

# Teaching mathematical modelling: a research based approach

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**Paper Type:** Practitioner and Postgraduate Papers

## Abstract

A collaborative, research based laboratory experiment in mathematical modelling was included in a bioprocess engineering laboratory module, taught as part of an interdisciplinary program in biotechnology. The class was divided into six groups of three students and given the task of investigating a novel diafiltration process that is currently the focus of international research. Different aspects of the problem were assigned to each group and inter-group communication *via* email was required to ensure that there was a coherent set of objectives for each group and for the class as a whole. The software package, Berkeley Madonna, was used for all calculations. As well as giving the students an introduction to mathematical modelling and computer programming, this approach helped to illustrate the importance of research in bioprocess engineering.

In general, the experiment was well received by the students and the fact that they were discovering new knowledge generated a degree of enthusiasm. However, many students were consumed by the technical demands of computer programming, especially the attention to detail required. Thus, they did not think too deeply about the physical aspects of the system they were modelling. In future years, therefore, consideration will be given to giving the student prior instruction in the use of the software.

Administration of the experiment was not excessive but this was partly due to the small class size. Larger class sizes could place significant time demands on the instructor.

**Keywords:** Modelling, Research, Collaborative learning, Berkeley Madonna, Computer programming

## 1. Introduction

The teaching of mathematical modelling within conventional laboratories is a feature of many engineering and physics-based degree programs ([Ingham et al, 2007](#)). In such degree programs, the process of generating the mathematical model is given equal emphasis to the numerical solution and investigation of the predictions of the model equations. In more biology-focused programs, model formulation is often seen as beyond the mathematical abilities of the students and more emphasis is on using simulation to

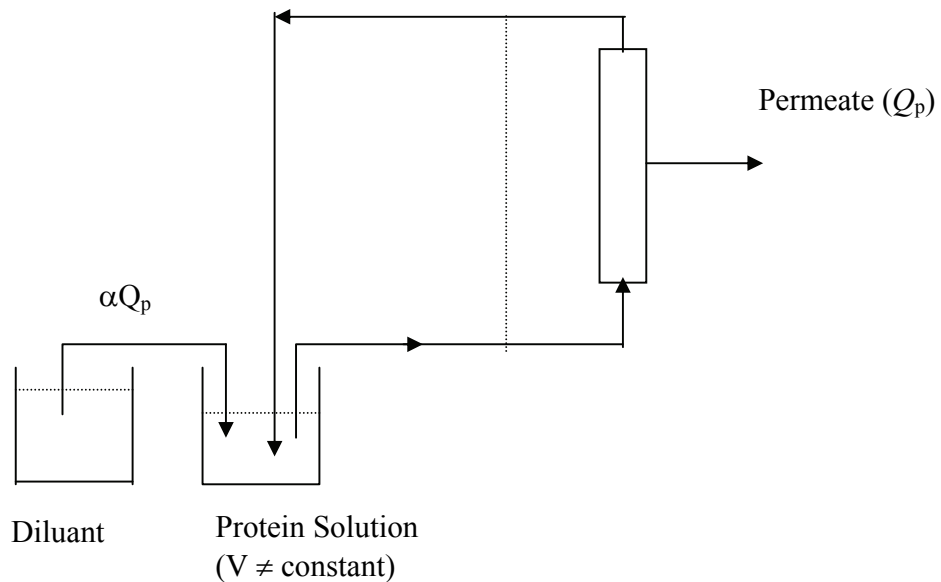
explore the model predictions ([Allman and Rhodes, 2004](#)). This has led to development of some software packages, such as STELLA where the underlying mathematics is hidden and modelling is done through the construction of block diagrams.

The focus of this paper is introducing students on an interdisciplinary biotechnology program to the concept of a numerical experiment, i.e., investigating the implications of a mathematical model using computer simulation. A typical modelling or simulation 'experiment' involves coding a mathematical model of a system in the required language, running the model for range of scenarios and generating a number of graphs and tables to represent system behaviour. However, unless the system chosen shows unanticipated or non-intuitive behaviour, the student response at the end of this type of exercise is often one of indifference. Often the student knows what to expect and the modelling exercise is used to confirm what the student already knows. The key to a good modelling exercise is that the problem is new to the student (and instructor), open-ended, that it yields non-obvious results which stimulate thinking and that it is sufficiently complex to develop the students' technical skills. A good way to devise a suitable modelling exercise, therefore, is to find a genuine research problem that has not been completely solved and is preferably of interest to researchers in the field ([Brew, 2003](#)). The problem described here was designed as a collaborative class project in which student groups would contribute individual aspects of the problem to create an overall result for the class.

## **2. Problem Design**

This year, students were presented with a mathematical model of a novel diafiltration process which is currently the subject of international research. It should be pointed out that generation of a mathematical model from first principles is beyond the capability of this student group and thus the focus was in using *computer simulation* to investigate the implications of the mathematical model. The first task was to code the model in the simulation language Berkeley Madonna. This is an easy-to-use simulation package for solving systems of ordinary differential equations. It is user-friendly and accessible to students who have only a small amount of experience in mathematics and computer programming. Coding is very similar to how one would write the equations by hand and a unique and useful feature of Madonna is that the order in which the equations are written does not matter. Results from simulations are easily visualised and exportable into Excel if required.

The diafiltration process studied involved comparing a novel process termed 'ultrafiltration with variable volume diafiltration' (UFVVD) with the more established technique of 'ultrafiltration with constant volume diafiltration' (UFCVD). Diafiltration is a process in which fresh water is added to a tank of fluid (typically a protein solution) which is pumped through a membrane filtration unit as in [Figure 1](#). The effect of simultaneous water addition and filtration leads to low molecular weight impurities being flushed out of the original solution. In general, water is added at a rate  $\alpha Q_p$ , where  $Q_p$  is the permeate flowrate and  $\alpha$  is a parameter with value  $0 \leq \alpha \leq 1$ . The difference between UFCVD and UFVVD relates to how  $\alpha$  varies with time but a detailed explanation is beyond the scope of this paper ([Foley, 2006](#)).



**Figure 1** General process flowsheet for concentration-diafiltration process

In our approach, the class is divided into six groups of three students. Each group is then given a specific aspect of the model to investigate. One group works on the model in any given laboratory session. Inter-group transfer of results (*via* email) is required to ensure that the class as a whole undertakes a coherent research project. No laboratory report was required but the students were sent a summary document at the end of the module to give them an overview of what the class as a whole had discovered. Material in this document formed the basis of short questions in an end of module examination.

### 2.1 Student Exercises

UFVVD is a two-step process involving an ultrafiltration step followed by a variable volume diafiltration step. The first task of all groups was to determine when the diafiltration step should commence in order to minimise the total time required. The final task of all groups was to investigate any aspect of the problem that they, as a group, thought would be interesting. The specific exercises required for each group are outlined in [Table 1](#). The combined result of the class effort is a clear comparison between UFVVD and UFCVD on the basis of process time, water usage and process cost for cases where membrane fouling is significant and where it is not.

**Table 1** Student tasks and inter-group communication

<b>Group</b>	<b>Task</b>	<b>Communication with Other Groups / Instructor</b>
<b>All</b>	Optimise UFVVD	Email to instructor
<b>C</b>	Compare times for UFVVD and UFCVD	Email Results to Group A cc to instructor
<b>E</b>	Calculate water usage for UFVVD	Email results to Group F cc to instructor
<b>F</b>	Calculate water usage for UFCVD and compare with results sent by Group E	Email results to Group A cc to instructor
<b>A</b>	Compare the economics of UFVVD and UVCVD using results sent by Group C and Group F	Email results to instructor
<b>B</b>	Modify original UFVVD analysis done by all groups to take account of membrane fouling	Email results to Group D cc to instructor
<b>D</b>	Do the UFCVD analysis done by Group C but modify to take account of membrane fouling. Compare with results sent by Group B	Email results to instructor

### 3. Results and Discussion

This approach was introduced in the current academic year for the first time and due to the small class size was not excessively demanding on instructor time. The main source of workload was keeping track of the emails that were sent or copied to the instructor. Inevitably, there were instances when a group had to be reminded to email their results to the next group as required (see [Table 1](#)).

In general, students reacted positively to the challenge of investigating a new subject and discovering new knowledge. They had to make decision for themselves as opposed to following a recipe as is often the case in conventional laboratories. Students did show a tendency to be consumed by the technical demands of computer programming. In particular, they found the attention to detail required to be daunting. While this could be seen to be a valuable exercise for the students in itself, it did detract somewhat from the students understanding of the physics of the system they were studying. Indeed, the problem chosen for this experiment, namely the novel diafiltration process, was perhaps a little subtle and may not have encouraged thinking to a degree that would be desirable.

#### **4. Conclusions**

In general, the approach used helped to create enthusiasm and promote independent thinking in the students. Future application of this approach will focus on giving the student prior instruction in computer programming and consideration will be given to choosing a problem with which they might better engage.

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