week to the song. The familiar song becomes the tool to support memory. The sole focus of this exercise was on the naming of 'today'. No further reference to everyday events was made to anchor children's understanding of how specific days were related sequentially to one another. No connections to events in children's everyday lives were made to support their understanding of the sequencing of days.

7.2.2.2 Months of the Year

The months of the year were introduced to the children using a similar format to the teaching of the days of the week, in that a list of the months in order from January to December was displayed in a line adjacent to the CT display board, with an arrow pointing to the current month. As noted earlier, this CT segment also had an accompanying song.

January, February, March and April
May, June, July, August and September
October, November and December
These are the months of the Year
(sung to the tune of Michael Finnegan)

Sinéad usually began this segment, similar to that of the days of the week, by asking the question: "Who can tell me what month it is?" Again, like the days of the week described above, children often shouted out random guesses. Similarly, to the days of the week above, despite having sung the song repeatedly over the course of the year, some children still struggled to identify the current month. For example, Audra, a child who spoke English as an additional language, confused the days of the week with the names of the months during these circle time discussions.

> Sinéad: What month are we in? Audra offers "Tuesday". This is said somewhat unconfidently; she did not shout it out but rather said the word quietly, but not so quietly that Sinéad could not hear her. Sinéad supported her by saying, "Do you remember what month your birthday is in?". This prompt seemed to help, as Audra replied, "May". "Yes", said Sinéad. "We are in the month of May, can everyone say,

May?". The group chorused "May" quite loudly and proceeded to sing the months of the year song. (FN 5).

Gifford (2005) notes the significance of birthdays in children's lives, and birthdays featured in CT discussions on the current month, whether this was Sinéad using birthdays as a prompt for month recognition or individual children commenting on their forthcoming birthday. On one of the days, during a discussion on what month it was, John shouted out, "It's almost my birthday" (looking at the birthday display). Sinéad said, "Yes, your birthday is in July", and showed John where July was on the display (FN 2). Here, Sinéad used the display to stimulate thinking in a relevant and meaningful way, and as a means for children to make mathematical connections to their real lives. This is in contrast to discussions reacting to the days of the week above.

Other significant events in children's lives were also discussed during this part of CT. Field notes demonstrate children's use of the CT display during discussions on the current month. For example, with the arrow on May, Seán noted that "there's not much months". He had observed that there were not many months left until he was going to school. Sinéad responded, "You're right, not many months left until September" (FN 1). Again, in contrast to the days of the week conversations, a significant, meaningful and relevant situation in children's lives was raised through the discussion of the current month.

Towards the end of the study, there were signs that some of the children were able to state the month, and so this segment of CT became quite short with little prompting from Sinéad or the song. Field notes show that Sinéad simply asked the question, received the correct answer and moved on to discussing the current season.

7.2.2.3 Seasons of the Year.

As with the previous time-related segments of CT, Sinéad asked the question, "What season are we in?" On the first observation day, the children chorused "Spring". This was acknowledged by Sinéad as being correct, and a discussion followed as to the signs of Spring. Children offered answers such as, "there are buds on the trees", "birds are making nests", "it is getting warm", and on other occasions, variations of these ideas are observed by the children, "the leaves are green, and in Autumn they were a different colour". This point was confirmed by Sinéad, "yes, in Spring and Summer the leaves are green, and they change colour in the Autumn" (FN 2). By the fifth day, the group had decided that [6]Summer had arrived, and Sinéad asked the children if they knew what happened in Summer. Some of the children referred to the bees on the hand-written sign for Summer on the CT board (Figure 6.2). Seán remarked how bumblebees like flowers. Sinéad responded by telling the group that "they collect pollen from the flowers and make honey". There was no further discussion on this topic. This part of CT was generally short with stating the season and one or two facts about the season being discussed.

7.2.3 Measures During Whole Group Time (WGT): Weight and Capacity

There were no observations of the children weighing things during free play, even with a variety of scale types available to them. However, the children did participate in an adult-led session about weighing during whole group time. At the end of CT, Sinéad concluded her explanation of what a weighing scale does (referring to a picture of one on the A3 whiteboard) and explained to the group that they were going outside to do some weighing and decide which items were heavier using the large bucket balance. She explained about the two buckets and that when one object is heavier than the other, that bucket goes down and the

[6] Traditionally Spring in Ireland began on the 1st February, with Summer beginning on 1st of May. This is in contrast to Meteorological seasons where Spring begins on 1st of March and Summer on 1st of June.

bucket with the lighter object goes up. Filip asked if he could bring a small (but relatively heavy!) dinosaur to weigh. He was allowed. The children lined up by the classroom door to go outside.

> Once outside, the children were asked to sit on the grass in front of the large wooden bucket scales. They agreed that the buckets were empty before Sinéad asked them what would happen when she put the dinosaur in one of the buckets. The children predicted that the bucket with the dinosaur would be heavier. When the bucket went down, they screamed in delight. "It's like a see-saw", commented Chandra. Sinéad continued: "So, when it goes down, it means it is?" "Heavier", chorused the children. Sinéad brought a selection of items for weighing. Each child got an opportunity to add an item to the buckets. The dinosaur was removed before John added a rock to the bucket. It went down. "It's heavier", shouted the children. They carried on with items such as a small plastic doll, a wooden dice and then…Sinéad emptied both buckets and said, "I will put some water into one bucket and the rock in the other, which one do you think will be heavier?" Will the rock be heavier? 'NO', cry the children. Will the water be heavier? 'Yes', cried the children. The water was heavier and the children were delighted – they were right. "I wonder how we can make this even, make it the same weight?" pondered Sinéad aloud. "Filip suggested that they 'take some water away". They tried this, taking water away, little by little, until both buckets were balanced. Sinéad concluded the session by saying, "We know they weigh the same because the buckets are the same height". (FN 15).

Children's active participation in this activity is supported through the use of questioning and encouraging making predictions. Questions serve to support children's attention to the important aspects of the activity to be learned, in this case, understanding that heavy items cause a bucket to move downwards and so can be described as 'heavier'. (Rogoff, 1990). The precursor activity, demonstrating what a scale is used for and the introduction to the associated vocabulary, enabled the children to fully participate when outside. Using the dinosaur selected by Filip represents Sinéad's flexibility and willingness to follow individual children's interests and ideas. At the beginning of this FN excerpt, Sinéad asks the children to predict which items will be heavier or lighter. The focus was on making a direct comparison (Brownell et al., 2014), and on the acquisition of relevant vocabulary – heavier and lighter, and to experience first-hand what happens to a bucket scale when heavy items are added to one of the buckets. Sinéad does not follow up with an open-ended question

to support children's articulation of their reasoning behind their predictions, however. Open-ended questions such as 'how do you know' or 'why do you think that' would support mathematical processes of reasoning, justifying and communicating (Clements & Sarama, 2021).

However, at the end of the excerpt, Sinéad did pose a divergent question to stimulate children's thinking about equivalence and offered a chance to articulate this thinking " *I wonder how we can make this even, make it the same weight?"* Filip's idea of taking water away from one of the buckets is an example of this thinking and communication. His ideas are explored. Sinéad concludes her 'lesson' by explaining that both buckets weigh the same because they are at the same height.

Figure 7.6

Example of a large bucket balance, similar to that used by Sinéad



Picture sourced from: creative-activity.com

During another WGT session, Sinéad's focused on capacity as she demonstrated how to make a lava lamp. She shared an example that she had made earlier and explained to the children how she made it. She explained how the oil and water did not want to mix, so the bubbles of oil made their way to the top to "get away from the water". She explained the steps involved in making one, then invited the children to sit at the table and gave each child

a glass jar, each identical in size and shape. The children were fascinated by her example and were keen to make their own. The children moved their chairs from the linear position for CT back to the tables.

> Once the children were seated, Sinéad gave each child a clear jar and lid and placed a large jug of water (already prepared) on one of the tables. She took one of the jars and said, "Everyone, put your hand sideways and put it halfway up the jar". She modelled how to do this. The children copied her. Once Sinéad was happy that all children had copied her correctly, she said, "Now put your hand up a little higher, it's three-quarters the way up. This is where you need to fill your water up to".
>
> Each child then got a chance to fill their jar three-quarters full. They did this very carefully and waited patiently for their turn. When Seán poured his, he compared the amount in his jar to Arjaat's and said, "You get more and I get less". After Priya filled her jar, Filip looked at the jug on the table and claimed, "It's empty" (FN 11).

The choice of the lava lamp was successful in gaining children's attention and in stimulating their interest in the mathematics and science concepts embedded within making one. In this activity, Sinéad modelled the language of capacity and provided physical and visual aids to estimate where halfway and three-quarters were on the jar. These, accompanied by her verbal instructions, supported the children, through observation and imitation (Rogoff, 2003), in gaining a sensorial impression of where each of these measurements lay on their individual jars. The active, hands-on experience of pouring the water into clear jars afforded the making of comparisons, as shown in Arjaat's comment, 'you get more and I get less'.

Other opportunities for exploring capacity were present during WGT. For example, during a sand/water activity (FN 2), Sinéad provided some yoghurt pots and spoons for the sand activity. Some children filled the pots using a spoon. This was done in silence. No capacity-related language was uttered during this time by either Sinéad or the children. Children were exploring capacity at a sensorial level. For the water play, plastic (fillable) sea creatures and a toothbrush for washing them were provided. Another example of informal engagement with capacity occurred at the end of this session when Seán filled his dolphin up

with water, poured it out, laughed and said to Sinéad, “Teacher, look, teacher, look what he can do”. To which Sinéad replied, “That’s very good, Seán”. Here, Seán was exploring the capacity concepts of full and empty; however, no mathematical talk ensued.

This was the children’s only experience with capacity over the fieldwork period. There was no continuous provision of sand or water daily in the room. I found this strange as in my experience, sand and water play support the acquisition and use of related vocabulary, both modelled by the educator and practised by the children. In general, sand and water trays are quite large, and perhaps there was no room for them in this ECCE room. However, I would have expected that the small sand tray used by Sinéad on this occasion might have been freely available to the children during free play, but it was not. Access to these materials occurred only during WGT and with an adult-determined goal.

7.3 Shape and Space

Observations of children’s involvement in spatial and shape-related activities were fewer in number than those documenting participation in number and measure activities. Chandra, in particular, was very interested in circles, and FN excerpts below illustrate this. Other than the Unit blocks, children’s exploration of 3D shape took the form of 3D construction using resources such as straws, circles and connectors, as discussed below. 3D shape was not discussed during CT.

7.3.1 Shape in Free Play

Over my time in the setting, Chandra showed a continuous interest in circles, her interest even revealing itself at snack time as she noticed circles in Priya’s lunch box, saying, “You have a circle” when pointing to Priya’s mini cheddar crackers. She identified them, drew them, connected them and made them, as evidence from my FNs shows:

> Chandra: I'm going to make a circle. She put her straws together with three connectors and made a circle. "That's too small", she said, "I'll make a bigger one". She made another circle – it was the same size as the one she had made previously. She put the circles side by side and said, "That's the baby one and this is the mommy" (FN4)

Here, Chandra made circles using construction straws. She had a plan to make a family of circles. She made two circles, each the same size, despite wishing to make one bigger than the other. She did not compare the size of the circles, seemingly happy to have made two and labelled them 'baby' and 'mommy'. The making of the circles was more important than their size. Chandra's interest in circles continued over time, and I observed her exploring putting circles together to make various items:

> Chandra fetched the basket of circles and connectors and sat at the table with them. She joined two circles together with a long connector, "I'm making a long one", she said as she added pieces to her work. A few minutes later, Chandra was still working with the connectors; she joined many pieces together to make a 'swing'. Chandra then made 'glasses' with the circles and connectors. She tried her glasses on. They were too big to stay on her nose as the connecting piece used for the bridge was too long. I made a pair, using two circles for the eyes, a shorter connector for the nose piece, two longer connectors to go to my ears and two circles to go over my ears. Chandra smiled and copied me. (FN 7).

In the excerpt above, Chandra explored putting shapes, specifically, circles together to make objects, a swing and a pair of glasses. For her, there was joy in working with circles. She knew that circles often form part of a pair of glasses and tried to make a pair. Although the original pair did not fit her face, she selected the correct shapes, circles for eyes and straight connectors for the bridge and temples. Chandra had replicated her knowledge of shapes in the environment in her construction work. The cultural philosophy of the setting, in providing daily extended periods of time to explore, coupled with the freedom to reconnect with concepts and repeat explorations over a long period of time, served to enable children to follow their individual mathematical interests and engage in deep, exploratory learning.

Figure 7.7

Circles and Connectors



Image source: Kaplan.com

7.3.1.1 3D Shape in FP

The following FN excerpt, captured during free play, shows John playing alone with the construction straws while Sinéad is engaged in an organisational task. Chandra is also playing with the straws, but not engaging with John. It was selected to show Sinéad's strategies to support John in his exploration of his 3D shape construction while engaged in another task.

> John has been playing with the construction straws for the past five minutes and has made a 'bridge' in the form of a triangular prism. He has been communicating his endeavours to Sinéad, who was filling paint pots at the sink. John calls out to Sinéad, saying, "Look, it's all triangles" he puts his large triangular prism on its side. It is quite tall now as he has added more sections to it. Sinéad responds from her position at the sink, "Do you know what it's called when triangles are together? It's a prism.". Meanwhile John has turned his construction on its side and says, "mine is bigger than Chandra's'. Sinéad asks, from across the room, "can you see the other shapes in it?". John shouts back, "triangle, triangle, triangle, triangle". Sinéad is trying to draw John's attention to the squares on the underside of his triangular prism. She leaves her prep activity and comes over to John and says: "put your finger on here, and she guides his finger around a square face of the triangular prism. "A square", exclaims John excitedly. (FN 4)

In the extract above, John brought his mathematical observations to Sinéad's attention. Sinéad stopped what she was doing to interact with John. In this joint attention

episode (Rogoff, 1990), Sinéad guided John through exploring some of the properties of 3D shapes. She asked if he knew the name for the shape he made by drawing his attention to the properties of his construction, the shapes that had been combined to make it. She knew that he did not know the name of the 3D shape he made, so she quickly gave him the term, using correct mathematical terminology. When Sinéad asked John what other shapes he could see in his prism, he began to point out all the triangles, naming each one as he did so. In this regard, Sinéad's question was successful in engaging John in looking closely at his construction and considering it in a mathematical way. However, further action was needed on Sinéad's part for John to see other shapes in his construction. She left her activity and joined him on the mat and instructed him, with guidance, to feel around the shapes on the underside of his construction. John's delight in the discovery of the square is noteworthy and is indicative of the joy that such discoveries can bring to a young child.

As noted in Chapter Six, Sinéad only intended to focus on 2D shapes during CT (FN 16). On the final day, as a thank-you gift, I had brought a set of wooden 3D solids; John was particularly taken by the 3D shapes and sat close to Sinéad as soon as she opened the bag. In the extended excerpt from my field notes for that day, the sustained joint interaction (Rogoff, 1990) between Sinéad and John is documented, providing an insight into how Sinéad explained the properties of some 3D shapes to John and how she encouraged him to explore them.

> John took the sphere into his hand. "That's a ball", Seán said. Sinéad did not share the mathematical term, sphere, with Seán. She replied, "See if you can roll it". He rolled it across the floor and laughed. John took the wooden 3D shapes to the table, and Sinéad sat beside him. She had the tablet in her hand. John had two shapes, one in each hand – a hexagonal prism and an octagonal prism. He placed them face-to-face. Sinéad asked, "Do they match up? See, that one has six sides and this one has eight sides. Do you remember, we did it in a 2D shape during circle time?" I noticed that Sinéad googled 3D shapes and had a page with 3D shapes open on the tablet. John did not answer but began to stack a cylinder, a hexagonal prism and a hemisphere.

> Sinéad then showed John a picture of some 2D shapes and showed him how the cube's sides are all squares "When it has all the squares together, it is called a cube". She then took a magnetic square from the basket of magnetic shapes and placed it flat on the table in front of John. John looked puzzled. Sinéad continued, "See, this is flat" (magnet), and this is 3D, you can feel it in your hand". She proceeded to make a cube with the magnetic squares.
>
> Sinéad then showed John a pyramid, "Look, it has four triangles and a square base". John looked briefly, then tried to pick up the cone, but it kept slipping out of his hand. Sinéad showed him how to pick up the cone, using his pincer grip. She then turned to another shape, "This is a hemisphere", and this is a "cuboid". (FN 16).

John was intrigued by the new material and keen to explore it. Rather than allowing him to explore the shapes in his own time and space, Sinéad decided to introduce the names of the new shapes to him. She commented on his activity of matching the two 2D faces, asking him if the 2D faces matched up and drew his attention to the number of sides, while making a connection to their learning of a hexagon during CT. Despite John's apparent disinterest in her talk, evident by his lack of verbal engagement and stacking activity, Sinéad explained how a cube is constructed using the square magnetic tiles. John was fond of playing with these, and here Sinéad was using a resource that John liked to play with to try and engage him further with exploring the composition of a cube. Sinéad continued to try and gain John's interest by showing him a square-based pyramid, explaining its composition to him, before showing him how to pick up the cone and introducing the terms hemisphere and cuboid. Sinéad's verbal explanations of 3D shapes are supported by her choice of materials, to further support her unsolicited explanations. Her use of the tablet and Google, perhaps, demonstrates her need to be sure of the correct terminology before communicating the correct mathematical terms when explaining the composition and attributes of 3D shapes to children. It was a way to support her own developing knowledge of this area, also.

7.3.2 Shape in CT

2D shape was formally introduced to the children during CT. 2D shapes were introduced in a similar way to numbers, in that there was a 'Shape of the Week' (see Chapter Six). Over the fieldwork period, the children said goodbye to the heart shape (FN 1,4, 9 & 13) and looked at the rectangle, circle, star and square. The discussion focused on counting the sides of each shape with no other attributes of each shape being considered. Properties of shapes, such as the length of sides relative to each other or the number of angles, were never discussed. An example of this can be found below:

> Sinéad moves to the circle time chart: "What shape are we saying goodbye to?" (pointing to a heart). 'Heart', chorused the children. Sinéad produced a rectangle from the small wicker basket and asked, "What shape are we saying hello to?" Some children say 'rectangle' and Sinéad pins the shape to the board, proceeds to count the sides, pointing to each side as she does so, then asks, "How many sides does it have?" "four". "Four", say the children. (Field Notes FN 1)

This was revision as the children had been introduced to these shapes during CT over the ECCE year. I asked Sinéad if this was the second time around for the selected shapes during CT (RC1), and she informed me that it was. However, initially, at the start of the year, she spent longer, maybe two to three weeks on each shape (RC1). This reflects Siobhán's (owner/manager) observations that, "staff tackled the 'basics' of shape fairly well", but that, "some of the more complex aspects of shape, such as angles, sides and position and…mirror images…that we would like to be teaching to these age groups, was not tackled as well". This statement reflects Siobhán's understanding of the need to expose young children to a wider range of mathematics than is generally taught (Clements & Sarama, 2021), as well as her noticing that her staff engaged in the 'basics of shape' with young children. Sinéad, however, proved to be a staff member staff who engaged children in lower levels of mathematical exploration and thinking in relation to shape, particularly during CT.

This routine was repeated over the three days of the first week; the focus was on shape recognition and the number of sides. Shapes were presented using prototypical examples (Clements & Sarama, 2021); no variations or orientations of individual shapes were shown to the children. There was no discussion as to the length of the sides, identifying the shorter or longer sides or making comparisons between this week's shape and last week's. Shapes presented were not shown in non-prototypical orientations; therefore, the children were not taught to recognise that it is the same shape regardless of orientation. Researchers (e.g. Clements & Sarama, 2021) claim that preschoolers should be shown this aspect of shape, however, they also note that this is rarely a feature of preschool mathematics (Clements & Sarama, 2021). Sinéad's teaching strategy was performed repeatedly with each shape reviewed. The exception was in week two, when the circle was revisited as the following vignette taken from the FNs shows:

Sinéad moved on to the shape of the week and said, "Say goodbye to…" "rectangle", shouted the children and "say hello to…circle". The rectangle is replaced by a circle on the CT board. Chandra got up from her seat and found a plastic ring – "This is a circle", she exclaimed. Other children followed her lead, left their seats and found other items in the room that were circular in shape. After affirming all the circles, Sinéad instructed, "Everyone, put your things back, five, four, three, two, one". When all children were back on their chairs, Sinéad said, "Everyone get out their magic wands, and we'll draw a circle". She modelled drawing a circle, saying "go around" as she drew a circle in the air. The children copied her. (FN 4)

Perhaps, emboldened by the events of the previous day, Chandra led the activity again the following day (FN 5) as she once again led the other children on a circle hunt. Sinéad allowed this to happen, acknowledged the items were indeed circular, asked the children to replace their selected circular objects and began counting backwards, this time from ten to

zero. The usual air drawing of a circle followed. It could be that Sinéad had planned to discuss the properties of the circle, just as she had done with the rectangle, star and square and decided to follow Chandra's lead and allow the circle hunt to take place. The FN excerpts above show the power of individual children to lead an activity; this agency can only be enabled by an adult who is willing to support this (Rogoff, 2003). However, Sinéad limited the activity by placing a time limit on replacing the examples of circles. This could have been developed further by exploring the various examples of circles chosen by the children. For example, noting the similarities and differences of the selected circles, what makes them all circles, could they be compared in some way, size, colour, materials, etc? As noted previously, the setting's CPOP policy advocated using children's activities and interests observed during FP to inform discussions during CT. During RC1, however, Sinéad acknowledged that she had not previously considered this approach, revealing a discrepancy between the policy's intentions and its implementation in practice. Had this strategy been employed in this situation, extended mathematical talk and discussion might have occurred, enriching children's mathematical thinking.

7.4 Space

The incidences of children engaging with shapes and understanding spatial language. recorded here reflect the only observations of Sinéad intentionally supporting spatial activity and thinking over the data collection period. While making their Hulk House (see 7.2.1.3), the boys considered the location of the furniture items in their house and where certain items should be placed in relation to each other. An example of deliberate exposure to spatial language occurred when Sinéad played a YouTube video of an action song during WGT (FN 5). The song, reflecting the theme of Under the Sea, instructed children to engage in various movement types. Instructions included: jump over the shell, duck under the bird, swim around the shark, and lie under the jellyfish. Once the song had played through, Sinéad

supported the children by modelling the movements and restating the instructions given in the video.

During another WGT session, Sinéad asked Filip to hand out glue spreaders, located on the shelf behind the tables. Filip had difficulty finding them. Sinéad noticed this and said, "Filip, the sticks are in the silver bucket on your right". And when Filip looked confused, she added, "They are next to the brushes, behind the bigger brushes". Sinéad's instructions helped Filip to locate the glue sticks (FN 6). Here, Sinéad gave explicit spatial directions, modelling spatial language.

7.5 Data Handling

As with spatial awareness above, observed and documented incidents of children's participation in data-related activities were limited; the only episode being observed during FP time. There were no documented incidents of Sinéad intentionally engaging the children in data activities. The foundations of data handling lie in the processes of classification, sorting and quantifying (Clements & Sarama, 2021). Sorting, matching and classifying are important mathematical skills, and are often naturally incorporated into routine preschool activities, such as tidying up (Montague-Smith & Price, 2018). The following FN excerpt demonstrates children's participation in sorting and classifying in their own self-directed activity, and in the tidy-up process.

7.5.1 The Garden

In this excerpt, the boys took a selection of natural materials and arranged them on the car mat in front of the shelf to make a garden:

> The boys took out the baskets of natural materials (cones, shells, tree bark, wooden discs, conkers, wooden/metal curtain rings and wooden spools). They emptied the

> contents of the baskets all over the car mat. There appeared to be some sort of order as all the spools are together, conkers are together, etc. They were making a 'garden'.
>
> [Sinéad was at the sink, writing] Arjaat took the curtain rings away from the collection of objects – "we don't need these circles", he said. The others disagreed; they wanted to cover the entire rug with items. Seán disagreed the most and left to do another activity. The others continued to add items to the mat. Sinéad tried to take a picture, but Filip shouted at her, "No, not yet!", so she backed away. Arjaat found three shells on the mat and shouted across the room to Sinéad, "Teacher, they are the same". He looked in the basket of shells and shouted, "Teacher, I've got one more the same". Sinéad replied, "How many are the same?" Arjaat didn't answer. Then, Sinéad, distracted by Chandra crying, went to sit with her. A dispute occurred at the colouring table, and she needed to attend to it. Five minutes later, the boys decided to stop making their garden and began to tidy up. Each type of object needed to go back into its own basket. They all helped to sort the objects into the original baskets. [Sinéad was still at the colouring table, settling an issue.] (FN 12)

In this excerpt, while creating their garden, the boys sorted and classified the loose parts materials into sets, the beginnings of data handling. In the beginning, they worked collaboratively (Rogoff, 2003); however, Arjaat felt the need to remove the curtain rings, saying 'We don't need these circles". Arjaat can recognise and name circles in the environment. He also engaged in noticing similarities between objects and recognising items that are the same, and was keen to share his observations with Sinéad (Rogoff, 1990). He understood that objects that share common attributes can be grouped (Brownell et al., 2014). There was no evidence of his understanding that objects in a collection can be sorted in different ways (Brownell et al., 2014). Furthermore, there was no evidence of Arjaat's understanding that sets can be compared and ordered to determine which set has more or fewer than another (Brownell et al., 2014). However, Arjaat did explore quantity at some level as he understood he had 'one more', another shell, similar to the others. Although he brought this observation to Sinéad's attention by calling out to her, he did not reply when she asked, "How many are the same?" The question was, perhaps, to Arjaat, pointless. Arjaat was not interested in how many there were altogether. For him, it was the similarity of the objects that were of interest to him at this time.

Sinéad was interested in their activity and was ready to document it. However, her participation was short-lived, and she did not get an opportunity to extend this conversation, as she was called away to settle a dispute. I wondered if she would have continued her questioning regarding the quantification of the sets, leading the children to compare the quantities. The excerpt concluded with the boys tidying away their garden. The organisation of the shelving into baskets of specified items aided in developing sorting skills, as they collaboratively resorted the items into individual baskets.

Figure 7.8

The Loose Parts Shelf Used by the Boys to Make Their Garden



Photo source: researcher's own

The following selected FN excerpt shows again how commercial games served to support children's participation in mathematics (Balladares et al., 2024). The game of Dobble supports children's observation and matching skills. The game is presented in a tin containing 55 circular cards. Each card has eight pictures on it. Two cards are placed on the table, and children need to find the matching pictures, developing the skills of noticing similarities and differences in the objects presented on the cards. Matching items are the same but differ in format. For example, each card may have a blue snowflake, but on one card, the

snowflake will be bigger than the snowflake on the other card. The game supports the foundational skills of comparison and matching, the beginnings of data handling. Speed is prized in this game.

> Filip approached me and said, 'Let's play Dobble and he talked me through how to play the game. He expertly split the pack of cards into two piles and gave one to me and one to himself. 'We need to find the same, OK?...one two three go'. He quickly found two items that matched on the first set of cards drawn and laid the set in front of him, laughing as he did so. We drew a second set of cards and again, Filip was quick to spot the matching pair. 'You're too slow, ' he said, laughing and added the matching cards to his pile. I asked Priya, who was observing the activity, if she would like to play instead of me. She did and sat opposite Filip. Filip met his match. She, too, was quick to spot the matching pairs. They continued for ten minutes, laughing for much of it (FN 14)

Although Dobble was the most observed game played during FP time, it was often hard to capture an episode as play was over very quickly. I sensed that the children were practised in the game (Rogoff, 1990) and probably knew the content of the cards at this point in the year. This was probably why Filip and Priya were so adept at identifying the matching cards. Their visual perception had been well-developed through their interaction with the game over the year. Their laughter indicates the enjoyment experienced when playing the game.

Figure 7.9

Dobble: the shared element is fire



Picture Source: ***https://boardgamereview.co.uk***

7.6 Pattern

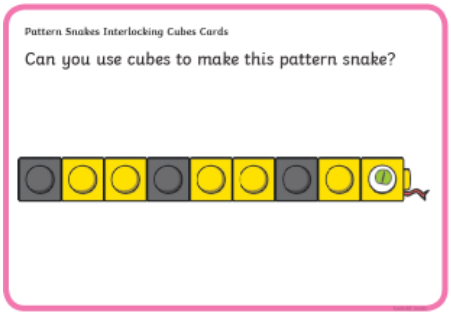
As with the two previous big ideas (space and data), there were relatively few incidents of children's self-directed participation in patterning activities observed during the fieldwork period. As noted elsewhere in this thesis (Section 7.2.2), the children did engage with cyclical patterns associated with time (days of the week, months and seasons) during CT, and there was also evidence of children interested in finger patterns during this time (Seán). The learning environment was seeded (Wager, 2013) with resources to support children's participation in patterns. There were dominoes (dot patterns), commercial games (dot patterns), hundred squares (number patterns), pattern snake cards (simple pattern copying/extension) and items for threading (making patterns). Sinéad noticed that the children were not using the threading beads to make patterns and moved the small threading beads to a more prominent position to encourage pattern making. She believed this move to be beneficial in promoting engagement in this activity as in her first interview (RC1) she commented, "with the beads as well, cos I know now a lot of them will actually take them out, pour it onto the little white lid, to make necklaces, they would count to make it, and I know Aparna has done one where she has tried to make pink/orange/pink and she's counting them out to make sure she had enough to make the right pattern in that order to make the necklaces or bracelets".

As noted in Section 7.1.1.1, Sinéad made some resources in response to her observations of individual children's interests. For example, she downloaded and made some pattern cards (Figure 7.10). However, although the cards are for copying patterns, the purpose for Sinéad was to support John's interest in numbers and counting.

> I printed these off for John. It's like a snakes pattern thing; for him to count out each of the blocks was so he could focus on doing something else. That was an add-on to the actual activity. So there's about ten of them or something, and that shows them that they can do one brown and two yellow. He can count out how many he needs to actually build that snake. And there's loads of different kinds of patterns. (RC1)

Figure 7.10

Snake Pattern Cards



Source: Twinkl.com

Figure 7.10 is an example of the card Sinéad provided for John to support his counting skills. In the FN excerpt above, Sinéad alludes to the different types of patterns on the cards for children to copy, and she describes this example as an ABBABB type pattern. This demonstrates a strong level of MCK in the area of pattern. Sinéad did not allude to where she learned this terminology; however, this information accompanies the cards on the commercial website from where they were sourced, so perhaps Sinéad acquired this knowledge there. While Sinéad's focus for John was on counting, she knew he was also developing his understanding of the different types of patterns. Observations show that the pattern cards influenced Arjaat and Seán to make snakes, though they neither copied nor extended the patterns on the cards. For Arjaat, the fun came with naming the snake rather than completing a pattern:

> Arjaat took the basket of snap cubes and laminated pattern cards to the table where Sinéad was sitting. Seán followed Arjaat to the table. Arjaat showed Sinéad one of the pattern cards. Sinéad talked through the pattern on the card, pointed her finger at each colour as she did so – red/blue/red/blue, but this is not what Arjaat wanted. He wanted a name, the name of a snake, so he asked her, "What's it called?" Sinéad replied, "A garden snake". Arjaat laughed and took another card from the basket and asked, "What's it called?" Sinéad laughed and said, "A north pole snake". Arjaat laughed harder and said, "You're joking me". Shortly after, Sinéad shouted, "Five minutes to tidy up time" as the assistant manager came in to relieve Sinéad for her break.

> Meanwhile, Seán had made a 'rainbow' snake with 18 cubes: three red, three white, three yellow, three brown, three green, and three black. He then broke his snake into each of the coloured components and put it back together again. (FN 10)

This interest in snakes developed from a suggestion made by Sinéad of a sea creature children could make with playdough, "you could even make a sea snake. I saw it in my book, you can have snakes that swim in the ocean, in warm oceans, not cold like the Atlantic". The boys, particularly Arjaat, were captivated by the idea of snakes in the ocean. Sinéad then shared the pictures of sea snakes in the book with the boys and told them the names of the snakes in the pictures, for example, a sea krait. (FN 7). Both snakes shared with the children were striped, representing an ABAB pattern. This was not commented on by either Sinéad or the children. This was an ad hoc comment by Sinéad; it had not been pre-planned, and with only one colour of playdough being provided that day, the potential to replicate the patterning of the sea snakes was limited. This is an example of adaptive teaching (Gibson & Ross, 2016), where Sinéad responds to Arjaat's interest in the snake by sharing information gleaned from her book. There was a missed opportunity to make the connection between the pattern of the sea krait and the ABAB pattern on the card, making the mathematics in both explicit in the process.

Seán engaged in his own patterning activity of making a 'rainbow snake' with three cubes of each colour. He broke it up into its individual components before remaking it exactly as it had been before. Arjaat did not make a snake; his interest lay in the patterns of the snakes on the pictures. The boys were exposed to the patterns on sea snakes and used resources in their environment to explore this at their own level of interest.

Examples of perceptual patterns (e.g. dot patterns, finger patterns) have been discussed elsewhere in this chapter. Dot patterns were evident in many of the resources in the room, from dominoes, dice and on the weekly number worksheet provided on Fridays. Dot

patterns also feature on some of the commercial games. All these varied dotted representations of numbers support number pattern recognition as well as developing the skill of subitising (Clements & Sarama, 2021).

The children experienced number patterns during their counting to the number of the week. The field note below demonstrates that Arjaat and Adomas were interested and capable of counting beyond ten and were familiar with the pattern for numbers 21-29, 31-39, etc., struggling only with the decade numbers.

> Sinéad had gone on her break, and the assistant manager had begun the tidy-up process for CT. The boys had finished tidying away their activity and were sitting on the mat. Arjaat shouted across to me, "me and Adomas are good at counting to 100". Adomas had begun to count and stopped at 19. Seán observed, "But that was not long, it was too short", as if to suggest that when you count to 100, you need to count for a longer period of time. Adomas started to count again and got as far as 16 when Arjaat, distracted by the other children singing Humpty Dumpty and doing the actions, left the mat. This was short-lived, and he was back on the mat in a couple of seconds. I can hear Adomas "41,42,43,44,45,46,47,48,49, 50,51,52,53,54,55,56,57,58,59…he paused, he did not seem to know what came next. I said "60", Adomas nodded at me and continued "60, 61,62…..69 and paused again. I said "70". Adomas resumed "70, 71,72,73,74,75,76,77,78,79, paused" Looked at me, I said "80" and he carried on from 80-89, paused, looked at me again, "90" I said and Adomas continued "91-99, pauses, I said "100" – he smiled and said, "100". Then Arjaat, who was listening intently said, "101, 102, 103, 104, 105". Seán pipes in "count to 200 and fifty two thousand" and laughs. (FN 4).

This FN excerpt shows that two of the boys, Adomas and Arjaat, were interested in verbal counting far beyond ten, as had been their experience during CT. Séan's observation that counting to 100 takes a lot longer than counting to 16 shows that he understood that counting to higher numbers takes longer than counting to smaller ones. Adomas's ability to verbally count the number sequence to 50 demonstrated that he knew that the number pattern 1-9 was repeated after the decade label. He needed support with the remaining decade labels (60-100), which I was happy to give him in Sinéad's absence. Arjaat, too, showed an understanding of the number pattern 1-9 as he counted on from 100 to 105, before being interrupted by Seán.

7.7 Chapter Summary

This chapter explored the varied ways in which the children engaged with the Big Ideas of early mathematics (Brownell et al., 2014), drawing upon data gathered through systematic observational field notes recorded in my capacity as a participant observer, alongside a semi-structured interview with Siobhán, as well as reflective conversations with Sinéad on her self-captured video and photographs of resources to support ECME during the study. This approach facilitated an in-depth understanding of both children's mathematical interactions and the pedagogical practices shaping their experiences. Findings presented here are important for the Irish preschool educational context, as this study is the first to document how young children experience and participate in mathematical activities and how their participation in the cultural activity that is mathematics is facilitated and supported by their educator, Sinéad.

A range of pedagogical strategies to support children's participation in mathematics was observed and documented. These include seeding a well-resourced environment (Wager, 2013) with a wide range of materials to support participation in the Big Ideas of Early Mathematics (Brownell et al., 2014), the use of songs to support learning of temporal aspects of time and spatial awareness and related prompts to recall the sequencing of the days of the week. Numeral recognition was supported through games, CT and observation and imitation of how to form numerals through air-writing in CT, as well as through engagement with weekly number-focused worksheets. CT provided opportunities to participate in guided verbal and physical counting to support learning of counting one to ten. Participation in the learning of shapes was supported through visual representations of 2D shapes, with some consideration of the basic attributes of each shape learned. Construction play supported engagement with constructing 2D and 3D shapes, with block play offering experiences to sensorially

The children chiefly participated in number-related activities, with aspects of number learning permeating across each part of the daily ECCE routine observed. This was followed by measure and shape. Participation in the mathematical ideas of spatial reasoning, data handling and pattern was less frequently observed. Despite the learning environment being richly seeded to promote engagement with these aspects of mathematics, children did not participate in them as frequently as number, measures or shape. Again, one could question the role of the adult during free play. If Sinéad had a more active role, she might have planned some intentional learning activities to support these aspects of mathematics with the children. Findings suggest a reimagining of how powerful CT could be if Sinéad, as suggested in the CPOP, used this space to talk about, nurture, and enhance aspects of children's mathematical activity or interests that occurred during free play. Caution must be exercised, however. This study represents a four-week snapshot of practice, and findings reflect observations captured during this period. As indicated in the interview data, children may have engaged in further activities to develop early spatial, data and pattern skills prior to the study taking place

Children's participation in mathematical learning was observed both in free play and in adult-led learning sessions such as CT and WGT. The dedicated hour of free play afforded children choice in their morning's activity and time not only to become deeply immersed in their play but also to revisit ideas over an extended period. Both commercially produced and teacher-designed games emerged as highly engaging activities, providing a playful context in which children explored early number concepts, including numeral recognition, subitising, and counting. Analysis indicates that four of the five counting principles were evident in the play, suggesting that such activities can support not only skill development but also conceptual understanding.

Extended time in FP provided the opportunity for prolonged engagement with block play, particularly for Arjaat, Adomas, Filip and Seán. Time to engage with shape, space and measure on a deep level. However, with the boys dominating the unit blocks and Sinéad's commitment to refraining from influencing children's choice of activity, the girls were not encouraged to play with the unit blocks, thereby placing them at a disadvantage in terms of developing the required spatial reasoning skills needed for future mathematical understanding and development.

Some individual mathematical interests were observed, facilitated and enhanced by Sinéad. Chandra's interest in circles, and John's interest in number operations were obvious, and both were facilitated by Sinéad, albeit in different ways. In the instance of Chandra, Sinéad allowed her to change the CT routine, leave her seat, to find examples of circles. The environment also enabled Chandra to identify and make circles when she wished to. Despite Sinéad's claim that she refrained from interacting in children's self-chosen activities unless asked to do so, she deliberately sought out John at times to participate in number activities she thought he would enjoy. This she did for John alone. There were no other recorded observations of Sinéad asking children to join her in an activity. Sinéad's observations of John as a child who was 'ready for the next step' show Sinéad's understanding of John and his interests, and of the relevant equipment in the room to support him in developing his calculation skills. As noted earlier, number operations are not mentioned in Aistear (GoI,2024c) and are part of the Primary Mathematics Curriculum (DE, 2023a). Despite not being listed in Aistear, Sinéad commits to supporting John in his interest, although at times she is afraid of doing too much, in case he 'acts out' in primary school. This will be considered further in Chapter Eight.

In line with recommendations in the literature (Klibanoff et al., 2006), the setting's EYMTL policy called for the use of 'maths talk' in the classroom. Maths talk is defined in

the PMC (DE, 2023a) as a 'collaborative process where children's thinking, strategies and ideas are expressed, shared and/or exchanged' (p.32). Consistent with previous studies (e.g. Klibanoff et al., 2006), analysis revealed that a lack of adult engagement in *math talk* was evident across the FN collection, which was notably absent during FP. Sinéad did make the odd comment in passing, such as guiding John through the shapes in his prism. However, this was not part of normal practice. Again, the mantra of not interfering with children's self-selected activity and the perceived need to wait until asked may play a part here. When Sinéad did offer a mathematical observation, comment or question, it generally went unanswered by the child it was intended for. Opportunities for Sinéad to respond to children's thinking arose during CT, but these were not taken up by Sinéad, who always seemed to be under time pressure to get through her CT content. She never deviated from the routine content and questions. CT is an ideal time to share ideas, to explore children's observations, queries and comments further (Bustamente et al., 2018), to discuss children's activities conducted during FP, and provide the necessary vocabulary to mathematise these.

Chapter Seven presented findings to answer the questions,

1. What types of mathematics do children participate in?
2. In what ways does the adult guide support this participation?

Chapter Nine will further discuss findings presented in this chapter, as well as consider implications for policy, practice and further research. Chapter Eight details findings in answer to the question: What are the challenges in supporting children's participation in ECME in this setting?

CHAPTER 8: CHALLENGES TO SUPPORTING ECME

This chapter addresses the final sub-question: What are the challenges in supporting children's participation in ECME in this setting? Findings presented in this chapter are chiefly underpinned by Bronfenbrenner's *Bioecological Systems Theory,* and his interpretation of culture, which posits that humans are both products and producers of culture (see Chapter Four). Findings show how government policy (macrosystem) and a person's characteristics (Bronfenbrenner & Morris, 2006) influence practice in the ECEC setting and in the ECCE classroom.

This chapter details the challenges identified by Siobhán and Sinéad in their interviews, reflective and casual conversations, as well as those identified by me through FN analysis. Findings suggest several challenges existed in this setting in relation to supporting ECME. These ranged from structural constraints, such as insufficient non-contact time and other required non-teaching duties, to fluctuating break times. These constraints limited Sinéad's presence in the ECCE room during Free Play time, thereby reducing the time available to support children if they required it. Further challenges identified include having to attend to several children at once, leaving interactions to solve conflicts, as well as Sinéad's beliefs about her role during free play. Sinéad's understandings of and beliefs about the primary school curriculum were also identified as a potential challenge to supporting ECME in this setting. This section begins by considering the structural constraints affecting children's participation in ECME in this setting, before discussing challenges to supporting ECME in Little Ones identified by Siobhán (owner/manager). This chapter concludes by considering challenges identified in the ECCE classroom by Sinéad.

8.1 Structural Constraints

8.1.1 Non-Contact time

As noted in Chapter Six, the ECCE day ran for three hours. Sinéad's assigned non-contact time to complete organisational and administrative tasks was scheduled to take place from 8.30 to 9.15 each morning (FN 2) before the children began to arrive at 9.20. This was time spent preparing materials for art and craft activities, photocopying (Fridays), completing pedagogical documentation such as *Transition Flowers* (Appendix A), or adding items to the children's scrapbooks, writing up observations on the required App, completing planning records and graduation certificates. Often, when I arrived at the setting, Sinéad was deeply involved in these tasks, and I frequently felt uneasy asking her questions during this time. From my time as a preschool educator, I understood the demands of the required paperwork, particularly towards the end of the year, as assessment documentation necessary for children's transition to school became an increasing burden. Sinéad (FN 2) informed me that she was often unable to get everything done in the allocated time. This explained why she frequently used FP and Snack time to engage in organisational and administrative tasks. Table 8.1 below details one day's observation and illustrates how Sinéad used FP time to complete these and other classroom-based tasks. This data is illustrative of the typical routine tasks observed during the fieldwork period.

Table 8.1

Sinéad's Activity During Free Play Time (Example Taken from Field Notes).

Date: 3/5/23	Activity
Time: 9.30-9.38	Cutting out crabs for art activity
9.38-9.40	Taking photos of children's play activity using IPad
9.40	Back cutting out shapes
9.45	Sitting with one child, playing Dobble
9.47	Asks another child to take her place. Moves to sink to pour paint into pots for art activity
9.50	Back at Dobble table, scanning the room, not interacting with children playing Dobble
9.52	Shouts across room to girls playing with straws: , "what are you making?" Daisy smiles. S "you don't know yet? That's OK".
9.54	"How long have you made it before?" [suggesting that this is not the first time the girls have engaged in this activity?] "It was taller than me, I think". The two girls giggle. Sinéad continues, " did it go from the floor to the ceiling?". Lucy replies, "ya".
9.55	Sinéad at Dobble table, gets up and walks to the nearby shelf, takes a tub of writing pencils and seems to sort through them. Two girls continue to play Dobble.
9.56	Sinéad joins in playing Dobble (can't hear the conversation). But she then says to Daisy & Lucy, "wow, you're getting superlong – I wonder how many is in it? "What does it look like? How many pieces are in it? That seems like too many.
10.02	The three boys are still in the home corner/builders shop. Sinéad from the Dobble table to the boys: "boys, what's going on down there?" Boys reply in unison, "nothing'

Table 8.1 above illustrates how Sinéad spent the first half hour of this day (FN1). Approximately 15 minutes were spent cutting out shapes for the day's art activity, three minutes were spent pouring paint, again in preparation for the intended art activity, and another minute was spent sorting pencils. In total, nearly 20 minutes of FP were taken up by

organisational duties. When interpreted through Bronfenbrenner's Bioecological Systems Theory (Bronfenbrenner & Morris, 2006), these findings underscore the significance of proximal processes—the reciprocal, enduring interactions between children and adults that form the core mechanism of development within the microsystem (Bronfenbrenner & Morris, 2006). Systemic factors at the exosystem or macrosystem level—such as administrative workload- can indirectly shape the frequency and quality of these interactions by influencing the educator's availability and full presence in the classroom. Thus, in these situations, educators are ill-positioned to notice and respond to children's mathematical explorations, observations and thinking (Dockett & Goff, 2013; Lee, 2017), or to offer assistance when requested (Rogoff, 1990; 2003). These findings highlight the need for the allocation of a longer non-contact time to prepare for daily activities and to complete administrative duties.

8.1.2 Non-Teaching Related Duties

One of Sinéad's responsibilities was to meet children and their parents at the building entrance to welcome them to the setting. This duty was assigned to staff on a rota basis. Sinéad's turn to engage in this task occurred during week three. This impacted Sinéad's presence in the room during FP. Sinéad could be delayed at the door if a parent had something to communicate to her. During this week, findings show that on some days, Sinéad missed up to 25 minutes of FP time, between 15 minutes for her break and delays at the door. Again, impacting the time spent with the children. This reduction in classroom presence had a direct impact on the time available for engaging in sustained interactions with children. Interpreted through a Bioecological Systems lens (Bronfenbrenner & Morris, 2006), this situation illustrates how processes at the *exosystem* level—such as institutional routines, including parental engagement procedures—can indirectly influence the *microsystem*, where the most significant learning interactions occur. Notwithstanding the need to communicate strong parent-educator interaction, the temporary removal of the LE from the immediate

learning context, the ECCE room, reduced available time for Sinéad to engage in the proximal processes to support early mathematical learning. Consequently, Sinéad had fewer opportunities to observe children's activity and learning, limiting her capacity to engage in pedagogical processes that foster mathematical development. Such processes—including responsive assistance, the provision of scaffolding (Rogoff, 2003), and the facilitation of mathematical talk and discussion (Klibanoff et al., 2006)—are central to supporting children's active participation in mathematical thinking. The reduction of these opportunities limited the potential for rich, sustained interactions that can shape and extend children's mathematical understanding.

8.1.3 Fluctuating Break Times

Under the Organisation of Working Time Act (1997), Sinéad is entitled to a 15-minute break as well as a 30-minute lunch break during the working day. Analyses of FN show that the timing of Sinéad's 15-minute break time differed each day. During a conversation (FN 3), Sinéad mentioned that both assistant managers were engaged in non-managerial activities, such as cooking meals, as the setting was in the process of recruiting a cook. Sinéad informed me that this was the reason for the differing break times. Fluctuating break times impacted Sinéad's potential interactions with the children during free FP, as she commented that she did not really want to start something with a child until her break was over, as she did not want to disrupt the flow of an activity (FN 3).

Often, when Sinéad left the room to take her break, the children were generally busy, occupied with their own self-chosen/self-directed activities. However, on one occasion, her break did interfere with a sustained interaction she had been having with Filip. In this interaction, Sinéad showed an unsolicited interest in Filip's activity:

> Sinéad moves to sit with Filip, who is at the table playing with the Magnetic building blocks and asks, "What are you building?" Filip does not reply. Sinéad takes two

large hexagons and puts them together, and said, “I’ve made a clam, open/shut, open/shut. This is a reference to the Under the Sea song they learned last week. Filip chooses a small triangle and lays it down on the table. “Triangle”, he says, and turns it around. Sinéad answers: “It’s still a triangle no matter which way you turn it”. Filip does not reply and looks for other triangles in the basket and joins them together (a repeat of yesterday). He counts 11 triangles, adds two more and says “13”, takes two more and says, “15”, he finds another triangle in the basket and says “16” …” I’ll get to 17” and looks in the basket “actually there’s no more, there’s only 17”. Sinéad says “we can check, count them, how many triangles are there?” They count together “18”. “You were right”, comments Sinéad, to which Filip replied: “No, I said 17

Filip: “I liked that game”. Sinéad: “Which game?” Filip: The one where you count and count again. Sinéad: Was that a worksheet? Filip: No. Sinéad: “It is in this room? Show me”. Filip: “No, I’m doing this *now”* (raised voice). Sinéad makes a cube with the Magnatiles. Filip: There’s 22 triangles. I had 21 and I found 1 more, I’m still making it bigger. Filip to Sinéad: “I’m making it longer than the table now” He has connected the triangles so that they span the entire length of the table. Sinéad asks, “Is it longer than the table?” Filip “Yeah”. 10.05 Sinéad has to leave this interaction as she is relieved for her break by the assistant manager.

Figure 8.1

Magnetic building blocks used by Filip



Image source: WorldofWonderToys.ie

This sustained interaction lasted ten minutes before Sinéad had to leave to take her break. The dialogue within the FN excerpt demonstrates Rogoff’s (2003) emphasis on mutual engagement and shifting roles within guided participation. Sinéad joins Filip’s activity, through her making of a clam, she playfully modelled exploration and her comment, ‘It’s still a triangle no matter which way you turn it’ had the potential to develop Filip’s conceptual understanding of a triangle (Weisberg et al., 2013; Clements & Sarama, 2021). However,

Filip's focus was on connecting the triangles and counting them, not on the triangles themselves. The FN excerpt shows that Sinéad did not fully tune in to Filip's self-driven objectives. It is only towards the end of the first part of the excerpt that there is evidence of Sinéad trying to join in Filip's counting activity. However, again, she did not listen to his talk and his statement that he had counted 17 and therefore he had incorrectly counted his triangles. I have interpreted Sinéad's lack of intersubjectivity (Rogoff, 1990) due to her anticipating her break and not wanting to become fully engaged in Filip's activity.

However, the episode could equally be reflective of Sinéad's belief that she should not interfere in children's activity and of her noticing of mathematics in children's free play more generally. It could be argued that while Sinéad did notice Filip's counting activity, she did not realise *his* mathematical objective. Therefore, she did not fully understand his activity and did not attend to, or respond to, the activity in a way to enhance Filip's mathematical thinking (Ball, 2011; Lee, 2017).

Similar to section 8.2.3 above, Bronfenbrenner's bioecological model (Bronfenbrenner & Morris, 2006) highlights how *exosystemic* factors can support or constrain desirable proximal processes. In this study, the institutional routines interrupted the flow of proximal processes. Sinéad's departure for her break (10:05) abruptly terminated a rich mathematical interaction that might otherwise have extended Filip's exploration and sustained his engagement. This exemplifies how organisational structures within the exosystem indirectly shape the quality and continuity of interactions in the microsystem, thereby influencing learning. The legal requirement for staff breaks, although operationally necessary, inadvertently limits opportunities for deep engagement in mathematical reasoning and the co-construction of knowledge.

8.1.4 Missed Opportunities

The inconsistent nature of the timing of Sinéad's break often led to her missing opportunities to observe or support the children's engagement in mathematical activity. By the end of week three, the children began to address me as 'Teacher' and see me as someone they could approach if they needed help. A consequence of this occurred when Sinéad was on her break, when children sought my support in their activities. An example follows below:

> Arjaat takes a measuring tape and says, "Hammerhead sharks are 4 metres". He lays a measuring tape (1 metre) on the ground. I watch from the table as he adds another tape. He notices that I am watching but says nothing; he seems to be waiting for me to say something. "This is 2 metres", I say. Hammerheads are twice this long. Arjaat brings me another tape. He doesn't say anything, but it seems he would like me to be part of his activity – this one is made of paper and has been ripped into several pieces. It appears that some of the pieces are missing. I explain that we can't use it. I suggest that we start at the divider that marks the start of the home corner, and we lay one tape down and add the other at the end of this one. We then move the first and add it to the end of the second. Then we move the second and place it at the end of the third. "Now, a hammerhead is the same length as from the home corner to the wall here", Arjaat says, "wow, look how long a hammerhead shark is…that's so long". This evolves into a discussion about the different types of sharks in a book with Arjaat. This discussion draws Adomas, Filip and Seán to us. Meanwhile, the assistant manager, who is covering Sinéad's break, continues to play the Catch and Count game with the other children. (FN 8)

As a participant observer, it can be difficult to remain on the sidelines, simply observing and not interacting. The children viewed me as an adult in the room, someone who was there to support them in their interests and inquiries. I took this as an indication that the children were comfortable with my presence (Roberts-Holmes, 2014). I understand that this position of 'other adult' may have compromised my role as researcher. However, in this instance, as I was viewed as another 'teacher' by the children, and the other available adult was busy, I had a duty to support the children when they sought my help. The other adult in the room, the assistant manager, was engaged with other children and was unavailable to Arjaat at this time. Arjaat's interest in the length of a hammerhead shark was stimulated by a discussion during CT about the length of a hammerhead shark (FN 7). As the assistant

manager was busy with the other children and I appeared to be interested in his activity, Arjaat approached me to help him. Arjaat expects that the adults in the room will engage with him if he needs it. It is the accepted norm for behaviour during FP time. As I was now part of the culture of the ECCE room, it was expected that I, too, would support the children when approached to do so. In this part of the routine, children have the power, the autonomy to follow their interests with or without the support of the adults in the room. They know and understand this. The well-resourced, mathematically rich learning environment facilitated this line of inquiry and Arjaat's wonder at the length of a hammerhead. The activity was relevant and meaningful for him, and formed part of a sustained interest in measure, particularly length and height, over the observation period. In this example, there is evidence that children engaged in mathematical thinking that was supported and extended by me as a knowledgeable adult (Rogoff, 2003), which Sinéad did not have the opportunity to notice due to her break. However, given the evidence presented elsewhere of her reluctance to interfere in children's play, it could be considered whether Sinéad had been in the room, would she have noticed and enhanced Arjaat's line of inquiry, particularly with her belief that she should not interfere in children's play unless asked to do so.

8.1.5 Temporal Impacts on CT

As noted in Chapter Six, CT was scheduled from 10.30 to 11.00; however, there were instances where it did not begin until 10.40 due to Sinéad's late return from her break. This fluctuating break time often resulted in Sinéad constantly looking at her watch, rushing through CT content, leaving little to no time to notice and/or respond to children's ideas or interests.

As noted in Chapter Four, a person brings to any situation a set of personal characteristics. Person characteristics, as conceptualised within Bioecological Systems Theory (Bronfenbrenner & Morris, 2006), interact with contextual and temporal dimensions

of classroom and setting culture to shape the quality and occurrence of proximal processes in this ECCE classroom. Sinéad's force (dispositional) characteristics, such as a strong sense of professional responsibility and adherence to structure, appear to have influenced the way she managed CT. Her disciplined approach to maintaining the same routine and content, even under time pressure, reflects a disposition toward order and predictability. While such characteristics can contribute positively to classroom consistency and behavioural management, in this instance, they inadvertently constrained the flexibility needed to notice and engage responsively with children's mathematical observations and ideas (Dockett & Goff, 2013; Lee, 2017). Sinéad's commitment to the teaching of certain 2D shapes and numbers to ten may reflect an inner belief that privileges the learning of certain types of mathematical concepts and practices over others (Wager, 2014), and her reluctance to acknowledge children's mathematical observations reflects a form of inequitable noticing (Wager, 2014). The comments recorded in the FNs ("We need to move quickly today…" FN 4) and Sinéad's own reflections on time pressure ("sometimes the time gets away from you" RC1) indicate that her disposition towards task completion and routine adherence overshadowed her responsiveness to children's contributions, thereby limiting opportunities for effective engagement in the proximal processes required for learning.

Sinéad's resource characteristics, including her pedagogical knowledge, experience, and capacity for reflection, impacted her practice during CT. Gibson and Ross (2016) assert that the noticing process is one of adaptive teaching; however, there was limited evidence of 'in the moment responses' (Gibson & Ross, 2016) or any adaptation to her practice during CT in the FNs. This suggests that her reflective resources and capacity to adapt the routine and respond to children's comments and observations may have been constrained by situational stressors (e.g., time pressure and institutional scheduling demands). Contextual constraints (e.g. time pressure and institutional routines) operating at the exosystem level

indirectly shaped her capacity to enact proximal processes at the microsystem level, particularly in her interactions with children during CT (Bronfenbrenner, 1979).

Furthermore, the temporal component of the PPCT model (Bronfenbrenner & Morris, 2006) is evident in how repeated scheduling pressures over time appeared to reinforce a pattern of hurried, rigid practice. Over time, these routinised practices contributed to a classroom culture characterised by efficiency rather than responsiveness, thereby limiting the developmental potential of CT sessions as rich contexts for mathematical exploration and discussion (Cheeseman, 2015).

8.1.6 Managing Several Children at Once

Another challenge to supporting ECME in the ECCE classroom is that Sinéad was often called upon to support several children at once, having to divide herself between two or three competing individual interests. This challenge was acknowledged by Siobhán in her interview (INT1) (see section 8.1.4). The following field note excerpt demonstrates how Sinéad manages the interests of two children, Chandra and John.

> 10.12 Priya brought the counting card game to the table, where Sinéad and Chandra were completing a matching animal puzzle together. Priya worked independently. She is not interested in the other activity. Sinéad, however, was watching Priya work while playing with Chandra. This was not easy, as Chandra wanted Sinéad's full attention. She asked Sinéad, 'Where does this one go?' and Sinéad reverted her attention to Chandra. I noted that Priya made a counting error. She counted eight rather than ten chickens on the card. This was missed by Sinéad, as her attention was now fully on Chandra.
>
> 10.20 John approached the table. He brought the Game of Ladybirds and wanted Sinéad to play with him. He was asked to sit on the mat and play alone. This he did. However, after a minute, he called to Sinéad, "I want to play with you". Sinéad acknowledged his request by saying, "I'll be there in two minutes", and soon joined John on the mat. After a while, Sinéad returned to Chandra and the matching game, after asking John to continue on his own for a bit. John was not happy with this arrangement; his face was sad, and he twisted around on the mat and did not seem interested in continuing the game. Sinéad to John from the table – she watched him while playing with Chandra, and asked, "do you want to put this away or continue? John: "I want to play with you!". Sinéad: "OK, but I need to go back and forth to Chandra". Sinéad sat by John again on

> the mat, looked at the pairs made by John and said, "do you want them on top of each other or beside each other?". The game continued (FN 3).

In the FN excerpt above, Sinéad's interactions with Chandra, Priya, and John illustrate the challenges of sustaining high-quality proximal processes (Bronfenbrenner & Morris, 2006) when competing demands are placed upon an educator within an educational microsystem. While Sinéad endeavoured to support Chandra's engagement with the puzzle and simultaneously observe Priya's independent counting activity, her attention was divided. As a result, she missed a critical learning opportunity when Priya miscounted. MacDonald (2018) contends that noticing is a required skill in ECEC environments. It enables educators to know and understand each child in the class. However, this excerpt illustrates the challenges that emerge when several children simultaneously seek the educator's attention, thereby reducing an educator's capacity to notice and respond to each child's individual mathematical explorations, developing skills, and misconceptions.

The person characteristics (Bronfenbrenner & Morris, 2006) of both Sinéad and the children are key to interpreting this episode. Sinéad's force or (dispositional) characteristics - including attentiveness and a strong sense of pedagogical responsibility - are evident in her attempts to include and respond to multiple children at the same time. However, these positive dispositions were constrained by the working context of the situation: multiple demands from individual children and a high ratio of children to adults (one to eleven, Walsh et al., 2024). Meanwhile, the demand characteristics of the children (e.g. Chandra's need for exclusive attention and John's desire for shared play) exerted a strong pull on Sinéad's ability to respond adequately to each child. These characteristics demanded particular responses from Sinéad, shaping how the interactions developed and the extent to which proximal processes could be sustained. This divided attention weakened the processes of reciprocity

and continuity essential to the effective engagement in proximal processes (Bronfenbrenner & Morris, 2006; Rogoff, 2003). This FN excerpt highlights the necessity to reconsider the structural supports available in ECEC settings, such as adequate staffing, to enable educators to engage in sustained, high-quality interactions necessary to facilitate and encourage children's participation in mathematics. Section 8.2 reports on Siobhán's perceived challenges in supporting ECME in this setting.

8.2 Siobhán's Perspective on the Challenges in Supporting ECME in the Setting

8.2.1 Staffing Issues

During her interview (Int 1), Siobhán identified several challenges for her staff supporting ECME in the learning rooms. Siobhán spoke about issues concerning staff turnover. Staff retention had become a major issue in recent years. Indeed, Siobhán had a 'completely new team' since she had instigated a course for staff in ECME in 2021. Unfortunately, she had not had the time to run this course for the current cohort. Siobhán valued the learning and expertise that graduate staff can bring to the EC classroom; however, she commented that graduates do not always apply for positions offered.

8.2.2 The Education Gap and Staff Qualifications

Siobhán spoke about the lack of knowledge of staff, particularly those with level 5 and 6 qualifications in ECEC, in terms of mathematics and how to recognise the potential for mathematics in resources and in supporting children's engagement in ECME, as she noted:

> "…not recognising what was maths equipment, like the snap cards and the Lotto games, and you know, like the matching games and those kind of things, but you know that obviously shows an education gap as well"

Siobhán stated that "not all ECEC degrees cover maths and numeracy". Recent research (O'Neill et al., 2022) affirms this statement. As noted in Chapter Six, Sinéad was

the LE, a graduate of a level 8 course and had completed a module titled 'Literacy and Numeracy through Play' as part of her training. A review of module contents shows that numbers, pattern, counting, measures, shape and space were part of the curriculum. Despite this, she stated that she had not considered the potential for mathematics in some of the resources in her room until prompted to do so by being asked to take photographs of resources with mathematical potential for this study. As she shared a picture of a wooden birthday cake and described the potential for mathematics, she commented, "I didn't think of that until I was looking around for things to take…and I just didn't think it was for numeracy' (RC2). This suggests that all pre-service ECEs be alerted to the potential for children's engagement with mathematical concepts in common resources in an ECEC room. Siobhán highlighted the lack of opportunities for PD for ECME. She remarked that there was PD material on the Aistear / Síolta website, but questioned whether her 'team would actually voluntarily go and engage with that', suggesting that unless the training was mandatory, staff would not engage.

Siobhán's observations highlight differences in staff qualifications and educational preparation - specifically, how educators with Level 5 and 6 qualifications may lack the subject knowledge and pedagogical awareness necessary to identify mathematical opportunities within everyday play resources. These resource characteristics (knowledge, skill, and experience) (Bronfenbrenner & Morris, 2006) directly affect educators' ability at the microsystem level to initiate and sustain reciprocal proximal processes that support children's mathematical learning. Similarly, Sinéad's reflection that she had not recognised the mathematical potential in common play materials (e.g., a wooden birthday cake) until prompted suggests that even higher-qualified educators may possess resource gaps, particularly in the application of theory into practice.

The perspectives of Siobhán and Sinéad illustrate how contextual factors shape opportunities for professional development. The limited integration of ECME in further education and undergraduate courses (Gillic et al. 2026, under review), as commented on by Siobhán, highlights broader issues within the macrosystem, where mathematics is not a compulsory part of initial professional qualifications, and little to no mathematics-focused PD is provided by the state. This lack of ECME training within pre-service and in-service courses, identified by Siobhán, impacts future interactions and proximal processes that support children's participation in ECME in the microsystems of preschool classrooms. Government policy must make mathematics mandatory in initial EC professional courses, and robust PD should be provided for educators.

8.2.3 Staff Confidence with Mathematics

Siobhán noted that staff with level 5 and 6 qualifications often feel less confident or nervous with mathematics than with other areas of child development, such as literacy or social and emotional development. However, there were some aspects of mathematics that Siobhán felt staff were more confident in exploring with young children, such as the temporal aspects of measure (days of the week, months), as well as 'the basics of shape'. However, during her interview, Siobhán said she needed to be careful in the ways she communicated mathematical ideas to staff and that she tried to avoid using technical terms such as "subitising", as she felt such language could potentially scare staff. She explained, "I try to dumb it down, I try and make it more accessible by not using the mathematic [*sic*]words if I can avoid them". Siobhán's observations, experiences and actions of her staff reflect a broader educational field where educators report mathematics anxiety (Jenβen, 2022), low self-efficacy beliefs (Ginsburg et al., 2008), often as a result of previous negative experiences (Opperman et al., 2016), and low levels of MCK, often due to lack of training in ECME (Blömeke et al., 2017).To support her staff, Siobhán asserted that it was her role to give staff

confidence and support with ECME in their practice, and support staff to recognise the importance of ECME, but that at the time of the study, neither she nor her managerial staff had the time to do this effectively. Siobhán's assertion that she, as curriculum leader, lacked time to carry out duties aligned with the role raises questions about government plans to develop pedagogical leadership in ECE settings (GoI, 2024b; GoI, 2021). This finding supports a call for dedicated time and space to be allocated for curriculum/pedagogical leaders to review and support the teaching and learning of mathematics in preschool settings.

8.2.4 Time and Space

Practical issues such as lack of time to support and extend children's mathematical interests, as also identified by Siobhán, as a challenge to supporting ECME in EC settings. She commented:

> "I suppose, sometimes, the ceiling would be our abilities to, you know, that's being really blunt. Sometimes our abilities and our capacity to have the time and space, cos you don't always have the time or space to continue, you know, extend interests and develop ideas and concepts with children, so really that would probably be the real limiting step there"

She further commented:

> "I think running a room…it's just go, go, go all the time, all day, every day…and there's some days your plans don't work out, and those days aren't just rare, they're common...I think a fair amount of fire-fighting happens in early years education, that means that some of the planned stuff doesn't happen, either you know"

Drawing on her experience as an ECE, Siobhán defends staff capabilities to have ample time and space to support children's emerging interests. Siobhán recognised the considerable challenges educators face when managing 11 children independently, highlighting how the demands of classroom management can constrain the delivery of planned activities and reduce opportunities for rich, responsive interactions. This observation highlights the systemic and structural factors inherent in early childhood settings that can

impede the enactment of pedagogically optimal practices, rather than reflecting limitations of individual practitioners. Siobhán's comments align with and serve to triangulate my own observations and study findings, which provide many examples of this.

8.3 Sinéad's Perspectives on Practice

This section considers Sinéad's perspectives on practice, her beliefs towards mathematics in preschool and the early years of primary education, as well as her understanding of her role during FP.

8.3.1 Transition to Primary School

The topic of 'doing too much' arose during RC2 with Sinéad. When we discussed John and Sinéad's support in developing his interest in numbers and number operations, she confessed to being afraid to engage children in concepts traditionally taught in Junior Infants (JI), the first year of primary school. Sinéad used the phrase 'too well ahead' in our discussion about her previous experiences in a Montessori preschool setting. And when I asked what she meant by 'too well ahead', she explained:

> "Like we did the sounds, letters, counting. They knew all the stuff for junior infants' level, that I was afraid that they were going to be bored, cos I see the homework that the schoolies, junior infants like, and it's based on worksheets or just counting stuff and some of them are like sheets that I would put out, that my kids are able to do. So, they're not actually like, em, stimulating their learning at all because they're just kinda doing what they already know… I feel that's when they might act out, then"

There is some overlap between the Montessori 3-6 mathematics curriculum and the infant mathematics curriculum in primary school. I was intrigued by how Sinéad perceived Montessori-educated preschool children to be "ahead", as there is evidence to suggest that she engages with the same content, including number operations (albeit not through the Montessori method), with her current preschool class. As in the case with John, Sinéad had branched out into Senior Infant curriculum content (GoI, 1999c) in doing number operations

with numbers six to ten and beyond into the teens. She supported his current interest in this kind of number work. There appears to be a tension in her words and actions, however. On the one hand, she followed the accepted EC mantra to attend to the child in the here and now, to celebrate the being, and on the other, she fears that in doing so, the child may be bored and 'act out' as a result in primary school. The quote illustrates the complex and seemingly conflicted nature of Sinéad's beliefs towards pedagogical practice in primary school. These beliefs are possibly the result of past experiences in Montessori ECCE practice and in a JI classroom. In her opinion, her 'children are very well prepared to go into JI, like a lot of them, through play and stuff, and then they're back to square one in Junior Infants' (RC1).

This statement reflects Sinéad's belief that the children in her care have mathematical knowledge (like numbers 1-10) and that this knowledge, gained through playful activity, will serve them well as they transition to primary school. Her experience, however, informs her other belief that the children will relearn numbers as part of the JI curriculum (GoI, 1999c). There is a sense of resentment in Sinéad's statement; she has spent time teaching the children their numbers, and to what end. Sinéad's frustration at the lack of collaborative, cross-sectoral thinking around the early childhood mathematics curricular agenda in both sectors is highlighted in the FN excerpt below:

> "There's no lead in, and that annoys me; there's no lead in from preschool to primary. They do everything again when they go to school. When I was in college, I did a placement in a primary school, and they were doing number four. We do that here, why do they need to do it again?... We need some thinking around that". (FN1)

As detailed in Chapter Seven, Sinéad provided a worksheet for children to practice number recognition and counting, based on the number of the week. Is this what she understood by 'doing the number four'? Findings in Chapter Seven also showed that Sinéad did not pick up on the opportunity to explore the structural aspect of four, when Seán

represented four with two fingers on each hand. Such structural explorations are a key feature of the Infant mathematics curriculum (DE, 2023a) and are explored at the preschool level internationally (e.g., Björklund et al., 2021). Sinéad's observations of practice represent a snapshot of mathematics curriculum content and pedagogy in the JI classroom. As noted in Chapter Three, number operations are not included in the list of mathematical concepts in Aistear (GoI, 2024c) that young children in preschool settings should explore. Further research is needed to explore how ECEs in Ireland understand the concept of number for young children.

In other discussions, upon observing that some of the children were interested in counting on past ten to one hundred and beyond, I asked Sinéad if she would continue past five. She said she would go up to ten during CT. Sinéad acknowledged that there were children in the group who were ready for the teen numbers, but she felt she needed to focus on the majority who were not (FN1). However, observations show that six of the eight regular attenders (Seán, Filip, Arjaat, Adomas, Chandra, and Priya) could recite, identify and count up to ten, with the boys regularly wanting to count beyond that to 100.

Sinéad's resource characteristics (Bronfenbrenner & Morris, 2006) (her degree qualification, practicum experience in a JI class, and her professional experience in a Montessori classroom) contribute to the conflict and frustration she experiences. This mesh of experience shapes her engagement in guiding children's participation in ECME. Her choice to limit counting to ten, despite some children being interested and able to progress further, reflects how these previous experiences and beliefs arising from them influence her current pedagogical practices (Anders & Rossbach, 2015; Chen et al., 2014; Platas, 2015).

Sinéad's frustration at the lack of cross-sectoral cohesion in terms of curricular content and pedagogy for mathematics is clearly stated in the above FN excerpts. This

indicates a lack of interconnections between the microsystems of preschool and primary school, a characteristic of Bronfenbrenner's (1979) mesosystem, and is a source of professional tension for Sinéad. The 2009 edition of Aistear (NCCA, 2009) asked that children 'develop counting skills' (p. 35), and given that this edition was produced for both preschool and early primary settings, it is no wonder that counting is practised in both of these educational settings. Counting to five is a category on the HSE's Transition Flower document (Appendix A), completed by Sinéad. The children in Sinéad's class participate in counting activities to ten. This is a challenge for ECEs. Young children are interested in numbers and counting (Clements & Sarama, 2021) and if one is to follow the early childhood mantra of 'follow the child', then numbers and learning to count, and exploration of the structural aspects of number, will be experienced in a preschool room.

This highlights the confusion that a curricular framework can produce. A lack of clear guidelines or set expectations can lead to uncertainty as to what learning is expected to take place, something that a set curriculum would potentially solve. Until recently (2023), Aistear (NCCA, 2009) acted as a shared curricular framework across the preschool and infant classes of primary school. The absence of shared cross-sectoral curricular guidance produces ambiguity around pedagogical expectations. Such guidance, a potential product of government agencies in the exosystem, would support ECEs and JI teachers to fully support individual children's mathematical transition from preschool to primary education. I argue that a boundary object[7], such as a set curriculum for mathematics for the 3-6 age range, outlining expectations for learning and responsive pedagogical practices, spanning the ECCE and JI classes, may help to alleviate this uncertainty across both ECE sectors. Alternatively, additional mathematics content to include the full range of the 'Big Ideas of ECME' (see

[7] Boundary Object: 'A *boundary object* is something that people on different sides of a 'boundary' use in varying, yet sufficiently coordinated, ways to enable them to accomplish something together' (Blåsjö & Christensson, 2018, p.85)

Chapter Two) and guidance on appropriate pedagogical strategies to support ECME should be added to the existing EC framework, Aistear, to guide ECEs in supporting children's participation in these.

8.3.2 Sinéad's Perceived Role During Free Play

Sinéad's belief that she should not comment/interact with the children during FP, unless asked to do so by a child, influenced her perception of her role during this period of the daily routine. She acknowledged that she adopted a laid-back role during this time; however, she did want to get involved. This extended to not wishing to take children away from their self-selected activity to do a planned activity, even if the activity was aimed at a specific child and their mathematical interest:

> "I don't like pulling him aside when he's doing an activity, so lately, if I find him not doing something, I've pulled him aside to do maths, but when he's lovin' what he's doing, I'm like, Oh just leave him, he's having fun…so I do try and do that, but if he's busy, then I feel bad for pulling him away from them and he's having so much fun with his activities" (RC1).

This further demonstrates Sinéad's belief that children should be in charge of their activities during FP time. It was not her role to take them away from it, particularly when they appeared to be deeply engaged and enjoying their play. To take them away from this would go against the belief that FP was a time for the children to direct proceedings. This belief that she must not interfere with children's self-selected activity and the philosophy that FP belongs to the children facilitated children's non-engagement with mathematics.

Girls' non-engagement with construction /block play has been discussed in Chapter Seven. Some children, like Audra, preferred to spend most of their time in activities such as colouring in photocopied pictures. This is how Audra preferred to spend most of FP time.

Field observations show that Audra engaged in only one explicitly mathematical activity during FP over the data collection period:

> Audra has taken the small attribute box, and is piling circles of various sizes in front of her – categorising? She then names a circle, a triangle and a square. She asks me what a rectangle is – I tell her. She then makes piles consisting of random shapes turns away from me. I take this as a sign she doesn't want me there anymore and I move table. (FN 7).

Audra could identify and name the basic 2D shapes of circle, triangle and square. She had been absent the previous week when Sinéad and the children had been looking at rectangles during CT. Given findings alluded to in this chapter, I propose that Audra's non-attendance is not the only contributing factor to her not knowing what a rectangle is; rather, it is the result of several factors. Sinéad's beliefs about the primacy of FP, combined with her limited CT routine, have inadvertently resulted in inequity of opportunities for Audra (and other girls) to learn important mathematical content (Wager, 2014), which may have implications for the future. This is particularly relevant given a) the importance afforded to early shape learning in the literature (Hawes et al., 2015; Clements & Sarama, 2021) for later STEM education, and b) the importance of early mathematics learning more generally to long-term academic achievement (Duncan et al., 2007; Watts et al., 2014)

8.3.3 Children's Non-attendance

Sinéad stated that non-attendance had been an issue over the course of the ECCE year and mentioned Audra and suggested this negatively impacted Audra's learning of numbers (FN 4). Aparna was also an irregular attender, attending twice a week on average. Sinéad felt that these were her intended audience during CT; these were the children who needed to learn their numbers. Those who already knew them were not her concern.

However, the content and format of the CT routine remained unchanged regardless of children's attendance or prior mathematical understanding. Consequently, children who attended regularly and were already proficient with numbers up to ten were required to engage in the same repetitive activities, despite their evident readiness for greater challenge. This lack of differentiation constrained the quality of the proximal processes (Bronfenbrenner & Morris, 2006). Over time, the repetitive and undifferentiated nature of these interactions reduced developmental potential for more capable learners, as the absence of novelty and cognitive challenge limited opportunities for sustained engagement, exploration, and conceptual growth (Bustamente et al., 2018; Fleer, 2010; Lundquist et al., 2023). Over time, (chronosystem) the continued enactment of this CT routine across weeks further compounded its cognitively limiting effects, perhaps negatively shaping children's learning by reinforcing predictable and non-cognitively challenging interactions rather than promoting the dynamic, progressively more complex exchanges that foster deep mathematical understanding (Bustamante et al., 2018; Bronfenbrenner & Morris, 2006; Rogoff, 2003; Zhang & Quinn, 2018).

Research suggests that engaging in mathematics in a second language is challenging for children and often results in an achievement gap. (Arizmendi et al., 2021). Clements and Sarama (2021) observe that poor proficiency in the language of instruction hinders children with an additional language from participating in mathematics. Further, they suggest teaching the child in their first language. This was not possible in this setting, with the lead educator, Sinéad, teaching through the medium of English only, and many languages represented in the group. Ten of the 11 children attending the ECCE class were learning English as an additional language. However, the majority were regular attenders; only Audra and Aparna attended infrequently. Findings indicate that the regular attenders had a greater command of English and mathematical vocabulary in English than Audra. Rogoff (1990) positions

language competency as a necessary tool for participating in cultural settings. Relying on direct instruction during CT only was not enough for Audra to acquire the vocabulary and concepts. Sinéad's commitment to non-intervention in FP also facilitated Audra's non-participation in mathematical activity and potential language acquisition. For Audra, the language barrier and mathematical gap were widened as a result of irregular attendance, freedom to opt out of mathematical activity during free play, and a lack of dedicated adult intervention to help support her development.

Doabler et al. (2016) found that a specific mathematics curriculum, with explicit sequenced instruction using concrete manipulatives and targeted vocabulary learning, was of benefit to young EAL learners. This approach could be adopted in this ECCE room, with a commitment to intentionally working with children, either on a one-to-one or group basis, to develop key mathematical vocabulary and concepts through play, during FP time. Having a second adult in the room would support this intervention, with one adult intentionally working with children and the other free to supervise and respond to the needs of the children not involved in the intervention.

8.4 Chapter Summary

This chapter highlighted some of the challenges identified and experienced by Siobhán and Sinéad in their efforts to support young children's engagement in ECME. Other challenges were observed by me and recorded in my field notes. Challenges include structural constraints such as staff recruitment and retention, the length of non-contact time, the fluid timing of Sinéad's break time and duties such as greeting parents, which hinder Sinéad's ability to notice, interpret and enhance children's mathematical experiences and learning during FP time. All of these activities take Sinéad away from potential interactions in the ECCE classroom.

Other challenges identified include a lack of staff confidence and knowledge (particularly those with level 5 and 6 qualifications), how resources in the classroom can be used to support ECME through play, as well as the lack of training in ECME available for ECEs generally. Sinéad's belief that she should not do too much in terms of mathematics limits her practice in some respects, as she refrains from engaging in numbers past ten during CT. Additionally, her focus on numbers one to ten for her irregular attenders during CT may be a result of this belief. This belief is in opposition to her personal belief (and that of Siobhán) that ECEs should follow the child in whichever direction they choose to go.

Temporal constraints such as fluctuating breaktimes and resultant contribution to the rushed feel of the circle time routine were noted in this chapter. Such constraints negatively impacted on time spent attending to children's mathematical observations during circle time. The busyness of the ECCE day in general was also commented upon as it influenced the time Sinéad could give to each child, especially when having to manage several children at once. Other issues, such as children's non-attendance and Sinéad's commitment to supporting these children in learning numbers, were considered as challenges.

Sinéad's belief that she should not interact with the children during FP unless asked to do so, was also noted and presented as a challenge to supporting mathematics during FP time. This chapter has highlighted the need for intentional support for children with EAL who irregularly attend.

Chapter Nine provides a discussion on key findings presented in Chapters Seven to Eight. Implications for future research, policy and practice are made. My contributions to knowledge, along with their limitations, are acknowledged. Recommendations for future research, practice, and policy in relation to facilitating and supporting young children's participation in early mathematics in preschool settings.

CHAPTER 9: DISCUSSION AND CONCLUSION

The Sky's the Limit - Or is It?

This micro-ethnographic study explored how a group of young children in a preschool classroom (microsystem) in Ireland were supported in their participation in early childhood mathematics education (ECME). It also reviewed the development of Irish educational policy in relation to ECME and the ECE sector (Chapter Three). This final chapter reminds the reader of the research questions and the selected methods for answering these before presenting a summary and discussion of the key findings of the study with regard to the research literature and Irish educational policy. The limitations of the study are then detailed. This is followed by reflections on the research process. The chapter concludes by discussing the contribution this study makes to the Irish policy and practice landscape in terms of ECME in early childhood settings. This is followed by suggested recommendations for future research, policy and practice.

9.1 Overview of the Study

This study aimed to explore how children's participation in ECME was supported within the context of preschool education in Ireland. The following sub-questions were devised to aid in answering the main research question:

- What types of mathematics do children participate in?
- In what ways do the adults directly and indirectly guide or support this participation?
- What are the challenges in supporting children's participation in ECME in this setting?

As noted in Chapter One, there is a lack of empirical research exploring how early mathematics education is supported in early childhood settings (Jegodtka et al., 2022; Marks

et al., 2019). Using an ethnographic approach, this study went deeper than previous Irish studies conducted by Neylon (2014) and O'Dwyer (2021). As a participant observer, I observed, documented and analysed practice. I documented the mathematics the children engaged with during their FP and the pedagogical interactions used to support their mathematical learning. Furthermore, I observed and analysed how aspects of mathematics were directly taught during specific adult-directed whole-group time. Relevant setting policies and procedures were also analysed to explore how ECME was supported at the setting level.

9.2 Addressing the Research Questions

In this section, I discuss findings that serve to answer the three research questions. I discuss each in turn. Answers to these questions support our understanding of how young children experience ECME in their preschool class and how the challenges their educators face in facilitating early mathematical learning experiences.

9.2.1 RQ 1: The Types of Mathematics the Children Participated In

Analysis of FNs and other data, such as participant captured video, photographs and reflections on these, revealed that the children engaged with all the Big Ideas of Early Mathematics (Brownell et al., 2014) as well as Bishop's (1988) six universal mathematical activities: counting, locating, measuring, designing, playing and explaining throughout the data collection period. This section begins by discussing children's engagement with number. Consistent with Worthington and van Oers' (2016) findings, children's engagement with number-related activities was most frequently observed in this study. This is then followed by discussions relating to their engagement with measures, shape, space, data, and pattern.

9.2.1.1 Number

Traditionally, there has been a debate about whether young children should be taught number concepts in preschool settings (Brownell et al., 2014; Sarama & Clements, 2009). In this setting, formal teaching of mathematical concepts, such as numeral recognition, counting and number operations, was observed. This intentional teaching of number chiefly occurred during CT (FNs 1-15) (counting and numeral recognition); however, examples of this were also observed during FP (section 7.1.1.5). In response to her noticing of John's interest in number operations, Sinéad intentionally engaged him in exploring basic addition calculations (FN 5, RC2 – reflection on self-captured video). This demonstrates, for John, that Sinéad engaged in a responsive pedagogy of noticing, interpreting, and enhancing John's mathematical interests and activity (Lee, 2017; MacDonald, 2018). As noted in Chapter Three, number operations are not included in the list of mathematical concepts that young children should experience in Irish preschool settings in Aistear (GoI, 2024c); rather, they are included in the mathematics curriculum for the infant classes (DE, 2023a), as well as in the current iteration of the NLNDLS (GoI, 2024a). Sinéad was willing to support John in participating in an activity deemed suitable for a primary school child, willing to follow his interests and in this way following the setting's philosophy. She enabled John to be a mathematician (Lembrer & Meaney, 2015). She had set John a problem to be solved and guided him through the process of solving it. John was an apprentice (Rogoff, 1990), learning how to complete calculations under the guidance of a more experienced other. Providing appropriate equipment in the room (Wager, 2013) to support John's interest in number operations, together with allocating time for one-to-one engagement with Sinéad, facilitated his developing understanding of the cultural mathematical activity of counting (Bishop, 1988), addition, subtraction, and the structural properties of number. Like John, Seán too had an interest in the composition of numbers, demonstrating his understanding that two and two

made four during CT. Furthermore, his actions attracted the interest of Adomas, who imitated his actions and speech (Rogoff, 1990). In this instance, Seán's interest in the structural aspect of number was not responded to by Sinéad during CT, perhaps because she did not recognise the value of this for his learning of number.

FP also offered children an opportunity to experience numbers without adult guidance. As with many children of this age, the children in this setting demonstrated a love of counting (Clements & Sarama, 2021), an interest which was particularly evident in the boys. Findings show that (section 7.1.1.5), when asked to find the combined quantity of combined sets, John could count sets of up to fifteen objects accurately using the 'count all' strategy, which, as Clements and Sarama (2021) assert, is the dominant counting strategy used by most four-year-old children. As presented in the FN excerpts of Priya and the counting cards (section 7.1.1.1), John's playing of the Ladybird game (section 7.1.1.3), and the counting sticks (section 7.1.1.5), there was evidence that some children understood the first four counting principles: one-to-one correspondence, stable order, cardinality, and order irrelevance (Gelman & Gallistell, 1978). Sinéad did not stop the children exploring numbers to 20 through games during FP. This limitation appears to have applied only to direct instruction during circle time (CT) to support children whom Sinéad perceived not to have understood the counting sequence or process. Sinéad specifically identified Audra, an EAL student, for this intentional intervention (RC2). It suggests that children's individual interests were pursued during FP, whereas direct instruction on numbers up to ten was delivered collectively during CT.

Children's emergent understanding of written numerals was also evident. John and Filip were keen to learn how to recognise the teen numbers. Games such as Bingo (section 7.1.1.4) supported Filip's growing interest in numbers and his ability to recognise numbers up to twenty (Moomaw, 2017; Ramani & Eason, 2015). The focus for numbers appeared to be

recognising, writing and counting numbers one to ten. This was Sinéad's goal (RC1), and although the Bingo game supported Filip's interest in the teen numbers, no other structured activity supported children's participation in learning past ten. This despite the obvious interest of both Adomas and Arjaat in counting on past ten during CT (FN 1, FN15). Sinéad expressed a concern about going too far ahead in terms of teaching numbers past ten. This contrasts with Siobhán's perspective, who, in her interview (INT 1), when asked about limiting children's mathematical interests, stated, "No! I wouldn't, and I would be disturbed if I heard any of my staff stopping them at 10". There appears to be a discrepancy between the views of the classroom-based educator and those of the owner-manager. Sinéad did not wish to go past ten during CT, despite the obvious interest in doing so among the children. That said, the majority of the children observed could recognise numerals and count up to ten objects correctly, and some were interested in exploring numbers to ten at a deeper level (e.g., John, Adomas, Seán & Arjaat).

9.2.1.2 Measures

Findings in relation to children's participation in the cultural activity of measuring (Bishop, 1988) were presented in section 7.2. The most commonly observed aspect of measurement during FP time was that of making comparisons. Arjaat, in particular, had a prolonged interest in making comparisons and measurements in general over the four-week fieldwork period. He was particularly interested in making comparisons between his own height and his creations (section 7.2.1.3). He was also interested in making comparisons between the height of his collaborative creations and those of other people, myself included (section 7.2.1.1). This provided a relevant and meaningful context for him to explore this aspect of measure (Papandreou & Tsiouli, 2020).

The boys loved to work as a team, as evidenced by their constructing of the Hulk House, learning from each other (Rogoff, 2003) in joint problem-solving scenarios. Again,

the extended period of FP time of one hour enabled the boys to express their agency, to become deeply immersed in their creations, to identify problems (such as the roof not being high enough to cover the furniture) and to solve them (Pyle & Danniels, 2017). The use of the furniture facilitated rich mathematical discussion and collaborative problem-solving. This finding contrasts with those of Trawick-Smith et al (2017), who contend that children's use of other toys/resources can distract from the mathematical aspect of the block play. I argue that the furniture was positioned to represent the culturally accepted position in a room (Rogoff, 1990), most probably reflecting that of the arrangement of Arjaat's home. Such activity reflects Bishop's (1988) mathematical activity of *Locating,* knowing where things are and how they relate to each other. This positioning represents children's understanding of the use of space within a house, and is therefore a mathematical asset to the resultant play and thinking. Furthermore, the inclusion of the furniture in the play added another layer of complexity to the block building, particularly as the boys had to consider the need to make the walls higher, and the combining of two long, triangular-shaped blocks to make a rectangular shape to fit the space. In this case, the addition of the furniture created a problem to be solved, and this was done collaboratively (Gura, 1992; Rogoff, 1990). Conversely, this study does support other findings of Trawick-Smith et al. (2017) in that social, collaborative block play supported the complexity of this highly mathematical activity, as children jointly discussed and solved problems as they arose (Gura, 1992).

The routine content, songs and questions asked during CT, although well-intentioned, in that they were to regularly expose children to temporal aspects of measure (Bustamente et al., 2018; Haylock & Cockburn, 2017; Zhang & Quinn, 2018), were not fully engaging the children during the study. Perhaps this was the result of having sung the same songs and answered the same questions since September. The loud voices and behaviour, such as rigorous swinging on chairs, would seem to indicate that at this time of year (May), they

needed something new, a new challenge and would have benefited from a new way to present the planned CT content. This finding supports findings from Bustamante et al. (2018) in that CT was of poor quality, offering little to no challenge to the children. The setting's CPOP (P1) suggested that, "children plan their own activities, carry them out and are encouraged and supported to talk about them in 'circle/news time". I asked Sinéad if she had considered using this strategy. She replied that she 'hadn't thought about it' (RC1). Her comment suggests that she was unaware of this strategy and perhaps unfamiliar with strategies within the setting's EYMTLP (P2). Using this strategy would have given a renewed focus to CT, focusing on children's interests, drawing on funds of knowledge (Moll et al., 1992), and providing an opportunity to engage the group in sustained mathematical talk (Klibanoff et al., 2006), enabling Sinéad to model and use mathematical vocabulary relevant to aspects of children's play. This would have been particularly important for children like Audra, who, with EAL and infrequent attendance, would have benefited from intentional exposure to mathematical vocabulary used in context (Doabler et al., 2016; Arizmendi et al., 2021).

Despite the lack of math talk and participation in meaningful mathematical thinking during CT (Bustamante et al., 2018; Lundquist et al., 2023), Sinéad's weight-related activity reflected an ability to engage the children in mathematical thinking. Her pre-modelling (Rogoff, 1990) of relevant language before taking the children outside supported their correct use of it during the weighing session. This session was highly interactive, with children being allowed to choose items for weighing and to predict which item would be heavier or lighter, with Sinéad's questions guiding the thinking process.

A further observation worth considering is that of the boys dominating the Unit Blocks. None of the girls were observed in this area. This is not to say that the girls did not engage in construction play; they did, though with other construction resources rather than

with blocks. The Unit block area was the boys' domain. This finding is in line with the observations of Clements and Sarama (2021).

9.2.1.3 Shape and Space

Section 7.3 described observations of children engaging in learning activities that support the development of an understanding of shape. This study has shown that children were interested in shape, and that these interests were, in some cases, sustained over a period of time (e.g. Chandra, see sections 7.3.1 and 7.3.2). Seeding the learning environment (Wager, 2013) with different examples of shapes, such as circles, is critical for supporting children in identifying them in the environment. Findings also show that children's encounters with shape included both 2D and 3D during FP, but that shape learning was limited to 2D shapes during CT. Shape learning during CT centred around the number of sides of particular shapes and did not consider other properties such as the length of sides, orientation or angle. Shapes were presented in prototypical representations (Clements & Sarama, 2021), with no variants of each shape presented to the group. There was no discussion around comparing one 2D shape and another, or noticing similarities and differences between them. Discussions relating to different orientations of shapes did not take place. Speaking in a US context, Clements and Sarama (2021) claim this is representative of typical shape instruction in preschool settings.

Children's participation with 3D shape exploration occurred during FP, primarily through the use of resources such as construction straws (section 7.3.1.1), circles and connectors (section 7.3.1) or unit blocks (sections 7.2.1.1 and 7.2.1.3). Playing with such resources enables children to engage in the mathematical activities of locating and designing (Bishop, 1988). Research (e.g., Fisher et al., 2013) highlights the value of guided block play for developing early spatial reasoning skills and supporting children's acquisition of related vocabulary (Cohen & Emmons, 2016; Schmitt et al., 2023). Findings from this study show

little to no intervention from Sinéad during block play. She commented only when approached to do so, reflecting her belief that she should not interfere in children's free play (RC2) (McInnes, 2019). When intentional teaching of 3D shape (section 7.3.1.1) did occur with John, Sinéad used Google to identify the 3D shapes and magnetic tiles to make a net for a cube. By selecting these tools, she hoped to engage John at a deeper level; however, John was primarily interested in properties of the shapes, such as stacking and rolling, rather than learning their names or their composition.

Section 7.4. presented findings in relation to children's participation in spatial awareness/reasoning activities. Such activities are reflective of Bishop's (1988) mathematical activity of locating. Spatial awareness or spatial reasoning is an area of ECME research that has had little impact on practice (Verdine et al., 2014) and one where ECEs report a lack of confidence (Bates et al., 2023). Hawes et al. (2015) contend that spatial reasoning underpins other mathematical Big Ideas, such as algebra, measurement and pattern. Despite its importance, it is rarely afforded the level of attention typically given to number concepts in early childhood settings. Aistear (GoI, 2024c) now explicitly includes spatial awareness in its list of mathematical concepts to be explored in preschool settings. Its predecessor, Aistear (NCCA, 2009), did not. Children's participation with spatial concepts derived mainly from playing with construction equipment (Unit Blocks) is well-documented (Brownell et al., 2014; Hawes, 2015; Verdine et al., 2017). Block play offers numerous opportunities for children to represent and organise space, as illustrated by the construction of a "Hulk House" in which specific areas—such as a kitchen—were designated for particular functions. Such experiences contribute broadly to the development of spatial skills (Clements & Sarama, 2021). Research further highlights the significant role of peer interaction in fostering construction abilities, with children engaged in collaborative block play demonstrating more rapid skill development (Gura, 1992; Trawick-Smith et al., 2017). Considering the

acknowledged contribution of block/construction play to the development of spatial skills, the domination of the boys' group in the Unit Block area is concerning. Girls are often identified as a group that does not engage in construction/ block play as frequently as boys, and consequently, their spatial skill development is not as strong as a result (Clements & Sarama, 2021). Observational field notes from this study support this hypothesis. Sinéad's belief that her role during FP is that children are free to choose their own activity (as expressed in RC2), and that she should only interact when asked (McInnes, 2019) contributed to this issue. The girls were free to choose their own activity and could choose not to engage in block play. Sinéad believed that she should not interfere with children's self-selected activity unless asked to do so, nor was she to encourage children to try different activities during this time (RC2). As a result of this belief, the girls were not encouraged to play with the unit blocks. Findings from this study indicate that, at present, a stronger awareness of the significance of supporting all children, especially girls, in participation in spatial reasoning activities in both ECE policy and practice is warranted. This is particularly relevant considering the reported continuous underachievement and discrepancies in achievement by gender in this area in later years of school (McHugh et al., 2024) for Irish children, and the current policy emphasis on numeracy (GoI, 2024a), and growing the future STEM workforce (DES, 2017b).

9.2.1.4 Data Handling

Section 7.5 presented findings in relation to children's participation in data handling activities. Sarama and Clements (2009) assert that there is a paucity of research exploring children's early participation in data handling activities. In this study, incidences of children engaging in data handling activities were infrequently observed during the study. Documented activities stem from children's self-directed activities during free play. Aside from *The Game of Ladybirds* (section 7.1.1.3), there were no documented instances of Sinéad intentionally engaging children in foundational data-handling processes such as matching,

sorting, ordering, or classifying. However, the environment provided many resources to support this type of learning. In her first interview (RC1), Sinéad was asked to capture photographs of resources she had used in the past to engage children in mathematical activities. Sinéad captured one resource to support early understanding of data handling, compare bears: "the teddy bears, which is actually kinda the, it's kind of a colour sorting activity and a number sorting activity, so there's the bowls underneath where they can sort out the colours as well". The coloured bears provide opportunities for children to engage in sorting (by colour or size), matching (by colour or size), and ordering (by size). However, Sinéad did not refer to the latter two skills. Additional resources were also available, including number and shape matching games (e.g., *Dobble*), numerals that could be ordered by size, and stones that could be sorted by weight. Sinéad did not mention these materials during our reflective conversations, nor did she use the term *data handling*; instead, she referred only to the terms included in Aistear (GoI, 2024c) - sorting and matching.

9.2.1.5 Pattern

Section 7.6 presented findings in relation to children's participation in patterning activities. Within the research literature, pattern is widely recognised as a foundational aspect of early mathematical development and a critical predictor of later mathematical achievement (Rittle-Johnson et al., 2019). Despite this well-established evidence base, the most recent revision of *Aistear* (GoI, 2024c) does not include pattern within its list of mathematical concepts for early learning. In contrast, it was included in the previous iteration (NCCA, 2009); however, it was not included in either of the two lists of mathematical concepts provided under the theme of Exploring and Thinking, rather it sat outside this list under the aim Communicating (NCCA, 2009). Gripton (2023) asserts that this lack of prominence is common in curricular documents. If pattern is not explicitly mentioned in curriculum guidance, educators may be less likely to recognise, plan for, or intentionally scaffold (Lee,

2017) children's engagement with this mathematical concept. This is problematic given that the development of pattern awareness requires deliberate and explicit teaching (Gripton, 2023). These findings suggest that pattern should be afforded higher visibility within Aistear (GoI, 2024c) and be included in all pre-service and PD early mathematics programmes for the ECE sector.

The data set shows Sinéad's intention to support children's understanding of pattern. For example, in RC1 (Autophotography), she spoke of deliberately printing off a pattern snake resource (see Figure 7.10) for John. Analysis of FN excerpts (section 7.6) suggests that Sinéad intentionally attempted to promote Arjaat's awareness of pattern by explicitly highlighting it. However, her subsequent shift toward supporting Arjaat's imaginative play - specifically, his interest in inventing names for the snakes - indicates a reversion to a more playful, child-led approach (Sproule et al., 2017). While responsiveness to children's interests is a valued pedagogical stance in early childhood education, this example illustrates a potential missed opportunity for playful, responsive, intentional mathematical teaching (Björklund, 2014). Teaching only when children initiate engagement with a resource and following their self-directed goals may result in fragmented learning experiences, particularly in relation to mathematical concepts that benefit from explicit instruction, such as pattern (Gripton, 2023).

Sinéad's lack of intervention in Seán's activity, despite evidence that he was constructing a pattern, further underscores this tension between child-led exploration and educator-directed mathematical guidance (Skene et al., 2022). Clements and Sarama (2021) contend that perceptual patterns - such as those found in dominoes, finger configurations (see discussion on Seán's 'two and two' finger pattern, sections 7.1.2.1 and 9.2.1.1), cyclical patterns (e.g. days of the week), or number sequences - are frequently overlooked as legitimate mathematical structures in early childhood contexts. The findings from this study

reinforce this observation: Sinéad's conceptualisation of pattern, as reflected in her interview (RC2) responses, was confined largely to visible and replicable forms (e.g., making or copying patterns). This suggests a limited recognition of the more abstract or embedded forms of pattern that children encounter daily, highlighting the need for PD that broadens educators' understandings of mathematical structure and intentional teaching within play-based pedagogies (Grieshaber et al., 2021; MacDonald, 2018).

9.2.2 RQ 2: Strategies Employed to Support Children's Participation in ECME

This section, again situated within the microsystem of the ECE setting, answers the second research question: in what ways do the adults directly or indirectly guide or support children's participation in ECME? It begins by outlining Siobhán's setting-level contributions, including the development of the setting's Early Years Mathematics Teaching and Learning Policy and her support for staff training. The discussion then turns to the microsystem-level strategies implemented by Sinéad, such as the creation of a mathematically rich environment, intentional teaching, and structured guided participation.

9.2.2.1 The Influence of Setting Policy

This section considers Siobhán's interpretation and application of national policy documents, written within the exosytem, to the practice in the microsystem, her setting. This setting is unusual in that the manager has produced an Early Years Mathematics Teaching and Learning Policy. This demonstrated Siobhán's understanding of government policy regarding current ECE policy (e.g. DES, 2011; DES, 2017) regarding the importance of mathematics, her personal belief in the importance of ECME (Dunekacke et al., 2015), and her desire to be a pedagogical leader for ECME in her setting (DES, 2017; GoI, 2021). It showed her commitment to ensuring that each child received a holistic education when attending the setting. With a higher degree in ECE, Siobhán is confident in both her

mathematical and pedagogical content knowledge and noted in her interview (INT1) that she played a key role in 'giving confidence and support' to staff to support ECME in their practice.

However, findings indicate that Sinéad was unaware of several key procedures outlined in the EYMTLP, including the use of *Math Talk* and the discussion of children's mathematical activity during FP or CT. Although Siobhán attempted to influence pedagogical practice through the development of a written policy, the absence of formal training (as stipulated in the policy) and the lack of systematic monitoring limited the policy's practical impact on educators' implementation of ECME. Consequently, the existence of the policy alone was insufficient to ensure consistent and high-quality mathematical engagement across daily routines.

In her interview (INT1), Siobhán stated that she reviewed the mathematical resources in each room a couple of times a year, as she was in charge of sourcing resources for each of the rooms. However, she also alluded to the fact that she had handed over the daily management of the setting to two assistant managers, and that both managers found it hard to review and monitor practice in the room due to time constraints. The lack of time for managers to observe and monitor practice affects the implementation of setting policy, and consequently, the desired level of practice. Such observation of practice reflects a supportive and collaborative team approach to mentoring for professional learning. Such procedures can not only positively impact professional practice but can also contribute to levels of educator self-efficacy and confidence (Longley & Gilken, 2020) in supporting young children in mathematics.

Using the lens of Bronfenbrenner's Bioecological Systems Theory (Bronfenbrenner & Morris, 2006), this situation illustrates how interactions across multiple ecological systems

influence children's learning experiences. The EYMTLP represents an influence within the *macrosystem* - reflecting the broader policy environment and consequently the institutional ethos - while Siobhán's leadership and mentoring intentions operate within the *exosystem* and *mesosystem*, where administrative decisions and staff interactions shape practice at the *microsystem* level of the classroom. However, when mechanisms for communication, training, and monitoring are constrained—as Siobhán noted due to time limitations among assistant managers—the intended proximal processes that support children's mathematical learning are also limited.

9.2.2.2 Professional Development Support

This set of findings predominantly relates to EC policy development in the exosystem. It highlights that policy, while often well-intentioned, can fail to meet objectives set. This indicates that goals articulated within the exosystem cannot be achieved without careful action at the meso and micro levels. The current EC workforce development plan, *Nurturing Skills,* states that, 'managers should: Facilitate and support pre-service and in-service education and training of ELC practitioners' (GoI, 2021, p.133). The EYMTP explicitly states that 'training and development will be offered for ECEs if there are concerns or particular interests in early years mathematics'. In her interview, Siobhán stated that the setting would fund mandatory training (First Aid, Child Protection, and Manual Handling) and would fund any further training staff were interested in. However, she noted that staff often had to give up personal time to attend training, and this can be a barrier to engaging with non-mandatory training, and that staff need to be really interested in a course to commit to it. Siobhán further asserted that in her experience, staff prefer online training that they can do in their own time rather than completing face-to-face sessions. Siobhán further noted the current lack of mathematics-specific training opportunities available for ECEs. The NLDS (DES, 2011) stated that intended support for ECEs in ECME would include mathematics

educational content on all undergraduate degree programmes, as well the development of PD materials such as handbooks and online courses. Some PD materials, such as videos, are available on the Nurturing Skills website; however, to date, despite renewed calls for this in a subsequent review of the 2011 strategy (DES, 2017) and in later iterations of the strategy (GoI, 2024a & b), no state-funded formal PD course exists for ECME in the ECE sector. The remaining sections present and discuss the observed strategies used by Sinéad to support and guide children's participation in ECME in the ECCE classroom microsystem.

9.2.3 Adult Planned Strategies for Supporting Children's Participation in ECME

9.2.3.1 Provision of a Mathematically Rich Environment

The microsystem of the ECCE classroom can be identified as a social context for learning (Rogoff, 1990). The small room and small number of children provided a space for children to observe the activity of others, and to learn from this observation (Field notes & RC2). Free play supported this learning process; it provided a space for children to try things out, to get things wrong, and to work collaboratively on play projects, learning from each other as they did so (Gura, 1992; Rogoff, 1990). This was particularly evident when Arjaat copied Adomas in retrieving unit blocks to support his structure. Knowing that adult support was available as required was also significant for some children. For example, John in particular sought Sinéad's support regularly, and Sinéad served as a reference point for other children, as in the case of Arjaat and his measuring activity.

Resources such as commercial and teacher-made number games motivated children's participation in number activities (Moomaw, 2017: Rogoff, 1990). Advocated in the literature as a key strategy for developing early number sense (Moomaw, 2017), games have the potential to support developing social (Balladares et al., 2024) and mathematical skills (Ramani & Eason, 2015). Despite these positive outcomes, research suggests that games are

not generally a feature of preschool practice (Pyle & Danniels, 2017). Findings from this study suggest the opposite. The environment was well-seeded with a variety of commercial and teacher-made games, which children regularly chose and participated in. Moreover, findings indicated that the children enjoyed playing these games, either alone or with a small group. They were happy to learn in this way. It was fun. It was clearly evident that these games sparked joy for the children, whether they played alone or with others. From Priya's simple 'yes' when she had completed all of the counting cards, to the smiles when she finished *Catch and Count* with the emotional support of her classmates, to John's delight in tricking Sinéad at the end of the ladybird game, and his utter delight in engaging with number operations, to the collaborative joy when playing Bingo. Cohrssen and Niklas' (2019) study noted the efficacy of games for enabling children with EAL to participate in mathematics. Findings presented here show that Audra was keen to participate in the Bingo game with Sinéad and the other children. Her participation was facilitated by Sinéad through responsive assistance (Rogoff, 2003), giving prompts and calling out the numbers in different ways to enable children with other knowledge to take part, exposing her to mathematical vocabulary associated with number (Doabler et al., 2016).

Gripton (2023) advocates for a pattern-rich environment, and as noted in Chapter Three, in mathematical terms, pattern is understated in Aistear (GoI, 2024c). Observational field notes from this study show, that the learning environment, while well-resourced, was predominantly number-focused with opportunities for counting, numeral recognition and number operations. These observations are supported by Sinéad's presentation of resources (self-captured, RC1), which mainly focused on the learning of number and counting. Such findings align with those of Lee and Ginsburg (2009). Fewer resources overtly targeted children's patterning awareness and skills. Those that were available focused on dot patterns (dominoes) and the making of patterns (beads, compare bears, pattern snakes). At some level,

Sinéad understood the importance of children's participation in pattern activities as she had moved threading activities to a more prominent position to encourage children's participation in pattern-related activity (RC1), even if children did not participate in such activity regularly. This suggests that Sinéad had observed children's lack of engagement with some resources and planned an intervention to encourage participation with these, as in the case of the beads.

Resources to support early data handling skills (matching, sorting, ordering, and classifying) were present in the classroom. This finding is supported by data generated from RC1 (where Sinéad spoke about the small plastic bears) and field notes. Provision of loose part material (see Figure 7.8) supported the making of sets. The shelving layout was particularly powerful for supporting children's engagement in data handling processes of sorting and classification, with each resource having its own container and allocated space on the shelf (Montague-Smith & Price, 2018). Many of the shelves had pictures of individual resources to show children where these resources belonged, helping the tidying-up process and maintenance of the learning environment.

While it could be argued that the children in this setting are very well-supported in their exposure to a rich and varied mathematical environment, environments alone do not facilitate the teaching and learning of mathematics in early childhood settings (Moomaw, 2011; Trawick-Smith et al., 2016; Clements & Sarama, 2021). The adult plays a major role in supporting children's participation with mathematical ideas and concepts (Björklund, 2014; Klibanoff et al., 2006; MacDonald, 2018; Weisberg et al., 2016) through the processes of noticing, responding and enhancing (Lee, 2017).

9.2.3.2 Worksheets

There is a paucity of research in relation to the use of worksheets in EC settings. This study contributes to the small body of research on the use of worksheets in ECME, particularly in the Irish context. Worksheets have been identified as a part of preschool practice (Moloney, 2011), and are generally perceived as contentious in Irish early childhood circles. Conceived as premature formal teaching of mathematics and as a means of adults imposing mathematics on children (Ransom & Manning, 2013), worksheets are often considered more appropriate for primary school. Their effectiveness in terms of providing challenging learning experiences is debated in the literature (e.g., Ransom & Manning, 2013), and they are often perceived as an artefact of schoolification (Barnett & Jung, 2024) in early childhood contexts.

Rogoff (1990) asserts that adults often engage in a form of structured, guided participation that prepares children for school. It could be argued that by providing the worksheets, Sinéad was involved in such a process. However, I assert that Sinéad chose to do the worksheet solely as a means to consolidate children's learning of the number of the week, as a way to foster children's mathematical learning and development. The Friday worksheet had become part of the classroom culture. Field notes show that the children were not averse to completing the work at hand; no child objected to completing the work. In fact, completing the required tasks sparked joy and pride (FN 3, FN12). Furthermore, in RC2, Sinéad stated that she perceived worksheets as a strategy to consolidate and enhance number learning, particularly for John and had printed some out specifically for him to complete. I argue that when children readily participate in an activity and perceive it as interesting and challenging to them, it can be conceptualised as being developmentally appropriate, and not necessarily an indicator of schoolification or academic pushdown.

Sinéad planned to collect the completed worksheets each week with the intention of making a book to send home after children had completed a worksheet on number ten. However, Seán's enthusiasm to share his work with his mother prompted the introduction of a weekly routine in which worksheets were placed in children's bags—demonstrating how a child's agency can influence established practices (FN3). Some participants in Moloney's (2011) study suggested that worksheets proved to parents that learning is happening in the preschool settings. Aistear (GoI, 2024c) recommends documenting learning, for example, through learning stories, portfolios, photographs, and samples of children's work. In this study, Sinéad used the worksheets to share children's work on number with parents.

9.2.3.3 Structured Guided Participation

Evidence of structured guided participation (Rogoff, 2003) occurred when Sinéad engaged John in the number operations activity with the counting sticks. This activity was planned in response to observed interest and the identification of John as being ready for such an activity (FN1; FN3; RC 2). In this activity, Sinéad modelled how to combine numbers and where to place the symbols and sticks (FN 5, RC2, video). She allowed John to place the sticks on either side of the plus symbol, gradually allowing him to take responsibility for carrying out the activity. Sinéad consistently offered praise, motivating John's participation (Rogoff, 2003). Through this repeated activity, she bridged John's prior knowledge of numbers and their structure (Rogoff, 1990), enabling him to develop his knowledge and understanding. Findings show that Sinéad only did this for John. This has implications for equitable practice. Why was John's particular interest singled out? It could be that Sinéad chose to work with John, a child she deemed mathematically capable, to impress me, to ensure I captured his ability and interest. However, Seán and Adomas would also have benefited from such an activity, given their interest in the composition of numbers (FN 1; FN

15; RC1). Other children, such as Audra, would have gained from such intentional, focused attention, given her needs for extra support (Doabler et al., 2016)

Other examples of structured guided participation were evident in whole-group activities, such as those involving weight and capacity. For the weight activity, Sinéad ensured children's participation by modelling the required language before going outside, and then asking questions to develop the children's prediction skills (FN 15). During the focused capacity activity (FN 11), the processes of observation and imitation (Rogoff, 1990) were in evidence when Sinéad modelled the level to fill the jars with water, while also modelling the capacity-related language. In RC1 (autophotography), Sinéad spoke about the usefulness of the graduated jug in supporting children's learning of the vocabulary of capacity - full and half full. Sinéad's level of PCK (Lee, 2017) for these activities is high; she is familiar with the associated mathematical language and models it for the children. Further activities, such as having sand and water freely available daily, would support children in imitating (Rogoff, 1990) and using this vocabulary in everyday play activities.

9.2.3.4. Mathematical Talk

The importance of math talk (MT) is highlighted within the research literature (O'Neill et al., 2022; Dooley et al., 2014; Klibanhof et al., 2006), and in policy documents such as the Literacy, Numeracy and Digital Literacy Strategy (GoI, 2024a), and in the PMC (DE, 2023a) as a key pedagogical practice for ECME. It is not referred to in Aistear (GoI, 2024c). Clements and Sarama (2021) assert that 'the more teachers talk about math, the more their children develop math knowledge' (p.363). Siobhán was aware of the importance of math talk and had included it as a pedagogical strategy in the EYMTLP. However, observations showed little to no valuable math talk during adult-child interactions in the ECCE classroom. Examples of data where children offered ideas or knowledge (e.g., Seán and the composition of four) show that these were affirmed, but not responded to or enhanced

through discussion. This was the case during CT, which was often rushed and where the routine followed the same pattern every day.

Fieldnotes show that CT was characterised by a flow of closed questions, which took the same format every day. CT was a space where children were to listen and answer the same lower-order questions, such as What day is it today? What month are we in? What number are we saying hello to? In relation to the *Shape of the Week,* questions such as What shape is this? How many sides does it have? were the only questions asked about individual shapes. In line with findings from Bustamante et al.'s (2018) study, open-ended questions to stimulate thinking were largely absent. As alluded to above, there were signs of the children becoming restless during CT. This could be interpreted as the children signalling their need for a challenge during CT, something which the research states is often lacking in CT (Bustamante et al., 2018; Lundquist et al., 2023). Indeed, Sinéad created a situation similar to that she sought to avoid, in that children would misbehave as a result of knowing too much when entering primary school. In this instance, the children were acting out due to a lack of challenge and stimulation. I argue that being open to acknowledging and enhancing children's mathematical observations (as noted in the setting policy), adapting the CT routine by engaging in math talk (Cohrssen et al., 2014; Dunphy et al., 2014; Klibanoff et al., 2006), and asking open-ended questions would have challenged and extended children's thinking and perhaps gained more interest in CT content. Perhaps, targeted PD to support ECEs in the recommended playful, guided strategies to support, as noted in policy documents (DES, 2011, Des, 2017, GoI, 2024b) would assist ECEs in providing richer, deeper and more meaningful mathematical learning experiences as advocated in the literature (Clements & Sarama, 2021; Dooley et al., 2014; O'Neill et al., 2022).

Surprisingly, little sustained math talk between Sinéad and the children was observed during FP. Comments recorded were affirming in nature, 'good job ', or confirming in nature,

'yes, it is getting bigger'. There was evidence in the FNs for Sinéad to potentially actively participate in children's play. This was particularly evident during block play. Adult participation in these activities is effective in promoting children's levels of spatially related vocabulary (Ferrara et al., 2011; Weisberg et al., 2013). There was some evidence of Sinéad prompting children to examine their work mathematically, for example, when she asked John to look at the 2D shapes within his 3D construction.

However, FNs document that Sinéad was largely engaged in menial tasks, such as cleaning or administrative tasks, when a child brought something to her attention. During FP, children approached her and not the other way around. This finding is important as the literature highlights the importance of an active adult role (Fleer, 2019; Rogoff, 1990; Wood, 2022) and that "the frequency of mathematical interactions has an influence on the development of children's mathematical competence" (Jegodtka et al., 2022, p. 208). The importance of recognising mathematical moments and responding to them is of high significance (Dockett & Goff, 2013; Gasteiger & Benz, 2018; Jegodtka et al., 2022).

9.2.3.5. Noticing as Equitable Practice

As noted in Chapter Four, equitable educator noticing requires educators to consider the 'socio-political processes that privilege some forms of mathematical activity over others and position some students as mathematically capable and others incapable' (Hand, 2012, p.242). In this study, Sinéad identified John as a child with strong mathematical interests and abilities, and one who was 'ready for the next step' (FN 3; RC2). Sinéad responded to and attended to his learning needs. Other children also had strong mathematical interests and learning needs, e.g., Adomas counting to 100, Arjaat counting from 100 (FN15), and Seán in the structural aspects of four (FN1). However, although these interests and learning needs had been noticed and, on occasion, affirmed (FN1; FN15), they were not enhanced by Sinéad. Sinéad had noticed that, due to her poor attendance, Audra too required support. Sinéad's

strategy was to repeat circle time content to develop Audra's knowledge of numbers one to ten (RC2), rather than attend to her needs on a more personal level. These findings support those of Lunquist et al. (2023), where participants focused on supporting children in need of support during CT. Wager (2014) states that such non-equitable noticing and responding practices can negatively impact children's developing identities as mathematicians and their self-perceived capabilities of being mathematical.

Louie (2018) contends that noticing is inherently subjective. In preschool classrooms, it is the adult who decides what is worth observing and what warrants further pedagogical attention. From this perspective, Sinéad's belief that she should not intervene in children's freely chosen activities during free play - unless explicitly invited - may inadvertently limit some children's engagement in mathematical learning opportunities. This is particularly evident in relation to the girls' lack of participation in block play. Although Sinéad observed that certain children chose the same activity each day, her view that this preference constituted a child's right prevented her from encouraging exploration of alternative activities. The issue of girls' non-engagement with block play and its implications for spatial development has been discussed earlier in this chapter.

9.2.3.6. Intentional Teaching

Sinéad worked within the philosophy outlined in setting policies, and that of predominant EC culture, that a balance of child- and adult-led learning activities takes place over the course of the ECCE day (Pyle & Danniels, 2017; Zosh et al., 2018). Findings show that intentional teaching of mathematical concepts mainly took place during adult-led parts of the routine: CT and WGT. Sinéad held a belief that she should not interfere with children's self-selected activities during FP; therefore, few examples of intentionality were observed during this time (e.g. FN5; RC2- video). Sinéad's lack of intentional teaching during FP and her belief that she should not interfere in children's activities are problematic. This is a space

that could be used to intentionally engage children who were not as knowledgeable in mathematics, such as Audra, in playful mathematical activity (Doabler et al., 2016). Intentional, targeted activity has been proven beneficial to vocabulary development, particularly with EAL children (Arizmendi et al., 2021; Doabler et al., 2016; Clements & Sarama, 2021). There is also a case for intentionally taking small groups of children to develop skills, such as patterning (Gripton, 2023), a mathematical concept that Sinéad noticed regular non-participation in.

Bustamante et al. (2018) identify CT as a space for ECEs to engage in intentional teaching. As discussed earlier, Sinéad's input during CT, although intentional, was not challenging and certainly did not challenge the more able children in any way. This reflects Lundquist et al.'s (2023) findings. Sinéad's statement that she intentionally focused on supporting the weaker children during CT further supports findings by Lundquist et al. (2023); however, there was little evidence to suggest that it was beneficial for lower achievers who may have required further intentional hands-on activities to support knowledge and concept development, such as focused small group work (Doabler et al., 2016).

9.3 RQ3: Challenges Identified in Supporting Children's Participation in ECME

Previous sections have considered research questions 1 and 2, which were situated within the context of the microsystem of the ECCE classroom. This next section considers findings in relation to RQ 3. Findings discussed here chiefly relate to issues located within the broader ECE policy landscape of the macrosystem. Chapter Eight highlighted some of the challenges identified and experienced by Siobhán and Sinéad in their efforts to support young children's engagement in ECME. The first challenge to be considered is staff recruitment and retention. The Irish ECE sector is currently experiencing high levels of attrition, with many settings struggling to fill staff vacancies. Siobhán (INT1) noted that graduates do not

always apply for advertised positions, suggesting that graduates often seek employment outside of the ECE sector. This opinion reflects findings from Moloney and Pope's (2013) study, where ECE graduates often leave the sector to pursue careers elsewhere. Indeed, Sinéad left the sector the year following data collection for this study. Staff recruitment and retention remain a critical issue. This is reflective of wider issues within the broader government policy system, which directly impact ECE practice and young children's access to high-quality education. Moloney and Pope (2013) note that many ECE graduates seek further training in the Primary education sector, where pay and conditions are better. Similar sentiments are echoed by *Early Childhood Ireland (ECI)*, which further notes that staff turnover within the ECE sector now stands at a rate of 24% per annum (ECI, 2024). Furthermore, ECI (2024) asserts that the ECE system will require a further 20,000 ECE staff to educate and care for young children by 2028. Therefore, action must be taken to secure graduate positions in the ECE sector. Continued investments in ECE qualifications and measures such as pay parity for ECE graduates with their primary education counterparts may help stem the movement of graduates to the primary sector. Moreover, this action is critical if the objectives set out in *Nurturing Skills* (GoI, 2021) - to increase the number of lead educators and graduate positions in ECE settings - are to be fulfilled.

In-house structural constraints can be directly addressed by setting management, subject to appropriate funding and ratios. These include staff non-contact time and 'door time'. Sinéad's allocated non-contact time was insufficient to attend to all of her preparatory and administrative tasks. Findings show that she was often engaged in preparing resources for WGT activities, cleaning resources, or completing observations on the app on the class tablet, during FP. This activity could also be interpreted as Sinéad feeling that children did not require her help at this time of year, when they were familiar with routine and resources. She could use this time to catch up with other tasks.

To allow classroom-based staff to be continually present in their classrooms, sufficient funding must be made available to release staff from non-teaching duties such as 'door time'. Temporal constraints such as fluctuating breaktimes and their regular contribution to the rushed feel of the CT routine were noted in Chapter Eight. Such constraints negatively impacted on time spent attending to children's mathematical observations during CT. The fluid timing of Sinéad's break, while a direct consequence of the lack of a cook, leaving the assistant managers to engage in non-managerial tasks of preparing meals, could be solved by employing a level 6 'floater' to cover staff breaks, or by giving Sinéad a full 45-minute break between sessions, rather than splitting her allocated break allowance into two. This would enable Sinéad to be present in the room for the duration of the ECCE session.

These structural and temporal constraints negatively affected Sinéad's capacity to engage meaningfully in interactions and processes, such as noticing, interpreting and enhancing (Lee, 2017) that support children's emergent mathematical understanding. From a bioecological standpoint, such disruptions to the proximal processes - the reciprocal interactions between educator and child that drive development - may diminish the quality and frequency of learning opportunities (Bronfenbrenner & Morris, 2006). The overall busyness of the ECCE day also constrained the depth of individualised attention Sinéad could offer, particularly when managing multiple children simultaneously. Findings suggest that the presence of an additional adult within the classroom could enhance the microsystem's capacity to foster rich, sustained learning interactions, consistent with socio-cultural theories of learning (Rogoff, 1990; Vygotsky, 1978).

Further challenges emerged at the macrosystem level, where broader issues such as workforce qualifications (Gasteiger & Benz, 2018), PD (O'Neill et al., 2023), and confidence in ECME (Blömeke et al., 2017) influence practice. Siobhán (INT1) highlighted the limited mathematical confidence and pedagogical knowledge among staff holding level 5 and 6

qualifications, a challenge echoed in the literature (Gasteiger & Benz, 2018; Jenßen, 2022). Such issues are frequently linked to educators' prior negative experiences with mathematics and to gaps in pre-service training (Lee & Ginsburg, 2007). Empirical evidence indicates that targeted professional development can positively transform these beliefs and practices (Johnston & Bull, 2021). Given the ongoing challenges in recruiting and retaining graduate educators (ECI, 2024) and the basic requirement for lead educators to hold a level 6 qualification (GoI, 2021), it is imperative that ECME training be mandated in all level 6 and pre-service early childhood education programmes. Policy-level interventions, combined with the appropriate financial and structural supports at the macrosystem, therefore, have the potential to cascade downwards, shaping the mesosystemic structures and microsystemic interactions that underpin children's learning experiences.

Sinéad's personal pedagogical beliefs appeared to significantly shape her interactions with children and the pedagogical opportunities afforded within her ECCE classroom (Chen et al., 2014; Platas, 2015), a key context for the occurrence of proximal processes (Bronfenbrenner & Morris, 2006). Within the *microsystem* of her ECCE classroom environment, her conviction that she should not interfere with children's play had direct implications for engaging in proximal processes (Bronfenbrenner & Morris, 2006) with the children in the room. Sinéad believed that during FP, her role was primarily observational; she did not plan structured activities, nor did she typically initiate interactions with children. While she responded when approached, she rarely engaged in sustained dialogue or co-constructed meaning intentionally. In doing so, opportunities for social learning through shared problem-solving and scaffolding (Rogoff, 1990; Vygotsky, 1978) were limited. Sinéad's description of her role as that of a "facilitator of a rich environment" reflects a belief that learning occurs autonomously when the environment is sufficiently stimulating. This is a recurrent theme in the research literature (Fisher et al., 2010; Clements & Sarama, 2021;

McCray & Chen, 2011; Pollitt et al., 2020). This interpretation aligns with aspects of the Montessori philosophy, in which the adult serves as a careful observer and curator of a prepared environment rather than an active instructor (Montessori, 1915/1997).

Viewed through a bioecological lens, these beliefs appear to have been shaped within Sinéad's *mesosystem*—the interconnections between her professional preparation, prior experiences, and current work context. Her college practicum and previous exposure to Montessori practice likely reinforced a conception of the educator as a non-interventionist guide. While it is not possible to generalise from this study, this type of practice is not out of line with my own experience of preschools and not unexpected given Sinéad's background. These formative experiences, situated within her *chronosystem* (Tudge et al., 2009) - that is, the period of time spanning her educational experiences—contributed to the development of a pedagogical practice that privileges observation over intervention. However, as Pollit et al. (2020) contend, such beliefs sit in tension with current *macrosystem* discourses in early childhood education, which emphasise sociocultural and relational approaches where intentional teaching and sustained interactions are fundamental to children's learning.

This tension is also evident in Sinéad's approach to mathematics. Her belief that she should "not do too much" in this area was reflected in her restraint from engaging with numbers beyond ten during circle time. This finding aligns with those of my previous study (Gillic, 2021a). On one level, this may demonstrate sensitivity to individual differences, particularly in relation to children who attend irregularly. However, it also suggests a personal concern that introducing more advanced mathematical content might cause behavioural difficulties in primary school (Gillic, 2021a). This reasoning illustrates how educators' beliefs are shaped not solely by immediate practice contexts but by broader cultural and institutional scripts in the exosystem (Bronfenbrenner, 1979) that inform what is

perceived as ‘developmentally appropriate practice’ concerning ECME (Lee & Ginsburg, 2007, 2009).

9.3.1 Transition to School

The implications of Sinéad’s experiences and beliefs discussed above influenced Sinéad’s inaction, choosing not to respond to or enhance children’s mathematical knowledge or thinking, and the fear of doing too much. This shows, perhaps, that Sinéad is limiting her program either out of her care for the children or from a lack of direction in terms of curricular expectations. Sinéad’s comments do not reflect the thinking of Siobhán, however. In her interview, when asked what her beliefs towards ECME in preschool were, Siobhán stated that ‘the sky’s the limit. Get them doing maths as far as they can take it…I would be absolutely talking to them about all maths concepts that are relevant to the play that’s going on’. Aistear (GoI, 2024c) outlines a list of mathematical concepts to be explored and experienced; however, no guidelines exist as to how to do this or what potential limits are. The waters are thus muddied, leading to frustration and a lack of understanding in terms of mathematical learning for preschool settings. Limiting children’s mathematical experiences for fear of future behavioural issues goes against the rights-based mathematics education advocated by Cohrssen and Page (2016), and may negatively impact children’s interest in mathematics and perceptions of themselves as mathematicians.

Sinéad’s comments regarding children entering primary school with too much knowledge are echoed in the Children’s School Lives Report on children’s transition to school (Sloan et al., 2022), where some primary teacher participants stated that children were entering school ‘being too numerate’, and that they would prefer that the preschool programmes focus on personal, social and emotional development, and independence rather than on literacy or mathematics. Sinéad felt that the children in her care were well-prepared

for the JI curriculum. These findings suggest the need for further conversations with both sides of the ECE sector (preschool and primary) as to what is expected in each phase and to consider the possibility of developing a guiding curricular document for the 3-6 age range.

9.4 Connections to the Policy Context

Findings from my exploration of the microsystem of this ECE setting should be interpreted in light of, and have the capacity to inform policy developed at macro and exosytem levels. As noted in Chapter Two, Bishop (1988) asserts that mathematics education experienced in any given society is determined by local cultural and political understandings of mathematics and the societal value attributed to it.

As documented in Chapter Three, before the establishment of the preschool sector, young children encountered ECME in primary school, where primary teachers were trained in mathematical content and in child-centred, inquiry-based teaching practices (Murphy, 2006). While Aistear (NCCA, 2009) continued to advocate for a child-centred approach to pedagogy for preschool settings and set out a list of mathematical concepts that young children should experience, those working in the ECE sector were not afforded training in ECME despite repeated specific actions in successive Literacy and Numeracy strategies and implementation plans (DES, 2011; DES, 2017; DE, 2023) calling for a highly trained workforce. This lack of coherent government policy and strategy has left ECEs without the necessary MCK and PCK to support children's high-quality participation in ECME in ECE settings. The impact of this limited policy delivery is felt directly in the microsystem and was observed and experienced by Siobhán, the setting manager, where in her interview, she stated that not all ECE degrees contain a mathematics component (Gillic et al., 2026, under review) and that ECEs with level 5 and 6 qualifications in particular lack the mathematical content knowledge to note the

mathematical aspects of children's play (Dockett & Goff, 2013) and in classroom resources. This finding is concerning as ECEs with level 6 qualifications are entitled to hold the position of Lead Educator, responsible for leading pedagogy in an ECCE classroom (DECDYI, 2021).

In the context of limited sectoral training, explicitly identifying mathematical concepts in Aistear (GoI, 2024c) appears particularly important to guide all ECEs in their work in supporting ECME in preschool; however, without the supports such as the promised guides (handbooks/online courses) to support ECME (DES, 2017; GoI, 2024a; GoI, 2024b), the stated aim of ensuring a highly-trained workforce capable of supporting young children's participation in high-quality ECME remains elusive. This lack of clarity regarding which mathematical concepts are appropriate for ECE settings impacted aspects of Sinéad's practice. Findings show that Sinéad was conflicted. She engaged John in number operations (calculations) with numbers up to 15 (FN5 & RC2 on video); she confessed to feeling uneasy engaging him in mathematical content (RC2) deemed more appropriate for children in Junior or Senior infants. In addition, the framework for DE inspectors calls for members of the inspectorate to seek evidence of children's understanding and use of mathematical symbols. This could be interpreted as children understanding numerical symbols as well as '+' and '=', as in the case of John (FN 5/RC 2 on video); however, with no reference to number operations in Aistear (GoI, 2024c), upon which the inspection tool is based (DE, 2025), this is subject to personal interpretation by individual inspectors. This finding suggests the need for a more cohesive approach to policy from strategy, to implementation, to the development of inspectorate guidelines and curricular development than is currently evident.

A further observation is the varying lists of mathematical concepts deemed appropriate for young children to experience in ECE settings noted in various policy

documents. For example, the NLNDLS (GoI, 2024a) included 'doing calculations…creating patterns…time' (p.31) for ECME in preschool settings. These mathematical concepts and processes are included in the PMC (DE, 2023a) but are either absent or muted in Aistear (GoI, 2024c). This incoherent approach can confuse ECEs as to which mathematical concepts are appropriate for the children in their care and lead to a limited range of mathematical concepts experienced by children in preschool settings. In this study, as noted in other studies (e.g. Lee & Ginsburg, 2009; Trawick-Smith et al., 2016), the children largely engaged in activities supporting early number, measure and 2D shape development, with data, spatial reasoning and pattern experiences to a lesser extent (Gripton, 2023). This research indicates the need for a more coherent and connected policy landscape, which produces a clear message that a broader ECME curriculum (as suggested in the NLNDLS (GoI, 2024a) is required to fully fulfil the aims of the strategy to ensure that numeracy skills are nurtured and enhanced from birth.

9.5 Limitations of the Study

The justification for my choice of micro-ethnography has been noted in Chapter Five. Several limitations were identified with this micro-ethnographic research design. Limitations of generalisability, reliability, and trustworthiness are discussed in Chapter Five. Cohen et al. (2018) maintain that ethnographic studies are strong in reality, but are not generally replicable as they are bound by time, location, and setting characteristics. Ethnographic studies are also high in researcher bias/subjectivity. This is a characteristic of ethnographic FN, where participant observation can lead to selective observation and documentation on the part of the researcher. FNs are subjective; what is captured is driven by researcher-selected topics, questions and what the researcher deems important to answer the research questions. To answer my research questions, I chose to focus on adult strategies that support or limit children's participation in ECME. This heightened focus may have led me to unconsciously

seek evidence that supported my pre-existing ideas or hypotheses, while overlooking or giving less consideration to evidence that contradicted them. As noted in Chapter Five, I maintained a reflective journal in which I engaged in reflexive analysis of the literature reviewed, the methods employed, the data collection instruments used, the analytical procedures undertaken, and my interactions with participants. University supervisors reviewed the research as it evolved, checking if the case was over- or understated by me. The verification of the data and its analysis was further supported through the use of multiple data collection methods, which facilitated effective triangulation.

As noted in Chapter Five, I deliberated whether to conduct the study over the period of a year (attending the setting one day a week) or over a shorter period of one month (attending five days a week across the month). However, had I chosen to attend one day a week over the course of the year, I may have witnessed Sinéad's ways of working over an extended period of time and gleaned an insight as to how certain practices were enacted over the course of the preschool year (e.g. CT). This alternative approach may also have offered an understanding of how children's play choices, play routines and mathematical learning were supported over time. A further consideration in relation to the timing of the study is that the study took place in May, a time when children had covered much already. This may explain why interactions during FP were not as frequent as children were familiar with the routine and materials, so Sinéad's support was not as necessary as it may have been earlier in the year.

My professional experience as a preschool teacher must also be acknowledged. As noted in Chapter Five, I regarded this experience as a methodological strength, particularly in my role as a participant observer. It can also, however, be perceived as a limitation of the study. It should be acknowledged that at times there were occasions where the thin line between educator, teacher and researcher may have been crossed. This is evident in the field

notes where I interacted with children, during block play, or during counting activities, perhaps limiting the free flow of children's emerging ideas and thinking. In such instances, my teacher instinct kicked in, and my immediate response was to support the children in their endeavours. I endeavoured to keep my personal and professional biases in check by keeping a research journal.

Another limitation stemming from my active engagement with the children as both researcher and participant was the challenge of simultaneously observing and documenting their activities and taking part in them. To minimise data loss, I documented the episode in as much detail as possible as soon as I could after the event, taking time away to write if necessary. Inevitably, however, in some instances, some data, particularly examples of children's language, may not have been documented verbatim or may not have been included in my field notes.

9.6 Reflections on the Research Process

The research journey was characterised by ongoing reflection and adaptation. While the study began with a clearly articulated research question and anticipated theoretical and analytical frameworks, the complexity of the research context necessitated a more flexible and iterative approach. Over time, a range of analytical tools, including the ACT framework (Fraivillig et al. 1999), the Knowledge Quartet (Rowland et al., 2009), and Situational Analysis (Clarke et al., 2018), as well as theoretical perspectives such as Foucauldian lenses, were explored and set aside. This process deepened my understanding of research as a non-linear endeavour, requiring reflexivity, openness to uncertainty, and careful consideration in the selection of analytical and theoretical approaches.

As I went from data collection to the analysis and write-up phase, I experienced conflicting feelings, particularly when reporting on observed practices. I entered this study

filled with a conviction that I would observe high-quality ECME in practice. I believed this would happen due to Siobhán's particular interest in ECME and her previous efforts to improve practice among her staff. The presence of a specific ECME policy fuelled this conviction. While the ECCE room was well-resourced, with plenty of materials to support mathematical exploration, the prevailing philosophy of non-adult interference in children's freely chosen play limited Sinéad's willingness to initiate interactions during this time. I had not anticipated the impact of fluctuating break times on the potential for interactions, as this had not been my own personal experience. Nor had I expected to witness the rushed event that was CT, with its consistent line of questioning and repetitious activity during circle time, and little to no response to children's mathematical observations or thinking. These reflections on findings reflect the reality of the situation. This study has shown that, despite a manager's specific ECME knowledge and enthusiasm to support it in their setting, difficulties arise in implementing strategies to support learning. Structural limitations, such as staff recruitment and retention, inevitably impact daily events, namely, break times and educator presence in the room. Managers and staff do their best under difficult conditions.

To establish trustworthiness (Chapter Five), I wanted to share my findings with my participants. Findings were shared with Siobhán. While no dispute was raised about my analysis, no meaningful feedback was received. This may reflect the busyness of the setting and Siobhán's lack of time to respond. It may be argued that when she agreed to participate in the study, Siobhán anticipated a more positive outcome, as she had made specific efforts to improve ECME in the setting and was unhappy with some of my findings. I was saddened to hear that Sinéad had not only left the setting but also the ECEC sector post-analysis, and therefore, I could not share my work with her. Sinéad's departure from ECEC practice further highlights the exodus of graduate leads from the ECEC sector. Perhaps, in some way,

participating in this study may have hastened her departure. Such reflections serve to illustrate the complexity and nuances of conducting an ethnographic study.

9.7 Contribution to Knowledge

This ethnographic study, underpinned by an ecological approach, explored young children's participation in ECME in a preschool setting. The study explored how ECME is supported in the Irish educational policy domain (macrosystem) and provided empirical evidence of the pedagogical strategies employed by an ECE to facilitate and guide early mathematical learning in the microsystem that is the ECCE classroom. The study shows the mathematical ideas and concepts explored by children in their preschool environment. The study tells the story of how one Irish preschool setting endeavours to support early mathematical learning at the setting policy and classroom levels. This contribution is timely, set against a backdrop of rapid and successive government policy initiatives (e.g. DES, 2017b; GoI, 2024a; GoI, 2024b) which emphasise the importance of early mathematics education in ECEC settings. The study contributes to the small bank of international studies exploring pedagogical practice for ECME in preschool settings (e.g. Lundqvist et al., 2023; Trawick-Smith et al., 2016; Wickstrom, 2019, 2023) and is a significant contribution to the small bank of Irish research in the field of ECME (e.g. Gillic, 2020; Neylon, 2014; O'Dwyer, 2021). It also contributes to studies on circle time (e.g. Bustamante et al., 2018; Lundquist et al., 2023; Zhang & Quinn, 2018) and mathematical talk (e.g. Cohrssen et al., 2014; Klibanoff et al., 2006), demonstrating how, in this case, little challenge was afforded to children's mathematical thinking during circle and free play sessions. Findings from this study suggest the inclusion of ECME in all professional development programmes, both pre- and in-service, as a matter of urgency. This, in turn, will enable preschool settings to understand the significance of ECME and the importance of engaging children in a wide range of mathematical activity (Bishop, 1988; Clements & Sarama, 2021; Perry & Dockett, 2008) in

preschool, to acknowledge ECME as a child's right (Cohrssen & Page, 2016), and provide ECEs with the knowledge to engage in equitable noticing practices (Hand, 2012; Wager, 2015), to enhance children's mathematical explorations and thinking (Dockett & Goff, 2013; Lee, 2017), and provide specific support for children with EAL (Arizmendi et al., 2021; Baker et al., 2018; Doabler et al., 2016).

This study provides an important empirical contribution to the understanding and beliefs of an ECE about their role in the teaching and learning of ECME during periods of children's free play, as well as their main ECME focus: number to ten, counting, and measuring. Findings support those of previous studies (Gillic, 2021a; Gillic, 2021b; McInnes, 2019; Pyle & Danniels, 2017; Skene et al., 2022) in relation to the above. This is an important contribution to future policy development in Ireland, where current strategy documents (GoI, 2024a) advocate for a broad range of mathematical concepts to be consistently explored in preschool settings. Findings from this study suggest that this is not the experience of children in this preschool setting.

Importantly, this study demonstrated that young children are drawn to and are interested in the mathematical aspects of their world. They are being mathematicians (Fosse & Lossius, 2016). This finding is an essential one. It adds further weight to the argument for the inclusion of early mathematics education in preschool settings. It highlights the necessity for the mathematical competency of 'being mathematical', currently listed as one of seven key competencies in the PCF (DE, 2023b), and the statement *every child is mathematical* in the PMC (DE, 2023a), to be included in future iterations of the Aistear (GoI, 2024c), to ensure that children are seen as competent and capable mathematicians in their preschool classrooms.

This study has identified the challenges experienced by staff in this EC setting in guiding and supporting ECME in practice. Wider macro-systemic challenges include staff

recruitment and retention (ECI, 2024; Moloney & Pope, 2013), as well as a lack of PD in ECME for both pre- and in-service ECEs (INT 1). Micro-system challenges include in-house structural constraints, such as insufficient non-contact time for ECEs to complete administrative and preparatory tasks outside of the ECCE routine, and fluctuating break times and setting routines such as 'door time', which take ECEs out of the ECCE room during the morning. These structural constraints impact the potential for ECEs to be present in the room, ready to support and enhance children's learning as opportunities arise (Dockett & Goff, 2013; Gasteiger & Benz, 2018; Lee, 2017).

Methodologically, this micro-ethnographic study provides an in-depth account of how a group of preschool children's participation in ECME was facilitated by their lead educator and shaped by the broader framework of the setting's policy (Bronfenbrenner & Morris, 2006). The narrative accounts of the preschool day, presented in Chapters Six and Seven, offer a detailed portrayal of how the early childhood educator planned for and enacted children's participation in ECME during adult-led activities, as well as how such participation was either supported or constrained during periods of free play (Rogoff, 1990, 2003).

9.8 Recommendations

The ethnographic approach adopted in this study proved to be a valuable research methodology and has the potential to inform future research investigating how ECEC practices for ECME are enacted within naturalistic settings. Drawing on the findings of this study, a set of recommendations for policy, practice, and future research has been developed. The findings highlight the importance of recognising and responding to young children's emerging mathematical interests in preschool contexts, rather than constraining their mathematical learning opportunities by privileging the anticipated learning demands of primary schooling. While further PD is necessary to emphasise the importance of providing

all children with high-quality early mathematical learning experiences, an equity-focused perspective highlights the critical need for educators to intentionally support and promote young girls' engagement across a broad range of mathematical activities, as well as to provide appropriate support for children with EAL in early mathematical learning. Each recommendation will be discussed in turn.

9.8.1 Recommendations for Policy

These macro-level recommendations are intended for future educational policy development. Findings show that at the time of the study, Sinéad experienced a fluctuating break time due to assistant managers having to take on non-managerial roles. This often impacted Sinéad's ability to become deeply involved with children's mathematical activity, or having to rush through her CT content as a result. Having a second adult in the room would enable one staff member to take a group during FP, while the other adult supervised and attended to the other children. This would mean amending the current ratio stipulation of one adult to eleven children in an ECCE classroom (GoI, 2016; Walsh et al., 2024).

The literature identifies the necessity for EC staff to possess a knowledge of relevant mathematical content (Brownell et al., 2014; Sarama & Clements, 2009) and pedagogical content knowledge (Lee, 2017; McCray & Chen, 2012). At present, not all Irish ECEC pre-service courses include a mathematics module (O'Neill & Gillic, 2024). Findings indicate that EC staff, particularly those with level 5 and 6 qualifications, experience mathematics anxiety and demonstrate a lack of both MCK and PCK to support EMCE in the EC classroom. To support ECEs' professional preparation for practice, government intervention is required to mandate the inclusion of ECME modules in all further education (levels 5 and 6) and undergraduate ECEC programmes. Including pre-service educators at all levels is critical, as ECs with level 6 qualifications are eligible to work as lead educators in ECCE classrooms (GoI, 2021).

Findings from this study suggest a need to problematise the conceptualisation of children being 'over-prepared' upon entering primary school (Sloan et al., 2022) and the resistance to providing challenge at the end of the ECCE year (Gillic, 2021a). To this end, I recommend a national, cross-sectoral conversation between ECEs, JI teachers, relevant EC education providers and government bodies to develop a common understanding of how children's individual mathematical interests, abilities and knowledge are supported in both sectors and upon transition to primary school.

This study suggests there may be merit in developing a specific curriculum for the 3-7 phase (ECCE to the end of SI), enabling a more fluid approach to mathematics education (and education more broadly) across the transition from ECCE to primary school. Traditionally, the two 'infant' classes acted as a 'pre-school' (Murphy, 2006), and as such, they were a child's first introduction to formal schooling. This historical legacy can still be felt within the education system as it continues to grapple with how the preschool sector should relate to the Infant classes. The previous iteration of Aistear (NCCA, 2009) acted as a boundary object, providing some cross-sectoral continuity and listing mathematical concepts to be explored from birth to six years. The current version (GoI, 2024c) falls within the remit of preschool only. The exclusion of certain mathematical concepts in the current version, such as pattern or number operations, can lead to confusion about how ECEs should engage children with mathematics. An approach similar to the English Early Years Foundation Stage (EYFS) framework (DE, 2025), which includes preschool and the first year of primary school, could be adopted here. The EYFS outlines specific learning outcomes for children to reach by the end of the phase (0-5 years), leading to a more flexible approach to the teaching and learning of mathematics across the early years phase of education. In turn, this would support the rights of every child to a broad early mathematical education as advocated by Cohrssen and Page (2016).

9.8.2 Recommendations for Practice

Strategies such as math talk (Cheeseman, 2015; Klibanoff et al., 2006; Priestley, 2022; Purpura et al., 2011), games (Cohrssen & Niklas, 2018; Ramani & Eason, 2015), and explicit teaching can be used to intentionally support all children's ECME learning journey (Doabler et al., 2016). Direct, intentional and explicit teaching is particularly effective for EAL learners (Doabler et al., 2016). Findings from this study show that due to Sinéad's belief that she should not interfere in children's free play, some EAL learners did not receive intentional support during FP.

Both Bronfenbrenner (1979) and Rogoff (1990; 2003) advocate for high-quality interactions to support early learning and development. PD should also consider the role of the adult during FP, noticing, responding to, and enhancing mathematics observed through high-quality interactions (Dockett & Goff, 2013; Lee, 2017). In this regard, engagement with the continuum of play, such as that proposed by Pyle and Danniels (2017), would support educators in developing a more nuanced understanding of the varying roles adults may assume across different play contexts, ranging from child-directed free play to more guided and teacher-facilitated playful learning experiences (Zosh et al., 2018). In this regard, investment in ECME-focused PD may support a broader pedagogical shift within the sector, strengthening educators' confidence and capacity to recognise, extend, and intentionally scaffold mathematical thinking across everyday contexts (Dockett & Goff, 2013; Gasteiger & Benz, 2018; Lee, 2017). The need to encourage girls to participate in construction/block play should also be included (Clements & Sarama, 2021).

Circle time has been identified as a space to facilitate intentional teaching (Bustamante et al., 2018). Findings from this study show that CT was a space where intentional teaching occurred. However, in line with findings from other studies (Bustamante et al., 2018; Lundquist et al., 2023), the mathematical content was repetitive and

unchallenging and did not offer new, rich learning experiences for the children in terms of mathematics. I recommend a reconsideration of the mathematical content presented to children during CT, and that ECEs afford time and space to respond to and enhance children's own mathematical knowledge and observations during this time, facilitating richer mathematical talk and discussion and opportunities to engage in deeper mathematical thinking. Such a reconceptualisation of CT practice has the potential to significantly influence educator pedagogy by repositioning educators as active co-constructors of mathematical meaning with children, rather than transmitters of routine content.

Findings from this study echo those of Pollitt et al. (2020) in that the belief that the adult should refrain from engaging in children's freely chosen activity is firmly held by Sinéad. This cultural legacy impacts on educator–child interactions, limiting the mathematical learning experience. FP enables the child as an active agent in their own learning, whose learning thrives when they are actively listened to by responsive and interested adults (Carruthers, 2015). This study highlights the importance of tuning in to every child's mathematical interests (Dockett & Goff, 2013), offering responsive assistance (Rogoff, 2003, and not waiting for a child to ask for help. Adults should respond to observed interests with intentionality (Dockett & Goff, 2013; Lee, 2017). They should refer to and incorporate children's mathematical observations, ideas, and interests not only into the learning environment but also into aspects of the daily routine, such as CT. Such actions will ensure meaningful learning experiences that motivate participation in ECME. Furthermore, these findings point to the need for a cultural and pedagogical shift within early childhood practice, whereby educators recognise intentional interaction within FP not as a disruption to children's autonomy, but as a vital mechanism through which mathematical thinking can be extended and deepened.

9.8.3 Recommendations for Future Research

This doctoral study has provided the first Irish ethnographic study of how preschool children are supported in their participation in ECME. It adds to the small, emergent research literature in an Irish context (e.g. Gillic, 2020; Neylon, 2014; O'Dwyer, 2021). This study also sits within a government policy context, with its increasing emphasis on the inclusion of mathematics in ECEC (DES, 2011; DES, 2017; DES, 2020; GoI, 2024a).

Research (Cohrssen et al., 2014; Dunphy et al., 2014; Klibanoff et al., 2006; Priestley, 2022) advocates for children's engagement in high-quality mathematical talk and discussion to engage children in the mathematical activity of explaining (Bishop, 1988) and in the processes of reasoning, arguing, justifying and articulating mathematical thinking (Dooley et al., 2014). Findings from this study indicate a lack of quality mathematical talk and discussion during the ECCE sessions observed. To this end, I suggest that observational studies be conducted with several partnering preschools to investigate the type and quality of math talk that arises during the preschool day, with particular attention to talk during interactions that occur within periods of free play and circle time. Follow-up intervention-based studies would support ECEs in their modelling of mathematical language and using a 'math-talk' approach to support young children's mathematical thinking (Dooley et al., 2014).

Given the reported perspectives of JI teachers that some children are entering primary school as 'too numerate' (Sloan et al., 2022), and findings from this study which indicate that Sinéad refrained from engaging the children with numbers past ten during CT, I recommend a study be conducted exploring the perspectives of ECCE educators and JI teachers regarding the expected mathematical knowledge of preschoolers upon entry to primary school. Their perspectives could provide a starting point for a national conversation in relation to this issue. Furthermore, longitudinal research following young children's mathematics education

journey from their final year of ECCE to the end of their JI year would provide an insight as to how children's mathematical abilities, interests and development are attended to across time and in differing classroom contexts (Bronfenbrenner & Morris, 2006).

This study has highlighted the need for targeted, intentional support for young EAL learners and early mathematics. Research (Baker et al., 2018) reports a gap in understanding how best to support young EALs and mathematics, particularly in supporting young EAL learners to develop their mathematical vocabulary and knowledge. A research-informed curriculum/strategy should be developed to support ECEs in enabling these children to fully participate in mathematical activities. To underpin this, I recommend that research be carried out exploring ECEs' current knowledge of pedagogical strategies to support EALs' knowledge and understanding of early mathematical concepts and associated vocabulary, as well as desk-based research on how best to support young EAL learners in learning mathematics. Such research is timely, given the inclusion of targeted supports for EALs in ECEC settings in the NLNDLS implementation plan (GoI, 2024b).

A final recommendation for further research is that a study be conducted exploring ECEs actions and interactions during free play time. This study could also focus on an ECE's 'playfulness' (Sproule et al., 2017) and how this is utilised to support early mathematical learning. Exploring this aspect of practice may help us to understand how the perceived divide between free play and intentional, adult-led teaching is experienced by ECEs and children in an ECCE classroom.

9.9 Conclusion

van Oers (2010) asserts that children's participation in ECME is a culturally guided process, a process where mathematical exploration and problem-solving are supported by a more knowledgeable other. This study is rooted in the understanding that young children

have a right to high-quality early mathematics education (Munn, 2005; Cohrssen & Paige, 2016). In my Montessori classroom, children's fascination with and enjoyment of participating in mathematical activity, from wanting to build a dodecahedron to measuring me with Unifix cubes, to simply wanting to make the highest tower possible, always brought me joy. Observing the children and their educator engaging in mathematical activity in this study also brought joy. My experiences in the classroom have led me to believe that young children are capable of being mathematicians. My time in this setting confirmed this belief; the children were interested in mathematical ideas, and their participation in such activity brought them joy. The study shows the commitment of the adults in supporting children's holistic development and in providing a warm, caring and positive learning environment. I hope I have done their mathematical knowledge and interests justice, and made what is normally invisible, visible.

This study concludes that guiding children's participation in ECME is a complex and multifaceted occupation. It requires a range of knowledge, skills and beliefs. Educators need an in-depth understanding of every child in their care. They need to know their individual interests, abilities and capabilities as well as their funds of knowledge. They need to know who likes to work alone, with the adult or with other children. They need to use this knowledge to devise tailored, interesting, and appropriate learning opportunities in response to observation of individual children's learning needs. Having a richly-resourced learning environment facilitates this tailoring of individual learning experiences.

The 11 children participated in all of the 'Big Ideas of Mathematics' across the data-collection period. In line with previous studies (Gillic, 2021a; Ginsburg et al., 2008), number activities were the most observed, followed by measures, shape and space, data and pattern. Clearly, more work needs to be done in ECE circles to highlight the importance of all aspects of mathematics in children's early years. There is a necessity to question the efficacy of

Aistear (GoI, 2024) in terms of curricular guidance for ECEs. While many of the 'Big Ideas' are listed, some, like pattern, are not. A more research-informed approach is needed if ECEs are to support children's participation in a broader range of mathematical ideas and concepts.

In terms of knowledge, this study proposes that a knowledge of mathematical learning environments, how to create one, and how to seed it with interesting and challenging materials is critical in guiding children's participation in ECME. To do this effectively, educators must understand the potential of each piece of equipment in the classroom to engage children in mathematical thinking and learning. It can be argued that in such a richly seeded mathematical learning environment that children in this study had no choice but to engage in mathematical exploration. However, as noted above, not all children regularly engaged with the available mathematical resources; some preferred to sit and colour or draw during FP.

There is a need to problematise the ever-present cultural ECE belief that adults should not interfere with children's freely chosen activity during free play, and the belief that providing a math-rich environment will facilitate mathematical learning. The research is replete with studies emphasising the need to engage children in mathematical talk and discussion, to guide them in developing their capacity to participate in mathematical processes of communicating, reasoning, justifying, problem-solving and connecting (see Chapter Two) (Dooley et al., 2014). Taking a small group to intentionally, yet playfully, teach mathematical concepts and vocabulary would support all children in acquiring such skills.

As noted in Chapter One, in Ireland, we are in interesting times in terms of ECME. Our government policies extoll the importance of high-quality mathematics education from birth; however, the intended accompanying plans are slow to come to fruition. A national conversation is necessary to determine what ECME in Irish ECE settings should look like and

the professional development required to facilitate this. Policy documents use the language of speed when describing implementation plans, yet fail to consider the conditions necessary to implement targeted plans within the indicated time frames. I call for a suite of research reports to map early mathematical learning from preschool to primary, drawing on evidence of how this is done in other jurisdictions.

Policy documents extol the importance of girls participating in STEM careers, and yet, findings from this study show that only two of the girls (Chandra and Priya) showed any regular interest in mathematical games or some kind of construction play. Their lack of participation in block play is concerning from both a mathematical and STEM point of view. Equally concerning was Sinéad's lack of inclination to encourage the girls' participation in such activity, as her belief in free choice prevailed.

Findings presented in this thesis highlight the need for further research. As our policy documents indicate that ECME become a key feature of EC practice, further research is required into how young children are guided in their participation in ECME in Irish ECEC settings. Research is particularly needed to explore the nature of adult-child interactions during children's free play, mathematics presented in CT, ECEs' understanding of the Big Ideas, and their perceived importance of number concepts over other mathematical areas, such as pattern and data. This study is a good start, but as the traditional Irish proverb states, '*tús maith, leath na hoibre'* – a good start is half the work. Much remains to be done.

REFERENCES

Ahmed, S.K. (2024). The pillars of trustworthiness in qualitative research. *Journal of Medicine, Surgery, and Public Health, 2,* https://doi.org/10.1016/j.glmedi.2024.100051

Albon, D. & Huf, C. (2021). What matters in early childhood education and care? The contribution of ethnographic research. *Ethnography and Education 16*(3), 243-247, https://doi.10.1080/17457823.2021.1916978

Altheide, D.L., (1987). Reflections: ethnographic content analysis. *Qualitative Sociology, 10*(1), 65-77, https://doi.og/10.1007/bf00988269

Altheide, D.L., & Schneider, C.J. (2013). *Qualitative media analysis.* (2nd ed.). Sage.

Amador, J.M., Bragelman, J., & Castro Superfine, A.C. (2021). Prospective teachers' noticing: A literature review of methodological approaches to support and analyze noticing. *Teacher and Teacher Education, 99,*103-256. https://doi.org/ 10.1016/j.tate.2020.103256

Anders, Y. & Rossbach, H.-G. (2015). Preschool teachers' sensitivity to mathematics in children's play: The influence of math-related school experiences, emotional attitudes, and pedagogical beliefs. *Journal of Research in Childhood Education, 29* (3), 305-322. https://doi.org/10.1080/02568543.2015.1040564

Andrews, M. (2012). *Exploring play for early childhood studies.* Learning Matters.

Arizmendi, G. D., Li, J. T., van Horn, M. L., Petcu, S. D., & Swanson, H. L. (2021). Language-focused interventions on math performance for English learners: A selective meta-analysis of the literature. Learning Disabilities Research & Practice, 36(1), 56-75. https://doi.org/10.1111/ldrp.12239

Aubrey, C. (1997). Re-assessing the role of teachers' subject knowledge in early years mathematics teaching. *Education 3-13, 25*(1), 55-60, https://doi.org/10.1080/03004279785200121

Aubrey, C., Godfrey, R. & Dahl, S. (2006). Early mathematics development and later achievement: Further evidence. *Mathematics Education Research Journal, 18*(1), 27-46, https://doi.org/10.007/bf03217428

Australian Government Department of Education [AGDE] (2022). *Belonging, being and becoming: The early years learning framework for Australia* (*V2.0*). Australian Government Department of Education for the Ministerial Council.

Baker, D.L., Richards -Tutor, C., Sparks, A. & Canges, R. (2018). Review of single subject research examining the effectiveness of interventions for at-risk English learners. *Learning Disabilities Research & Practice, 33*(2), 64-74 https://doi.org/10.1111/ldrp.12160

Balfanz, R. (1999). Why do we teach young children so little mathematics? Some historical considerations. In J.V. Copley (Ed.) *Mathematics in the early years.* (pp. 3-10). NCTM/NAEYC.

Ball, D.L. (2011). Foreword. In M.G. Sherin, V.R. Jacobs & R.A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes.* Routledge.

Ball, D.L., Ben-Peretz, M., & Cohen, R.B. (2014). Records of practice and the development of collective professional knowledge. *British Journal of Educational Studies, 62*(3), 317-335, https://doi.org/10.1080/00071005.2014.959466

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407, https://doi.org/10.1177/0022487108324554

Balladares, J., Miranda, M., & Cordova, K. (2024). The effects of board games on math skills in children attending prekindergarten and kindergarten: A systematic review. *Early Years, 44*(3-4), 710-734, https://doi.org/10.1080/09575146.2023.2218598

Barnett, S.W., & Jung, K. (2024). Auspice and other policy-related variations in preschool practice in the United States: Have public preschool programs been more academic? *International Journal of Child Care and Education Policy, 18*(3), https://doi.org/10.1186/s40723-024-00139-6

Basargekar, A., & Lillard, A. S. (2021). Math achievement outcomes associated with Montessori education. *Early Child Development and Care, 191*(7-8), 1207–1218. https://doi.org/10.1080/03004430.2020.1860955

Bates, K.E., Williams, A.Y., Gilligan-Lee, K.A., Gripton, C., Lancaster, A., Williams, H., Borthwick, A., Gifford, S., & Farran, E.K. (2023). Practitioners' perspectives on spatial reasoning in educational practice from birth to 7 years. *British Journal of Educational Psychology, 93*(2), 571-590 https://doi.org/10.1111/bjep.12579

Beauchamp, T.L. & Childress, J.F. (2008). *Principles of biomedical ethics.* (6th ed.). Oxford.

Becker, H.S. & Greer, B. (1960). Participant observation: The analysis of qualitative field data. In R. Adams & J. Preiss (Eds.), *Human organisation research: Field relations and techniques.* Dorsey. (pp. 267-289).

Bell, J. (2009). *Doing your research project: A guide for first-time researchers in education, health and social science* (4th ed.). Open University Press.

Beneke, S.J., Ostrosky, M.M., & Katz, L. (2008). Calendar time for young children: Good intentions gone awry. *Young Children*, *63(*3).12-16. https://jstor.org/stable/42730993

Benz, C. (2012). Maths is not dangerous – Attitudes of people working in German kindergarten about mathematics in kindergarten. *European Early Childhood Research Journal*, 20(2), pp.249-261. https://doi.org/10.1080/1350293x.2012.681131

Bishop, A. (1988). *Mathematical Enculturation: A Cultural Perspective on Mathematics Education.* Springer.

Björklund, C. (2014). Powerful teaching in preschool—A study of goal-oriented activities for conceptual learning. *International Journal of Early Years Education, 22*(4), 380– 394. https://doi.org/10.1080/09669760.2014.988603

Björklund, C., & Barendregt, W. (2016). Teachers' pedagogical mathematical awareness in Swedish early childhood education. *Scandinavian Journal of Educational Research, 60(*3), 359-377, https://doi.org/10.1080/00313831.2015.1066426.

Björklund, C., Magnusson, M., & Palmér, H. (2018). Teachers' involvement in children's mathematizing – beyond dichotomization between play and teaching. *European Early Childhood Education Research Journal, 26*(4), 469-480 https://doi.org/10.1080/1350293X.2018.1487162

Blömeke, S., Gustafsson, J.E., & Shavelson, R.J. (2015). Beyond dichotomies: Competence viewed as a continuum. *Zeitschrift für Psychologie, 223*(1), 3-13, https://doi.org/10.1027/2151-2604/a000194

Blömeke, S., Thiel, O., & Jenβen, L. (2017). Before, during and after examination: Development of prospective preschool teachers' mathematics-related enjoyment and

self-efficacy. *Scandinavian Journal of Educational Research, 63*(4), 506-519 https://doi.org/10.1080/00313831.2017.1402368

Bodrova, E. & Leong, D. (2024). *Tools of the mind: The Vygotskian approach to early childhood education*. (3rd ed.). Routledge.

Bogdan, R.C. & Biklen, S.K. (2003). *Qualitative research for education: An introduction to theory and methods* (4th ed.). Pearson Education Group.

Bolton, G. (2010). *Reflective practice: Writing and professional development*. Sage.

Braun, V. & Clarke, V. (2022). *Thematic analysis: A practical guide.* Sage.

Brewer, J.D. (2000). *Ethnography.* Open University Press.

Brinkmann, S., & Kvale, S. (2018). *Doing interviews.* (2nd ed.). Sage.

Bronfenbrenner, U. (1979). *The ecology of human development: Experiments by nature and design.* Harvard.

Bronfenbrenner, U. (1989). Ecological systems theory. In R. Vasta (Ed.) *Annals of Child Development. Six theories of child development,6,* (pp.187-249). JAI Press.

Bronfenbrenner, U. (2000). Ecological systems theory. In A.E. Kazdin (Ed.) *Encyclopedia of Psychology, Vol. 3.* (pp. 129-133). Oxford University Press.

Bronfenbrenner, U. & Morris, P.A. (2006). The bioecological model of human development. In W. Damon, & R.M. Lerner (Eds.), *Handbook of Child Psychology: Vol 1. Theoretical Models of Human Development* (6th ed.) (pp.793-828). John Wiley.

Brosterman, N. (1997). *Inventing kindergarten*. Harry N. Abrams.

Brown, E.T. (2005). The influence of teachers' efficacy and beliefs regarding mathematics instruction in the early childhood classroom. *Journal of Early Childhood Teacher Education, 26(*3), 239-257, https://doi.org/10.1080/10901020500369811

Browne, L. & Wong, K.S-R. (2016). Transnational comparisons of teacher expectation of mathematical functional ability in early years and key stage 1 pupils – a study undertaken in Hong Kong and England. *Education 3-13, 45(*4), 504-515.

https://doi.org/10.1080/03004279.2016.1140798

Brownell, J., Chen, J., Ginet, L., Hynes-Berry, M., Itzkowich, R., Johnson, D. & McCray, J. (2014). *Big ideas of early mathematics: What teachers of young children need to know.* Pearson.

Bruce, T. (2001). *Learning through play: Babies, toddlers and the foundation years.* Hodder & Stoughton.

Bruce, T. (2021). *Friedrich Froebel A critical introduction to key themes and debates.* Bloomsbury.

Bruner, J. (1996). *The culture of education.* Harvard University Press.

Brunner, E. (2022). Outlook: context and its consequences - In S. Dunekacke, A. Jegodtka, T. Koinzer, K. Eilerts & L. Jenßen (eds.) *Early childhood teachers' professional competence in mathematics*. (pp. 218 – 240). Routledge. https://doi.org/10.4324/9781003172529-14

Bryman, A. (2016). *Social research methods* (5th ed.). Oxford University Press.

Bustamante, A.S., Hindman, A.H., Champagne, C.R., & Wasik, B.A. (2018). Circle time revisited: how do preschool classrooms use this part of the day? *The Elementary School Journal*, 118(4), https://doi-org.dcu.idm.oclc.org/10.1086/697473

Carr, M., & Lee, W. (2012). *Learning Stories: Constructing learner identities in early education.* Sage.

Campbell, C., & Speldewinde, C. (2022). Early childhood STEM education for sustainable development. *Sustainability,14*, 3524. https:// doi.org/10.3390/su14063524

Centre for Early Childhood Development and Education (CECDE)(2006). *Síolta the national quality framework for early childhood education.* CECDE.

Cheeseman, J. (2015). Mathematical conversations that challenge children's thinking. In B. Perry, A. MacDonald & A. Gervasoni (Eds.), *Mathematics and transition to school* (pp.273-293). Springer.

Chen, J-Q, McCray, J., Adams, M., & Leow, C. (2014). A survey study of early childhood teachers' beliefs and confidence about teaching early math. *Early Childhood Education Journal, 42,* 367-377. https://doi.org/10.1007/s10643-013-0619-0

Chesworth, L. (2016). A funds of knowledge approach to examining play interests: listening to children's and parents' perspectives. *International Journal of Early Years Education, 24*(3), 294-308 https://doi.org/10.1080/09669760.2016.1188370

Child Care Act, (1991). https://www.irishstatutebook.ie/eli/1991/act/17/enacted/en/html

Clarke, A.E., Friese, C., & Washburn, R.S. (2018) *Situational Analysis: Grounded Theory after the Interpretive Turn*, (2nd ed), Sage.

Clements, D.H. & Sarama, J. (2014a). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). Routledge.

Clements, D.H. & Sarama, J. (2014b). The importance of the early years. In R. E. Slavin (Ed.), *Science, technology & mathematics (STEM)* (pp. 5-9). Corwin.

Clements, D.H. & Sarama, J. (2021). *Learning and teaching early math: The learning trajectories approach* (3rd ed.). Routledge.

Cohen, L.E. & Emmons, J. (2016). Block play: spatial language with preschool and school-aged children. *Early Child Development and Care, 187,* (5-6), 967-977, http://dx.doi.org/10.1080/03004430.2016.1223064

Cohen, L.E. & Uhry, J. (2011). Naming block structures: A multi-modal approach. *Early Childhood Education Journal, 39* (1), 79-87 https://doi.org/10.1007/s10643-010-0425-x

Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). Routledge.

Cohrssen, C., Church, A., & Tayler, C. (2014). Purposeful pauses: teacher talk during early childhood mathematics activities. *International Journal of Early Years Education, 22*(2), 169-183 https://doi,org/10.1080/09669760.2014.900476

Cohrssen, C., & Niklas, F. (2019). Using mathematics games in preschool settings to support the development of children's numeracy skills. *International Journal of Early Years Education, 27*(3) 322-339 https://doi.org/ 10.1080/09669760.2019.1629882

Cohrssen, C., & Page, J. (2016). Articulating a rights-based argument for mathematics teaching and learning in early childhood education. *Australasian Journal of Early Childhood, 41(*3), 104-108. https://doi.org/10.1177/183693911604100313

Cohrssen, C., & Tayler, C. (2016). Early childhood mathematics: A pilot study in preservice teacher education. *Journal of Early Childhood Teacher Education, 37*(1), 25-40 https://doi.org/10.1080/10901027.2015.1131208

Copley, J.V., & Padron, Y. (1998). Preparing teachers of young learners: Professional development of early childhood teachers in mathematics and science. *Forum on Early Childhood Science, Mathematics, and Technology Education.* Washington: American Association for the Advancement of Science. Retrieved from: https://files-eric-ed-gov.dcu.idm.oclc.org/fulltext/ED416992.pdf

Copur-Gencturk, Y., Plowman, D., & Bai, H. (2019). Mathematics teachers' learning: Identifying key learning opportunities linked to teachers' knowledge growth. *American Educational Research Journal* 56(5): 1590–1628. https://doi-org.dcu.idm.oclc.org/10.3102/0002831218820033.

Copur-Gencturk, Y., and T. Tolar. (2022). Mathematics teaching expertise: A study of the dimensionality of content knowledge, pedagogical content knowledge, and content-specific noticing skills. *Teaching and Teacher Education, 114* https://doi-org.dcu.idm.oclc.org/10.1016/j.tate.2022.103696.

Costa, H.M., Outhwaite, L.A., & Van Herwegen, J. (2025). Early years practitioners' training, beliefs and practices concerning mathematics: implications for education and practice. *International Journal of Early Years Education, 33*(3), 470-487 https://doi.org/10.1080/09669760.2023.2254822

Crawford, L.M. (2020). Conceptual and theoretical frameworks in research. In Burkholder, G.J., Cox, K.A., Crawford, L.M., & Hitchcock, J.H. (Eds.) *Research design and methods*: *an applied guide for the scholar practitioner*. Sage.

Creswell, J.W. (2007). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*, (3rd ed.). Pearson Education.

Creswell, J.W. (2013). *Qualitative inquiry & research design: Choosing among five approaches* (3rd ed.). Sage.

Creswell, J.W. & Poth, C.N. (2018). *Qualitative inquiry & research design: choosing among five approaches* (4th ed.). Sage.

Dahlberg, G., Moss, P., & Pence, A. (1999). *Beyond quality in early childhood education and care*: *Postmodern perspectives*. Taylor & Francis.

Denny, J.H. (2009). *The relationship between preschool teachers' beliefs and school readiness and classroom practice in Tennessee childcare programs*. [Doctoral Dissertation, University of Tennessee]. Retrieved from http://trace.tennessee.edu/utk_graddiss/32/

Denscombe, M. (2010). *The good research guide for small-scale social research projects*. (4th ed.) Open University Press.

Denzin, N.K. (2012). Triangulation 2.0. *Journal of Mixed Methods Research, 6*(2), 80-88. https://doi.org/10.1177?1558689812437186

Denzin, N.K. & Lincoln, Y.S. (2000). *Handbook of qualitative research* (2nd ed.). Sage.

Department of Children, Equality, Disability, Integration and Youth (DCEDIY) (2022). *Annual early years sector profile report* 2020-2021. Pobal.

Department of Children, Equality, Disability, Integration and Youth (DCEDIY)(2024). *Early years recognised qualifications: Ireland.* https://assets.gov.ie/static/documents/dcediy-early-years-recognised-qualifications-september-2024.pdf

Department of Education (DE) (1971). *Primary school curriculum: Teacher's handbook part 1*. Department of Education.

Department of Education (DE) (2022). *A guide to early years education inspection.* DE.

Department of Education (DE). (2023a). *Primary mathematics curriculum for primary and special schools.* NCCA.

Department of Education (DE) (2023b). *Primary curriculum framework for primary and special schools.* NCCA.

Department of Education (DE) (2025). *A guide to early years education inspection.* DE.

Department for Education (2025). *Early years foundation stage statutory Framework for group and school-based providers: Setting the standards for learning, development and care for children from birth to five*. DE.

Department of Education and Skills (2011). *Literacy and numeracy for learning and life The national strategy to improve literacy and numeracy among children and young people 2011-2020.* DES

Department of Education and Skills (DES)(2017a). *National strategy: Literacy and numeracy for learning and life interim review: 2011-2016 New targets: 2017-2020*. DES.

Department of Education and Skills (DES) (2017b). *STEM Education Policy Statement 2017–2026.* DES

Department of Education and Skills (DES)(2018). *A review of early years education focused inspection: April 2016 – June 2017 Insights and future developments*. DES.

Department of Education and Skills (DES)(2019). *Professional award criteria and guidelines for initial professional education (Level 7 and Level 8) degree programmes for the early learning and care (ELC) sector in Ireland*. DES.

Department of Education and Skills (DES)(2020). *STEM Education 2020; Reporting on Practice in Early Learning and Care, Primary and Post-Primary Contexts*. DES.

Department of Education and Youth (DEY) (2025). *Science, technology, engineering and mathematics (STEM) education specification: for primary and special schools.* NCCA.

Dibley, L. (2011). Analysing narrative data using McCormack's lenses. *Nurse Researcher, 18*(3), 13-19. https://doi.org/10.7748/nr2011.04.18.3.13c8450

Doabler, C.T., Nelson, N.J., & Clarke, B. (2016). Adapting evidence-based practices to meet the needs of English learners with mathematics difficulties. *Teaching Exceptional Children, 48* (6), 301- 310. https://doi.org/10.1177/0040059916650638

Dockett, S. & Goff, W. (2013). Noticing young children's mathematical strengths and agency. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th Annual Conference of the Mathematics Education Research Group of Australia).* MERGA.

Dockett, S., Einarsdóttir, J., & Perry, B. (2012). Young children's decisions about research participation: opting out. *International Journal of Early Years Education, 20(*3), 244-256, https://doi.org/10.1080/09669760.2012.715405

Dooley, T. (2019). *Learning and teaching primary mathematics An addendum to NCCA research reports 17 and 18.* NCCA.

Dooley, T., Dunphy, E., Shiel, G., Butler, D., Corcoran, D., Farrell, T., NicMhuirí, S., O'Connor, M., & Travers, J. (2014). *Mathematics in early childhood and primary education (3-8 years): Teaching and learning Research Report No. 18.* National Council for Curriculum and Assessment.

Dossey, J.A. (1992). The nature of mathematics: Its role and its influence. In D.A. Grouws (Ed.) *Handbook of research on mathematics teaching and learning* (pp.39-48). MacMillan.

Duignan, M. & McDonnell, F. (2022). An overview of the development of government regulation and inspection in the early childhood education and care sector in Ireland, 1921-2021. In N. Hayes & T. Walsh (Eds.), *Early childhood education and care in Ireland: Charting a century of developments (1921-2021).* (pp.203-228). Peter Lang

Duncan, G. J., Dowsett, C.J., Claessens, A., Magnuson, K., Huston, A.C, Klebanov, P., Pagani, L.S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology 43* (6), 1428– 1446. https://doi.org/10.1037/0012-1649.43.6.1428.

Dunekacke, S., Jenβen, L., & Blömeke, S. (2015). Effects of mathematics content knowledge on pre-school teachers' performance: A video-based assessment of perception and planning abilities in informal learning situations. *International Journal of Science and Mathematics Education, 13,* 267-286 https://doi.org/10.1007/s10763-014-9596-z

Dunphy, E., Dooley, T., Shiel, G., Butler, D., Corcoran, D., Ryan, M. & Travers, J. (2014). *Mathematics in early childhood and primary education (3-8 years) Definitions, theories, development and progression Research report no.17.* National Council for Curriculum and Assessment.

Dunphy, E. (2012). Perspectives on early learning. In M. MhicMhathúna & M. Taylor (Eds) *Early childhood education and care* (pp. 203-213). Gill & MacMillan.

Dunphy, E. (2017). Transition from preschool to primary school. Optimising opportunities for mathematics learning. *Education Matters Yearbook 2017-2018* (105-110).

Dunphy, E., & MhicMhathúna, M. (2022). The history and work of the Centre for Early Childhood Development and Education (CECDE) (2001-2008). In N. Hayes & T. Walsh (Eds.) *Early childhood education and care in Ireland: Charting a century of developments (1921-2021).* (pp.149-176). Peter Lang.

Early Childhood Ireland (2024). *Policy in Action 12 November 2024.* Retrieved from: https://www.earlychildhoodireland.ie/policy-in-action-12-november-2024/#:~:text=Despite%20increased%20investment%20and%20a,waiting%20lists%20throughout%20the%20country.

Edwards, S. (2017). Play-based learning and intentional teaching: Forever different? *Australasian Journal of Early Childhood*, *42*(2), 4–11. https://doi.org/10.23965/AJEC.42.2.01

Egan, A. & Garrity, S. (2022). The legislative and regulatory landscape for early childhood education and care in Ireland. In N. Hayes & T. Walsh (Eds.), *Early childhood education and care in Ireland: Charting a century of developments (1921-2021).* (pp. 177—202). Peter Lang.

Elgavi, O., & Hamo, P. (2024). Math on the brain: Seven principles from neuroscience for early childhood education. *Early Childhood Education Journal, 53, 2837-2848* https://doi.org/10.1007/s10643-024-01656-2

Elia, I., Mulligan, J., Anderson, A., Baccaglini-Frank, A., & Benz, C. (2018). Research in early childhood mathematics education today. In I. Elia et al. (eds.), *Contemporary research and perspectives in early childhood mathematics education, ICME-13, Monographs* (pp.1-14). Springer. https://doi.org/10.1007/978-3-319-73432-3_1

Emerson, R.M., Fretz, R.I. & Shaw, L.L. (2011). *Writing ethnographic field notes.* (2nd ed.). University of Chicago Press.

English, L.D. (2023). Linking probability and statistics in young students' reasoning with chance. *Statistics Education Research Journal*, *22*(2), 1-13. https://doi.org/10.52041/serj.v22i2.418

Epstein, A. S. (2007). *The intentional teacher: Choosing the best strategies for young children's learning*. National Association for the Education of Young Children.

Ernest, P. (2016). An overview of the philosophy of mathematics education. In Ernest, P., Skovsmose, O., van Bendegem, J. P., Bicudo, M., Miarka, R., Kvasz, L., & Moeller, R. (Eds.). *The Philosophy of Mathematics Education* (1st ed.) Springer International Publishing. https://doi.org/10.1007/978-3-319-40569-8

Fenech, M., Blaxland, M., Woodrow, C., & Jones, A. (2025). A funds of knowledge approach to supporting children's academic learning: Complexities and enablers in the Australian early childhood context. *Australasian Journal of Early Childhood, 0,* 1-3 https://doi.org/10.1177/1836939125133789

Ferrara, K., Hirsh-Pasek, K., Newcombe, N.S., Golinkoff, R., & Lam, W.S. (2011). Block talk: Spatial language during block play. *Mind, Brain and Education, 5* (3), 143-151, http://dx.doi.org/10.1111/j.1751-228X.2011.01122.x

Fields, D.A. & Kafai, Y.B. (2009). A connective ethnography of peer knowledge sharing and diffusion in a tween virtual world. *International Journal of Computer Supported Collaborative Learning, 4*(1), 47-69. https://doi.org/10.1007/s11412-008-9057-1

Fillis S. (2018). *Professional award criteria and guidelines for initial professional education (level 7 and level 8) degree programmes in early childhood education and care (ECEC) in Ireland: An evaluation of the consultation findings*. https://www.education.ie/en/The-Education-System/Early-Childhood/evaluation-report-consultation-on-draft-criteria-and-guidelines.pdf

Fisher, K.R., Hirsh-Pasek, K., Newcombe, N. & Golinkoff, R.M. (2013). Taking shape: supporting preschoolers' acquisition of geometric knowledge through guided play. *Child Development, 84*(6), 1872-1878, https://www.doi.org/ 10.1111/cdev.12091

Fives, H., & Buehl, M. M. (2012). Spring cleaning for the "messy" construct of teachers' beliefs: What are they? Which have been examined? What can they tell us? In K. R. Harris, S. Graham, T. Urdan, S. Graham, J. M. Royer, & M. Zeidner (Eds.), *APA educational psychology handbook, Vol. 2. Individual differences and cultural and contextual factors* (pp. 471–499). American Psychological Association. https://doi.org/10.1037/13274-019

Fleer, M. (2010). *Early learning and development: Cultural-historical concepts in play.* Cambridge University Press.

Fleer, M. (2018). Conceptual playworlds: the role of imagination in play and learning. *Early Years, 41*(4), 353-364 https://doi.org/10.1080/09575146.2018.1549024

Fleer, M. (2019). Foreword. In N. Pramling, C. Wallerstedt, P. Lagerlöf, C. Björklund, A. Kultti, H. Palmér, M. Magnusson, S. Thulin, A. Jonsson, & I. Pramling Samuelsson (Eds.) *Play-responsive teaching in early childhood education* (pp v-vii). Springer.

Flewitt, R. (2006). Using video to investigate preschool classroom interaction: education research assumptions and methodological practices. *Visual Communication, 5* (1), 25-50, https://doi.org/10.1177/1470357206060917

Flewitt, R. & Ang, L. (2020). *Research methods for early childhood education.* Bloomsbury.

Fosse, T., & Lossius, M. H. (2016, July). *Norwegian Kindergarten teachers' work with mathematics*. Paper presented at the 13th International Congress on Mathematical Education, Hamburg, Germany.

Foucault, M. (1977). *Discipline and punish: The birth of the prison.* Penguin.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing Children's Mathematical Thinking in Everyday Mathematics Classrooms. *Journal for Research in Mathematics Education*, *30*(2), 148–170. https://doi.org/10.2307/749608

French, G. (2013). *A Journey Without a Roadmap.* From Aistear, May 2013, Retrieved from: https://arrow.tudublin.ie/aaschsslrep/22/

French, G., & McKenna, G. (2022). *Literature review to support the updating of Aistear the early childhood curriculum framework*. NCCA.

Fusch, P.I., Fusch, G.E., & Ness, L.R. (2017). How to conduct a mini-ethnographic case study: A guide for novice researchers. *The Qualitative Report, 22*(3), 923-941. https://doi.org/10.46743/2160-3715/

Gasteiger, H. (2015). Early mathematics in play situations: Continuity of learning. In: Perry, B., MacDonald, A., Gervasoni, A. (Eds) *Mathematics and Transition to School. Early Mathematics Learning and Development*. Springer, https://doi-org/10.1007/978-981-287-215-9_16

Gasteiger, H., & Benz, C. (2018). Enhancing and analyzing kindergarten teachers' professional knowledge for early mathematics education. *The Journal of Mathematical Behaviour, 51,* 109-117, https://doi.org/10.1016/j.jmathb.2018.01.002

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications.* American Psychological Association. https://doi.org/10.1037/10163-000

Geertz, C. (1973). *The interpretation of cultures: selected essays.* Basic Books.

Gelman, R. & Gallistel, C.R. (1978). *The child's understanding of number.* Harvard University Press.

Ghodsee, K. (2016). *From notes to narrative: Writing ethnographies that everyone can read.* Chicago.

Gibson, S.A. & Ross, P. (2016). Teachers' professional vision. *Theory into practice,* 55, 180-188. https://doi.org/http://dx.doi.org/10.1080/00405841.2016.1173996

Gifford, S. (2003). How should we teach mathematics to 3-and 4-year-olds? Pedagogical principles and practice for the foundation stage. *Mathematics Teaching,* 184, 33-38

Gillic, C. (2020). Exploring a progression continua approach to developing spatial awareness in preschool-aged children. *An Leanbh Óg the OMEP Journal of Irish Early Childhood Studies, 13*(1), 65-86

Gillic, C. (2021a). 'They'd floor you, they really would...they are capable of learning anything' - Irish preschool practitioners' self-reported practices on young children learning mathematics. *An Leanbh Óg the OMEP Ireland Journal of Irish Early Childhood Studies, 14*(1), 103-122.

Gillic, C. (2021b). Responding to children's (mathematical) thinking in preschool. Eighth conference on research in mathematics education in Ireland MEI 8, Dublin, Ireland.

Gillic, C., O'Neill, S., O'Reilly, N., O'Donoghue, M. & Winget-Power, J. (2026). "The more you know, the more you realise there is much more to learn". Early childhood lecturers' experience and attitudes towards teaching early childhood mathematics. [Manuscript submitted for publication].

Gilpin, J. (2020). Exploring the impact of Aistear on Montessori. *Student Teacher Educational Research E-Journal, (3),* 27-35, ISSN:2712-0201 [www.ster.ie]

Ginsburg, H.P. (2006). Mathematical play and playful mathematics: A guide for early education. In D.G. Singer, R.M. Michnick Golinkoff, & K. Hirsh-Pasek (Eds.), *Play*

= *Learning: How play motivates and enhances children's cognitive and social-emotional growth* (pp. 145-165). Oxford University Press.

Ginsburg, H.P. (2016). Helping early childhood educators to understand and assess young children's mathematical minds. *ZDM Mathematics Education, 48*, 941-946 https://doi/org/10.1007/s11858-016-0807-7

Ginsburg, H.P., Lee, J.S., & Stevenson-Boyd, J. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report, 22(*1), https://doi.org/10.1002/j.2379-3988.2008.tb00054.x

Glaw, X., Inder, K., Kable, A., & Hazleton, M. (2017). Visual methodologies in qualitative research: Autophotography and photo elicitation applied to mental health research. *International journal of qualitative methods, 16,* 1-8*,* https://doi.org/10.1177/1609406917748215

Goodwin, C. (1994). Professional Vision. *American Anthropologist, 96*(3), pp. 606-633.

Gore (1995). On the continuity of power relations in pedagogy. *International Studies in Sociology of Education, 5* (2), 165-188.

Gough, A. (2015). STEM policy and science education: scientistic curriculum and sociopolitical silences. *Cultural Studies of Scientific Education* 10, 445–458. https://doi.org/10.1007/s11422-014-9590-3

Government of Ireland (1997). *Child Care (Pre School Services) Regulations 1996 and Child Care (Pre School Services) (Amendment) Regulations 1997.* The Stationery Office.

Government of Ireland (GoI)(1999a). *Ready to learn White paper on early childhood education.* The Stationery Office.

Government of Ireland (GoI) (1999b). *Primary school curriculum.* GoI.

Government of Ireland (GoI)(1999c). *Primary school curriculum mathematics.* GoI.

Government of Ireland (GoI) (2018). *First 5 A whole-of-government strategy for babies, young children and their families 2019-2028.* GoI.

Government of Ireland (GoI) (2021). *Nurturing skills: The workforce plan for early learning and care (ELC) and school-age childcare (SAC) 2022-2028*. GoI.

Government of Ireland (GoI) (2024a). *Ireland's literacy, numeracy and digital literacy strategy 20242033: Every learner from birth to young adulthood.* GoI.

Government of Ireland (GoI) (2024b). *Ireland's literacy, numeracy and digital literacy strategy 2024-2033: Every learner from birth to young adulthood Implementation plan to 2028.* GoI.

Government of Ireland (2024c). *Aistear the early childhood curriculum framework*. NCCA.

Greig, A., Taylor, J., & MacKay, T. (2013). *Doing research with children: A practical guide* (3rd ed.). Sage.

Grieshaber, S., Krieg, S., McArdle, F., & Sumison, J. (2021). Intentional teaching in early childhood education: A scoping review. *Review of Education, 9*(3), 1-28 https://doi.org/10/1002/rev3.3309

Gripton, C. (2022). Pattern in early years mathematics curriculum; a 25-year review of the status, positioning and conception of pattern in England. *Research in Mathematics Education, 25*(1), 3-23. https://doi.org/ 10.1080/14794802.2021.2010237

Gripton, C. (2023). Developing mathematical patterning in ECE classrooms: participatory research with teachers of 3–5-year-olds. *European Early Childhood Education Research Journal, 31*(3), 326–342 https://doi.org/10.1080/1350293X.2022.2108097

Gura, P. (1992). *Exploring learning: Young children and blockplay*. Paul Chapman.

Gurhy, A.M. (2017). Using Assessment for learning to enhance the teaching and learning of mathematics in one primary school: A lesson study approach. [Doctoral thesis, Dublin City University] Retrieved from: https://doras.dcu.ie/21958/

Hammersley, M. (2013). *What is qualitative research?* Bloomsbury Academic.

Hammersley, M. (2018). What is ethnography? Can it survive? Should it? *Ethnography and Education, 13*(1), 1-17, https://doi.org/10.1080/17457823.2017.1298458

Hand, V. (2012). Seeing culture and power in mathematical learning: Toward a model of equitable instruction. *Educational Studies in Mathematics, 80*, 233-247 https://doi.org/10.1007/s10649-012-9387-9

Hassinger-Das, B., Hirsh-Pasek, K. & Golinkoff, R.M. (2017). The case of brain science and guided play: A developing story. *Young Children, 72* (2), 45-50.

Hawes, Z., Tepylo, D., & Moss, J. (2015). Developing spatial thinking: Implications for early mathematics education. In B. Davis and Spatial Reasoning Study Group (Eds.), *Spatial reasoning in the early years, principles, assertions and speculations* (pp.29-44). Routledge.

Hayes, N. (2013). *Early years practice: getting it right from the start.* Gill & MacMillan.

Hayes, N., O'Toole, L., & Halpenny, A.M. (2017). *Introducing Bronfenbrenner: A guide for practitioners and students in early years education*. Routledge.

Hayes, N. (2022). Mapping Irish policy development in early childhood education and care: A century of change? In N. Hayes & T. Walsh (Eds.). *Early childhood education and care in Ireland: Charting a century of developments (1921-2021).* (pp. 35-56). Peter Lang

Haylock, D. & Cockburn, A.D. (2017). *Understanding mathematics for young children: A guide for teachers of children 3-7,* (5th ed.). Sage.

Hedges, H. & Cullen, J. (2005). Subject knowledge in early childhood curriculum and pedagogy: beliefs and practices. *Contemporary Issues in Early Childhood, 6*(1), 66-79, https://doi.org/10.2304/ciec.2005.6.1.10

Hojnoski, R., Polignano, J., & Colomba, H.L. (2016). Increasing teacher mathematical talk during shared book reading in the preschool classroom: A pilot study. *Early Education and Development, 27*(5), 676-691 https://doi.org/ 10.1080/10409289.2016.1102018

Hollingsworth, H.L. & Winter, M.K. (2013). Teacher beliefs and practices relating to development in preschool: importance placed on socioemotional behaviours and skills, *Early Development and Care, 183*(12), 1758-1781.

Holloway, I., Brown, L., & Shipway, R. (2010). Meaning not measurement: Using ethnography to bring a deeper understanding to the participant experience of festivals and events. *International Journal of Event and Festival Management*, *1*, 74-85. https://doi.org/10.1108/17852951011029315

Hunting, R.P., Mousley, J.A., & Perry, B. (2012). A study of rural preschool practitioners' views on children's mathematical thinking. *Mathematics Education Research Journal, 24,*39-57. https://doi.org/10.1007/s13394-011-0030-3

Imenda, S. (2014). Is there a conceptual difference between theoretical and conceptual frameworks? *Journal of Social Sciences, 38*(2), 185-195 https://doi.org/https://doi.org/10.1080/09718923.2014.11893249

Irish National Teachers Organisation (INTO) (1995). *Early childhood education issues and concerns.* INTO.

Jacobs, V.R., Lamb, L.C. & Phillip, R.A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), pp.169-202.

Jegodtka, A., Hosoya, G., Szczesny, M., Jenβen, L., & Schmude, C. (2022). The relative frequency of mathematical learning opportunities in the morning circle in relation to the development of children's mathematical competencies. In S. Dunekacke, A. Jegodtka, T. Koinzer, K. Eilerts and L. Jenβen (Eds.) *Early childhood teachers' professional competence in mathematics.* (pp 206-217). Routledge.

Jenβen, L. (2022). A math-avoidant profession? Review of the current research about early childhood teachers' mathematics anxiety and empirical evidence. In S. Dunekacke, A. Jegodtka, T. Koinzer, K. Eilerts and L. Jenβen (Eds.) *Early childhood teachers' professional competence in mathematics.* (pp 97-98). Routledge.

Jewitt, C. (2012). *An introduction to using video for research*. National Centre for Research Methods Working Paper 03/12. National Centre for Research Methods.

Johnston, K., & Bull, R. (2021). Understanding educator attitudes towards and perceptions of mathematics in early childhood. *Journal of Early Childhood Research*, 20(3), 341-356. https://doi.org/10.1177/1476718X211052785

Kagan, D.M. (1992) Implication of research on teacher belief, *Educational Psychologist, 27(1), 65-90*

Kennedy, E., Shiel, G., French, G., Harbison, L., Leahy, M., Ó Duibhir, P., & Travers, J. (2022). *Towards a new literacy, numeracy and digital strategy: A review of the literature*. Department of Education.

Kernan, M. (2007). *Play as a context for early learning and development. A research paper.* NCCA.

Kingston, M. (2024). Investigating the probabilistic thinking of young children: A design-based research study to develop frameworks of growth points. [Doctoral Thesis, Dublin City University] https://doras.dcu.ie/30098/

Klibanoff, R.S., Levine, S.C., Huttenlocher, J., Vasilyeva, M., & Hedges, L.V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology, 42*(1), 59-69 https://doi.org/10.1037/0012-1649.42.1.59

Knaus, M. (2017). Supporting early mathematics learning in early childhood settings. *Australasian Journal of Early Childhood, 42*(3), 4-13, https://doi.org/10.23965/AJEC.42.3.01

Knoblauch, H. (2005). Focused ethnography. *Forum: Qualitative Social Research*, *6* (3), 123-141. http://www.qualitative-research.net/index.php/fqs/article/view/20/43

Kolb, D.A. (1984). *Experiential learning: Experience as the source of learning and development*. Prentice Hall.

Köngäs, M. & Määttä, K. (2023). Ethnography in early childhood education and care. *International Journal of Research in Education and Science (IRJES), 9*(3), 787-801, https://doi.org/10.46328/ijres.3215

Lake, V. E., & Kelly, L. (2014). Female preservice teachers and mathematics: Anxiety, beliefs and stereotypes. *Journal of Early Childhood Teacher Education*, *35*(3), 262-275. https://doi.org/10.1080/10901027.2014.936071

Lareau, A. (2021). *Listening to people: A practical guide to interviewing, participant observation, data analysis and writing it all up.* Chicago Press.

Lee, J. (2010). Exploring Kindergarten teachers' pedagogical content knowledge of mathematics. *International Journal of Early Childhood, 42,* 27-41 https://doi.org/ 10.1007/s13158-010-0003-9

Lee, J.E. (2017). Preschool teachers' pedagogical content knowledge in mathematics. *International Journal of Early Childhood, 49, pp.* 229-243. https://doi.org/10.1007/s13158-017-0189-1

Lee, J. & Ginsburg, H.P. (2007). What is appropriate mathematics education for four-year-olds? Pre-kindergarten teachers' beliefs. *Journal of Early Childhood Research, 45*(1), 2-31 https://doi.org/10.1177/1476718X07072149

Lee, J.S., & Ginsburg, H.P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, *34*(4), 37-45. https://doi.org/10.1177/183693910903400406

Lembrér, D., & Meaney, T. (2015). Being and becoming as socialisation in a mathematical activity in preschool. *Nordic Early Childhood Education Research Journal. 11(*5), 1-14.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.) *Multiple Perspectives on Mathematics Teaching and Learning.* Ablex Publishing.

Lerman, S. (2002). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics, 46,* 87-113 https://www.jstor.org/stable/3483241

Li, L., Disney & L. (2023). Young children's mathematical problem solving and thinking in a playworld. *Mathematics Education Research Journal*, *35*, 23–44. https://doi.org/10.1007/s13394-021-00373-y

Liljedahl, P. & Oesterle, S. (2020). Teacher beliefs, attitudes, and self-efficacy in mathematics education. In S. Lerman (Ed.) *Encyclopaedia of Mathematics Education*. (2nd ed). Springer. (pp. 825-828) https://doi.org/10.1007/978-3-030-15789-0_149

Linder, S. M., & Simpson, A. (2018). Towards an understanding of early childhood mathematics education: A systematic review of the literature focusing on practising and prospective teachers. *Contemporary Issues in Early Childhood, 19*(3), 274-296. https://doi.org/10.1177/1463949117719553

Lincoln, Y., & Guba, E. G. (1985). *Naturalistic inquiry*. Sage.

Longley, J.M., & Gilken, J.M. (2020). Mentoring in early childhood settings: Elements of effective relationships. *Dimensions of Early Childhood, 48,* (1), 17- 21

Louie, N.L. (2018). Culture and ideology in mathematics teacher noticing. *Educational Studies in Mathematics, 97,* 55-69 https://doi.org/10.1007/s10649-017-9775-2

Lundqvist, J., Franzén, K., & Munter, A.C. (2023). Early childhood mathematics: A case study. *Early Years, 43,* (4-5), 763-777 https://doi.org/10.1080/09575146.2021.2014404

MacDonald, A. (2018). *Mathematics in early childhood education.* OUP Australia.

MacDonald, A. (2020). Mathematics education beliefs and practices of under 3s educators in Australia. *European Early Childhood Education Research Journal, 28*(5), 758-769 https://doi.org/10.1080/1350293X.2020.1817246

MacDonald, A., & Carmichael, C. (2018). Early mathematical competencies and later achievement: insights from the Longitudinal Study of Australian Children. *Mathematics Educational Research Journal, 30,* 429-444 https://doi.org/10.1007/s133394-017-0230-6

MacDonald, A., & Murphy, S. (2019). Mathematics education for children under four years of age: a systematic literature review. *Early Years,41*(5) 522-539

https//: doi.org/10.1080/09575146.2019.1624507

Mac Naughton, G. (2005). *Doing Foucault in early childhood studies.* Routledge.

McCray, J.S. & Chen, JQ. (2011). Foundational mathematics: A neglected opportunity. In B. Atweh, M. Graven, W. Secada & P. Valero (Eds.) *Mapping equity and quality in mathematics education*. Springer.

McCray, J.S. & Chen, JQ. (2012). Pedagogical content knowledge for preschool mathematics: Construct validity of a new teacher interview. *Journal of Research in Childhood Education, 26*(3), 291-307 https://doi.org/10.1080/02568543.2012.685123

McHugh, G., Denner, S., Clerkin, A., Piccio, G., & Pitsia, V. (2024). *TIMSS 2023: Insights into mathematics and science achievement in Ireland.* Educational Research Centre.

McInnes, K. (2019). Playful learning in the early years – through the eyes of children. *Education 3-13 International Journal of Primary, Elementary and Early Years Education, 47*(7), 796-805 https://doi.org/10.1080/03004279.2019.1622495

Marcus, A., Perry, B., Dockett, S., & MacDonald, A. (2016). Children noticing their own and others' mathematics in play. In B. White, M. Chinnappan & S. Trenholm (eds.) *Opening up mathematics education research: Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia* (pp. 437-444). MERGA.

Marginson, S., Tytler, R., Freeman, B., & Roberts, K. (2013). *STEM: country comparisons: international comparisons of science, technology, engineering and mathematics (STEM) education. Final report.* Australian Council of Learned Academies.

Retrieved from: http://hdl.handle.net/10536/DRO/DU:30059041

Marks, R., Foster, C. Barclay, N., Barnes, A., & Treacy, P. (2019). A comparative synthesis of UK mathematics education research: what are we talking about and do we align with

international discourse? *Research in Mathematics Education 23*(1), 39-62 https://doi.org/10.1080/14794802.2020.1725612

Marshall, C. & Rossman, G. (2016). *Designing qualitative research* (6th Ed.). Sage.

Mason, J. (2002). *Researching your own practice: the discipline of noticing.* Routledge.

Mason, J. (2011). Noticing: roots and branches. In M.G. Sherin, V.R. Jacobs & R.A. Philipp (Eds.), *Mathematics Teacher Noticing: Seeing Through Teacher's Eyes.* Routledge.

Maxwell, J.A. (2013). *Qualitative research design: An interactive approach*. Sage.

Milinković, J. & Bogavac, D. (2011). Montessori Method as a basis for integrated mathematics learning. *Metodički obzori, 6*(1), 135-143

Mills, S. (2003). *Michel Foucault*. Routledge.

Min, K.C., Rashid, A.M. & Nazri, M.I. (2012). Teachers' understanding and practice towards thematic approach in teaching integrated living skills (ILS) in Malaysia. *International Journal of Humanities and Social Science, 2*(23)*, 273-281*

Ministry of Education (2017). *Te Whāriki Early Childhood Curriculum.* Ministry of Education, New Zealand.

Mizzy, V. (1964). *The Addams family* [Song]. Mizzy.

Moll, L.C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into Practice, 31*(2), 132-141 https://doi.org/10.1080/00405849209543534

Moloney, M. (2011). Locating quality in early childhood care and education discourse in Ireland: pre-school and infant classrooms as a crucible for learning and development.

[Doctoral thesis, University of Limerick]. Retrieved from: https://dspace.mic.ul.ie/handle/10395/1928

Moloney, M. & Pope, J. (2013). Where to now for early childhood care and education (ECCE) graduates? A study of the experiences of Irish BA ECCE degree graduates. *Education 3-13: International Journal of Primary, Elementary and Early Years, Education,43(2), 142-152.* https:/doi.org/ 10.1080/03004279.2013.782327

Moomaw, S. (2011). *Teaching mathematics in early childhood.* Brookes.

Moomaw, S. (2017). Not all preschool math games are created equal. *Young Children, 72*(3), 14-21, https://www.jstor.org/stable/90013677

Montague-Smith, A., Cotton, T., Hansen, A. & Price, A. (2018). *Mathematics in early years education.* (4th ed.). Routledge.

Montessori, M. (1915/1997). *The discovery of the child.* Clio Press.

Montessori, M. (1949/1988). *The absorbent mind.* Clio Press.

Montessori, M. (1964/1988). *The Montessori method.* Schoken.

Moss, P. (2001). *Beyond early childhood education and care.* Paper presented at the Early Childhood Education and Care Conference, Stockholm 13-15 June 2001.

Moyles, J. (1990). *Just playing? The role and status of play in early childhood education.* Open University Press.

Munn, P. (2005). Young children's rights to numeracy. *International Journal of Early Childhood*, *37*(3), 61-77

Mukherji, P. & Albon, D. (2015). *Research methods in early childhood: An introductory guide* (2nd ed.). Sage.

Murphy, D. (1995). *Comenius: A critical reassessment of his life and work*. Irish Academic Press.

Murphy, B. (2006). Child-centred practice in Irish infant classrooms: A case of imaginary play? *International Journal of Early Childhood, 38*(1), 112 -124

National Council for Curriculum and Assessment (NCCA)(2009). *Aistear the early childhood curriculum framework*. NCCA.

National Council for Curriculum and Assessment (NCCA)(2023). *Draft updated Aistear: the early childhood curriculum framework for consultation.* NCCA.

Navarro, J.L., & Tudge, J.R.H. (2023). Technologizing Bronfenbrenner: Neo-ecological theory. *Current Psychology, 42* (22)*,* 19338-19354

https://doi.org/10.1007/s12144-022-02738-3

Neylon, G. (2014). An analysis of Irish pre-school practice and pedagogy using the early childhood environmental four curricular subscales. *Irish Educational Studies,* 33(1), 99-116, https://doi.org/10.1080/03323315.2014.888237

Nic Mhuirí, S. (2020). *Shape and space in the senior primary classes: commissioned research paper.* NCCA.

Norwegian Directorate for Education and Training (2017). *Framework Plan for Kindergartens.* Ministry of Education and Research.

Nowell, L.S., Norris, J.M., White, D.E., & Moules, N.J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods, 16*, 1-13, https://doi.org/10.1177/1609406917733847

Nunes, T., & Bryant, P. (2022). *Using mathematics to understand the world How culture promotes children's mathematics.* Routledge.

O'Dwyer, L. (2021). *Teaching and Learning Early Year's Mathematics in Irish Preschools*. [Masters Dissertation, Maynooth University]. Retrieved from: Teaching and Learning Early Year's Mathematics in Irish Preschools - MURAL - Maynooth University Research Archive Library

O'Neill, S. & Gillic, C. (2024). *Conceptualising early mathematics: Report from the PLÉ early childhood mathematics special interest group*. PLÉ, the Association of Academics in Early Childhood Education and Care in Higher Education in Ireland.

O'Neill, S., Gillic, C. & Kingston, M. (2022). *Pedagogical strategies, approaches and methodologies to support numeracy in early childhood: A review of the literature.* Department of Education.

O'Neill, S., Gillic, C., & Winget Power, J. (2022). Using Communities of Practice to Support STEM Education in Early Childhood Care and Education: A Proposal. *National Early Years Research Day Proceedings* (3), 26-31. Early Childhood Ireland.

O'Neill, S., Gillic, C., & O'Reilly, N. (2023). Hungry for more: early childhood educators' perspectives on STEM education, teaching and professional development. *Irish Educational Studies, 42(*4), 971-993 https://doi.org/10.1080/03323315.2023.2261903

O'Reilly, K. (2012). *Key concepts in ethnography.* Sage.

Oppermann, E., Anders, Y., & Hachfeld, A. (2016). The influence of preschool teachers' content knowledge and mathematical ability beliefs on their sensitivity to mathematics in children's play. *Teacher and Teacher Education, 58,* 174-184 https://doi.org/10.1016/j.tate.2016.05.004

Organisation of Working Time Act, (1997). https://www.irishstatutebook.ie/eli/1997/act/20/enacted/en/html

Palaiologou, I., Needham, D. & Male, T. (2016). *Doing research in education. Theory and practice.* Sage.

Papandreou, M. & Tsiouli, M. (2020). Noticing and understanding children's everyday mathematics during play in early childhood classrooms, *International Journal of Early Years Education, 30*(4) 730-747. https://doi.org/10.1080/09669760.2020.1742673

Parks, A. N. & Wager, A.A. (2015). What knowledge is shaping teacher preparation in early childhood mathematics? *Journal of Early Childhood Teacher Education, 36*(2), 124-141, doi:10.1080/10901027.2015.1030520

Perry, B. & Dockett, S. (2008). Young children's access to powerful mathematical ideas. In L.D. English (Ed.) M. Bussi, G. Jones, R. Lesh, B. Sriraman & D. Tirosh (Ass. Eds.) *Handbook of international research in mathematics education* (2nd ed.) (pp.75-108). Routledge.

Perry, B., & Dockett, S. (2013). Reflecting on young children's mathematics learning. In L. D. English & J. T. Mulligan (Eds.), *Reconceptualising early mathematics learning* (pp. 150-161). Springer.

Perry, B., & Dockett, S. (2018). Using a bioecological framework to investigate an early childhood mathematics education intervention. *European Early Childhood Education Research Journal, 26*(4), 604-617 https://doi.org/10.1080/1350293X.2018.1487161

Philipp, R.A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. (pp. 257-315). National Council of Teachers of Mathematics.

Platas, L.M. (2015). The mathematical development beliefs survey: Validity and reliability of a measure of preschool teachers' beliefs about the learning and teaching of arly mathematics. *Journal of Early Childhood Research 13*(3) 295–310. https://doi.org/10.1177/ 1476718X14523746

Pollitt, R., Cohrssen, C., & Seah, W.T. (2020). Assessing spatial reasoning during play: Educator observations, assessment and curriculum planning. *Mathematics Education Research Journal,* 32(2)*,* 331-363.

Pramling Samuelsson, I., & Johansson, E. (2006). Play and learning – inseparable dimensions in preschool practice. *Early Child Development and Care, 176*(1), 47-65 https://doi.org/10.1080/0300443042000302654

Pramling, N., Wallerstedt, C., Lagerlöf, P., Björklund, C., Kultti, A., Palmér, H., Magnusson, M., Thulin, S., Jonsson, A., & Pramling Samuelsson, I. (2019). *Play-responsive teaching in early childhood education.* Springer.

Priestley, K. (2022). How do adults support children's mathematical talk, thinking and mastery? In D. Chilvers (Ed.) *How to recognise and support mathematical mastery in young children's play. Learning from the 'Talk for Maths' initiative.* David Fulton.

Professional Association for Childcare and Early Years (PACEY) (2013) What does 'school ready' really mean? PACEY.

Retrieved from: https://www.pacey.org.uk/Pacey/media/Website-files/school%20ready/SchoolReady-Report.pdf

Purpura, D.J., Hume, L.E., Sims, D.M., & Lonigan, C.J. (2011). Early literacy and early numeracy: The value of including early literacy skills in the prediction of numeracy

development. *Journal of Experimental Child Psychology, 110*(4), 647-658 https://doi.org/10.1016/j.jcep2011.07.004

Purpura, D.J., Napoli, A.R., Wehrspann, E.A., & Gold, Z.S. (2017). Causal connections between mathematical language and mathematical knowledge: A dialogic reading intervention. *Journal of Research on Educational Effectiveness, 10*(1), 116-137, https://doi.org/10.1080/19345747.2016.1204639

Pyle, A. & Danniels, E. (2017). A continuum of play-based learning: The role of the teacher in play-based pedagogy and the fear of hijacking play. *Early Education and Development*, *28(*3), 274-289. https://doi-rg.dcu.idm.oclc.org/10.1080/10409289.2016.1220771

Ramani, G.B., & Eason, S.H. (2015). It all adds up: Learning early math through play and games. *Phi Delta Kappan, 96*(8), 27-32

https://doi-org.dcu.idm.oclc.org/10.1177/0031721715583959

Ransom, M., & Manning, M. (2013). Teaching strategies: worksheets, worksheets, worksheets. *Childhood Education, 89*(3), 188-190,

https: www.doi.org/10.1080/00094056.2013.792707

Rashid, M., Caine, V., & Goez, H. (2015). The encounters and challenges of ethnography as a methodology in health research. *International Journal of Qualitative Methods,* 14(5),1-16, https://doi.org/10.1177/1609406915621421

Reikerås, E., Løge, I. K., & Knivsberg, A.-M. (2012). The mathematical competencies of Toddlers expressed in their play and daily life activities in Norwegian Kindergartens. *International Journal of Early Childhood*, *44*(1), 91–114. https://doi.org/10.1007/s13158-011-0050-x

Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly, 46*, 166–178. https://doi.org/10.1016/j.ecresq.2018.03.006

Roberts-Holmes, G. (2014). *Doing your early years research project A step-by-step guide* (3rd ed.). Sage.

Robson, S. (2014). The analysing children's creative thinking framework: Development of an observation-led approach to identifying and analysing young children's creative thinking. *British Educational Research Journal, 40*(1) 121-34

Rogers, D.L. (2006). Relationships between block play and the social development of young children. *Early Child Development and Care*, *20*(4), 245-261 https://doi.org/10.1080/0300443850200403

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context.* Oxford University Press.

Rogoff, B. (1995). Observing sociocultural activity on three planes: participatory appropriation, guided participation, and apprenticeship. In J.V. Wertsch, del Rio, P., & A. Alvarez (Eds.) *Sociocultural studies of mind.* Cambridge University Press. (pp.139-164)

Rogoff, B. (2003). *The cultural nature of human development.* Oxford University Press.

Roper, J.M., & Shapira, J. (2000). *Ethnography in nursing research.* Sage.

Rose, M. (2022). Friedrich Froebel. In A. Bradbury & R. Swailes (Eds.) *Early childhood theories today.* Sage.

Roβbach, H.G. (1993). Analyse von messinstrumenten zur erfassung von qualitätsmerkmalen frühkindlicher betreuungs- und erziehungsumwelten. Münster: Isititut für Sozialwissen-schäftliche Forschung.

Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching.* Sage.

Ruscoe, A., Lavina, L., Barblett, L., & Boylan, F. (2025). Funds of knowledge and the transition to school: A scoping review. *Review of Education,13*(2), https://doi.org/10.1002/rev3.70092

Sadek, E. & Sadek, J. (1996). *Good practice in nursery management.* Nelson Thornes.

Saracho, O.N. & Spodek, B. (2008). History of mathematics in early childhood education. In O.N. Saracho & B. Spodek (Eds.), *Contemporary perspectives on mathematics in early childhood education*, (pp. 1-20).

Sarama, J. & Clements, D.H. (2009). *Early childhood mathematics education research: Learning trajectories for young children.* Routledge.

Sawer, R. (2004). Creative teaching: Collaborative discussion as disciplined improvisation. *Educational Researcher, 33,* (2)12-20 http://www.jstor.org/stable/3699971?origin=JSTOR-pdf

Schmitt, L., Weber, A., Weber, D., & Leuchter, M. (2023). First insights into preschool teachers' instructional quality in block play and its associations with children's knowledge, interest, academic self-concept and cognitive aspects. *Early Education and Development, 35*(7), 1501-1523 https://doi.org/10.1080/10409289.2023.2233879

Schön, D. (1983). *The reflective practitioner: How professionals think in action.* Basic Books.

Schweinhart, L. J., Montie, J., Xiang, Z., Barnett, W. S., Belfield, C. R., & Nores, M. (2005). *Lifetime Effects: The High/Scope Perry Preschool Study Through Age 40.* National

Institute for Early Education Research. Retrieved from: https://nieer.org/research-library/lifetime-effects#

Seo, K.H. & Ginsburg, H.P. (2004). What is developmentally appropriate in early childhood mathematics education? In D.H. Clements, J. Sarama & A.M. DiBiase (Eds.) *Engaging young children in mathematics* (pp. 91-104). Lawrence Erlbaum Associates.

Shonkoff, J.P. & Phillips, D.A. (2000*). From neurons to neighborhoods: The science of early childhood development*. The National Academies Press. https://doi.org/10.17226/9824

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14 https://doi.org/10.3102/0013189X015002004

Silber, K. (1965). *Pestalozzi: the man and his work* (2nd ed.). Routledge.

Skene, K., O' Farrelly, C. M., Byrne, E.M., Kirby, N., Stevens, E.C., Ramchandani, P.G. (2022). Can guidance during play enhance children's learning and development in educational contexts? A systematic review and meta-analysis. *Child Development*. *93*(4), 1162-1180 https://doi.org/10.1111/cdev.13730.

Sloan, S., Devine, D., Martinez Sainz, G., Symonds, J. E., Crean, M., Moore, B., Davies, A., Farrell, E., Farrell, J., Blue, T., Tobin, E. & Hogan, J. (2022). *Children's school lives: The preschool to primary school transition in Ireland, Report No. 4*. University College Dublin.

Smidt, S. (2009). *Introducing Vygotsky: A guide for practitioners and students in early years education.* Routledge.

Smidt, S. (2011). *Introducing Bruner: A guide for practitioners and students in early years education.* Routledge.

Sproule, L., Walsh, G. & McGuinness, C. (2021). Signalling playfulness; disguising work as play in the early years classroom. *International Journal of Play, 10*(3), 228-242 https://doi.org/10.1080/21594937.2021.1959232

Standing, E.M. (1957). *Maria Montessori her life and work.* Plume.

Sumpter, L. (2019). Preschool educators' emotional directions towards mathematics. *International Journal of Science and Mathematics Education, 18,* 1169-1184 https://doi.org/10.1007/s10763-019-10015-2

Sylva, K., Siraj-Blatchford, I., Taggart, B. (2011) *ECERS-E – The four curricular subscales extension to the early childhood environment rating scale (ECERS-E)* (4th ed.). Teachers College Press.

Takunyaci, M., & Takunyaci, M. (2014). Preschool teachers' mathematics teaching efficacy belief. *Procedia – Social and Behavioural Sciences, 152,* 673-678

Taylor, H. (2013). How children learn mathematics and the implications for teaching. In H. Taylor & A. Harris (Eds.) *Learning and teaching mathematics 0-8.* (pp.3-19). Sage.

Temple, J. A. & Reynolds, A.J. (2008). *Benefits and costs of investments in preschool education: Evidence from the child-parent centers and related programs. Economics of Education Review, 26*(1), 126-144.

Tenzek, K.E. (2018). Field Notes. In B.B. Frey (Ed.). *The Sage encyclopedia of communication research methods.* Sage.

https://doi.org/10.4135/9781483381411.n20

Thiel, O. (2010) Teachers' attitudes towards mathematics in early childhood education, *European Early Childhood Research Journal*, 18(1), 105-115. https://doi.org/10.1080/13502930903520090

Thiel, O. & Jenssen, L. (2020). Affective-motivational aspects of early childhood teacher students' knowledge about mathematics. In O. Thiel & B. Perry (Eds.) *Innovative approaches in early childhood mathematics.* Routledge. (pp.50-72).

Torre, D. & Murphy, J. (2015). A different lens: using photo elicitation interviews in education research. *Educational Policy Analysis Archives, 23*(111), 1-26

Trawick-Smith, J., Swaminathan, S., & Liu, X. (2016). The relationship of teacher–child play interactions to mathematics learning in preschool. *Early Child Development and Care, 186*(5), 716–733. https://doi.org/10.1080/03004430.2015.1054818

Trawick-Smith, J., Swaminathan, S., Baton, B., Danieluk, C., Marsh, S. & Szarwacki, M. (2017). Block play and mathematics learning in preschool: The effects of building complexity, peer and teacher interactions in the block area, and replica play materials. *Journal of Early Childhood Research*, *15*(4) 433–448 https://doi.org/https://doi-org.dcu.idm.oclc.org/10.1177/1476718X16664557

Tudge, J.R.H., & Mokrova, I., Hatfield, B.E., & Karnik, R.B. (2009). Uses and misuses of Bronfenbrenner's bioecological theory of human development. *Journal of Family Theory & Review, 1* (4)*,* 198-210

Tudge, J.R. H. & Hogan, D. (2005). An ecological approach to naturalistic observations of children's everyday lives. In S. Greene & D. Hogan (Eds.) *Researching children's* e*xperience approaches and methods*. Sage.

United Nations (1989). *United Nations Convention on the Rights of the Child*. UN.

United Nations (2015). Sustainable Development Goals. https://sdgs.un.org/goals

United Nations Children's Fund (2019). *A world ready to learn: Global report on pre-primary education.* UNICEF.

Urban, M., Vandenbroeck, M., Lazzari, A., Van Laere, K., Peeters, J., (2011). *Competence requirements in early childhood education and care.* Universities of East London and Ghent.

Urban, M., Robson, S., & Scacchi, V. (2017). *Review of occupational role profiles in Ireland in early childhood education and care*. ECRC. Retrieved from: https://assets.gov.ie/24908/d9cb10bb2d9141f5b5288722ea13194c.pdf

van Es, E.A., & Sherin, M.G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571-596

van Es, E.A. & Sherin, M.G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education, 24* (2), 244-276.

Van Maanen, J. (2011). *Tales of the field, On writing ethnography* (2nd ed.). Chicago.

van Oers, B. (1996). Are you sure? Stimulating mathematical thinking during young children's play. *European Early Childhood Education Research Journal, 4* (1),71-87 https://doi.org/10.1080/13502939685207851

van Oers, B. (2001). Educational forms of initiation in mathematical culture. *Educational Studies in Mathematics, 46*(1-3), 59-85 https://doi.org/10.1023/A.1014031507535

van Oers, B. (2002). Teachers' epistemology and the monitoring of mathematical thinking in early years classrooms. *European Early Childhood Education Research Journal, 10(2),* 19-30 https://doi.org/10.1080/13502930285208931

van Oers, B. (2010). Emergent mathematical thinking in the context of play. *Educational Studies in Mathematics, 74,* 23-37 https://doi.org/10.1007s10649-009-9225-x

van Oers, B. & Poland, M. (2012). Promoting abstract thinking in young children's play. In B. van Oers (Ed.) *Developmental education for young children* (Vol. 7, pp.121-136). Springer.

Varol, F. & Farren, D.C. (2006). Early mathematical growth: How to support young children's mathematical development. *Early Childhood Education Journal, 33*(6), 381-387 https://doi.org/10.1007/s10643-0060-8

Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K. & Newcombe, N. (2014). Finding the missing piece: Blocks, puzzles, and shapes fuel school readiness. *Trends in Neuroscience and Education, 3*(1), 7-13, https://doi.org/10.1016/j.tine.2014.02.005

Verdine, B.N., Lucca, N., Golinkoff, R.M., Hirsh-Pasek, K., & Newcombe, N. (2017). Links between spatial and mathematical skills across the preschool years. *Monographs of the Society for Research in Child Development, 82* (1), 7-30, https://doi.org/10.1111/mono.12280

Vogt, F., Hauser, B., Stebler, R., Rechsteiner, K. & Urech, C. (2020). Learning through play – pedagogy and learning outcomes in early childhood mathematics. In O. Thiel & B. Perry (Eds.), *Innovative approaches in early childhood mathematics.* Routledge. (pp. 127-141).

von Marenholtz-Bülow, B. (1985/2007). *How Kindergarten came to America: Friedrich Froebel's radical vision of early childhood education.* The New Press.

Vygotsky, L. S. (1978). *Mind in society: Development of higher psychological processes*. Harvard University Press.

Wager, A.A. (2013). Practices that support mathematics learning in a play-based classroom. In L.D. English & J.T. Mulligan (Eds.) *Reconceptualising early mathematics learning.* (pp. 163-181). Springer.

Wager, A.A. (2014). Noticing children's participation: Insights into teacher positionality toward equitable mathematics pedagogy, *Journal for Research in Mathematics Education, 45*(3), 312-350. https://www.doi.org/ 10.5951/jresematheduc.45.3.0312

Wager, A.A. & Parks, A.N. (2016). Assessing early number learning in play. *ZDM Mathematics Education* (48) 991-1002 https://www.doi.org/ 10.1007/s11858-016-0806-8

Walsh, G., McGuinness, C., & Sproule, L. (2019). It's teaching…but not as we know it': using participatory learning theories to resolve the dilemma of teaching in play-based practice. *Early Child Development and Care, 189*(7), 1162–1173, https://doi.org/10.1080/03004430.2017.1369977

Walsh, G., Winter, F., Farrell, T., Orr, K., Hanna, K., McCarney, S., Ballentine, M., & Millar, A. (2024). *An independent review of the early childhood care and education (ECCE) Programme in Ireland.* Stranmillis University College/Dublin City University.

Walsh, T. (2016). Recent policy developments in early childhood education (ECE): A jigsaw with too many pieces? *An Leanbh Óg, 10.* pp. 69-94.

Watts, T.W., Duncan, G.J., Siegler, R.S., & Davis-Kean, P.E. (2014). What's past is prologue: relations between early mathematics knowledge and high-school achievement. *Educational Researcher, 43*(7), 352-360. https://doi.org/doi:10.3102/0013189X14553660.

Weisberg, D.S., Hirsh-Pasek, K. & Golinkoff, R.M. (2013). Guided play: Where curricular goals meet a playful pedagogy. *Mind, Brain and Education, 7*(2), 104-112. https://doi.org/10.1111/mbe.12015

Weisberg, D., Hirsh-Pasek, K., Golinkoff, R.M., Kittredge, A.K. & Klahr, D. (2016). Guided play: Principles and practices. *Current Directions in Psychological Science, 25(*3), 177-182 https://doi.org/0.1177/0963721416645512

Wickstrom, H.C. (2023). *Meeting in the Middle: Teacher-Facilitation of Play and Math Learning in Contemporary Kindergarten.* [Doctoral thesis, University of Toronto]. Retrieved from: Wickstrom_Hanna_Catherine_202306_PhD_thesis.pdf (utoronto.ca)

Wickstrom, H., Pyle, A., & DeLuca, C. (2019). Does theory translate into practice? An observational study of current mathematics pedagogies in play-based kindergarten. *Early Childhood Education Journal, 47*(3), 287–295. https://doi.org/10.1007/ s10643-018-00925-1

Williams, J.J., Tunks, J. & Gonzalez – Carriedo (2016). Supporting mathematics understanding through funds of knowledge. *Urban Education, 55*(3), https://doi.org/10.1177/0042085916654523

Wood, E. (2014). *Play, learning and the early childhood curriculum* (2nd ed.). Sage.

Wood, E. (2022). Play and learning in early childhood education: tensions and challenges. *Child Studies (1),* 15-26

Worthington, M. (2017). Funds of knowledge: children's cultural ways of knowing mathematics. In V. Kinnear, M.Y. Lai & T. Muir (Eds.), *Forging connections in early mathematics teaching and learning*. (pp.239-258). https://doi.org/10.1007/978-981-10-7153-9_13

Worthington, M. & van Oers, B. (2016). Pretend play and the cultural foundations of mathematics. *European Early Childhood Education Research Journal, 24*(1), 51-66. https://doi.org/10.1080/1350293X.2015.1120520

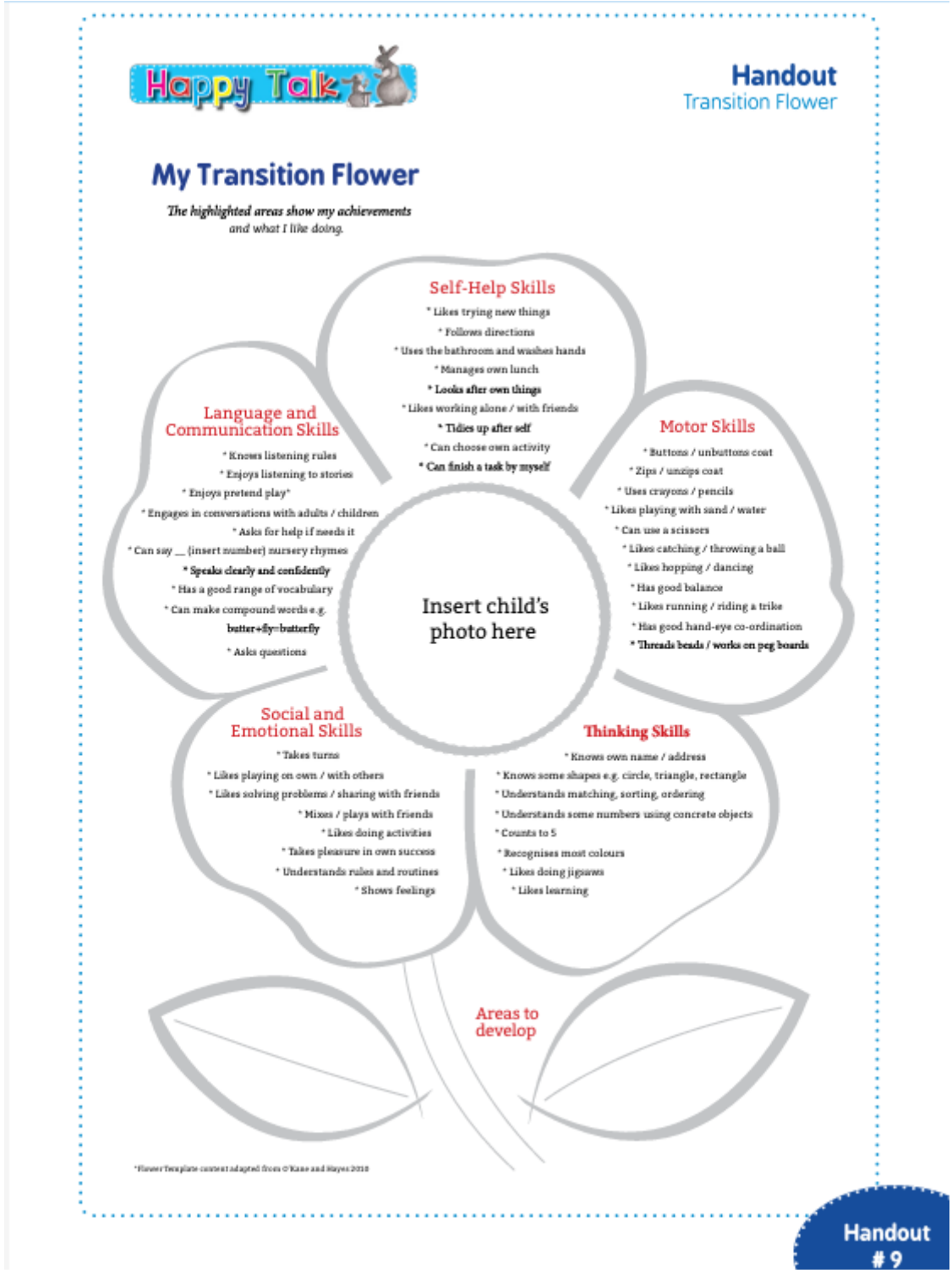
Youmans, A., Coombs, A., & Colgan, L. (2018). Early childhood educators' and teachers' early mathematics education knowledge, beliefs, and pedagogy. *Canadian Journal of Education, 41*(4), 1079 – 1104.

Zhang, C., & Quinn, M. (2018). Promoting early writing skills through morning meeting routines: guidelines for best practices. *Early Childhood Education Journal*, *46*(5), 547-556, https://doi.org/10.1007/s10643-017-0886-2

Zosh J.M, Hirsh-Pasek K, Hopkins, E.J., Jensen, H., Liu, C., Neale, D., Solis, S.L., Whitebread, D. (2018). Accessing the Inaccessible: Redefining Play as a Spectrum. *Frontiers in Psychology, 1124* https://doi.org/10.3389/fpsyg.2018.01124

APPENDICES

Appendix A My Transition Flower



Appendix B Personal Reflection

Primary School

As a shy, quiet child, who did not like speaking out in class, I was easily embarrassed, and found it hard to think on feet/under pressure. In 3rd class, I was put on the spot to justify reasoning behind my answer given to a question on angles. My teacher did not try and understand my perspective, persisted in questioning me, leaving me feeling flustered, and ridiculed. Similar incidents occurred in the following two years, and I started to convince myself I wasn't good at maths and tuned out. By 6th class I developed severe maths anxiety, compounded by being routinely put on spot in quickfire round to answer a question posed by other students. This resulted in becoming physically ill to avoid maths lessons.

Secondary School

The belief of 'I'm not a maths person' was well cemented, and I existed in maths classes. I had no interest in maths at this stage. Passed LC maths course.

Montessori Diploma

Engaging with the mathematics component of the Montessori teaching diploma, led to a change of belief. Coupled with a great instructor who had a love of the Montessori method and maths materials in particular. I felt this too! I observed the children in my classroom interacting with the math material and enjoying the experience. I cultivated a culture where it was OK to make mistakes.

Career Break

Time spent with my three girls, engaging in block play, supportive of playful early math. development at home. Using mathematics in everyday activities, modelling mathematical language, Language, plenty of spatial play, hands-on activities e.g. cooking & baking

BA ECSP (no maths content)

Despite the lack of mathematics education content, this degree supported confidence building in terms of academic ability. Interactions with tutors & literature. Importance of being policy savvy. My final year research project addressed the gap in Irish research regarding maths in preschool (published).

M Ed (with some ECME content)

Confidence and interest in early maths education continues to grow. Maths now firmly seen as being an important component in early child development and integral to preschool education. Playful pedagogy can spark joy and creativity in maths activity from an early age. Continued interactions with the policy, practice & research literature. Connections made with colleagues in Maths Ed through Maths4All. Masters study explored own practice in relation to developing spatial awareness in pre-schoolers. (published)

DCU Assistant Professor in Early Childhood Education.

Teaching on B Ed – early mathematics, children's participation in ECEC, Supervising final year research projects in ECME…Interactions with colleagues, course materials, research papers, policy development.

PLÉ SIG

Established and co-convened the Early Mathematics SIG for PLÉ. Now confident enough to work with others on Maths/STEAM related projects. Supporting others in their work. Interactions with teaching/research colleagues, early childhood educators & students of ECEC. Now recognised as someone who has a major interest in ECME among colleagues both in DCU and further afield.

Appendix C Interview Schedule – Manager

Initial questions:

How has your day been… tell me about your setting (no of staff/quals, no of children, ratios). How long established, services on offer etc.

Identity: on Wednesday, you introduced me as a 'teacher. just like yourself and Sinéad'. Why do you use this term here in this setting?

Beliefs and mathematics

How would you describe own beliefs towards mathematics generally? What has influenced these beliefs?

How would you describe your beliefs towards mathematics in preschool education? How should mathematics be taught to young children? How should they experience mathematics in preschool?

Would you put a ceiling on what young children learn mathematically here in this setting? (BA some participants reported a fear of doing too much as children would be bored in PS/ developing bad relationships with local PS staff).

Mathematics in the setting

What prompted you to explore how early mathematical experiences were supported in your setting?

What were your findings?

What actions did you take? Have these actions been sustained over time? Why/Why not? How?

Supporting Staff

How do you support your staff in supporting mathematical experiences in the ECCE rooms? observation, resources, training, reading materials, co-exploration of ped doc?

How do staff feel about the maths focus?

What difficulties, if any, prevent staff from supporting maths in ECEC?

What challenges did you/do you currently face in terms of providing a mathematics curriculum in this setting? How can these be addressed? (staff turnover since Covid/RP)

What are the challenges for you as a curriculum leader in relation to mathematics?

Is there a policy regarding ECME in the setting? Would you mind sharing/discussing aspects of this document with me? Does this policy outline the role of the curriculum leader/room leader/ room assistants in relation to ECME? What about mathematics and babies/toddlers? How is this catered for?

Supports for ECME

What supports are you aware of for ECEs to develop their pedagogical skills in relation to mathematics in ECEC?

Appendix D Reflective Journal Entry 'The Coat'

Date: 4/5/23 (RJ2)	
Experience, Event or Dilemma	The children were asked to put their coats on before going outside to the garden. J was last to the coat rail. I picked his coat from the rail and tried to assist him putting it on. 'Hey' he shouted crossly. He shook my hand away, stamped his feet and said, 'I can do it myself'. I stood back, annoyed at myself – of course he was able to do it, he needed no help – why did I feel I should help?
Reflection and Observation	This is where the boundary between being a researcher and another adult in the room becomes blurred. I reverted back quite quickly into EYE mode, perhaps too quickly. I am not that long in the setting and therefore do not know these children well yet. Not well enough to know how my actions will be received by individual children. It will take time to get to know each child and for them to know me. This is not my space, it is theirs and I am a visitor.
Analysis	I should have known better! And perhaps, I am overthinking things. However, I do need to be mindful that although I am a qualified ECE, I am also a visitor in this setting. It has been relatively easy to build a rapport with Sinéad. As a former ECE, we have much in common and much to chat about. The children do not have this shared knowledge. On the preliminary visit Siobhán introduced me to the children as a teacher. This does not mean that they will accept me readily. It will take time to develop relationships with individual children. I wasn't helping – I was interfering and going against my own belief that children are capable, competent and at this age quite independent. Perhaps I was trying too hard to be friendly. This is an ethical issue.
Action Plan	I need to remember that I am a visitor in the children's space. I will need to observe the children as they play and interact. I need to take the time to observe their body language for any signs of discomfort in relation to my presence and act accordingly. Maintain a relative distance for now and when individual children seem comfortable, I can perhaps, interact with them – on their terms.
New Experience	

Appendix E Data Collection Record

Data Source	Abbreviation	Date Collected
Interview with Manager	INT 1	26/4/23
Reflective Conversations with Lead Educator	RC 1	8/5/23
	RC2	22/5/23
Field notes (including Casual Conversations)	FN1	3/5/23
	FN2	4/5/23
	FN3	5/5/23
	FN4	8/5/23
	FN5	9/5/23
	FN6	10/5/23
	FN7	11/5/23
	FN8	12/5/23
	FN9	16/5/23
	FN10	17/5/23
	FN11	18/5/23
	FN12	19/5/23
	FN13	22/5/23
	FN14	23/5/23
	FN15	24/5/23
	FN16	25/5/23
Reflective Journal Entries	RJ1	3/5/23
	RJ2	4/5/23
	RJ3	5/5/23
	RJ4	8/5/23
	RJ5	9/5/23
	RJ6	10/5/23
	RJ7	11/5/23
	RJ8	12/5/23
	RJ9	16/5/23
	RJ10	17/5/23
	RJ11	18/5/23
	RJ12	19/5/23
	RJ13	22/5/23
	RJ14	23/5/23
	RJ15	24/5/23
	RJ16	25/5/23
Curriculum Planning & Observations Policy	CPOP	26/4/23
Early Years Mathematics Teaching & Learning Policy	EYMTLP	26/4/23

Appendix F Reflective Journal Entry – Adult Support

Date: 24/5/23 (RJ15)	
Experience, Event or Dilemma	As the children were finishing their lunch, I reminded Sinéad that tomorrow will be my last day in the setting. Sinéad mentioned how this had come around so quickly and she commented that it had been nice to have had another body in the room with her, as she said, "it was lovely to have someone else in the room, someone qualified, and who knows what they are doing"
Reflection and Observation	I am happy that Sinéad accepted my presence in the room and that she didn't mind me stepping in at times, cleaning tables, helping to set the room for circle time, working with individual children etc. That said, I did check in with her regularly that it was OK to do so
Analysis	Over the past few weeks, I have seen Sinéad balance admin (taking photos of children at play, completing transition paperwork, ensuring learning portfolios are up to date), cleaning duties, as well as supervising and interacting with children during free play. This is a busy time of year in an ECCE class with much paperwork to be completed and with such little non-contact time, some of this is done during the ECCE session. I had an ECE on an AIM contract supporting me in my work with 11 children. Sinéad does not have this luxury and works within the legal ratio of 1:11. Even though this ratio seems low in comparison to a junior infant classroom in primary school, the challenges of getting all required paperwork, cleaning etc done within half an hour each day non-contact time can be challenging and I know I found myself completing learning portfolios, observations etc during the session while my colleague supervised free play or I stayed back after official work hours to get things done – seems to be the accepted norm within education generally. I valued my colleague's help and support. With Sinéad running two ECCE sessions, staying until near closing time in the evening, staying after a session is not an option for her. Therefore, I am not surprised that she appreciated the help of someone familiar with ECCE routines and who had experience of working with this age group.
Action Plan	not applicable in this case

Appendix G Ethical Approval

Ollscoil Chathair Bhaile Átha Cliath
Dublin City University

DCU

Ms Cora Gillic
School of School of STEM Education, Innovation & Global Studies

23th January 2023

REC Reference: **DCUREC/2022/255**

Proposal Title: **Supporting mathematics in preschool settings**
Applicant(s): **Ms Cora Gillic, Ms Siun Nic Mhuiri, Prof. Mathias Urban**

Dear Colleagues,

Thank you for your application to DCU Research Ethics Committee (REC). Further to expedited review, DCU REC is pleased to issue approval for this research proposal.

DCU REC's consideration of all ethics applications is dependent upon the information supplied by the researcher. This information is expected to be truthful and accurate. Researchers are responsible for ensuring that their research is carried out in accordance with the information provided in their ethics application.

Materials used to recruit participants should note that ethical approval for this project has been obtained from the Dublin City University Research Ethics Committee. Should substantial modifications to the research protocol be required at a later stage, a further amendment submission should be made to the REC.

Yours sincerely,

DCU
Research & Innovation

Taighde & Nuálaíocht Tacaíocht
Ollscoil Chathair Bhaile Átha Cliath,
Baile Átha Cliath, Éire

Appendix H Sample Codes

Big Ideas of Mathematics (Brownell et al., 2014)	Guided Participation Rogoff (1990; 2003)	Codes from Data Analysis
Number Numeral Recognition Counting Number and Quantity Counting Principles 1 to 1 correspondence Stable Order Cardinality Order-irrelevance	Observation (child) Imitation Listening Scripts of everyday life Observation (ECE) Observation of child's interest Modelling Mutual structuring of participation Apprenticeship Joint Attention Responsive Assistance Motivation to participate Praise Mock excitement Intentional teaching Limits participation Learnt from Siblings/HLE/ Funds of Knowledge	Construction play Block play Boys dominate unit blocks Socio-dramatic play Imaginative Play Free Play Problem-solving Creativity Being a mathematician Mathematically rich environment Environment facilitates different types of math learning Learning environment caters for JI children Mathematical Talk and Discussion Preparing for School Institutional Procedures Lack of Time Administrative Tasks Reporting to Parents (APP) Reporting to Parents (Transition Flower) Door Duty – time away from class Prepping for activities Non-contact time
Measures Time Days of the Week Months Seasons Telling the Time Measuring Weighing Capacity Length Making Comparisons		
Shape 2D 3D Number of sides		
Space Direction		
Data Handling Matching Ordering Sorting		Lower qualifications affect self-efficacy Lack of training impacts on knowledge Lack of training impacts on confidence
Pattern		

Appendix I Example of Coded Data (RC1)

Excerpt	Analytical Notes	Theoretical	Codes/Themes
S: ya, like that's what I'm saying they're, they are like very prepared to go into junior infants, like, a lot of them, throughout the play and stuff and then they're going back to square one in junior infants and I'm like…like John will get bored,	Playful learning prepares children for Primary school Thinks JI teacher will start at the beginning with every child, despite ability/knowledge The use of textbooks often supports Sinéad's observation that JI teachers start at number 1 with all children. This idea may come from her experience helping JI children with their homework in her afterschool session. John will act out if bored, get into trouble if he knows too much? Where does this stand from a rights-based perspective? Maybe, we need a broader maths curriculum, spanning the 3-6 age range? Sinéad mentioned that transition documents such as the transition flower are passed to parents, rather than school. If schools received this documentation, it would help in tailoring lesson plans to individual interests and abilities.	The PSC (GoI, 1999) – the one Sinéad is familiar with recognised each child has individual learning needs and teachers should plan for these in their lessons. The current PSCF (DE, 2023) acknowledges that learning in primary builds upon knowledge obtained in preschool and the HLE, and that individual needs should be catered for when lesson planning. Aistear (NCCA, 2009) does not include reference to Number operations – which J is interested in and is supported in by Sinéad…an issue with a framework rather than a curriculum? Stronger links between preschool and primary settings is required. Mo Scéal (NCCA) templates are available to use. One captures a very broad range of learning across the 4 themes of Aistear. The second – Likert type scale for shape, space, problem-solving & counting. Little detail required.	**Theme:** transition to school Preparedness for school Playful learning facilitates knowledge acquisition Primary school teachers do not meet children where they are at Believes Children all start at the same point in JI Knowing too much results in boredom in JI Lack of direct communication between preschool and primary teachers

Appendix J Plain Language Statement (Parents/Guardians)

Supporting Mathematics in an Irish Preschool Setting

This research study conducted by doctoral student, Córa Gillic (cora.gillic@dcu.ie), under the supervision of Dr Siún NicMhuirí (DCU SEIGs) and Professor Mathias Urban (DCU LLECE) explores how early mathematics education is supported in Irish preschool settings. It will focus on leadership, classroom interactions and participant beliefs about early childhood mathematics education.

The researcher will be present in the setting for five morning sessions for four weeks, working in the room alongside educators. Observations will be made of practice and interactions that support mathematics during every day routines of the setting.

Participant Involvement

Educator participants are invited to video record examples of children's engagement with mathematics during play. They will choose a recording to share with the researcher on a weekly basis for reflection on mathematics education in the setting.

Educator participants will also take photographs of resources that promote mathematical exploration and thinking. Children will not feature in these photographs. With your consent, your child's teacher may share examples of your child's learning portfolio with the researcher, to share aspects of mathematical learning in play. Your child may not wish to have their work shared, and this is OK. We will ask their permission first.

Child participants will engage in the normal play and routines of the preschool setting day. They will not be asked to do anything other than play and interact as normal. Your child's participation is voluntary and they will not be discriminated against, should they wish not to participate. If your child chooses not to participate, they will remain in the classroom but will not be recorded.

Benefits (Direct or Indirect to Participants)

Teacher reflections on practice offer a way to improve practice. Through dialogue with the researcher, informal professional development may occur during reflective conversations on videos, perhaps leading to improvements in practice with regard to early mathematics education with the setting.

Confidentiality and Use of Data

Data will be stored securely and kept private. Transcriptions from video footage, reflective conversations and interviews may feature in the final doctoral thesis. However, participants will be assigned pseudonyms and the setting will not be named either in the thesis or in any further publications based on it. The researcher and supervisors will be the only persons with access to the data. This is to ensure that research procedures such as data collection and analysis adhere to the highest ethical standards. Data will be held for three years post study completion. confidentiality of information can only be protected within the limitations of the law – i.e., it is possible for data to be subject to subpoena, freedom of information claim or mandated reporting by some professions. participants have the right to access their data and

can apply to the DCU Data Protection officer: Mr Martin Ward (data.protection@dcu.ie Ph: 01 7005118/ 7008257). Participants have the right of the individual to lodge a complaint with the Irish Data Protection Commission.

If participants have concerns about this study and wish to contact an independent person, please contact:

The secretary, Dublin City University Research Ethics Committee, c/o Research and Innovation Support, Dublin City University, Dublin 9. Tel 01-7008000, e-mail rec@dcu.ie

Appendix K Informed Consent Form (Parents/Guardians)

Dear parents / Guardians,

My name is Córa Gillic and I am a lecturer and doctoral student with the Institute of Education in DCU.

As part of my doctorate degree, I have to carry out a piece of research. My research focuses on how early mathematics education is supported in preschool settings, with a real interest in how your child's teachers support early mathematics in _____.

_____ (manager) of _____, has given me permission to conduct my research in the preschool classroom and I would like to invite your child to take part. I have enclosed an information leaflet explaining the research project and how your child will take part. If you wish for your child to participate in this research study, please sign the form below and return it to me by _____.

I look forward to carrying out the project in the summer term 2023. If you have any questions, you can reach me on 086 2323030 or via e-mail: cora.gillic@dcu.ie.

Consent form for Doctoral Research Study

□ I have read and understand the information leaflet on the above study.

□ I have had an opportunity to ask questions and have received satisfactory replies to those questions asked, in addition to any requests for further details about the study.

□ I understand that I (or my child) can withdraw from the study at any time without giving any further explanation.

□ I understand that this study has been reviewed by, and received ethical clearance through DCU Research Ethics Committee at DCU Institute of Education.

□ I understand how to contact the researcher/supervisor/ethical committee if I have any concerns.

□ I give permission for my child to take part in the above study.

□ I do not give permission for my child to take part in the above study.

Name of Child: _____

Name of Parent/Guardian: _____

Signature of Parent/Guardian: _____

Appendix L Plain Language Statement (Manager / Educator)

Supporting Mathematics in an Irish Preschool Setting

This research study conducted by myself, Córa Gillic (cora.gillic@dcu.ie), under the supervision of Dr Siún NicMhuirí (DCU SEIGs siun.nicmhuiri@dcu.ie) and Professor Mathias Urban (DCU LLECE) explores how early mathematics education is supported in Irish preschool settings. It will focus on leadership, classroom interactions and participant beliefs towards early childhood mathematics education.

I will be present in the setting for five morning sessions for four weeks, working in the room alongside educators. Observations will be made of practice and interactions that support mathematics during the everyday routines of the setting.

Participant Involvement

Should you choose to take part, you will be invited to take part in an audio-recorded interview (ca. 1 hour) prior to the commencement of data collection in the preschool (ECCE) room. The aim of this interview is to explore the beliefs of the manager towards mathematics and mathematics in preschool settings. Aspects of leadership for early childhood mathematics will also be discussed. You will be invited to share relevant policy documents that demonstrate how early mathematics education is supported in your setting.

Participation in this study is purely voluntary. Participants have the right to withdraw from the study at any time, for any reason. No penalty, including a negative impact on current EC STEAM network activities, will be applied for withdrawal at any stage of the research process.

Findings from this study will be shared with you via e-mail.

Benefits (Direct or Indirect to Participants)

Participants will have an opportunity to reflect on practice, to offer opinions on early mathematics and to share aspects of their practice in relation to early mathematics in their preschool setting.

Potential Risks to Participants

While there are no serious risks to participants, it is acknowledged that participants may feel uneasy sharing aspects of their practices. I am committed to developing an atmosphere of co-research with participants where opinions are shared, respected and valued.

Confidentiality and Use of Data

Data will be stored securely and kept private. Transcriptions from video footage, reflective conversations and interviews may feature in the final thesis, however, participants will be assigned pseudonyms and the setting will not be named either in the thesis or in any future publications based on it. The researcher and supervisors will be the only persons with access to the data (video/audio recordings, interview, reflective conversation transcripts and personal data). This is to ensure that research procedures such as data collection and analysis adhere to the highest ethical standards. Data will be held for three years post study completion. Confidentiality of information can only be protected within the limitations of the law - i.e., it is possible for data to be subject to subpoena, freedom of information claim or mandated reporting by some professions. Participants have a right to access their data and can apply to the DCU Data Protection officer: Mr Martin Ward (data.protection@dcu.ie Ph: 01 7005118/ 7008257). Participants have the right of the individual to lodge a complaint with the Irish Data Protection Commission.

If participants have concerns about this study and wish to contact an independent person, please contact:

The Secretary, Dublin City University Research Ethics Committee, c/o Research and Innovation Support, Dublin City University, Dublin 9. Tel 01-7008000, e-mail rec@dcu.ie

Appendix M Informed Consent Form (Manager)

Informed Consent Form Setting Managers

Research Study Title: Supporting mathematics in an Irish preschool setting.

Dear_____

As part of my doctorate degree (PhD) I have to carry out a piece of research. My research focuses on how early mathematics education is supported in preschool settings, with a real interest in how early childhood educators support early mathematics in preschool settings, and how they are supported by their managers in this process.

I hope to conduct this research study in May 2023. I would like to interview you about how early mathematics education is supported in your setting. I have enclosed an information leaflet explaining the research project and how you will be asked to participate in the study. If you wish to participate in this research study, please sign the form below and return it to me by _____.

I look forward to carrying out the project in the summer term 2023. If you have any questions, you can reach me on 086 2323030 or via e-mail: cora.gillic@dcu.ie.

Please circle Yes or No for each statement

I have read the plain language statement and understand the information provided.	Yes/No
I understand that I will be invited to take part in an interview about how mathematics is supported in my setting.	Yes/No
I understand that I may be asked to share any relevant policy with the researcher that demonstrates how early mathematics education is supported in my setting.	Yes/No
I understand that I can withdraw from the study at any point without penalty.	Yes/No
I understand that DCU is the data controller and that my personal data (name, occupation) will be processed by the researcher.	Yes/No
I understand that the findings of this research may be submitted to an academic journal, and that pseudonyms will be used in all published accounts.	Yes/No
I understand that all research data (e.g. video, audio recordings, field notes) and personal data will be destroyed three years after project submission.	Yes/No

I have read and understood the information in this form. My questions and concerns have been answered by the researcher, and I have a copy of this consent form. Therefore, I consent to take part in this research study.

Signature:_____

Name in Block Capitals:_____

Date:_____

Appendix N Informed Consent Form (Educators)

Dear early childhood educator,

My name is Córa Gillic and I am a lecturer and doctoral student with the Institute of Education in DCU.

As part of my doctorate degree, I have to carry out a piece of research. My research focuses on how early mathematics education is supported in preschool settings, with a real interest in how early childhood educators support early mathematics in preschool settings, and how they are supported by their managers in this process.

_____ (manager) of _____, has given me permission to conduct my research in the preschool classroom of your setting and I would like to invite you to take part. I have enclosed an information leaflet explaining the research project and how you will be asked to participate in the study. If you wish to participate in this research study, please sign the form below and return it to me by _____.

I look forward to carrying out the project in the summer term 2023. If you have any questions, you can reach me on 086 2323030 or via e-mail: cora.gillic@dcu.ie.

Please circle Yes or No for each statement

I have read the plain language statement and understand the information provided.	Yes/No
I understand that self-captured video of children engaging in mathematics will be shared with the researcher.	Yes/No
I understand that I will be invited to reflect on my chosen video clip with the researcher and that this conversation will be audio recorded.	Yes/No
I understand that I can withdraw from the study at any point without penalty.	Yes/No
I understand that DCU is the data controller and that my personal data (name, occupation) will be processed by the researcher.	Yes/No
I understand that the findings of this research may be submitted to an academic journal.	Yes/No
I understand that all research data (e.g.video, audio recordings, field notes) and personal data will be destroyed three years after project submission.	Yes/No

I have read and understood the information in this form. My questions and concerns have been answered by the researcher, and I have a copy of this consent form. Therefore, I consent to take part in this research study.

Signature:_____

Name in Block

Capitals:_____

Date:_____

Appendix O Plain Language Statement (Children)





Córa the Maths Detective





Plain Language Statement for Young Children (3-5 years). Córa Gillic (2022)

Hello!

My name is Córa and I am a researcher. My job is to explore and discover how people do maths in school. I'm a sort of detective. I look for maths everywhere.

I always use my magnifying glass, my notebook, my camera and my phone to help me do my job.



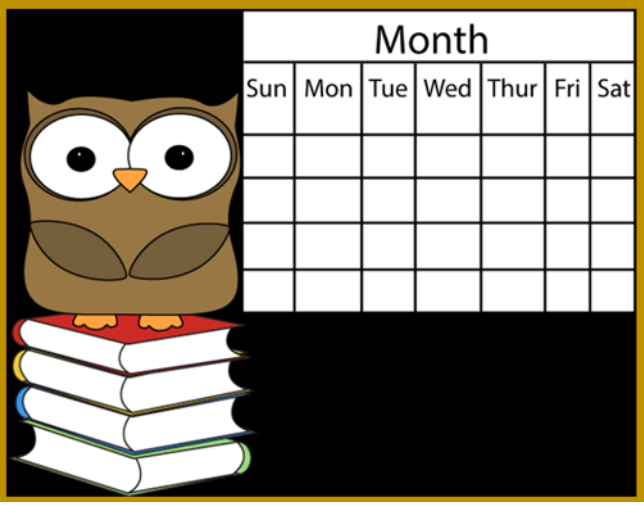
Sometimes I take photographs and videos of children in classrooms. I make notes in my notebook about how children and teachers work together with maths when they play.







I will be here every day for 4 weeks.



I would like to watch you all play and discover the maths you do. I will make notes, but your teacher (name) will take the videos and photos of your play. She's going to be my helper maths detective.



Together, myself and your teacher (name) will chat about the maths that we can see in this room.



But we need some help

We need to watch you play. I would like your teacher (name) to take videos of your play.



This means, your teacher (name) will be recording you. You might think that this is OK **or** you might not like that at all.

It is OK to say no! If you do not want to be in a video, you just need to tell us, and we won't do it.

You can also tell us by putting your name or photograph on a colour.

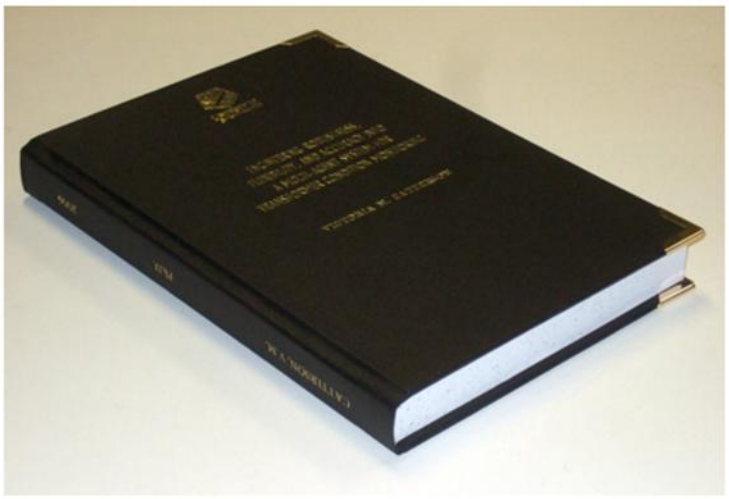


Green means yes.



Red means no.

When the research time is over. I will write about what I have seen and what your teacher (name) in my book, but I will never use your real name.



I will keep all the information that I collect in a locked drawer that no one else can open.



I will keep the information for three years in case I want to write about this research and share my ideas with other people. After this time, I will destroy my information.



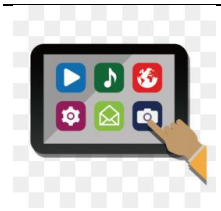
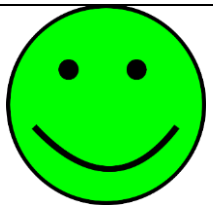
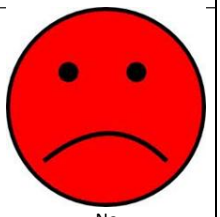


What do you think?

Do you have any questions for Córa the maths detective?

Appendix P Pictorial Assent Form (Children)

To be read to the children. Children will be asked to mark yes/no with a sticker or an X.

	The details of the research study were read to me.	 Yes	 No
 (pic: cleanpng.com)	I know that my teacher might take a video of me when I play.	 Yes	 No
	I know that my teacher might ask to show the researcher some pictures of my work.	 Yes	 No
 (image:freepik)	I know that it is OK if I do not want to take part.	 Yes	 No
	I would like to take part.	 Yes	 No